TTK4105 ØVING 4

Christian Le

$\boxed{1}$

a) $H_1(s)$: $\boxed{\begin{array}{ll}\text{Nullpunkt: } & s=2 \\ \text{Pol: } & s=-3\end{array}}$

$H_2(s)$: $s = \frac{-2 \pm \sqrt{4-16}}{2}$ $\boxed{\text{ingen reelle nullpunkt}}$

$(s+1)^2 - 4 = s^2 + 2s - 3$

Poler: $s = \frac{-2 \pm \sqrt{4+12}}{2} = \frac{-2 \pm 4}{2} = \boxed{\{-3, 1\}}$

$H_3(s)$: $\frac{s}{s+1} + \frac{1}{s+2} = \frac{s^2+3s+1}{(s+1)(s+2)}$

Nullpunkt: $s = \frac{-3 \pm \sqrt{9-4}}{2} = \boxed{\frac{-3 \pm \sqrt{5}}{2}}$

Poler: $\boxed{s = \{-2, -1\}}$

b) $H_1(t) = \mathcal{L}^{-1}\{H_1(s)\} = \mathcal{L}^{-1}\left\{\frac{s}{s+3} - \frac{2}{s+3}\right\}$

$\downarrow$ Delbrøk

$\frac{A}{1} + \frac{B}{s+3}$ $\quad sA + (3A+B) = s$

$A = 1 \quad B = -3$

$= \mathcal{L}^{-1}\left\{1 - \frac{5}{s+3}\right\}$

$= \boxed{\delta(t) - 5e^{-3t}}$

$$H_2(t) = \mathcal{L}^{-1}\{H_2(s)\} = \mathcal{L}\left\{\frac{s^2+2s+4}{(s+3)(s-1)}\right\}$$

Delbroksopps.

$$\frac{A}{1} + \frac{B}{s+3} + \frac{C}{s-1}$$

$$\frac{A(s+3)(s-1) + B(s-1) + C(s+3)}{(s+3)(s-1)} = \frac{s^2+2s+4}{(s+3)(s-1)}$$

$$s^2A + s(2A+B+C) + (-3A-B+3C) = s^2+2s+4$$

$A = 1$

$2A+B+C = 2$ $\qquad B = -C$

$-3A-B+3C = 4$ $\qquad 3C-B = 7$

$C = \frac{7}{4} \qquad B = -\frac{7}{4}$

$$\mathcal{L}^{-1}\left\{1 - \frac{7}{4}\cdot\frac{1}{s+3} + \frac{7}{4}\cdot\frac{1}{s-1}\right\}$$

$$=$$

$$H_2(t) = \boxed{\delta(t) - \frac{7}{4}e^{-3t} + \frac{7}{4}e^{t}}$$

$$H_3(t) = \mathcal{L}^{-1}\left\{\frac{s}{s+1} + \frac{1}{s+2}\right\} = \mathcal{L}^{-1}\left\{1 - \frac{1}{s+1} + \frac{1}{s+2}\right\}$$

$\downarrow$ Delbrøk

$$= \boxed{\delta(t) - e^{-t} + e^{-2t}}$$

$$\frac{A}{1} + \frac{B}{s+1}$$

$$sA + (A+B) = s$$

$$A = 1 \quad B = -1$$

$\boxed{2}$ a) Setter inn $\sin(\theta) \simeq \theta$ $\cos(\theta) \simeq 1$

(Altså $\lim_{\theta \to 0}$)

$$(m + M)\ddot{x} - Ml\ddot{\theta} + Ml\theta\dot{\theta}^2 = F \qquad (1)$$

$$Ml^2\ddot{\theta} - Ml\ddot{x} + Mgl\theta = 0 \qquad (2)$$

$$y = x - l\theta \qquad (3)$$

Finner likning for $\ddot{x}$ og setter inn i (1) og $(\ddot{3})$

$$\ddot{x} = l\ddot{\theta} + g\theta = 0 \qquad (2)^*$$

$$\boxed{\ddot{y} = \ddot{x} - l\ddot{\theta} = g\theta} \qquad (2)^* \rightarrow (\ddot{3})$$

$$(m+M)(l\ddot{\theta}+q\theta)-Ml\ddot{\theta}+Ml\theta\dot{\theta}^2=F \quad (2)^* \rightarrow (1)$$

$$\boxed{ml\ddot{\theta}+(m+M)q\theta+\underbrace{\cancel{Ml\theta\dot{\theta}^2}}_{\text{går mot } 0}=F}$$

$$F(s)=\mathcal{L}\{F\}=\left(s^2\theta-s\theta(0)-\dot{\theta}(0)\right)ml$$

$$+$$

$$(m+M)q\theta$$

$$H_1(s)=\frac{\theta(s)}{F(s)}=\frac{\theta}{\left(s^2\theta-s\theta(0)-\dot{\theta}(0)\right)ml+q\theta(m+M)}$$

