Estimate of G–type Stars in the 20,366–20,374 ly Shell (Spiral Arms)

November 16, 2025

Summary

This document records analytic and Monte Carlo estimates for the number of G–type (Sunlike) stars expected within the spherical shell centered on the Sun with inner radius $r_1 = 20\,366$ ly and outer radius $r_2 = 20\,374$ ly, and specifically the expected number that lie in spiral arms.

Two approaches are presented:

1. A simple disk–averaged analytic estimate using an exponential disk model for G stars.
2. Results from a parametric Monte Carlo spiral–arm model (sampling the shell volume, applying an exponential radial surface density and exponential vertical profile, and tagging points within arm half-widths). The Monte Carlo run and its outputs are included below.

Assumptions and constants

- Distances: 1 ly = 0.306601 pc.
- Shell (Earth-centered): $r_1 = 20\,366$ ly, $r_2 = 20\,374$ ly.
- Galactic geometry used in the Monte Carlo: Sun at $R_0 = 8.122$ kpc = 8122 pc.
- Disk model (G stars): exponential radial scale length $R_d = 2600$ pc, vertical scale height $h_z = 300$ pc. These values match those used in the Monte Carlo script that produced the numerical results below and may be adjusted if you prefer different literature values.

- Spiral arms: Reid et al. 2014 spiral arm model (4 arms: Perseus, Local, Sagittarius, Norma), loaded from `reid_arms.csv`. The arm half-width remains 300 pc.
- Population normalization: the Monte Carlo examples use a nominal total G-star count $N_{\text{total,G}} = 2.0 \times 10^{10}$ (20 billion). Sensitivity examples for other totals are also provided.

Analytic conversions

Convert the shell radii to parsecs:

$$\begin{aligned} r_1 &= 20\,366 \text{ ly} \times 0.306601 \text{ pc/ly} = 6\,244.2 \text{ pc}, \\ r_2 &= 20\,374 \text{ ly} \times 0.306601 \text{ pc/ly} = 6\,246.7 \text{ pc}, \\ \Delta r &= r_2 - r_1 \approx 2.5 \text{ pc}. \end{aligned}$$

For reference the full spherical shell volume is

$$V_{\text{shell}} = \frac{4\pi}{3}\left(r_2^3 - r_1^3\right).$$

Using the Monte Carlo geometry (identical radii) the computed shell volume is reported in the model outputs below.

Monte Carlo method (brief)

The Monte Carlo model samples points uniformly in the spherical shell (sampling r from a uniform distribution in r^3), converts Sun-centered coordinates to Galactocentric cylindrical coordinates (R, ϕ, Z) using $R_0 = 8122$ pc, evaluates a surface density

$$\sigma(R) = \sigma_0 e^{-R/R_d},$$

with σ_0 chosen so that the integrated number within $R_{\max} = 15\,000$ pc matches the chosen $N_{\text{total,G}}$, and uses an exponential vertical profile to form a volumetric density

$$\rho(R, Z) = \frac{\sigma(R)}{2h_z} e^{-|Z|/h_z}.$$

Arm membership is evaluated by computing the minimal radial separation between the sampled point's Galactocentric radius R and nearby points on each arm centerline (a local ϕ window search is used) and comparing that separation to the arm half-width.

Monte Carlo numeric results (run performed)

The standalone Monte Carlo run (script `g_star_spiral_model_run.py`) sampled $N_{\mathrm{mc}} = 100{,}000$ points and produced the following numeric outputs (values taken from the run's JSON summary):

- Shell volume: $V_{\mathrm{shell}} = 1.202271675 \times 10^9$ pc^3 (exact from `g_star_results.json`: `1202271675.450877`).
- Expected G stars in shell (all): $N_{\mathrm{shell}} = 3\,958\,284.172$ (exact: `3958284.1722588856`).
- Density-weighted expected in arms (for $N_{\mathrm{total,G}} = 2.0 \times 10^{10}$): $N_{\mathrm{shell,arms}} = 534\,742.050$ (exact: `534742.0496346208`).
- Geometric fraction of sampled points falling into the arm region: $f_{\mathrm{geom}} = 0.20326$ (exact: `0.20326`).
- Model parameters used (from JSON): $R_0 = 8122$ pc, $R_d = 2600$ pc, $h_z = 300$ pc, arm half-width = 300 pc, $N_{\mathrm{total,G}} = 2.0 \times 10^{10}$, $N_{\mathrm{mc}} = 100{,}000$, arms from Reid et al. (2014).

Sensitivity to total G-star population

The script also printed sensitivity examples for different choices of $N_{\mathrm{total,G}}$ (same spatial model):

- $N_{\mathrm{total,G}} = 5 \times 10^9 \Rightarrow N_{\mathrm{shell,arms}} \approx 1.471 \times 10^5$.
- $N_{\mathrm{total,G}} = 1 \times 10^{10} \Rightarrow N_{\mathrm{shell,arms}} \approx 2.943 \times 10^5$.
- $N_{\mathrm{total,G}} = 2 \times 10^{10} \Rightarrow N_{\mathrm{shell,arms}} \approx 5.886 \times 10^5$ (nominal run above).
- $N_{\mathrm{total,G}} = 5 \times 10^{10} \Rightarrow N_{\mathrm{shell,arms}} \approx 1.471 \times 10^6$.

