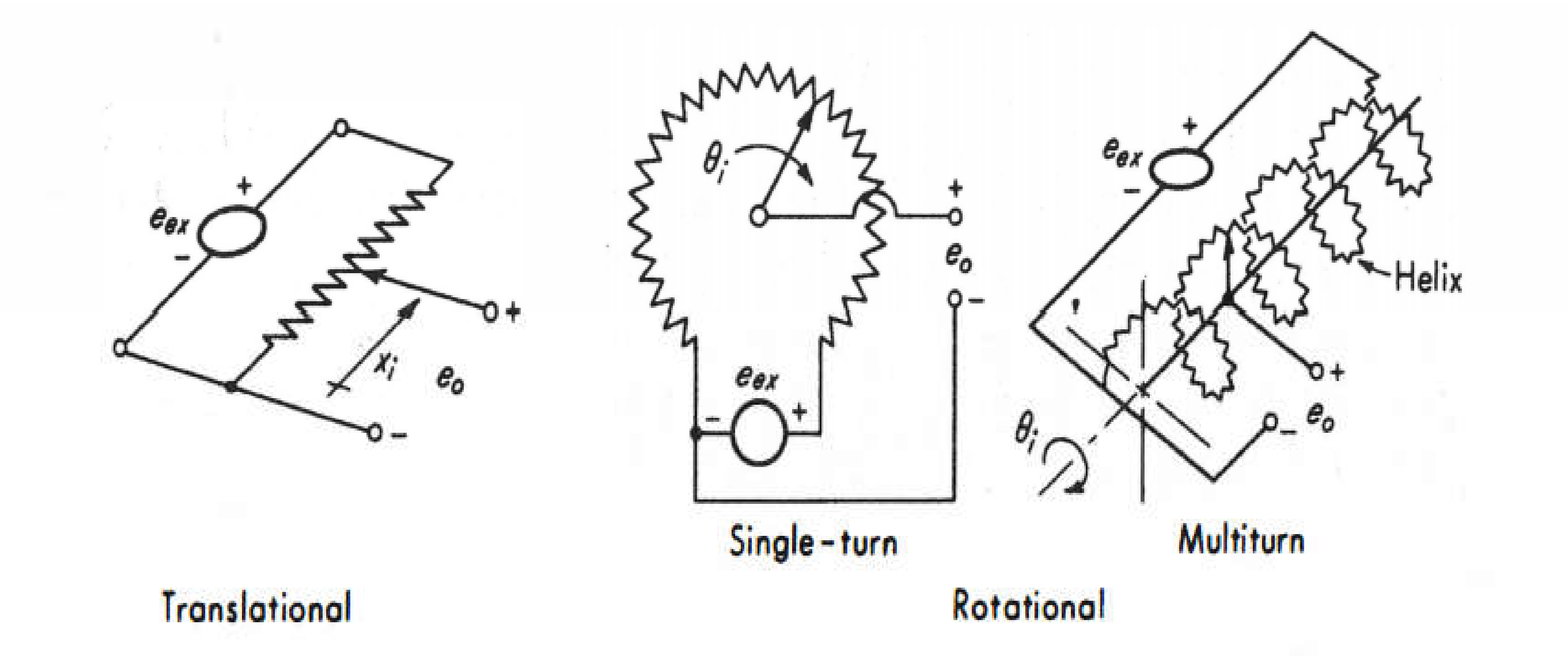
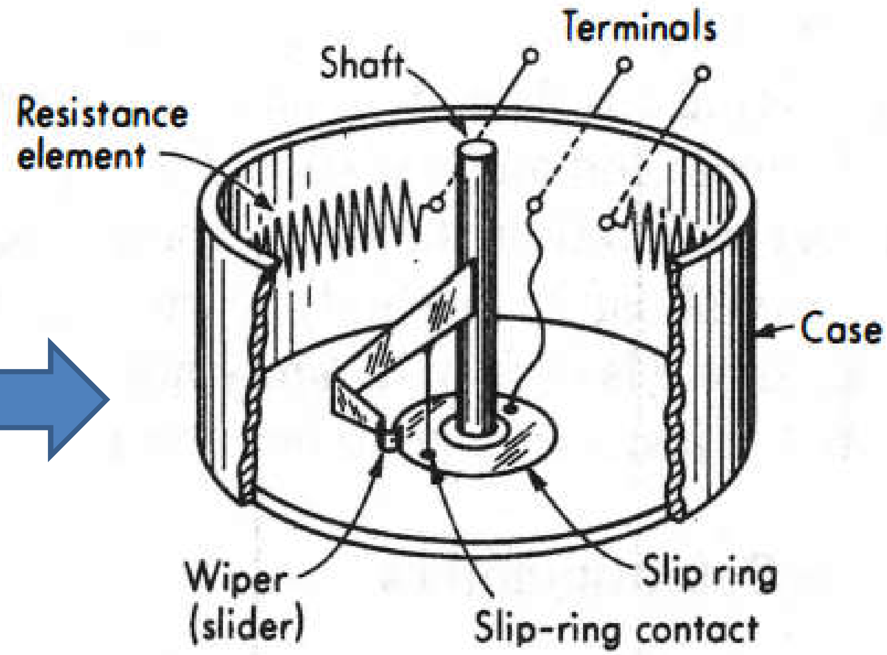
Theory of some important Transducers

Measurement of Displacement

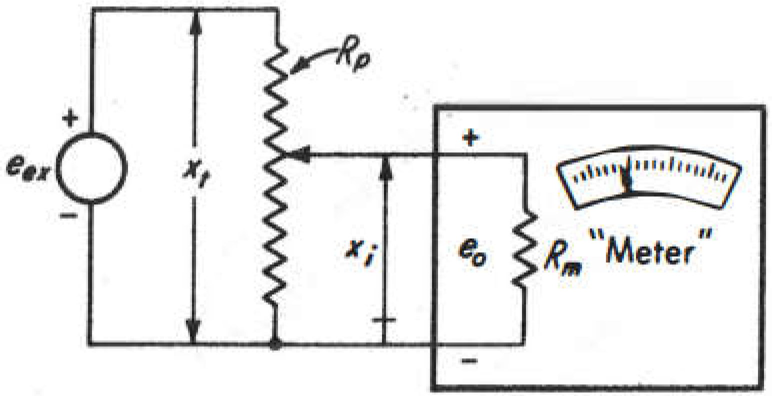
Measurement of Displacement by Potentiometer