Antar $\theta(0)=\dot{\theta}(0)=0$

$$H_1(s)=\boxed{\frac{1}{s^2ml+g(m+M)}}$$

$$\mathcal{L}\{\ddot{y}\}=s^2Y=g\theta$$

$$\frac{Y}{\theta}=\frac{g}{s^2}$$

(Antar $y(0)=\dot{y}(0)=0$)

$$H_2(s)=\frac{y(s)}{\theta(s)}=\boxed{\frac{g}{s^2}}$$

c) $\mathcal{L}\{ml\ddot{\theta} + (m+M)g\theta\}$

$$ml(s^2\Theta - s\Theta(0) - \dot{\Theta}(0)) + (m+M)g\Theta = 0$$

$$s^2 ml\Theta + (m+M)g\Theta = ml(s\Theta(0) + \dot{\Theta}(0))$$

$$\Theta(s) = \frac{ml(s\Theta(0) + \dot{\Theta}(0))}{s^2 ml + (m+M)g} = \frac{s\Theta(0)}{s^2 + \frac{(m+M)g}{ml}} + \frac{\dot{\Theta}(0)}{s^2 + \frac{(m+M)g}{ml}}$$

Uten å regne $\Theta(t)$ ser vi fra invers formel:

$$\frac{s}{s^2+\omega^2} = \cos\omega t \quad \text{og} \quad \frac{\omega}{s^2+\omega^2} = \sin\omega t$$

at $\boxed{\omega = \sqrt{\frac{(m+M)g}{ml}}}$ HVIS $\Theta(0)$ eller $\dot{\Theta}(0)$ er ulik 0

d) $\mathcal{L}\{\ddot{y}\} = s^2 Y = g\theta$

$$\left(\mathcal{L}^{-1}\left[\frac{n!}{s^{n+1}}\right] = t^n\right)$$

$$Y(s) = \frac{g\theta}{s^2}$$

$$Y(t) = \mathcal{L}^{-1}\{Y(s)\} = \boxed{g\theta_0 t}$$

$\boxed{3}$ $H(s) = \frac{y}{u}(s)$ (1)

$u = K(s)(r-y)$ (2)

a) $e = (r-y)$ $y = r - e$ (3) og $(3)^*$

$H(s) = \frac{y}{u}(s) = \frac{r-e}{K(s)(r-y)}$ $(2),(3)^* \rightarrow (1)$, (4)

$H(s) = \frac{r}{K(s)(r-y)} - \frac{\cancel{e}}{K(s)\cancel{(r-y)}}$ $(3) \rightarrow (4)$, (5)

$H(s) = \frac{1}{K(s)}\frac{r}{e} - \frac{1}{K(s)}$

$H(s)K(s) + 1 = \frac{r}{e}$

$$\boxed{\frac{r}{e}(s) = \frac{1}{1 + H(s)K(s)}}$$

b) $K(s) = 1$, $r(t) = 1$

$$r(s) = \frac{1}{s}$$

$$r(s) \cdot S(s) = e(s) = \frac{1}{s} \cdot \frac{1}{1 + H(s)}$$

$$= \frac{s+1}{s(s+1+1)} = \frac{s+1}{s(s+2)}$$

Delbrøk:

$$\frac{A}{s} + \frac{B}{s+2} = \frac{s+1}{s(s+2)} = \frac{1}{2} \cdot \frac{1}{s} + \frac{1}{2} \cdot \frac{1}{s+2}$$

$$A + \frac{sB}{s+2} = \frac{s+1}{s+2} = \frac{1}{2}$$

$$(s=0)$$

$$\frac{A}{s}(s+2) + B = \frac{s+1}{s} = \frac{1}{2}$$

$$(s=-2)$$

$$e(s) = \frac{1}{2}\left(\frac{1}{s} + \frac{1}{s+2}\right) \qquad \underline{s e(s) = \frac{1}{2}\left(1 + \frac{s}{s+2}\right)}$$

$$e(t) = \frac{1}{2}\left(1 + e^{-2t}\right) \qquad \boxed{e(\infty) = \frac{1}{2}}$$

