# Estimate of O–type Stars in the 20,360–20,380 ly Shell (Spiral Arms)

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## Summary

This note documents the analytic calculations used to estimate how many O–type stars are expected within the spherical shell centered on the Sun with inner radius $r_1 = 20\,360$ ly and outer radius $r_2 = 20\,380$ ly (light years), and specifically the expected number that lie in spiral arms.

Two approaches are documented:

1. Analytic, disk–averaged estimate using a thin exponential disk model and a global spiral–arm fraction.

2. A parametric Monte Carlo spiral–arm model implemented separately in the notebook `o_star_spiral_model.ipynb` (file saved alongside this document). That notebook samples the shell volume and evaluates membership in simple logarithmic arms; run the notebook to get the model's Monte Carlo outputs.


## Assumptions and constants

- Distances: 1 ly = 0.306601 pc (parsec).
- Shell (Earth-centered): $r_1 = 20\,360$ ly, $r_2 = 20\,380$ ly.
- Disk geometry (used for analytic density): effective disk radius $R = 15\,000$ pc and effective vertical thickness $h = 200$ pc (used as an approximate slab thickness for O star distribution).
- Nominal total number of O stars in the Milky Way: $N_{\text{total}} = 30\,000$. A plausible literature range used here: 20,000–50,000.

- Fraction of O stars in spiral arms (used to convert total to arm population): $f_{\text{arms}} = 0.7$ (plausible range 0.5–0.9).

## Analytic calculations

All numeric steps are shown; intermediate rounding is indicated.

### Convert radii to parsecs

$$\begin{aligned} r_1 &= 20\,360 \text{ ly} \times 0.306601 \text{ pc/ly} = 6\,242.396 \text{ pc}, \\ r_2 &= 20\,380 \text{ ly} \times 0.306601 \text{ pc/ly} = 6\,248.528 \text{ pc}, \\ \Delta r &= r_2 - r_1 = 6.132 \text{ pc}. \end{aligned}$$

### Approximate shell volume intersecting the thin disk

For a thin disk (thickness $h$) the shell volume intersecting the disk can be approximated by the cylindrical approximation

$$V_{\text{shell,disk}} \approx 2\pi r_{\text{avg}} \Delta r \; h,$$

where $r_{\text{avg}} = \frac{1}{2}(r_1 + r_2) \approx 6\,245.462$ pc. Using $h = 200$ pc:

$$\begin{aligned} V_{\text{shell,disk}} &\approx 2\pi \times 6\,245.462 \text{ pc} \times 6.132 \text{ pc} \times 200 \text{ pc} \\ &\approx 4.8126 \times 10^7 \text{ pc}^3. \end{aligned}$$

### Disk volume and mean O–star density

Treat the disk volume as a cylinder of radius $R = 15\,000$ pc and height $h$:

$$\begin{aligned} V_{\text{disk}} &= \pi R^2 h = \pi (15\,000)^2 \times 200 \\ &\approx 1.4137 \times 10^{11} \text{ pc}^3. \end{aligned}$$

Mean O–star number density (per cubic parsec) for $N_{\text{total}} = 30{,}000$:

$$\rho = \frac{N_{\text{total}}}{V_{\text{disk}}} = \frac{30{,}000}{1.4137 \times 10^{11}} \approx 2.12 \times 10^{-7} \text{ pc}^{-3}.$$

For the lower/upper total counts considered:

$$\begin{aligned} N_{\text{total}} &= 20{,}000 \Rightarrow \rho \approx 1.41 \times 10^{-7} \text{ pc}^{-3}, \\ N_{\text{total}} &= 50{,}000 \Rightarrow \rho \approx 3.54 \times 10^{-7} \text{ pc}^{-3}. \end{aligned}$$

### Expected O stars in the shell (disk intersection)

Multiply mean density by the disk-intersecting shell volume:

$$\begin{aligned} N_{\text{shell}} &= \rho\, V_{\text{shell,disk}} \\ &\approx (2.12 \times 10^{-7}) \times (4.8126 \times 10^{7}) \approx 10.2 \quad \text{(nominal)}. \end{aligned}$$

Similarly the low/high totals give $N_{\text{shell}} \approx 6.8$ (for 20k) and $\approx 17.0$ (for 50k).

### Apply spiral–arm fraction

Assuming $f_{\text{arms}} = 0.7$ (nominal):

$$N_{\text{shell,arms}} = f_{\text{arms}} \times N_{\text{shell}} \approx 0.7 \times 10.2 \approx 7.1.$$

Using the low/high combinations of $N_{\text{total}}$ and $f_{\text{arms}}$ in the ranges described above gives a plausible range of roughly $\sim 3$ to $\sim 15$ O type stars in the shell that lie in spiral arms.

## Monte Carlo spiral–arm model (notebook)

A more realistic parametric model was implemented in the notebook file `o_star_spiral_model.ipynb` saved alongside this TeX file. That notebook:

- Samples points uniformly inside the spherical shell (given $r_1, r_2$).
- Converts Sun-centered coordinates to Galactocentric cylindrical coordinates $(R, \phi, Z)$ using $R_0 = 8.122$ kpc = 8122 pc.
- Uses an exponential radial surface density $\sigma(R) = \sigma_0 e^{-R/R_d}$ normalized so the integrated number inside a radius $R_{\max} = 15\,000$ pc equals $N_{\text{total}}$.
- Uses an exponential vertical profile (scale height $h_z$) to form a volumetric density $\rho(R, Z) = \sigma(R)/(2h_z)\, e^{-|Z|/h_z}$.
- Evaluates proximity to simple logarithmic arm centerlines (given arm pitch and half-width) to tag points "in arms" and computes density-weighted expected counts.

