

**Problem Chosen**

**B**

**2021  
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Summary Sheet**

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## Fire prediction and monitoring

### Summary

In the 2019-2020 fire season, devastating wildfires occurred in every state in Australia. Fire prevention and control are essential. Drones for surveillance and situational awareness (SSA) have been used in Australia for several years. SSA drones carry high definition & thermal imaging cameras and telemetry sensors that monitor and report data from wearable devices on front-line personnel. This helps the Emergency Operations Center (EOC) to best command the firefighters for the best results and maximum safety. As a consultant, our team used drones and fires as research objects to create a model to determine the optimal numbers and mix of SSA drones and Radio Repeater drones to purchase for a proposed new division, "Rapid Bushfire Response", of Victoria's Country Fire Authority (CFA), analyze the model to adapt to the future. The change possibility of extreme fire events in ten years, optimize the position of drones to adapt to various situations and use this model as the basis to calculate the cost and provide a budget request.

The model was established based on the topographic characteristics of Victoria, comprehensively considered the model's economy, feasibility and safety. The model essentially explores the relationship model between the time of fire occurrence and the distribution of drones. The time of fire discovery should be roughly negatively correlated with the number of drones. In addition, the number of drones is also related to the terrain. Analyzing the problem, we need to establish a model to solve four small problems: fire monitoring problem, fire change trend and extreme fire treatment, terrain impact on fire, and budget application problem. We find out the internal logical relationship and restriction conditions and pass Linear programming yields model results. Finally, we fit the relationship between the number of UAVs and the longest response time through a quadratic function to verify the stability of the model.

**Keywords:** UAV; Nonlinear programming; Wild Fire fighting

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February 9, 2021

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# 1 Introduction

## 1.1 Background

The 2019-2020 fire season saw devastating wildfires in every Australian state, and fire control is critical. Unmanned aerial vehicle (uav) monitor and situational awareness (SSA) has been used for years in Australia, its carrying high definition and thermal imaging cameras and remote sensing sensor used for monitoring and reporting of frontline staff wearable equipment data, it helps to emergency operations center (EOC) can best command fire fighters, get the best effect and the maximum security. As consultants, our team with drones and fire as the research object, create a model used to determine the optimal number of SSA unmanned aerial vehicle (uav) and mixed purchasing radio relay uav for proposed new division, the analysis model to adapt to the changes in extreme fires events over the next decade, optimize the drone position to adapt to various situations and based on this model, the computational cost and provide a budget request.

## 1.2 Statement of the problem

Our team was commissioned by Victoria's Country Fire Authority (CFA), to create a UAV distribution model for its proposed new division, "Rapid Bushfire Response", to determine the appropriate procurement of UAVs. Everything we do is to achieve the ultimate goal that the Emergency Operations Centre (EOC) can be able to efficiently and safely direct fire fighting while SSA UAVs help monitor changing conditions.

The data and information we already have include information of UAV(Table 1) , a heat map of fires in Victoria, Australia(Figure 1), a topographic map of Victoria(Figure 2) and combination of UAV and signal.

Table 1: Information of UVA

| price         | maximum speed       | flight range |
|---------------|---------------------|--------------|
| \$10,000(AUD) | 20 m/s              | 30 km        |
| recharge time | maximum flight time |              |
| 1.75 hr       | 2.50 hr             |              |

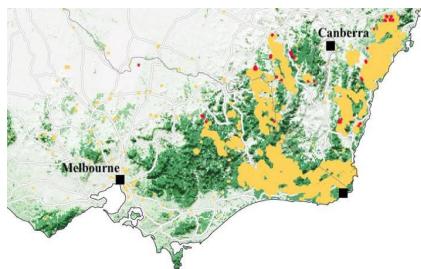


Figure 1: Heat map of fires in Victoria



Figure 2: Topographic map of Victoria

Hanging drones carrying repeaters have been used to dramatically extend the range of low power radios on the front lines. A 10-watt repeater, weighing 1.3 kg carried by a drone hovering well above ground level, can achieve a range of 20 km.

**Requirements of the client or requirements of this model:**

- \* Determine the optimal numbers and mix of SSA drones and Radio Repeater drones.
- \* Balance capability and safety with economics, as well as consider observational and communications mission needs and topography.
- \* Incorporate fire event size and frequency as parameters.
- \* Adapts to the changing likelihood of extreme fire events over the next decade and project what equipment cost increases will occur assuming the cost of drone systems stays constant.
- \* Optimize the locations of hovering VHF/UHF radio-repeater drones for fires of different sizes on different terrains.
- \* Prepare a one- to two-page annotated Budget Request supported by your models for CFA to submit to the Victoria State Government.

## 2 Assumption

The model establishment, model solution and thesis writing are based on the following assumptions:

- \* The UAV always maintains its maximum speed. Power consumption has nothing to do with driving direction, terrain, weather and other external factors.
- \* During the fire monitoring process, the distribution of front-line personnel was uniformly distributed locally, which generally depends on the characteristics of the regional fire frequency distribution.
- \* The number of firefighters in the area where the fire is found will increase in a short time to prevent the fire from developing. It is assumed that the increase in the number of firefighters is a specific function with time and there is no need to consider the impact of the error.
- \* The total area is divided into three terrains: forest, mountain and plain. The risk caused by the different fire intensity between terrains is different, and the risk coefficient of the model monitoring density will also change accordingly.
- \* The risk of the target area can be measured by terrain and the number of fires. Assume that the order of risk is:

$$\text{forest} > \text{mountain} > \text{plain}$$

- \* Do not consider the impact of extreme weather such as heavy rain and blizzard on the fire.