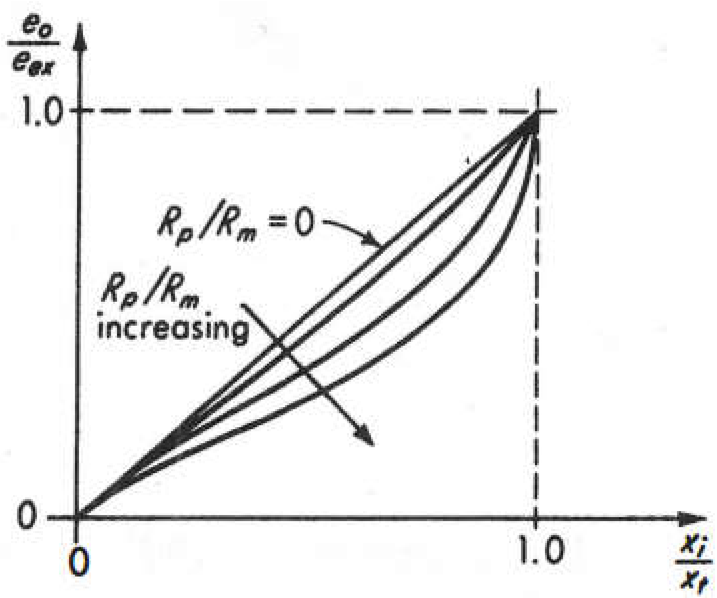


Potentiometer Construction



Analysis of Potentiometer Circuit





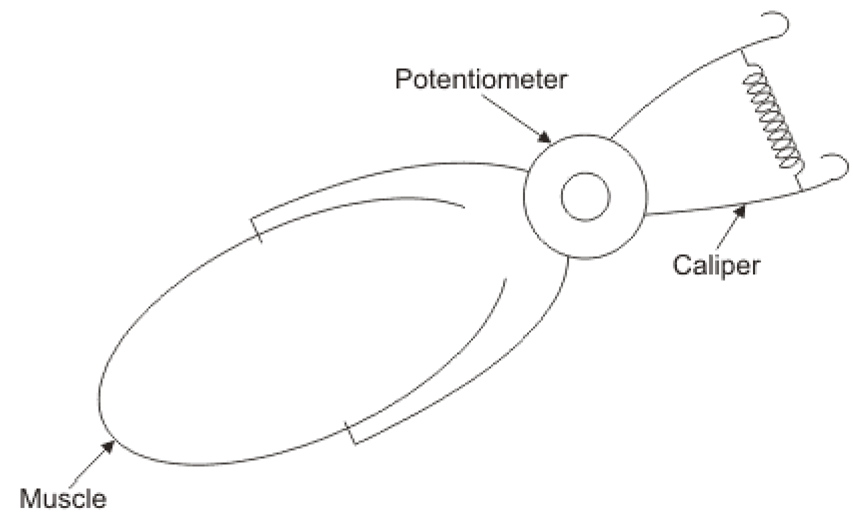
$$\frac{e_o}{e_{ex}} = \frac{1}{1/(x_i/x_t) + R_p/R_m)(1 - x_i/x_t)}$$

For an ideal meter, Rm will be infinite then one may have Rp/Rm will be almost zero

Then one may write

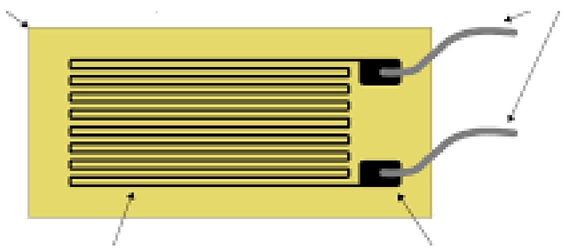
$$\frac{e_o}{e_{ex}} = \frac{x_i}{x_t}$$

Application of Potentiometer



To detect the changes in the chest (Thoracic) circumference, rotary potentiometer is attached to a chest band on a person.
In this setup, the transducer acts as a pivot to the caliper arms.

Strain Gauge



Measurement of Strain by Resistance Strain Gauge

Consider a conductor of uniform cross-sectional area A and length L, made of a material with resistivity ρ. The resistance R of such a conductor is given by

$$R = \frac{\rho L}{A}$$

Partially differentiating the above equation w.r.t ρ, L and A , we have

$$dR = \frac{L\,d\rho}{A} + \frac{\rho\,dL}{A} - \frac{\rho L\,dA}{A^2}$$

Dividing *dR* with *R* one may have

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

As $A = \pi D^2$

Thus one may write the above equation as

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - 2\frac{dD}{D}$$

Since the change is very small, one can modify the equation as

$$\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta L}{L} - 2\frac{\Delta D}{D}$$

