Manco il titolo mi ricordo

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Introduction

Scopo+descrizione sistema

1 Hydrostatic configuration

1.1 Physical model

The first part of the project is about understanding how the gas is distributed in a cluster, so to do a proper gravitational model is needed. We'll assume spherical symmetry, which will lead to all physical quantities depending only on r, the radial coordinate, which indicates the distance from the centre of the system. The gas is supported by thermal pressure and it's described by the hydrostatic equilibrium equation:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(<r)}{r^2}\rho_g(r), \tag{1}$$

which is a I order ODE. We have that in (1) $M(<r)$ indicates the total mass contained in a radius r, not just the mass of the gas, while P and ρ_g refer to the pressure and the density of the gas.
We have that the ICM is in equilibrium in the gravitational potential of the cluster, which is dominated by dark matter.
For the density profile of the latter we will assume a $Navarro-Frenk-White$ (NFW) $profile$, which takes the following form:

$$\rho_{DM}(r) = \frac{\rho_{DM,0}}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}, \tag{2}$$

where $\rho_{DM,0}$ and r_s are parameters which depend on the cluster mass. Integrating (2) we can calculate the mass profile, which as an analytical solution:

$$M_{DM}(r) = \int_0^r 4\pi r'^2 \rho_{DM}(r')\,\mathrm{d}r' = 4\pi\rho_{DM,0}r_s^3\left[\ln\left(1+\frac{r}{r_s}\right) - \frac{r/r_s}{1+r/r_s}\right]. \tag{3}$$

To have a more realistic model we will also need to consider the presence of the BCG. Its stellar mass is described by the $Hernquist\,profile$:

$$M_*(r) = M_{BCG}\frac{r^2}{(r+a)^2}, \tag{4}$$

where M_{BCG} is the mass of the elliptical central galaxy and a is a scale related to the half-mass radius $r_{1/2}$: $r_{1/2} = (1+\sqrt{2})a$.
Assuming the ICM is described by the perfect gas equation of state (i.e. $P = \frac{k_B\rho_g T_g}{\mu m_p}$), (1) becomes:

$$\frac{\mathrm{d}\ln\rho_g}{\mathrm{d}r} = -\frac{GM(<r)}{r^2}\frac{\mu m_p}{k_B T_g(r)} - \frac{\mathrm{d}\ln T_g}{\mathrm{d}r}, \tag{5}$$

where $T_g(r)$ is the gas temperature, while μ, m_p, and k_B are constants that represent the mean molecular weight, the proton mass and the Boltzmann constant respectively.

In the case of an isothermal gas and in the absence of the BCG eq.(5) has an analytical solution:

$$\rho_g = \rho_0 \exp\left\{-\frac{27}{2}b\left[1-\frac{\ln(1+r/r_s)}{r/r_s}\right]\right\} = \rho_0 e^{-27b/2}\left(1+\frac{r}{r_s}\right)^{27b/(2r/r_s)}, \tag{6}$$

$$with \quad b = \frac{8\pi G\mu m_p \rho_{DM,0} r_s^2}{27k_B T_{g,iso}}$$

1.2 The simulation

The first step is building two uniform grids, with one shifted by $\frac{1}{2}\Delta r$, where $\Delta r = r_{\rm j} - r_{\rm j\text{-}1}$. The maximum number of points in the grids is $j_{\rm max} = 5000$, while r ranges from $r_{\rm min} = 0Mpc$ to $r_{\rm max} = 3Mpc$. The points for the first grid are going to be

$$r_j = r_{min} + \frac{j-1}{j_{max}-1}r_{max}, \qquad 1 \le j \le j_{max} \tag{7}$$

so we'll use integer numbers to refer to them, while we'll use half-integers for the points in the second one. We'll thus need to define not only the points but also $r_{jmax+1/2}$, which we'll do as follows:

$$\begin{cases} r_{jmax+1/2} = r_{\rm jmax\text{-}1/2} + (r_{\rm jmax\text{-}1/2} - r_{\rm jmax\text{-}3/2}) \\ r_{\rm j+1/2} = r_{\rm j} + \frac{r_{\rm j+1} - r_{\rm j}}{2} \qquad 1 \le j \le j_{max} - 1 \end{cases} \tag{8}$$