### 3 Analysis of the Problem

#### 3.1 Fire monitoring problem

For Requirement 1, we need to obtain a drone monitoring model to coordinate the number and distribution of drones and repeaters, so as to realize the signal transmission from frontline personnel to EOC. The possibility of fire in this model area is a model parameter. The output of the model also takes security and budget as parameters, and we weigh the relationship between the two through the planning model.

#### 3.2 Fire changing trend and extreme fire treatment

Regarding the first point in Requirement I, we need to collect disaster data on past fires in the area and analyze the possibility of fires in a unit area during this period of time, so as to deduce possible future fire points. In addition, at the macro level, it is also possible to analyze the law of change in high-risk areas, and obtain a model of the degree of risk over time. Combining the above fire monitoring model can derive the changes of monitoring arrangement strategy over time.

Regarding the second point in Requirement II, we disaggregated the scope and cause of extreme fires, including the large fire at the time of discovery, multiple fires, and rapid fire spread. We have revised the initial model to prevent each situation by increasing surveillance density, regional division, and monitoring the direction of fire spread after a fire.

#### 3.3 Topography's influence on fire

For requirement three, by finding the influence of ground height and ground slope on the signal range of the UAV repeater, we added a terrain correction factor to the initial model to optimize the model's simulation in a complex terrain environment. In addition, by understanding the influence of terrain on fire, we have added terrain influence parameters to the fire spread model. By adjusting the parameters, it can better simulate the changes of the fire on the sloped ground.

#### 3.4 Budget application

For requirement four, by summarizing the above issues and finding relevant information, we connect the "project budget", "number of drones", "monitoring density and response time" and "disaster area and disaster impact" and obtain a final diagram between them. In addition, according to the actual situation, we input relevant data to perform parameter fitting, and get a more realistic budget application form.

### 4 Calculating and Simplifying the Model

#### 4.1 General model of drone monitoring

First, consider the general configuration model of UAV with repeater to explore in the risk area. This model aims to solve the optimal arrangement of UAV in flat and open space without interference from external factors, so as to expand the effective coverage area and achieve good communication effect. It should be noted that since this model is applied to a general model in a general situation, the model is constrained by the following conditions:

\* The area is large enough. When considering the influence of region boundary, the region

can be regarded as Continuous ring.

- \* Land of the region is empty and the terrain is flat without shelter.
- \* The characteristics of any subregion are consistent.

Under condition above, the optimal arrangement scheme of UAV is considered.

#### 4.1.1 Single UAV monitoring

First, consider the monitoring of a single drone within its flight range. A single UAV is limited by it's type. The monitoring Range has a maximum value, and the maximum monitoring area is a circle centered on the UAV base point. Due to the limitation of maximum speed and maximum flight time, the UAV cannot monitor every part of the maximum monitoring area at all times. Therefore, the model needs to design a scheme to balance the relationship between monitoring area and monitoring efficiency.

The observation quantities "Monitor Density" and "Response Time" are introduced below.

Monitoring density(Figure 3) is defined as the ratio of the area the UAV can monitor to the maximum monitoring area.

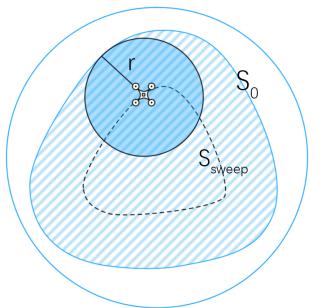


Figure 3: Monitoring density map

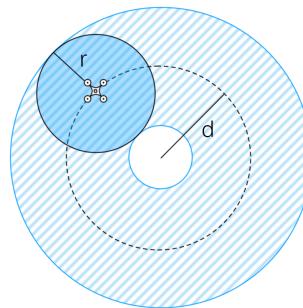


Figure 4: Flight path map

Because of the limited speed of the drone, there may be some locations that are not being detected at the moment in areas that a single drone can detect. In other words, when a front-line investigator sounds an alert in a surveillance area, it is not always immediately detectable by a drone covering the area. It usually takes some time before a drone enters the area and relays the signal via a repeater.

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In the case of a circular flight path(Figure 4), notice that the contradiction between monitoring density and response time revolves around the "path radius" observation. The path radius is used as a parameter in the input model, and the best value of the two is coordinated according to other parameters in the subsequent model. In addition, in order to increase the monitoring density, it is also a feasible solution to set up multiple UAVs in the same small area for joint monitoring. The monitoring plan of multiple drones in the unit area will be discussed later.

#### 4.1.2 Multiple drones array

The following discusses the arrangement of multiple UAVs. Assuming that the number of drones is fixed, and the characteristics of any sub-region of the region are consistent according to the constraints of the general model, the optimal arrangement scheme is unified everywhere. The above has determined that the periphery of the monitoring range of a single UAV is a circle, and the arrangement of multiple UAVs is the problem of filling several equally-sized circular areas. Among the various filling schemes, the hexagonal filling(Figure 5) has the highest filling density *cite there*, that is, the area of the unfilled part achieves the minimum value under the same number of arrangement units.