Gauge factor G can be defined as

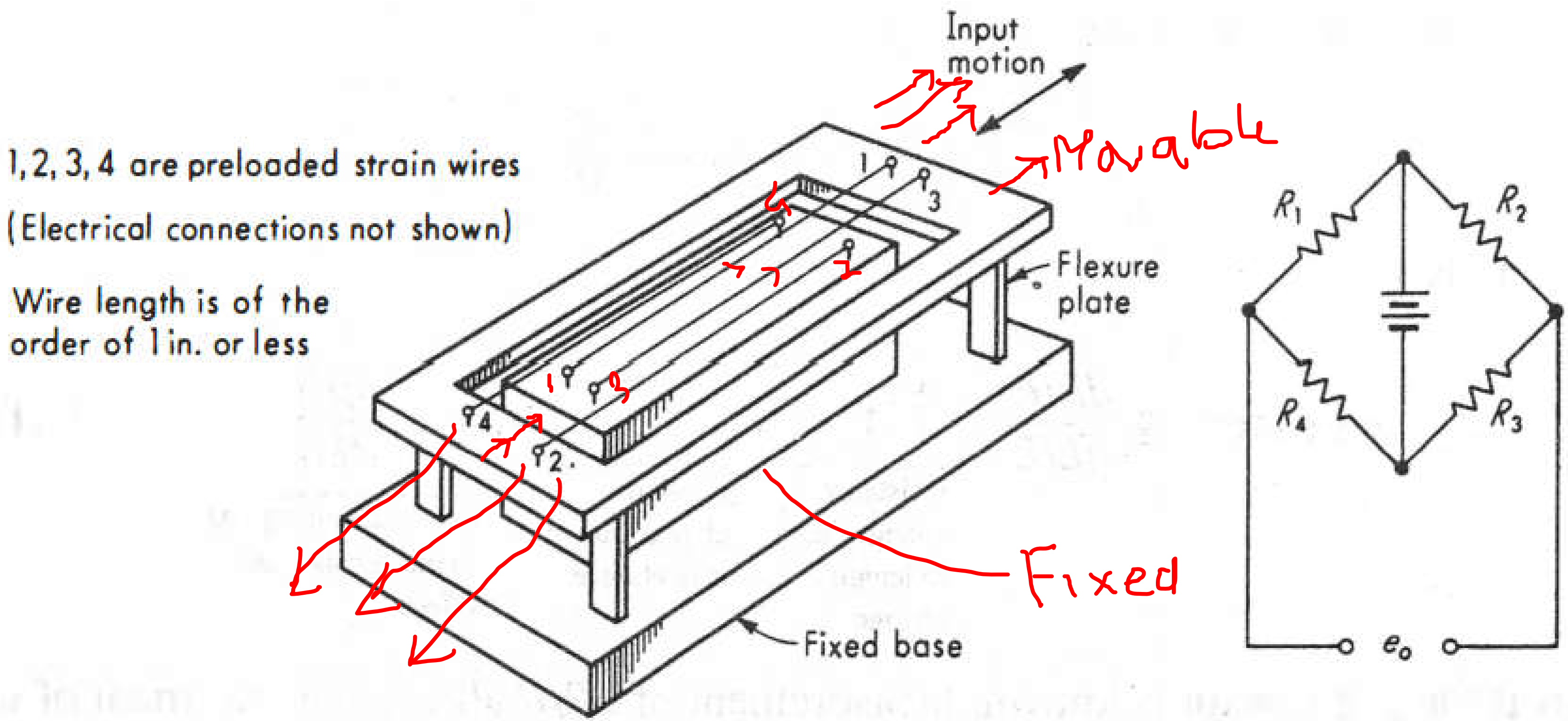
$$G = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = (1 + 2v)$$

Where

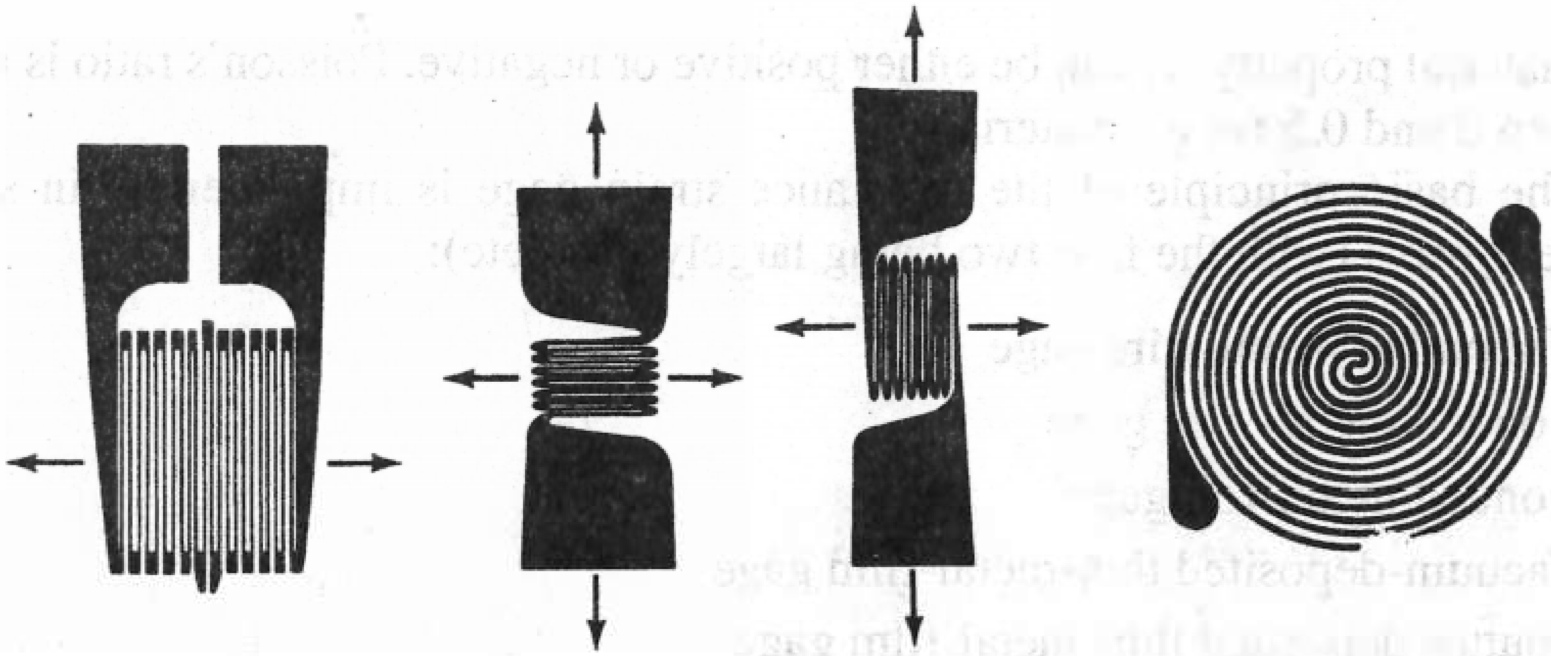
Poisson's Ratio $v = -\frac{\frac{\Delta D}{D}}{\frac{\Delta L}{L}}$