Both the temperature T and the density ρ are centered at $r_{\rm j+1/2}$, while we centered the mass to $r_{\rm j}$.
To proceed with the simulation, we first integrate numerically $\rho_{\rm DM}$ in order to get $M_{\rm DM}$. Assuming $M_{\rm DM,1} = 0$, we get the following expression:

$$M_{\rm DM,j} = M_{\rm DM,j\text{-}1} + \rho_{\rm DM,j\text{-}1/2}\Delta V_{\rm j}, \tag{9}$$

where the j-th volume element $\Delta V_{\rm j}$ is given by:

$$\Delta V_{\rm j} = \frac{4}{3}\pi(r_j^3 - r_{j-1}^3). \tag{10}$$

To check the integration we overplotted the results against the analytical solution found in (3).
Once obtained M_{DM} we can move on to the integration of (5). We consider three possible scenarios:

- isothermal gas without the presence of the BCG, where we have $\frac{\mathrm{d}\ln T_g}{\mathrm{d}r} = 0$ and $M_j = M_{DM,j}$;
- isothermal gas with the presence of the BCG, where we have $\frac{\mathrm{d}\ln T_g}{\mathrm{d}r} = 0$ and $M_j = M_{DM,j} + M_{*,j}$;
- non-isothermal gas with the presence of the BCG, where we have $\frac{\mathrm{d}\ln T_g}{\mathrm{d}r} \neq 0$ and $M_j = M_{DM,j} + M_{*,j}$;

We transform (5) from an ODE to a FDE (finite difference equation), with the derivative centered in j:

$$\frac{\ln\rho_{g,j+1/2} - \ln\rho_{g,j-1/2}}{r_{j+1/2} - r_{j-1/2}} = -\frac{\mu m_p}{k_B\overline{T}_{g,j}}\frac{GM_j}{r_j^2} - \frac{\ln T_{g,j+1/2} - \ln T_{g,j-1/2}}{r_{j+1/2} - r_{j-1/2}}, \tag{11}$$

where $\overline{T}_{g,j}$ is defined as $\frac{T_{g,j+1/2} - T_{g,j-1/2}}{2}$. The expression for $\ln\rho_{g,j+1/2}$ can then be easily found from (11):

$$\ln\rho_{g,j+1/2} = \ln\rho_{g,j-1/2} - \Delta r\frac{\mu m_p}{k_B\overline{T}_{g,j}}\frac{GM_j}{r_j^2} - (\ln T_{g,j+1/2} - \ln T_{g,j-1/2}). \tag{12}$$

To conclude the setting of the simulations, the following list contains the parameters value and the temperature profiles:

- for the NFW profile: $\rho_{DM,0} = 7.35 \times 10^{-26}$ g/cm^3 and $r_s = 435.7$ kpc;
- for the *Hernquist profile*: $M_{BCG} = 10^{12} M_\odot$ and $r_{1/2} = 12$kpc;
- the initial condition $\rho_{g,0}$ for the gas density is:
  - 4.1×10^{-26}g/cm^3 for the first scenario;
  - 8×10^{-26}g/cm^3 for the second;

- 1.5×10^{-25}g/cm^3 for the third;

- for both temperature pofiles we have $\mu = 0.61$, while their expressions are:
  - for the isothermal case: $T_g = T_{mg} = 8.9 \times 10^7$K;
  - for the non-isothermal one:

$$\frac{T_g}{T_{mg}} = 1.35 \frac{(x/0.045)^{1.9} + 0.45}{(x/0.045)^{1.9} + 1} \frac{1}{(1 + (x/0.6)^2)^{0.45}} \tag{13}$$

    where $x = r/r_{500}$, with $r_{500} \sim r_{vir}/2 = 1.4$Mpc.

To find the appropriate values for the initial condition of the gas density we first run the code with an initial guess with the expected order of magnitude, then we adjust rthe values in order to obtain a baryon fraction $f_b = \frac{M_* + M_{gas}}{M_* + M_{gas} + M_{DM}}$ close to the cosmic value (~ 0.16) at the virial radius of the cluster ($r_{vir} \approx 2.8$ Mpc)

1.3 Results and conclusions