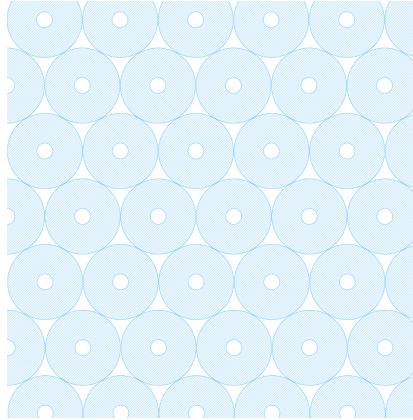


Figure 5: Hexagon fill map

Consider the total monitoring density of the area in the global arrangement. Similar to the situation of single drone surveillance, the total surveillance density of multiple drone surveillance areas can also be defined by the ratio of the area, that is, the sum of the area that can be monitored by each drone divided by the total area of the area. We introduce the observational "Monitor Density Factor", which is defined as the ratio of the monitoring density of a single UAV to the total monitoring density, which is used to measure the impact of the UAV arrangement on the monitoring density. Considering the area definition of UAV monitoring density, the monitoring density factor is the ratio of the sum of the area of each small area to the total area of the area. Due to the uniformity of the characteristics of any sub-region and the repeatability of the filling unit with the hexagonal filling, the monitoring density factor of the entire area can be equal to the monitoring density factor in the unit area. Therefore, the smallest arrangement unit filled with hexagons, namely the unit triangle(Figure 6 and 7), can be used as the calculation unit for monitoring the density factor.

The model first explores the situation where the number of drones in the monitoring unit is 1 in order to adapt to the most general situation.Taking into account the UAV's 2.5 h of battery life, 1.75 h of charging time, and regular arrangement, this model depicts a globally loose and

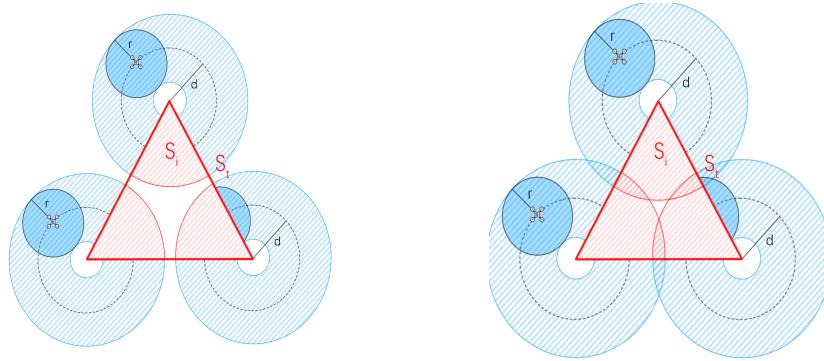


Figure 6: Unit triangle diagram map 1

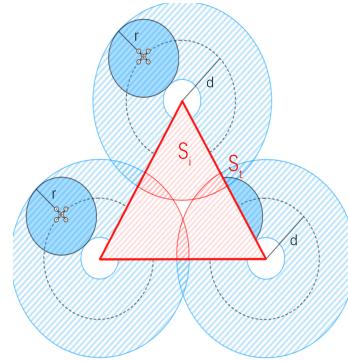


Figure 7: Unit triangle diagram map 2

locally concentrated model structure. In the time before the fire, the drones in the model are divided into "columns" according to the following intervals. At this time, the strategy is to take off separately, and the takeoff interval is 1.75h. When a fire breaks out in a certain place, all drones in that place will operate to increase the surveillance density in the fired area within a short period of time.

#### 4.1.3 Model parameter declaration and calculation

Table 2: Declared model parameter comparison table

| Parameters     | Description                                   |
|----------------|---|
| r              | The repeater monitors the signal radius       |
| R              | Maximum range of UAV                          |
| d              | The radius of the drone's orbit               |
| D              | Monitoring unit radius                        |
| $t_{interval}$ | Flight time interval                          |
| T              | Flight cycle                                  |
| $T_{max}$      | Maximum flight time                           |
| $\rho$         | Unit monitoring density                       |
| $\Omega$       | Total monitoring density                      |
| $\mu$          | Monitoring density factor                     |
| $t_s$          | The response time                             |
| $E(t_s)$       | The mathematical expectation of response time |
| k              | Center distance between units                 |
| v              | Fire frequency or risk                        |
| $t_{limit}$    | Maximum expected response time                |
| $\lambda$      | Risk factor                                   |
| $n_p$          | The number of drones in the unit circle       |
| $\Omega_k$     | The monitoring density in the kth time period |
| $\Omega_{n_p}$ | The monitoring density at this stage          |
| $\Omega'''$    | The average monitoring density of the process |

First calculate the monitoring situation for a single UAV. According to the above model graph relationship, we can get:

Table 3: Public parameter comparison table

| Parameters | Description                                  |
|------------|--|
| n          | The number of drones in the area             |
| $v_{fire}$ | The speed at which a fire spreads            |
| a          | Reverse acceleration due to fire suppression |
| $Q_{cost}$ | Fire loss                                    |
| $S_m$      | Area   |

Public parameters are parameters that also appear outside the model

$$\begin{cases} D = r + d \\ r < d < R \end{cases}$$

Defined by the monitoring density:

$$\begin{cases} \rho = \frac{S_{sweep}}{S_0} \\ S_{sweep} = \pi r^2 \\ S_0 = \pi D^2 \end{cases}$$

The result of the calculation is:

$$\rho = \frac{r^2}{D^2} \quad (1)$$

Consider the impact of the monitoring density factor on the overall monitoring density.  
From the factor definition:

$$\begin{cases} \mu = \frac{\sum_{adj(i)} S_i}{S_t} \\ S_i = \frac{1}{6} \times \pi D^2 \\ S_t = \frac{\sqrt{3}}{4} k^2 \end{cases}$$

The result of the calculation is:

$$\mu = \frac{\sum_{adj(i)} S_i}{S_t} = \frac{\frac{1}{2} \pi D^2}{\frac{\sqrt{3}}{4} k^2} = \frac{2\pi}{\sqrt{3}k^2} \quad (2)$$

according to (1) and (2) :

$$\Omega = \rho \mu = \frac{2\pi r^2}{\sqrt{3}k^2}$$

Mathematical expectation expression:

$$E(t_s) = \frac{T(1 - \Omega)^2}{2} = \frac{\pi d}{v_p} \left(1 - \left(\frac{r}{k}\right)^2 \times \frac{2\pi}{\sqrt{3}}\right)^2$$

