TTK4105 Øving 6

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$\boxed{1}$ a) $F_{hori}(t) = F_p \cdot \sin(\theta(t))$ (1)

$$F_{vert} = F_G = F_p \cdot \cos(\theta(t)) \quad (2)$$

Setter F_p i (2) inn i (1)

$$F_{hori}(t) = F_G \tan(\theta(t))$$

b) $J_\lambda \ddot{\lambda} = F_{hori}(t)\, a = F_G \dfrac{\sin(\theta(t))}{\cos(\theta(t))} a$

Når $\lim_{\theta \to 0}$ så er $\sin(\theta) \approx \theta$
$\cos(\theta) \approx 1$

$$J_\lambda \ddot{\lambda} = F_G \theta a \quad \text{(Bruker Laplace)}$$

$$s J_\lambda \dot{\lambda}(s) = F_G \theta(s) a$$

$$\dot{\lambda}(s) = \frac{a F_G}{s J_\lambda} \theta(s) = G_2(s)\theta(s)$$

c) Refererer til likning (3):

$$\theta(s) = G_1(s)\, u(s)$$

og gjør likning (4) til et uttrykk for $\lambda(s)$:

$$\lambda(s) = \frac{K_2}{s^2}\theta(s) = \frac{K_1 K_2}{s^4(\tau s+1)} u(s)$$

$$G(s) = \frac{\lambda(s)}{u(s)} = \frac{K_1 K_2}{s^4(\tau s+1)}$$

Vi har en pol av multiplisitet 4, som betyr 4-orders system

$\boxed{2}$ a) $\dot{\lambda}(s) = G_2(s)\,\theta(s)$ (4)

$$\theta(s) \approx \theta_r(s) \approx K_{p,\lambda}(\lambda_r(s) - \lambda(s))$$

$$s\frac{\dot{\lambda}(s)}{K_2} = K_{p,\lambda}(\dot{\lambda}_r(s) - \dot{\lambda}(s))$$

$$s\dot{\lambda}(s) = K_2 K_{p\lambda}(\dot{\lambda}_r(s) - \dot{\lambda}(s))$$

$$\ddot{\lambda}(t) = K_2 K_{p\lambda}(\dot{\lambda}_r(t) - \dot{\lambda}(t)) \qquad K_{p\lambda} = \frac{K_\lambda}{K_2}$$

$$K_{p,\lambda} = \frac{K_\lambda J_\lambda}{a F_g}$$

$$\Theta(s) \approx \Theta_r(s)$$

$$h(s) = \frac{\dot{\lambda}(s)}{\dot{\lambda}_r(s)} = \frac{(K_2/s)\,\Theta(s)}{(\Theta_r(s)/K_{p,\lambda}) + (\dot{\lambda}(s))}$$

$$\dot{\lambda}(s) = \frac{K_2}{s}\,\Theta(s)$$

$$\dot{\lambda}_r(s) = \frac{\Theta_r(s)}{K_{p,\lambda}} + \dot{\lambda}(s)$$

$$\frac{(K_2/s)}{(1/K_{p,\lambda}) + (\dot{\lambda}(s)/\Theta(s))} = \frac{(K_2/s)}{(1/K_{p,\lambda}) + (K_2/s)}$$

$$\frac{K_2}{((s/K_{p,\lambda}) + K_2)} = \frac{K_2 K_{p\lambda}}{s + K_2 K_{p,\lambda}}$$

b) $K_2 K_{p,\lambda} = \frac{1}{\tau} = 1$

Sjekk vedlegg

Vi når referanseverdien!

c) Vi opererer likt som i forrige oppgave, men istedenfor $\theta(s) \approx \theta_r(s)$

$$\theta(s) = G_3(s)\,\theta_r(s)$$

$$\dot{\lambda}(s) = G_2(s)\,\theta(s)$$

$$\dot{\lambda}(s) = G_2(s)\,G_3(s)\,\theta_r(s)$$

Setter inn denne i (a)

$$\dot{\lambda}_r(s) = \frac{\theta_r(s)}{K_{p,\lambda}} + \dot{\lambda}(s)$$

$$T(s) = \frac{G_2(s)\,G_3(s)\,\theta_r(s)}{\frac{\theta_r(s)}{K_{p,\lambda}} + \dot{\lambda}(s)} = \frac{G_2 G_3}{(1/K_{p\lambda}) + \frac{\dot{\lambda}(s)}{\theta_r(s)}}$$

$$= \frac{G_2 G_3}{1/K_{p\lambda} + G_2 G_3} = \frac{K_{p\lambda}\, G_2\, G_3}{1 + K_{p\lambda}\, G_2\, G_3}$$