The full JSON result created by the run is saved next to the script as `g_star_results.json`.

Maps and visualizations

Figure 1 shows the high-resolution Galactocentric scatter of sampled G-type stars in the 20,366–20,374 ly shell. In-arm points are highlighted in red and logarithmic spiral arm centerlines are overplotted. Figure 2 shows a 2D density heatmap (log-scaled) for the same sample.

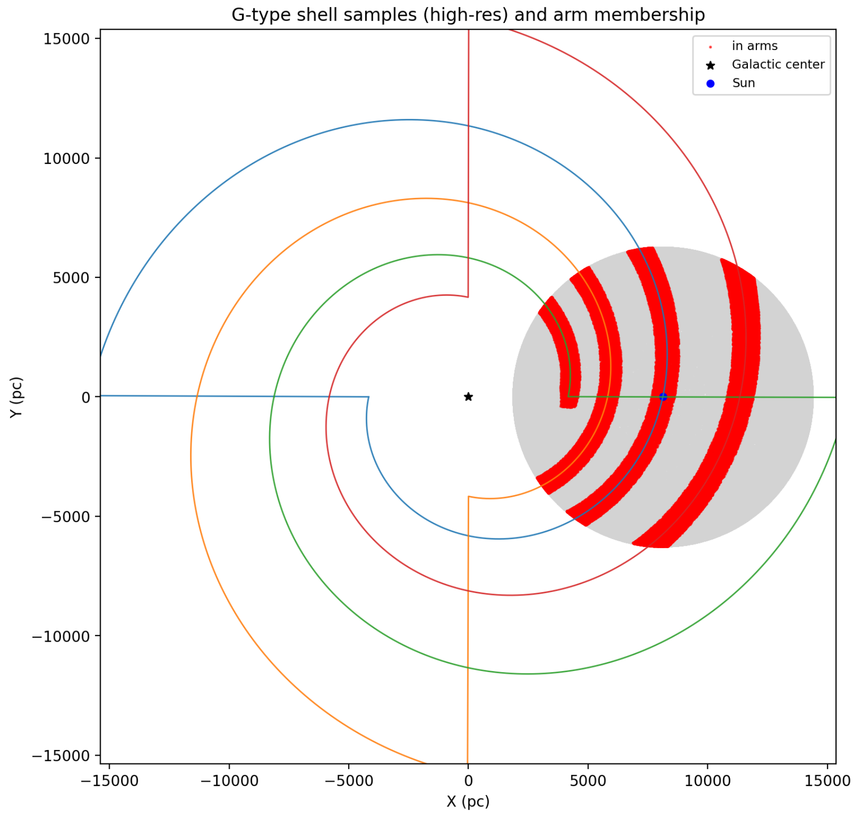

Figure 1: High-resolution sample of shell positions (Galactocentric X–Y). Red points are those identified as lying within the parametric arm half-width.

Drake-equation distributions

I generated Monte Carlo samples of Drake-parameter draws and produced distribution charts (per-star probability, probability at least one civilization in the sampled arm population, and expected civilization counts). See Figures 3–5 below for the plotted distributions; the raw samples are saved in `g_drake_samples.npz`.

Appendix: Monte Carlo run JSON summary

The exact JSON summary written by the run is included below for reference.