Since the drones operate at intervals to facilitate charging management at the beginning, the separation work is adopted. At this time, the total monitoring density of the drones used in the separation work is:

$$\Omega' = \rho\mu' = \frac{2\pi r^2}{\sqrt{3}(2k)^2}$$

Therefore, in a cycle, the surveillance density of drones is divided into two parts. The first part is  $t_{interval}$  when it runs separately. The second part is the running time  $T_{max} - t_{interval}$  together. In the absence of a fire, the overall surveillance density is the weighted average of these two parts:

$$\Omega'' = \frac{t_{interval}}{T_{max}} \times \Omega' + \left(1 - \frac{t_{interval}}{T_{max}}\right) \times \Omega$$

In this case, the mathematical expectation of finding time is:

$$E'(t_s) = \frac{T(1 - \Omega'')^2}{2} = \frac{\pi d}{v_p} \left(1 - 0.475 \times \left(\frac{r}{k}\right)^2 \times \frac{2\pi}{\sqrt{3}}\right)^2$$

**Convert the above conditions into a nonlinear programming model:**

$$\begin{aligned} & goal = \min(W_{cost}) \\ & s.t. \left\{ \begin{array}{l} n \times \frac{\sqrt{3}}{2} k^2 \geq S_m \\ E'(t_s) = \frac{T(1 - \Omega'')^2}{2} = \frac{\pi d}{v_p} \left(1 - 0.475 \times \left(\frac{r}{k}\right)^2 \times \frac{2\pi}{\sqrt{3}}\right)^2 \\ r \leq d \leq R \\ E'(t_s) < t_{limit} \\ \Omega \leq \lambda \times v \\ \Omega' = \rho\mu = \frac{2\pi r^2}{\sqrt{3}(2k)^2} \\ \Omega = 4 \times \Omega' \\ \Omega'' = \frac{t_{interval}}{T_{max}} \times \Omega' + \left(1 - \frac{t_{interval}}{T_{max}}\right) \times \Omega \\ W_{cost} = 10000 \times n \end{array} \right. \end{aligned}$$

It can be seen that the results of the above-mentioned nonlinear programming are determined by the four values of risk coefficient, expected response time upper limit, risk degree, and disaster area. We input these four values as external system parameters into the model, and get the following results:

**First, results of simulated forest:**

Qualification:

$$v = 0.6 \quad S_m = 1000000 km^2 \quad \lambda = 2.5$$

**Second, results of simulated mountain:**

Qualification:

$$v = 0.33 \quad S_m = 1000000 km^2 \quad \lambda = 2.5$$

Table 4: Forest simulation

| Expected $t_{limit}$ | Number of drones | k       |
|----------------------|------------------|---------|
| 0.5 hr               | 406              | 53.3456 |
| 0.45 hr              | 472              | 49.4536 |
| 0.4 hr               | 540              | 46.2361 |
| 0.35 hr              | 613              | 43.3707 |
| 0.3 hr               | 690              | 40.8808 |
| 0.25 hr              | 777              | 38.5298 |
| 0.2 hr               | 872              | 36.3696 |
| 0.15 hr              | 980              | 34.3188 |
| 0.1 hr               | 1108             | 32.2822 |
| 0.05 hr              | 1274             | 30.1022 |

In the planning results, d is always equal to 20

Table 5: Mountain simulation

| Expected $t_{limit}$ | Number of drones | k       |
|----------------------|------------------|---------|
| 0.5 hr               | 407              | 53.2767 |
| 0.45 hr              | 472              | 49.4616 |
| 0.4 hr               | 541              | 46.2037 |
| 0.35 hr              | 614              | 43.3574 |
| 0.3 hr               | 693              | 40.8184 |
| 0.25 hr              | 779              | 38.5098 |
| 0.2 hr               | 873              | 36.3625 |
| 0.15 hr              | 981              | 34.3129 |
| 0.1 hr               | 1108             | 32.2794 |
| 0.05 hr              | 1274             | 30.1022 |

In the planning results, d is always equal to 20

### Third, results of simulated plain:

Qualification:

$$v = 0.15 \quad S_m = 100000 \text{ km}^2 \quad \lambda = 1$$

## 4.2 UAV arrangement model under fire conditions

### 4.2.1 UAV layout adjustment

Once the fire signal discovered by the frontline personnel is transmitted to the EOC through the repeater mounted on the drone, the drone's monitoring will enter the fire state. In this case, in order to collect characteristic data such as the size, specific location, and trend of the fire, EOC will send more frontline investigators to the scene where the fire was discovered for further investigation. At the same time, firefighters will be dispatched and rushed to the scene. The drone monitoring in this process is even more critical. It is necessary to cooperate with the front-line personnel who rushed to further transmit the detected signals to ensure smooth communication. Note that the size and location of the fire are all obvious data, which can be received by further investigation. However, it is often difficult to predict the development trend

Table 6: Plain simulation

| Expected $t_{limit}$ | Number of drones | k       |
|----------------------|------------------|---------|
| 0.5 hr               | 41               | 52.9090 |
| 0.45 hr              | 52               | 47.0688 |
| 0.4 hr               | 54               | 46.1959 |
| 0.35 hr              | 61               | 43.3543 |
| 0.3 hr               | 69               | 40.8184 |
| 0.25 hr              | 78               | 38.5098 |
| 0.2 hr               | 87               | 36.3626 |
| 0.15 hr              | 98               | 34.3129 |
| 0.1 hr               | 111              | 32.2794 |
| 0.05 hr              | 127              | 30.1022 |

In the planning results, d is always equal to 20

and spreading direction of the fire. How to adjust the deployment strategy of drones has become a crucial issue.