Here Piezoresistive effect is neglected

Types of Strain Gauge

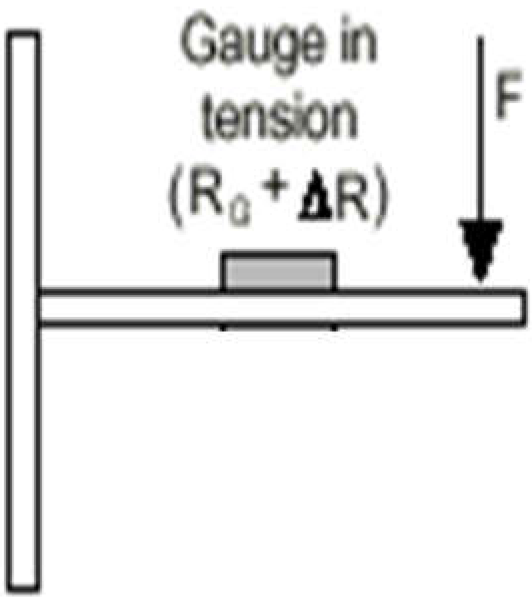


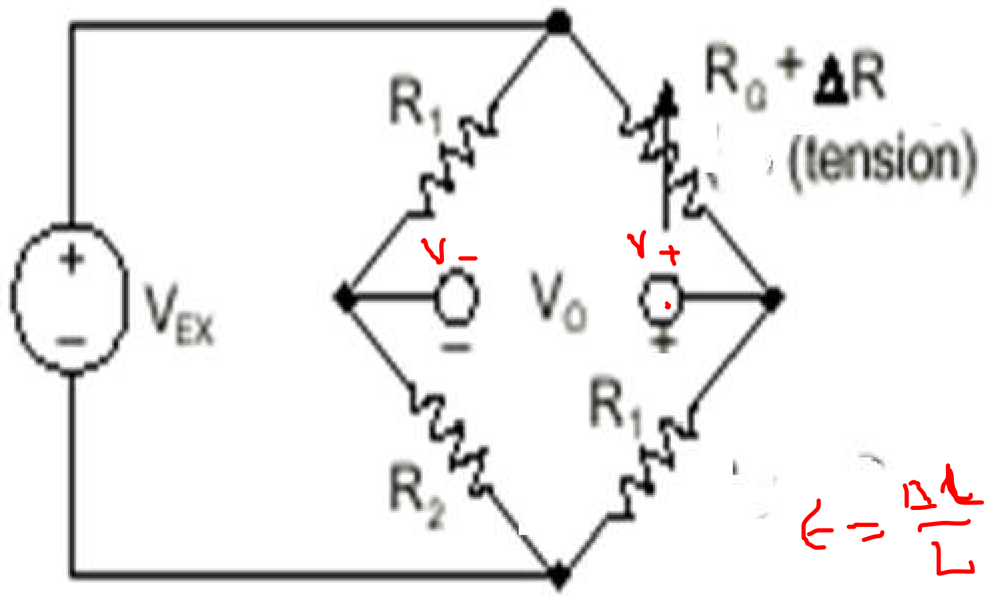
Unbonded strain gage



Foil strain gages.