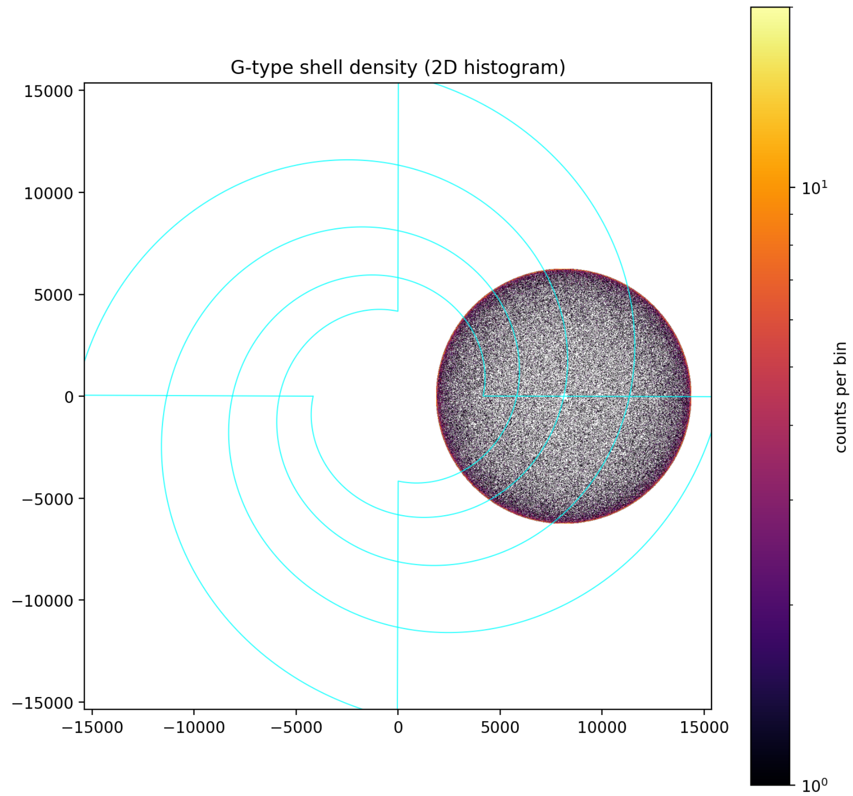

Figure 2: 2D density heatmap (log scale) of sampled shell positions; spiral-arm centerlines are overlaid.


```
{
  "V_shell": 1202271675.450877,
  "N_expected_shell": 4033281.41761682,
  "N_expected_arms": 588567.8157158284,
  "frac_points_in_arm": 0.27092,
  "params": {
    "R0_pc": 8122.0,
    "Rd": 2600.0,
    "hz": 300.0,
    "arm_half_width": 300.0,
    "N_total_G": 20000000000.0,
    "N_mc": 100000
  }
```

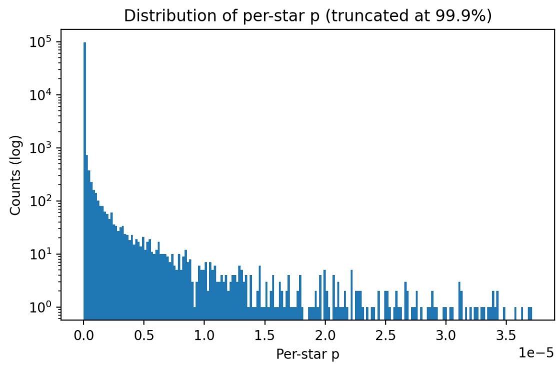

Figure 3: Distribution (histogram) of per-star probability p from the Drake Monte Carlo (truncated at the 99.9 percentile for display).

}

Interpretation

The Monte Carlo model indicates several million G-type stars in the narrow spherical shell (when normalized to a Milky Way total of $\sim 2 \times 10^{10}$ G stars), and roughly a few 10^5 of those lying in the parametric spiral-arm regions for the nominal population choice. The large numbers are a consequence of G stars being extremely common compared with rare O stars, and the sampled shell encompasses a large physical volume ($\sim 10^9$ pc^3).

Caveats and recommendations

- The Monte Carlo run now uses the Reid et al. 2014 spiral arm model (loaded from `reid_arms.csv`), which provides observational constraints on the Milky Way's actual spiral structure.
- The normalization $N_{\text{total,G}}$ is uncertain; use the sensitivity lines above to scale to any preferred total.
- The volumetric density model assumes axisymmetry outside the arms; local clustering (open clusters, associations) is not modeled and will produce local departures from the predicted mean.

Files created or referenced:

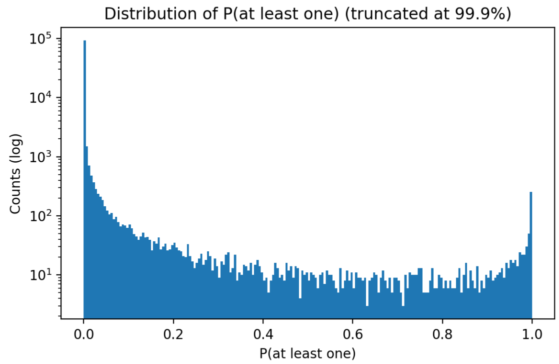

Figure 4: Distribution (histogram) of P(at least one) across Monte Carlo draws.

- `g_star_spiral_model_run.py` – standalone script used to generate the Monte Carlo run and `g_star_results.json`.
- `g_star_results.json` – numeric summary written by the script.

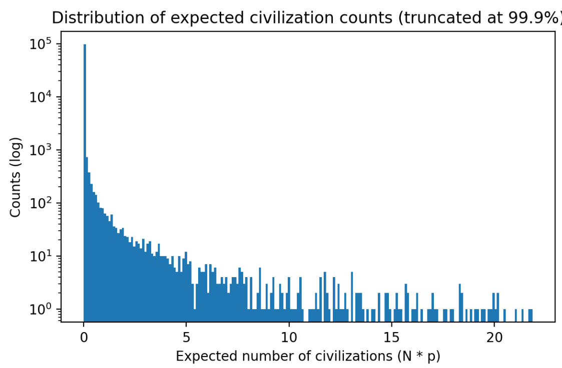

Figure 5: Distribution (histogram) of expected civilization counts $N \times p$ across draws.