As shown in the figure 8, suppose that a fire has been found in the red area, and the approximate range of the fire when found is as shown in the figure above. The original UAV monitoring arrangement is shown in the blue circular area in the figure, which is the hexagonal arrangement mentioned above. Assuming that the area of the drone is a tightly filled hexagon, and the shortest distance from the point to the center on the edge of the hexagon is half of the unit center distance, the fire situation can be measured by the area of the drone.

**In figure 9,notice that:**

- \* The area of the drone that already has a fire is the fire area.
- \* the area next to the fire area is called the dangerous area.

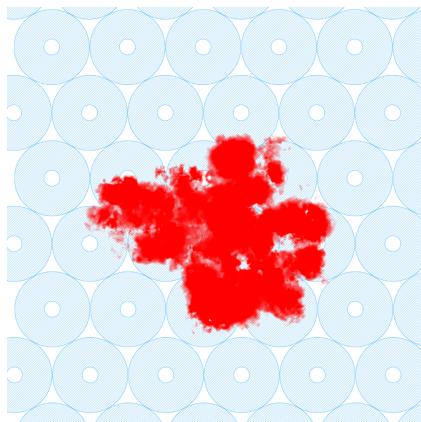


Figure 8: Fire simulation map

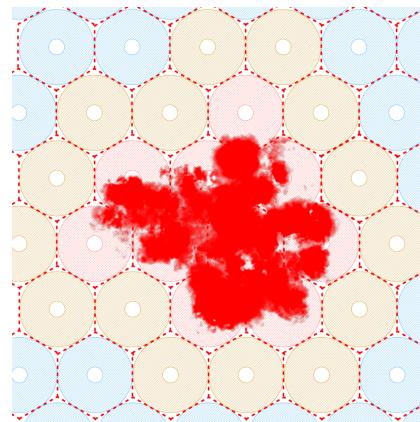


Figure 9: The fire zone map

In a fire situation, the focus should be on the dangerous area rather than the fire area. The hazardous area is located at the edge of the fire area, affecting the development trend and scope of the fire. As long as the spread of the fire in the dangerous area is controlled, the size of

the impact and the area affected by the fire can be controlled. A fire has already started in the fire area. Even if the fire is not extinguished in time, the fire will naturally go out as the combustibles burn out.

Therefore, drones in the fire area should fly away from the central fire area, and focus on monitoring the edge dangerous areas.

#### 4.2.2 Model optimization in extreme fire situations

In order to prevent major losses caused by extreme fires, this model changes the number of drones in the unit circle  $n_p$  and greatly increases the risk factor  $\lambda$  to achieve solutions to extreme fires and result verification.

At the same time we need to recalculate the monitoring density  $\Omega$

To calculate of  $\Omega'''$ : We divide a movement-charge cycle into  $2n_p - 1$  parts, and the first  $np - 1$  parts are used as the start-up phase of these  $n_p$  drones. The startup phase time is divided into  $n_p - 1$  time periods, and the length of each time period is  $\frac{1}{n_p-1} \times 1.75$ . In each time period we start a new drone, then there are k drones for reconnaissance in the kth time period. Therefore, the monitoring density in the kth time period ( $\Omega_k$ ) is:

$$\Omega_k = k \times \Omega$$

When the last drone takes off, the system enters the second phase, which is a 0.75-hour overall surveillance phase. The monitoring density at this stage is  $\Omega_{n_p}$ . Subsequently, the first aircraft began to enter the charging phase one after another, which is the reverse process of the starting phase, until the last aircraft entered the charging phase, and the first aircraft entered the starting phase again. This cycle achieves the maximum utilization of all aircraft. Therefore, the average monitoring density of the process ( $\Omega'''$ ) is:

$$\Omega''' = \frac{2 \sum_{i=1}^{n_p} \frac{1}{n_p-1} \times 1.75 \Omega_k + 0.75 \Omega_{n_p}}{1.75 + 2.5}$$

This model solves the charging problem by introducing multiple drones on the basis of Model 1. In this model, the drones no longer need to operate at intervals to achieve better results.

Due to the large losses caused by the fire, this model aims to find the location of the fire the first time to quickly prevent the fire from developing. So we let:  $\Omega''' \geq 1$

**Convert the above conditions into a nonlinear programming model:**

$$goal = \min(n \times n_p)$$

$$s.t. \begin{cases} n_p \geq 2 \\ \Omega''' = (\frac{5}{2}n - 3.5) \times \Omega \\ \Omega''' \geq 1 \\ \Omega = \rho\mu = \frac{2\pi r^2}{\sqrt{3}k^2} \\ \Omega \geq \lambda v \end{cases}$$

In order to reflect the characteristics of serious fire, the value of  $\lambda$  is as large as possible in the solution process to meet the requirements of disaster prevention. The result is as follows(List

1 and Picture 1) and the optimal solution for this situation is that the UAVs are placed in a unit circle with 10 UAVs, which are placed at 146 places in total.