Circuit with one active Strain Gauge





$\epsilon = \frac{\Delta L}{L}$

$V_0 = V_+ - V_-$

$= \left[\frac{R+\Delta R}{2R+\Delta R} - \frac{1}{2} \right] V_{EX}$

$= \frac{\Delta R}{4R+\Delta R} V_{EX}$

$= \frac{\Delta R}{4R} V_{EX}$ as $4R \gg \Delta R$

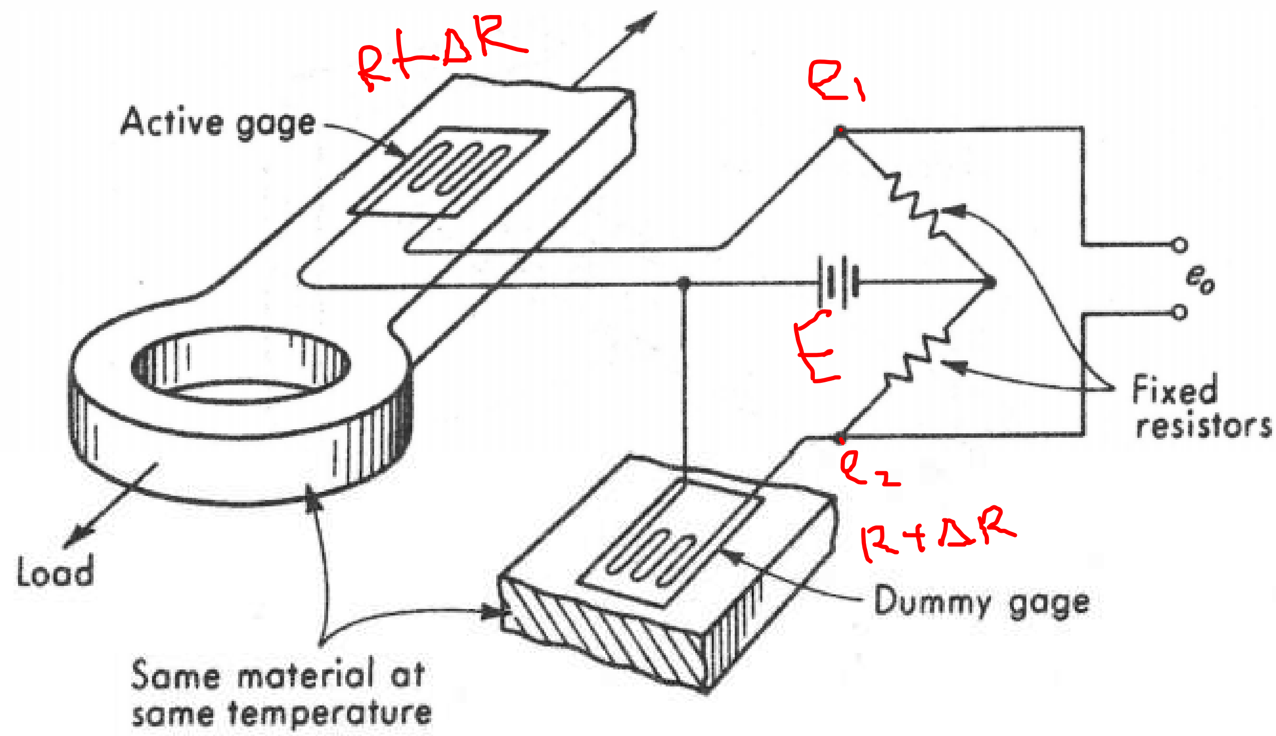
$= \frac{G_f \epsilon}{4}$

$$\frac{V_O}{V_{EX}} = \frac{GF \bullet \varepsilon}{4}$$

$\frac{\Delta R / R}{\epsilon} = G_f$

$\frac{\Delta R}{R} = G_f \epsilon$

Temperature Compensation

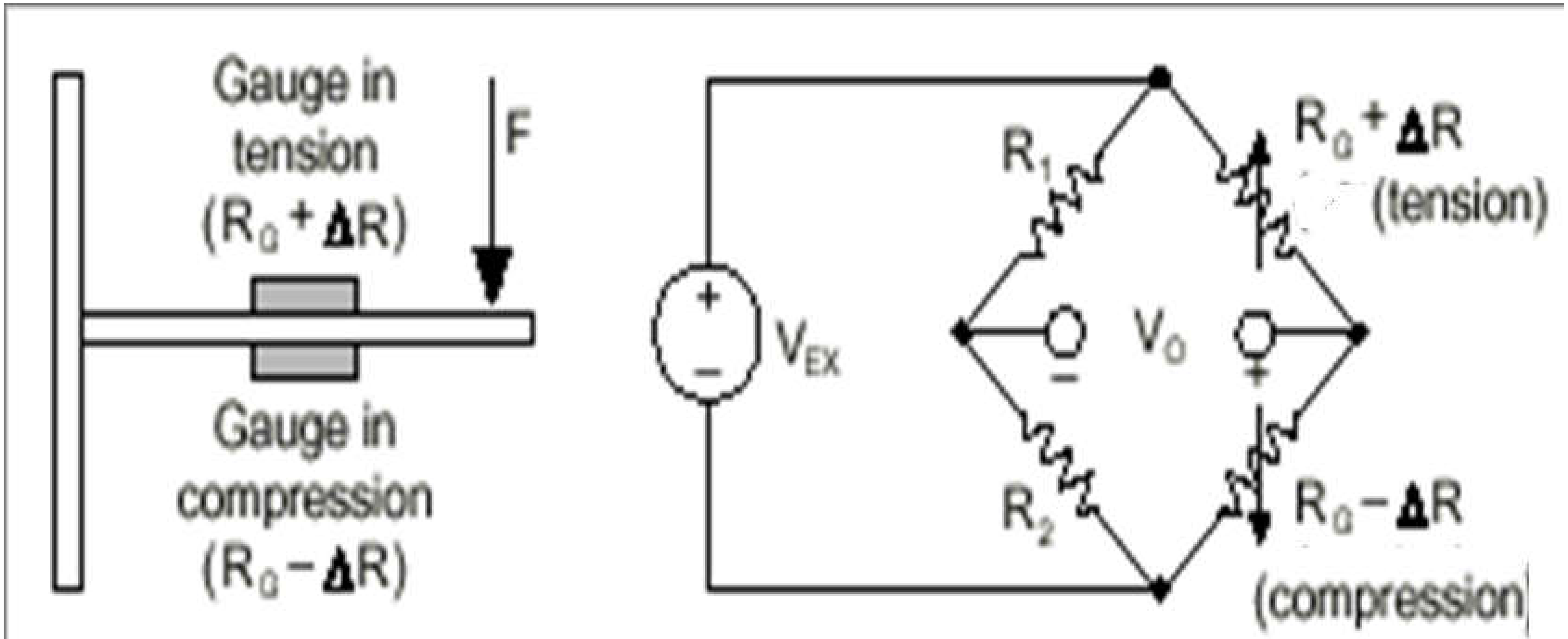


Strain-gage temperature compensation.

$$e_{o_{temp}} = \frac{R+\Delta R}{2R+\Delta R}\cdot E - \frac{R+\Delta R}{2R+\Delta R}\cdot E$$

$$= 0$$

Circuit with two active Strain Gauges



$$V_0 = \left[\frac{R+\Delta R}{2R+\Delta R} - \frac{R-\Delta R}{2R-\Delta R}\right] V_{EX}$$

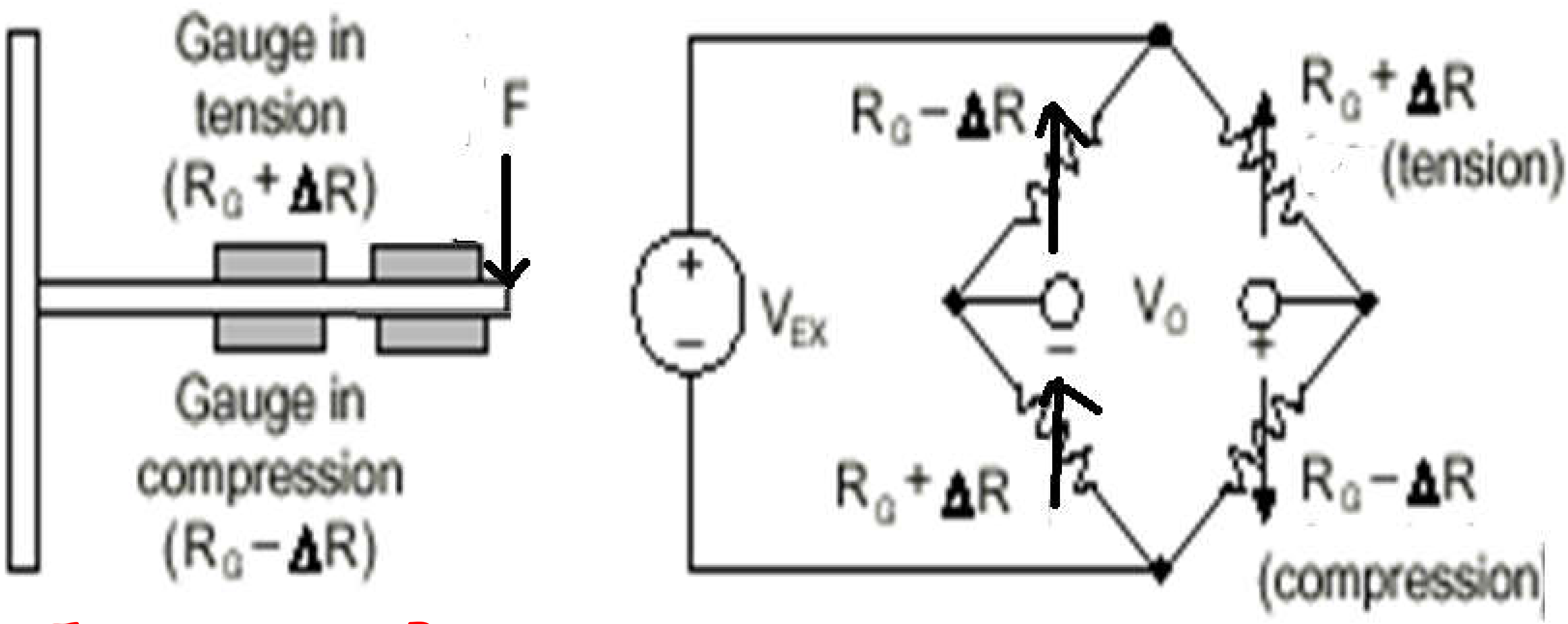
$$\frac{V_O}{V_{EX}} = -\frac{GF \bullet \varepsilon}{2}$$