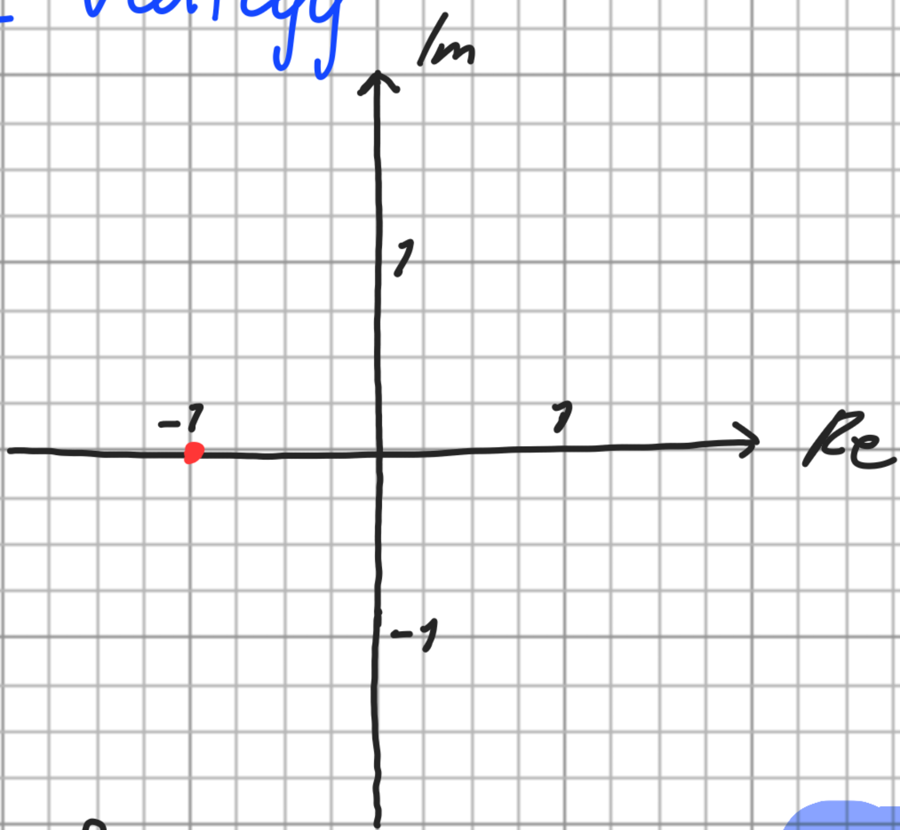
$$K_{p\lambda} G_2 G_3 = K_{p\lambda} \frac{K_2}{s} G_3(s)$$

$$L(s) = \left(\frac{\omega_n^2}{s(\tau s^3 + s^2 + 2\zeta \omega_n s + \omega_n^2)} \right) K_\lambda$$

d) Vedlegg viser at system blir ustabil for $K_\lambda > 1{,}8$ (Akkurat stabil ved $K_\lambda = 1{,}8$)

e) Sjekk vedlegg

$\boxed{3}$ a)

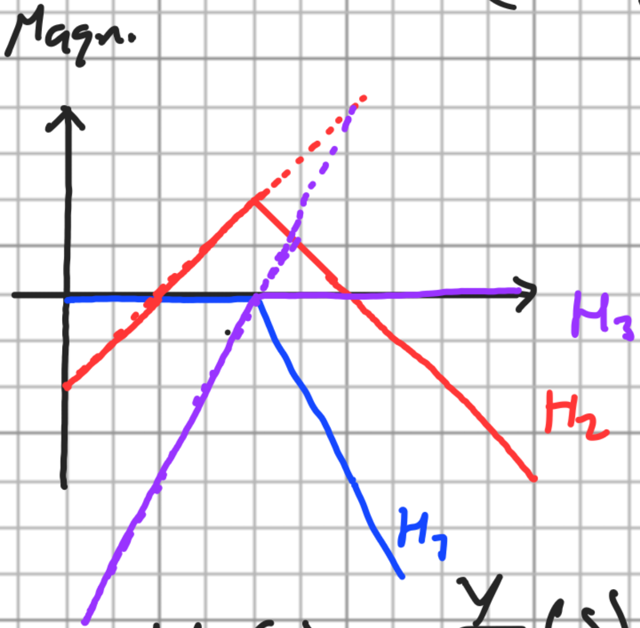


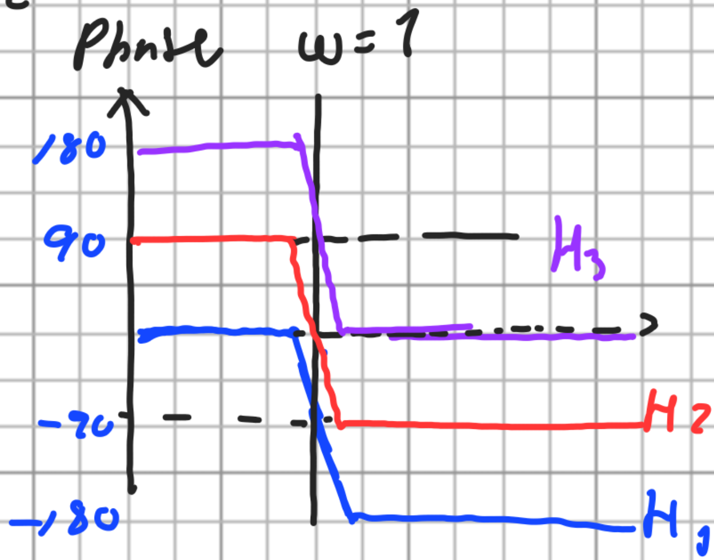
Alle tre filtre har pol ved $s = -1$

b) $H_1(s) = \dfrac{1}{s^2+2s+1} = \dfrac{1}{(s+1)^2}$

$H_2(s) = \dfrac{2s}{(s+1)^2}$

$H_3(s) = \dfrac{s^2}{(s+1)^2}$





c) $H(s) = \dfrac{Y}{U}(s) = \left(\dfrac{s^2+2s+1}{s^2+2s+1}\right) = 1$

d) A - m - 3 B - m - 2 C - h - 1

D - l - 2 E - l - 3 F - m - 1

G - h - 2 H - h - 3 I - l - 1