Qualification for List 1 and Picture 1:

$$v = 0.75 \quad S_m = 100000 \text{ km}^2 \quad \lambda = 25$$

Table 7: List 1

| $n_p$ | Number of drones | k       | d       |
|-------|------------------|---------|---------|
| 2     | 746              | 12.4410 | 29.8101 |
| 3     | 497              | 15.2370 | 29.3082 |
| 4     | 373              | 17.5942 | 20.0546 |
| 5     | 298              | 19.6709 | 29.9959 |
| 6     | 249              | 21.5484 | 29.9816 |
| 7     | 213              | 23.2749 | 20.0332 |
| 8     | 187              | 24.8819 | 20.1783 |
| 9     | 166              | 21.3913 | 20.0081 |
| 10    | 146              | 28.1060 | 29.9863 |
| 11    | 135              | 29.1766 | 20.7559 |
| 12    | 124              | 30.4740 | 20.1673 |
| 13    | 115              | 31.7184 | 29.5133 |
| 14    | 107              | 32.9135 | 19.9790 |
| 15    | 99               | 34.0710 | 20.2697 |
| 16    | 94               | 35.1884 | 21.5926 |

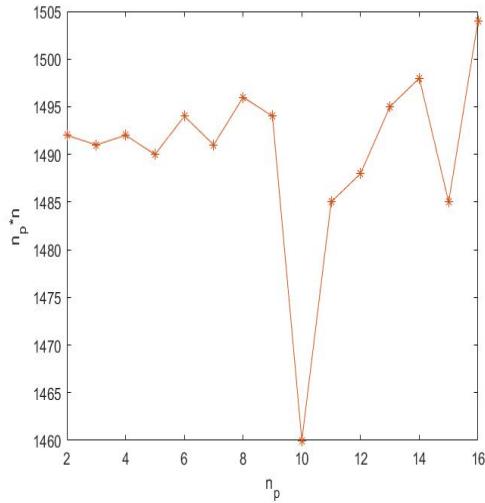


Figure 10: Picture 1

### 4.3 Risk over time model

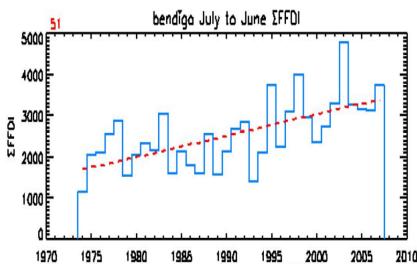


Figure 11: Bendigo July to June (FFID)

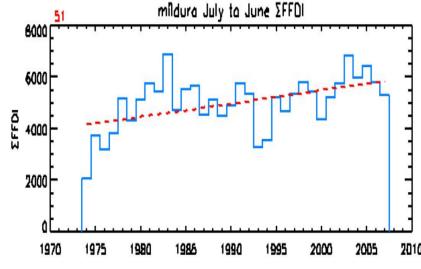


Figure 12: Mildura July to June (FFID)

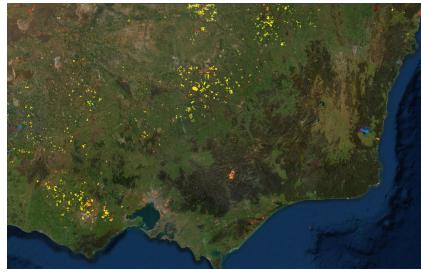


Figure 13: Fire in 2018

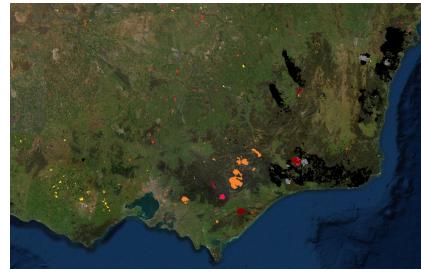


Figure 14: Fire in 2019

Here we introduce the MacArthur Forest Fire Hazard Index (FFDI) [2]. The MacArthur Forest Fire Hazard Index (FFDI) is based on four factors: temperature, rainfall (expressed as drought factor), humidity and wind speed to measure the degree of regional risk. When the FFDI is 25-50, the risk level is "very high". When the FFDI is greater than 50, the risk level is "extreme", and it usually states "total fire bans." The Black Friday Forest Fire of 1939 was used as an example of the 100 level. In the forest fires in 2009, the index reached more than 100 points in many places. In southeastern Australia, the total annual FFDI has been increasing for 35 years. Research conducted by CSIRO and the Bureau of Meteorology through the Australian

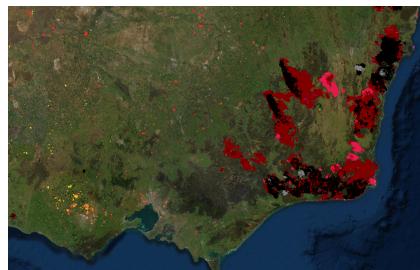


Figure 15: Fire in 2020

Weather and Climate Research Center and Bushfire CRC found that in many locations, the total annual FFDI increased rapidly from the late 1990s to the early 2000s (Lucas et al., 2007). In most locations, there has been an increase of 10- 40 between the 1980-2000 average and the 2001-2007 average. These increases are related to the increase in the number of "very high" and "extreme" fire hazard days. The figure shows the growth trend of the annual total FFDI of the four Victorian sites with high-quality data. The FFDI value was extremely high on February 7, 2009, indicating that FFDI continues to increase. Therefore, forests are likely to appear in the next ten years. The fire risk in Victoria is not optimistic.

## 4.4 Terrain optimization

### 4.4.1 Model optimization

The above general monitoring model reflects the arrangement of drones on open and flat ground. In actual problems, because the terrain will affect the effective distance of the signal, the flight path and the speed of fire spread, the terrain is an important factor that cannot be ignored in the model. If the above model is directly applied to the monitoring area, ignoring the influencing factors of these terrains will definitely affect the accuracy of the model. So now we optimize the above model for terrain features.