$$= \frac{2\Delta R}{4R} \cdot V_{EX}$$

$$4R >> (\Delta R)^2$$

$$\simeq \frac{GF\varepsilon}{2} \cdot V_{EX}$$

Circuit with four active Strain Gauges



$$V_0 = \left[\frac{R+\Delta R}{2R} - \frac{R-\Delta R}{2R}\right] V_{Ex}$$

$$= \frac{\Delta R}{R} V_{Ex}$$

$$\frac{V_O}{V_{EX}} = GF \bullet \varepsilon$$

$$= \epsilon G_F V_{Ex}$$

Application of Strain Gauge

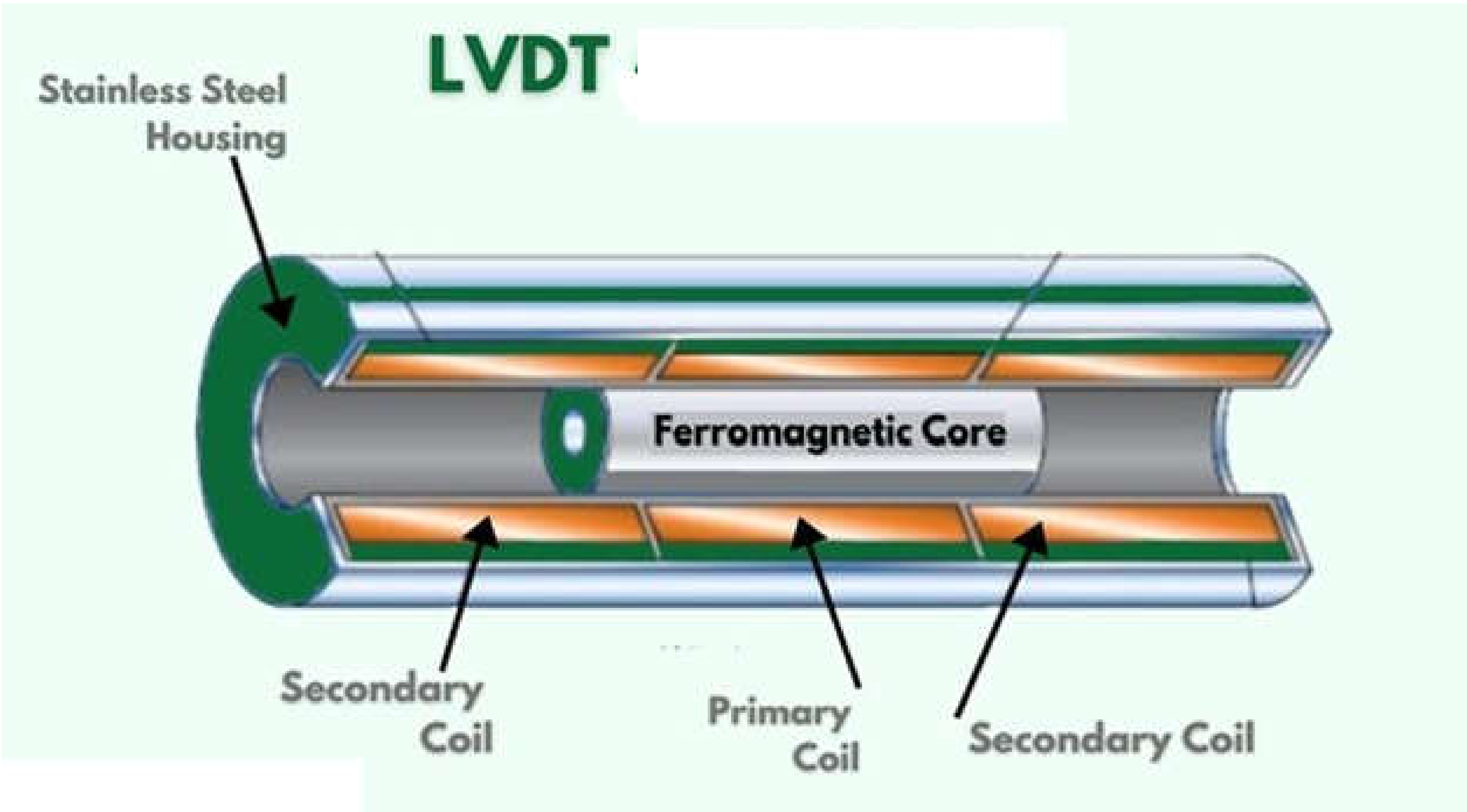
- Strain gauge measures the blood pressure inside the heart or blood vessels to diagnose cardiovascular abnormalities.