Vi ser via begge metodene at $e(\infty) \rightarrow \frac{1}{2}$

c) $e(s) = r(s)\, S(s) = \frac{1}{s} \cdot \frac{1}{1 + H(s)(1 + 1/s)}$

$$H(s) = \frac{1}{s+1}$$

$$s\,e(s) = \frac{1}{1 + \frac{1}{s+1}\left(1 + \frac{1}{s}\right)} = \frac{(s+1)s}{(s+1)s + s + 1}$$

$$= \frac{(s+1)s}{s^2 + 2s + 1} = \frac{(s+1)s}{(s+1)^2} = \frac{s}{s+1}$$

$$e(\infty) = \lim_{s \to 0} s\,e(s) = \frac{0}{0+1} = \underline{\underline{0}}$$

$\boxed{4}$ a) Egenverdier λ er gitt;

$$\det(A - I\lambda) = 0 = \begin{vmatrix} -1-\lambda & 2 \\ 0 & -4-\lambda \end{vmatrix}$$

$$= (-1-\lambda)(-4-\lambda) = (1+\lambda)(4+\lambda) = 0$$

$$\lambda = \{-1, -4\}$$

$$\boxed{\Lambda = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}}$$

b) Egenvektor for en gitt egenvektor er løsningen for nullrom av:

$$\left[\begin{array}{cc|c} -1-\lambda & 2 & 0 \\ 0 & -4-\lambda & 0 \end{array}\right]$$

$\lambda = -1$

$$\left[\begin{array}{cc|c} 0 & 2 & 0 \\ 0 & -3 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad \begin{bmatrix} x_1 \\ 0 \end{bmatrix} s$$

Egenvektor for $\lambda = -1$: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\lambda = -4$

$$\left[\begin{array}{cc|c} 3 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$3x_1 + 2x_2 = 0$

$3x_1 = -2x_2$

$x_1 = -\frac{2}{3}x_2$

Egenverdi for $\lambda = -4$: $\begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$$\boxed{Q = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}}$$

c) $\dot{x} = Ax + bu \qquad \dot{x} = Q\dot{\bar{x}} = AQ\bar{x} + bu$

$$\dot{\bar{x}} = Q^{-1}AQ\bar{x} + Q^{-1}bu \qquad y = cQ\bar{x}$$

$$Q^{-1}: \frac{1}{ad-bc}\begin{bmatrix} a & -b \\ -c & a \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$$

$$\bar{A} = Q^{-1}AQ = \begin{bmatrix} 1 & \frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}\left(\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 6 \\ 0 & -12 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -1 & 8 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -8 \\ 0 & -4 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}} = \Lambda$$

$$\bar{b} = Q^{-1} b = \begin{bmatrix} 1 & \frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}$$

$$\bar{c} = cQ = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & -2 \end{bmatrix}}$$

d) Bruker formel 4.77 s.135 Balchen

$$h(s) = c(sI-A)^{-1} b$$

$$(sI-A) = \begin{bmatrix} s+1 & 2 \\ 0 & s+4 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{(s+1)(s+4)} \begin{bmatrix} s+4 & -2 \\ 0 & s+1 \end{bmatrix}$$

$$h(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} & -\frac{2}{(s+1)(s+4)} \\ 0 & \frac{1}{s+4} \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} -\frac{6}{(s+1)(s+4)} \\ \frac{3}{s+4} \end{bmatrix} = \boxed{-\frac{6}{(s+1)(s+4)}}$$

$$\bar{h}(s) = \bar{c}(sI - \bar{A})^{-1}\bar{b}$$

$$(sI - \bar{A}) = \begin{bmatrix} s+1 & 0 \\ 0 & s+4 \end{bmatrix}$$

$$(sI - \bar{A})^{-1} = \frac{1}{(s+1)(s+4)} \begin{bmatrix} s+4 & 0 \\ 0 & s+1 \end{bmatrix}$$

$$\bar{h}(s) = \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+4} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} \frac{2}{s+1} \\ \frac{1}{s+4} \end{bmatrix}$$

$$= \frac{2}{s+1} - \frac{2}{s+4}$$

$$= 2\left(\frac{(s+4) - (s+1)}{(s+1)(s+4)}\right) = \boxed{\frac{6}{(s+1)(s+4)}}$$