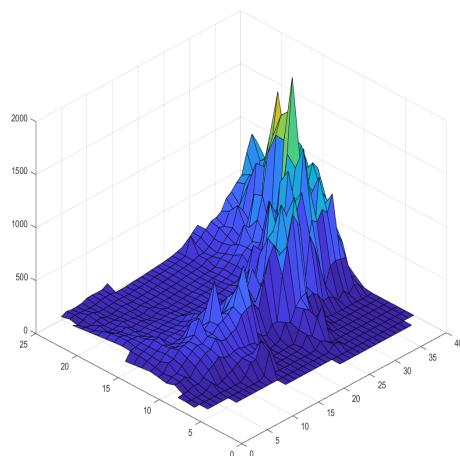


Figure 16: Schematic diagram of topographic features

In order to quantify the features of the terrain, the curved surface of the terrain is considered to be composed of many tiny planes. Observing a single plane, the distribution of frontline personnel is the same as the horizontal plane, so the monitoring range of the drone should also be above this plane. We apply the same flight path and arrangement rules as the horizontal plane.

As can be seen from the Figure 16, compared with flat ground, the number of drones is multiplied by a terrain correction factor after adding terrain parameters. The factor is equal to the square of the reciprocal of the cosine of the angle between the normal of the slope and the vertical. As follows:

$$\left\{ \begin{array}{l} \theta = \langle \vec{N}_P, \vec{N}_0 \rangle \\ k = \cos^{-1} \theta = \sec \theta \\ \frac{n'_p}{n_p} = k^2 \end{array} \right.$$

Can get this result:

$$\frac{n'_p}{n_p} = \cos$$

It can also be seen from the above formula that the number of drones on a slope is positively related to the slope of the slope. This result is in line with model expectations.

In order to consider how the terrain affects the model, first, the terrain will affect the UAV signal radiation range. On flat ground, the monitoring range of the drone is regarded as a circle, but in fact the signal from the repeater mounted on the drone is roughly spherical in space, and the signal strength is attenuated in these directions. In this model, it is assumed that the signal from the repeater in the space is spherical, and the radius is the same as the circular radius of the UAV monitoring range on flat ground. Same as on flat ground, after the frontline personnel standing on a mountain slope detect the fire, the signal sent out can also be transmitted within the monitoring range of the drone. In other words, the position of the frontline personnel in the space needs to be within the monitoring ball.

Secondly, the rate of fire spread will also be affected by the slope of the ground. Affected by terrain wind force and thermodynamic effects, the fire spreading speed is different from that in the flat ground model. When optimizing the model for the terrain, the fire spread speed can be regarded as a function of the ground slope as a parameter. By adapting the ground slope to affect the spread of fire, the regional risk and drone monitoring can be adjusted accordingly.

In addition, under actual conditions, the slope of the ground will also affect the risk of the area and the state of the fire in many ways. In this model, the optimization is mainly based on the above two points, and other influence methods are used to fine-tune the model by adjusting parameters.

#### 4.4.2 Optimization model calculation

When calculating the model, the terrain data should be processed first, and the terrain surface should be divided into planes per unit area. After the calculation, the area integration is performed to obtain the drone arrangement in the entire area.

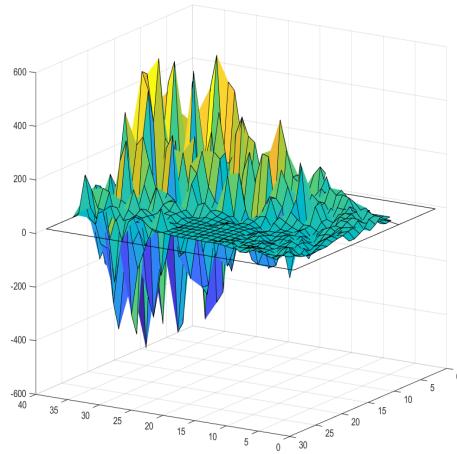


Figure 17: Result map of terrain influence factor

## 5 Validating the Model

We use some data for fitting to test the relationship between the longest response time and the number of drones. The result is shown in the figure, the corresponding function relationship is smooth and the model is relatively stable.

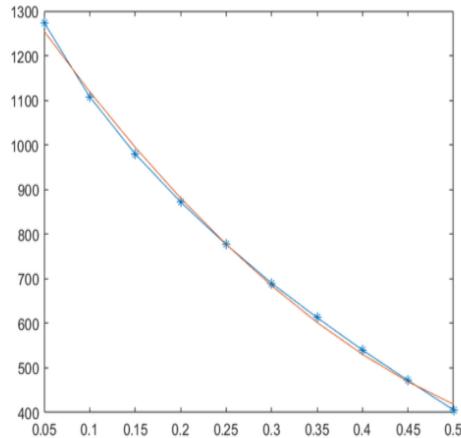


Figure 18: Relationship between the longest response time and the number of drones

## 6 Conclusions

Under normal circumstances, because the size of the fire and the regional risk are not extreme, the risk has a small impact on the model and the expected response time has a large impact on the model. Therefore, the model results vary according to the expected response

time. In the case of a total area of 1,000,000 square kilometers, an expected response time of 0.2 hours requires 873 drones. At the same time, it can be seen that the total area has a linear growth relationship with the number of drones. , When the total area drops to 10 times the original, the number of drones also drops to 10 times the original. All in all, under this model, there are not many drones that can effectively monitor large areas.

When an extreme fire situation occurs, due to the large loss caused by the fire, this model aims to increase the risk factor and increase the number of drones with a unit monitoring circle to effectively control high-risk areas. In a high-risk area with a risk factor of 25, an area risk degree of 0.75, and an area of 100,000 square kilometers, drones are placed with 146 centers and 10 units in the unit circle to achieve an efficient supervision mode. A total of 1,460 drones are required.

For extreme terrain models, we introduce slope parameters to measure the impact of terrain on the model, and get the "terrain correction factor". This factor explains to a certain extent the obstructive effect of terrain on helicopters. For areas with extremely steep slopes, the resulting structure is still acceptable.