- The strain gauge is attached to the tip of the catheter

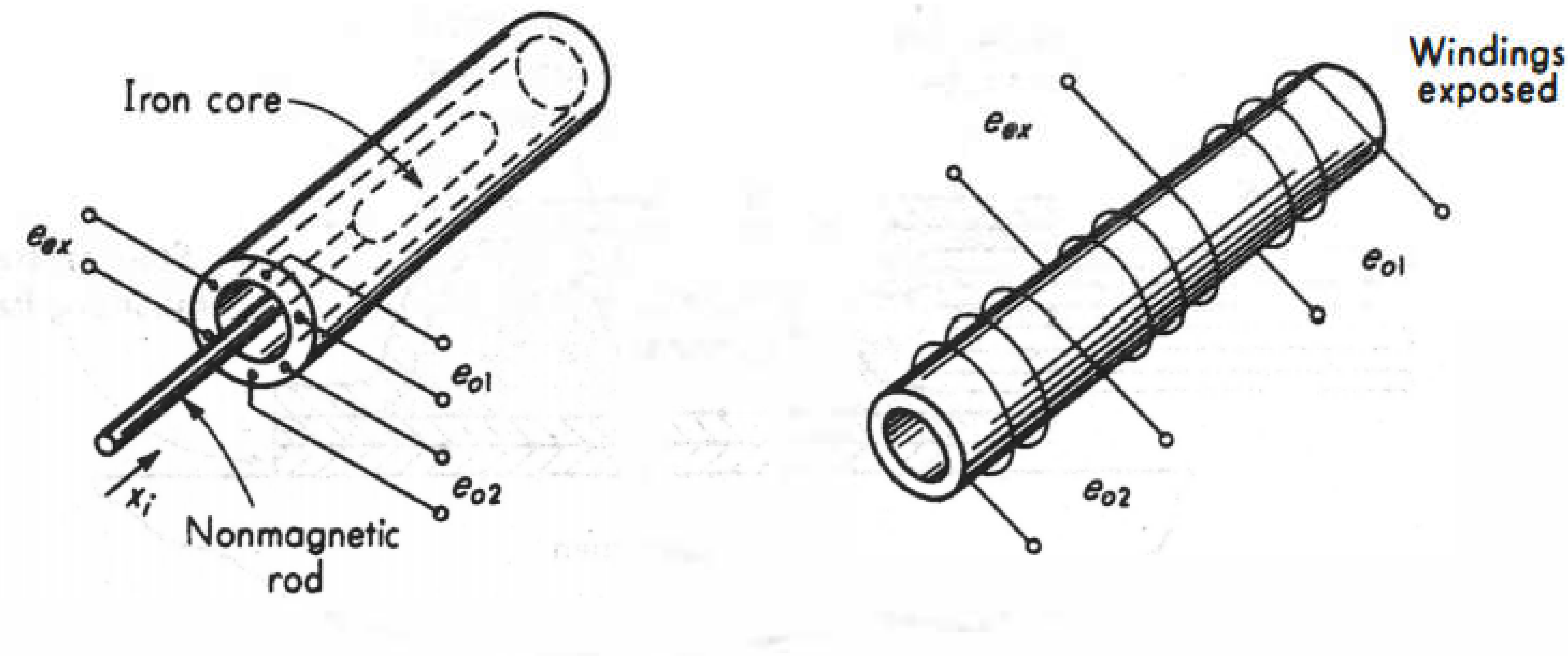
- The catheter is injected into the heart through veins

- On the front side of strain gauge diaphragm is mounted which undergoes deflection due to the applied force due to blood

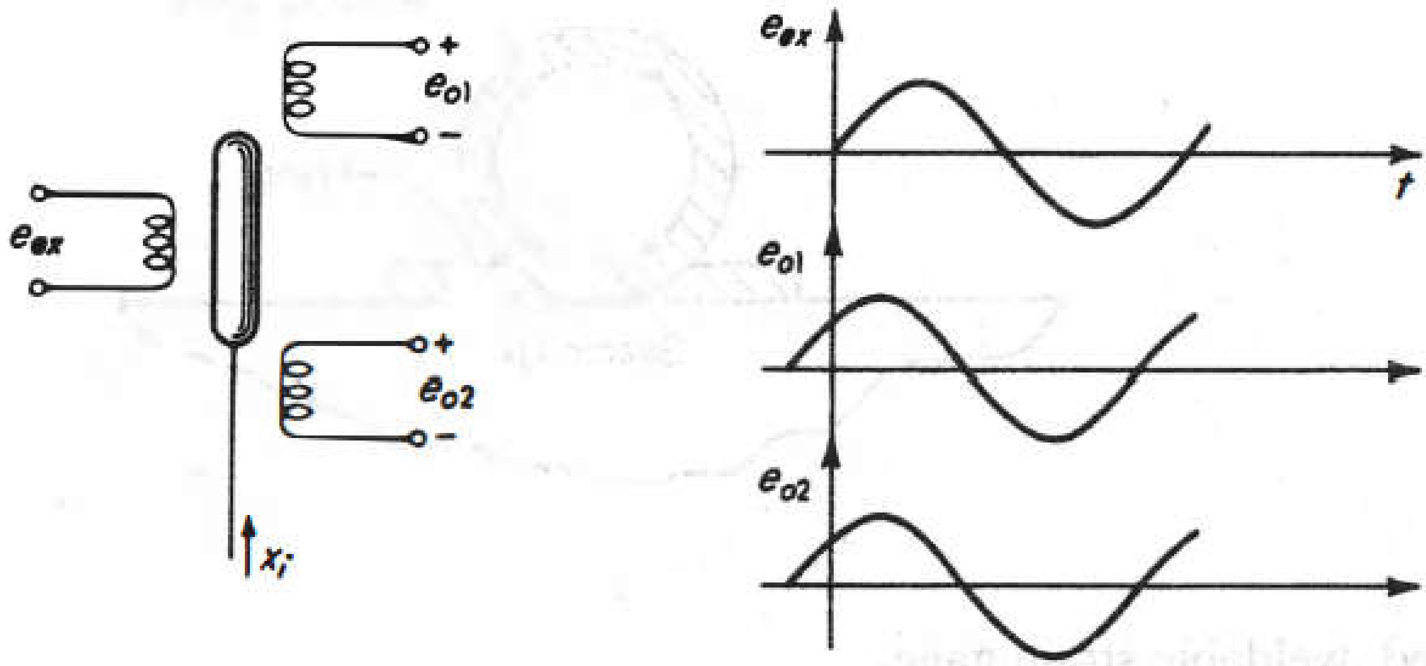
- Therefore, when blood pressure inside the heart varies, it deflects the diaphragm that in turn changes the strain gauge resistance.