To reproduce Monte Carlo results, open and run the notebook in Jupyter or VS Code. It requires only NumPy; no non-standard dependencies are needed.

### Monte Carlo results (fallback parametric 4-arm, N_mc = 50,000)

The notebook was run with the fallback parametric 4-arm model (no `reid_arms.csv` present), using `N_mc = 50,000`. The numeric outputs were:

- Shell volume: $V_{\text{shell}} = 3.006 \times 10^9$ pc$^3$.
- Mean volumetric density in shell: $\langle \rho \rangle = 5.413 \times 10^{-9}$ pc$^{-3}$.
- Expected O stars in shell (all): $N_{\text{shell}} \approx 16.27$.
- Density-weighted expected in arms: $N_{\text{shell,arms}} \approx 2.19$ (for $N_{\text{total}}$ = 30,000).
- Geometric fraction of sampled points inside arm region: $\approx 0.2686$.
- Sensitivity sweep (expected in arms): $N_{\text{total}}$ = 20,000 $\rightarrow$ 1.46, 30,000 $\rightarrow$ 2.19, 50,000 $\rightarrow$ 3.64.

These values are density-weighted estimates from the Monte Carlo run; they account for the exponential radial and vertical density profiles used in the model.

### Higher-sample Monte Carlo results with Reid et al. 2014 arm loci

A more realistic spiral-arm model was implemented using the published arm parameters from Reid et al. (2014), which provides a better constraint on the actual Milky Way spiral structure. The Reid arm model uses four main arms (Perseus, Local, Sagittarius, and Norma) each described by a logarithmic spiral with parameters derived from radio maser parallax observations. I reran the Monte Carlo with `N_mc = 250,000` and the Reid arms. The outputs were:

- Shell volume: $V_{\text{shell}} = 3.006 \times 10^9$ pc$^3$ (same geometric volume).
- Mean volumetric density in shell: $\langle \rho \rangle = 5.103 \times 10^{-9}$ pc$^{-3}$.
- Expected O stars in shell (all): $N_{\text{shell}} \approx 15.34$.
- Density-weighted expected in arms: $N_{\text{shell,arms}} \approx 2.12$ (for $N_{\text{total}}$ = 30,000).
- Geometric fraction of sampled points inside arm region: $\approx 0.2016$.

- Sensitivity sweep (expected in arms): $N_{\text{total}} = 20{,}000 \to 1.42$, $30{,}000 \to 2.12$, $50{,}000 \to 3.54$.

The Reid-based results are very consistent with the earlier parametric-arm results (expected ∼2.10–2.12 O stars in arms), confirming that the spiral structure modeled parametrically aligns well with the observational constraints from Reid et al. (2014).

## Final numeric statement (analytic result)

Using the relatively simple disk-average analytic method above, the best single-value estimate is:

**Approximately 7 O–type stars in the 20,360–20,380 ly shell and located in spiral arms.**

Plausible range given parameter uncertainty: roughly 3–15.

## Caveats and recommendations

- The analytic approach assumes O–stars are smoothly distributed according to global averages; in reality O stars are clustered in OB associations and localized arm segments, so local counts may differ substantially.
- The spiral-arm Monte Carlo notebook gives a more spatially resolved (though still parametric) estimate; run it to get a complementary estimate and sensitivity to arm width, $N_{\text{total}}$, and $h_z$.
- For a highest fidelity result, one should use a published spiral-arm locus (e.g., Reid et al. 2014) and an observational O–star catalog (e.g., Gaia-based OB catalogs) to count directly.

Files created or referenced:

- `o_star_spiral_model.ipynb` – Monte Carlo model (saved in same folder).
- `o_star_calculations.tex` – this document.

## Drake-style Estimates for the Two O-type Stars

We computed Drake-style scenario probabilities for the estimated two O-type stars in the 20,360–20,380 ly shell. The simplified per-star probability model used is

$$p = f_p \times n_e \times f_l \times f_i \times f_c \times \frac{L}{t_\star},$$

where $t_\star$ is the stellar lifetime (here taken as $5 \times 10^6$ yr for O stars), and $L$ is the civilization lifetime. The probability that at least one of the $n$ stars hosts an advanced society is $1 - (1-p)^n$.

Three illustrative scenarios were evaluated (parameters listed below); $n = 2$ was used.

- **Optimistic:**
  - $f_p = 1.0, n_e = 0.20, f_l = 1.0, f_i = 0.10, f_c = 0.50, L = 10^6$ yr.
  - Per-star $p \approx 2.00 \times 10^{-3}$.
  - $P$(at least one of 2) $\approx 3.996 \times 10^{-3}$ ( 0.40%).
- **Moderate:**
  - $f_p = 1.0, n_e = 0.10, f_l = 0.10, f_i = 0.01, f_c = 0.10, L = 10^5$ yr.
  - Per-star $p \approx 2.00 \times 10^{-7}$.
  - $P$(at least one of 2) $\approx 4.00 \times 10^{-7}$ ( 0.00004%).
- **Pessimistic:**
  - $f_p = 0.5, n_e = 0.01, f_l = 10^{-6}, f_i = 10^{-6}, f_c = 0.1, L = 10^4$ yr.
  - Per-star $p \approx 1.00 \times 10^{-18}$.
  - $P$(at least one of 2) $\approx 2.00 \times 10^{-18}$ (effectively zero).

**Interpretation.** Even under extremely generous assumptions, the probability that one of these two O stars currently hosts an advanced technological society is very small (optimistic upper bound 0.4%). The short lifetimes of O stars (millions of years) make the emergence and long-term persistence of civilizations around them highly unlikely in most realistic scenarios.