## 7 A Summary

Our team established a UAV detection model to solve the problem of UAV monitoring in the context of the Victorian fire situation, comprehensively inspecting Victoria's terrain, fire influence factors, UAV performance and the relationship between them. The logical relations are transformed into mathematical language and applied to solving nonlinear programming problems, and we use Matlab, Photo Shop, Python and other tools for data processing and model solving. The model is established under normal circumstances and optimized after introducing new parameters (new influencing factors) to obtain the monitoring situation of UAVs and the number and distribution of UAVs under various conditions. Finally, we wrote an annotated Budget Request supported by our model for CFA to submit to the Victoria State Government.

## 8 Evaluate of the Mode

### 8.1 Strengths

- \* The model adopts a regular and reasonable distribution form, focusing on the overall effectiveness of coverage and the actual function of the UAV. The final model realizes an effective monitoring mode.

### 8.2 Weaknesses

- \* Lack of consideration for complex terrain and differences in regional details.
- \* Parameter calculations are mostly replaced by representative mean values, which are not very accurate.

## References

- [1] Stephenson, C., Handmer, J., & Betts, R. (2013). Estimating the economic, social and environmental impacts of wildfires in Australia. Environmental Hazards, 12(2), 93-111.

- [2] Perrem, Kilian and Booth, Trevor H. CSIRO Submission 09/338[A].2009.
- [3] <https://firms2.modaps.eosdis.nasa.gov/map/>
- [4] <https://www.cfa.vic.gov.au/home>

# Appendices

## Appendix A First appendix

Dear, Victoria State  
Government

The Australian bushfires are a common natural disaster event in Australia's hot and dry season. In recent years, the scale of wild fires and the average area affected have both been on the rise. Large areas of wild jungle and land have been destroyed, causing a large number of casualties and hundreds of millions of property losses. Since November 2019 alone, bushfires that have lasted for more than half a year have greatly changed the land environment and have almost become the worst bush fire accident in history.

In order to effectively prevent the occurrence of bush fires, reduce the risk of fire occurrence, and reduce the scale of fire and disaster losses, the Victoria Fire Department sends frontline investigators to the bushes to monitor fire risks and investigate fire patterns every year during wildfire season. In order to cooperate with the work of frontline personnel, transmit detection signal data, and promptly guide the firefighters to extinguish the fire team to the forest to control the fire, we plan to send a UAV with a signal repeater as a signal transmission tool in the forest. After research and investigation, we plan to purchase a certain number of WileE-15.2X Hybrid Drone drones to participate in signal transmission.

After our mathematical analysis, we have drawn up a monitoring model for drones. This model can ensure that the number of drones is as small as possible while increasing the monitoring density of drones in jungle terrain, thereby shortening the response time for frontline personnel to issue fire alarms, and then quickly deploy operations in a short time. Control the fire at the beginning of combustion to achieve the purpose of reducing the disaster area and reducing fire losses. Simply put, this model can arrange drones to maximize their transmission effects, so that the fire can be effectively controlled in the early stage of a disaster, and the model has a certain ability to deal with extreme situations. Through calculations, our plan is to purchase 207 drones at a cost of \$2,070,000 AUD. According to the fire risk assessment in the Victoria area, a hexagonal arrangement is used in the high-risk areas to be distributed in the jungle. In the fire situation, there is an optimized distribution plan of drones to monitor the spreading trend of the fire, and effectively assist the frontline personnel and firefighters to control the fire.

At the same time, it should be noted that in order to deal with the extreme fire that may occur, it is necessary to appropriately increase the number of drones based on the above reference value. Increasing the number of drones helps to further shorten the signal response time and curb the spread of fire. In extreme cases, there is more room to move, and it will be easy to adjust the deployment model.

Sincerely yours,

Your friends

## Appendix B Second appendix

Here are simulation programmes we used in our model as follow.

### Input matlab source 1:

---

```
function [g, h] = ConstraintFunc(x)
    s = 100000;
    r = 20;
    vp = 72;
    t= 0.1;
    R = 30;
    v = 0.75;

    g(1) = s - x(1) * sqrt(3)/2 * x(2)^2;
    g(2) = r-x(3);
    g(3) = x(3) - R;
    g(5) = 25*v - x(4) * (r / x(2))^2 * 2 * pi / sqrt(3);
    g(6) = 2 - x(4);
    g(7) = 1 - 4*(5/8*x(4) - 0.875) * ((r / x(2))^2 * 2 * pi / sqrt(3))

    h = x(4)-16000
end

function f = GoalFunc(x)
    pcost = 10000;
    f =pcost * x(1) * x(4);
end

options = optimoptions('fmincon','MaxIter',10000000000)
[x,fval,exitflag,output] = fmincon('GoalFunc', rand(4,1), [], [], [], [], ...
    zeros(4,1), [], 'ConstraintFunc', options);
```

---

Here are simulation programmes we used in our model as follow.

### Input matlab source 2:

---

```
function f = GoalFunc(x)
    f = 10000*x(1)
end

function [g, h] = ConstraintFunc(x)
    s = 1000000;
    r = 20;
    vp = 72;
    R = 30;
    t = 0.5;
    v = 0.33;

    g(1) = s - x(1) * sqrt(3)/2 * x(2)^2;
    g(2) = r - x(3);
    g(3) = x(3) - R;
    g(4) = x(4) - t;
```

```
g(5) = 2.5*v - x(4) * (r / x(2))^2 * 2 * pi / sqrt(3);  
h = pi * x(3) / vp * (1 - (0.475 * (r / x(2))^2 * 2 * pi / sqrt(3)))^2 - x(4);  
end

---



```
options = optimset;  
[x, y] = fmincon('GoalFunc', rand(4,1), [], [], [], [], ...  
zeros(4,1), [], 'ConstraintFunc', options);
```



---


```