Kan ukke fortegn, vet ikke hvor fortegns feil kommer fra

$\boxed{5}$ $r(t) = \begin{cases} 25 & t < 60 \\ 40 & t \geq 60 \end{cases}$

(Implementert som step)

$$S(s) = \frac{e(s)}{r(s)} = \frac{r(s) - v(s)}{r(s)}$$

Bruker laplace på hastighetsmodell for å finne $v(t)$.

$$smV(s) - v_0 + (\rho A C_d v_0 + k_p) v(s) = k_p r(s)$$

$$v(s) = \frac{k_p r(s) + v_0}{sm + \rho A C_d v_0 + k_p}$$

$$S(s) = \frac{r(s) - \frac{k_p r(s) + V_0}{sm + \rho A C_d v_0 + k_p}}{r(s)}$$

$$= 1 - \frac{k_p}{sm + \rho A C_d V_0 + k_p} + \frac{v_0}{(\rho A C_d V_0 + k_p) r(s)}$$

$$e(s) = S(s) r(s)$$

$r(t) = 2 \quad + 15u(t-60)$

$$r(s) = \frac{2s + 15e^{-60s}}{s} \qquad sr(s) = 2s + 15e^{-60s}$$

$$e(\infty) = \lim_{s \to 0} se(s) =$$

$$s\left(r(s)\left(1 - \frac{k_p}{sm + \rho A C_d V_0 + k_p}\right) + \frac{V_0}{\rho A C_d V_0 + k_p}\right)$$

$$40\left(1 - \frac{k_p}{\rho A C_d V_0 + k_p}\right) \approx \boxed{1{,}33}$$

Dette stemmer overens med simulering!

(Sjekk vedlegg oppg 5a pdf)

b) Bruker laplace for å finne v(t) for ny dynamikk.

$$smv(s) - V_0 + (\rho A C_d V_0 + k_p)(v(s) - r(s)) = 0$$

$$v(s) = \frac{V_0 + r(s)(\rho A C_d V_0 + k_p)}{sm + \rho A C_d V_0 + k_p}$$

$$S(s) = \frac{e(s)}{r(s)} = \frac{r(s) - v(s)}{r(s)}$$

$$e(\infty) = \lim_{s\to 0} s e(s) =$$

$$s\left(r(s) - \frac{v_0 + r(s)(\rho A C_d v_0 + k_p)}{sm + \rho A C_d v_0 + k_p} \right)$$

$$40 - \frac{40\cancel{(\rho A C_d v_0 + k_p)}}{\cancel{\rho A C_d v_0 + k_p}} = \boxed{\underline{\underline{0}}}$$

Dette stemmer med simuleringen, men regulatoren håndterer ~~ikke~~ forstyrrelsen.

c) $\dot{e} = (\dot{r} - \dot{v}) = -\dot{v}$

$$m\ddot{e}(t) + (\rho A C_d v_0 + k_p)\dot{e}(t) + k_I e(t) = 0$$

$$\ddot{e}(t) + \left(\frac{\rho A C_d v_0 + k_p}{m}\right)\dot{e}(t) + \frac{k_I}{m} e(t) = 0$$

$$\omega_n = \sqrt{\frac{k_I}{m}} \qquad \zeta = \frac{\rho A C_d v_0 + k_p}{2m}$$

$$k_I = \omega_n^2 m \approx 31{,}248 \qquad k_p = \zeta 2m - \rho A C_d v_0 \approx 4{,}323 \cdot 10^3$$

Laplace transform av dynamikk:

$$smv(s) - v_0 + (\rho A C_d v_0 + k_p)(v(s) - r(s)) + \frac{k_I}{s}(v(s) - r(s))dt = 0$$

$$v(s) = \frac{v_0 + r(s)(\rho A C_d v_0 + k_p) + r(s) k_I / s}{sm + \rho A C_d v_0 + k_p + k_I / s}$$

$$e(s) = r(s) - v(s)$$

$$e(\infty) = \lim_{s \to \infty} s e(s)$$

$$= s\left(r(s) - \frac{v_0 + r(s)(\rho A C_d v_0 + k_p) + r(s) k_I / s}{sm + \rho A C_d v_0 + k_p + k_I / s}\right)$$

$$= 40 - \frac{40\cancel{(\rho A C_d v_0 + k_p)}s}{\cancel{\rho A C_d v_0 + k_p + k_I / s}\; s(\rho A C_d v_0 + k_p) + k_I/\cancel{s}} + \frac{40 k_I \cancel{/s}}{}$$

$$= 40 - 40 = \underline{\underline{0}}$$

Dette stemmer med simuleringen.

Modellen kan handere forstyrrelser gitt tid.