10.4.1

1) State vector: Input:

$$x = [\lambda \; r \; p \; \dot{p} \; e \; \dot{e}]^T \quad u = [p_c \; e_c]^T$$

For 4 first state we use same eq:

$$\begin{cases} \dot{\lambda} = r \\ \dot{r} = -K_2 p \\ \dot{p} = \dot{p} \\ \ddot{p} = -K_1 K_{pp} p - K_1 K_{pd} \dot{p} + K_1 K_{pp} p_c \end{cases}$$

and additionally

$$\dot{e} = \dot{e}$$

$$\ddot{e} = -K_3 K_{ep} e - K_3 K_{ed} \dot{e} + K_3 K_{ep} e_c$$

gives:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -K_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -K_1 K_{pp} & -K_1 K_{pd} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -K_3 K_{ep} & -K_3 K_{ed} \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_1 K_{pp} & 0 \\ 0 & 0 \\ 0 & K_3 K_{ep} \end{bmatrix}$$

2) Discretize w/ forward euler:

$$x_{k+1} = x_k + T\dot{x} = x_k + T(Ax_k + Bu_k)$$

$$= \underbrace{(I + TA)}_{A_d} x_k + \underbrace{BT}_{B_d} u_k$$

$$x_{k+1} = A_d x_k + B_d u_k$$

3) Ineq. constr: $e_k \geq \alpha \exp(-\beta(\lambda_k - \lambda_t)^2) \ \forall k \{1, \dots N\}$

Obj. func: $\phi = \sum_{i=0}^{N-1} (\lambda_{i+1} - \lambda_f)^2 + q_1 p_{ci}^2 + q_2 e_{ci}^2$

$\downarrow$

$q_1 = q_2 = 1$ $\qquad$ $\lambda_{i+1}^2 - 2\lambda_f \lambda_{i+1} + \lambda_f^2$

Check MATLAB

4) Implement (Not yet done)

5) Check LaC.