LVDT
Stainless Steel
Housing
Ferromagnetic Core
Secondary
Coil
Primary
Coil
Secondary Coil

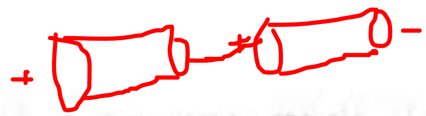
Measurement of displacement by LVDT

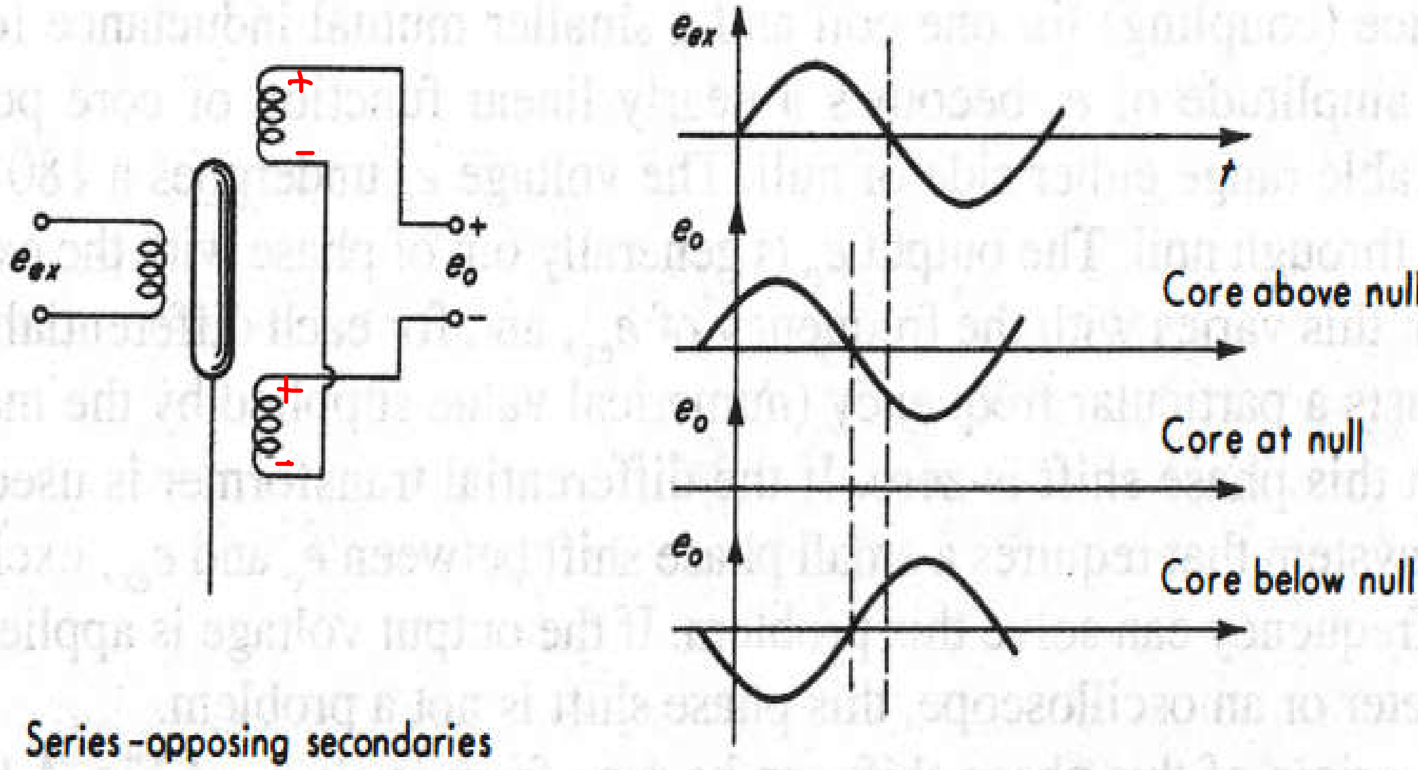


LVDT Construction

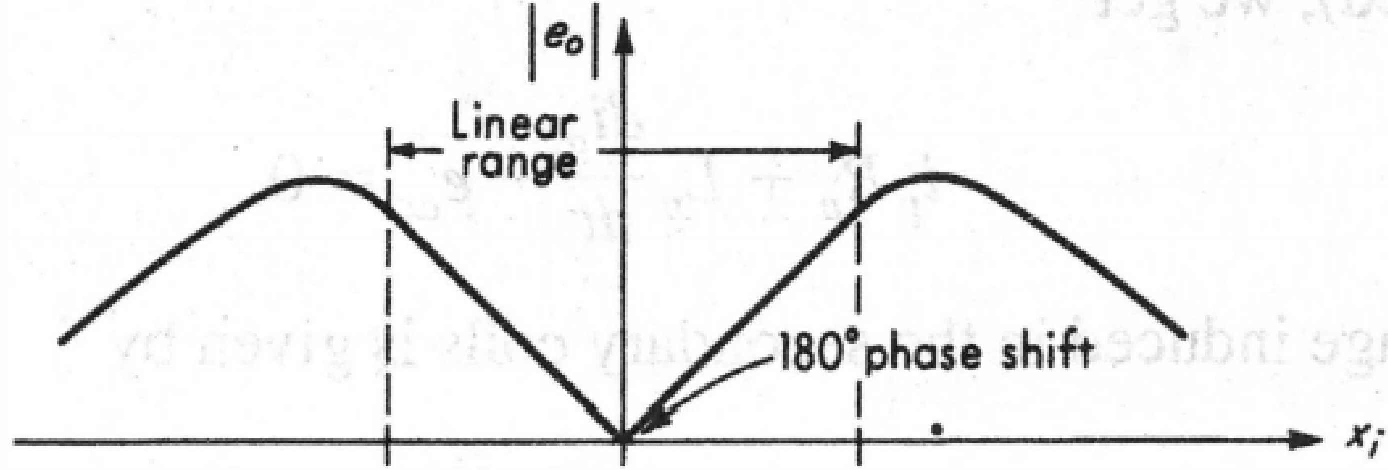


Core in null position



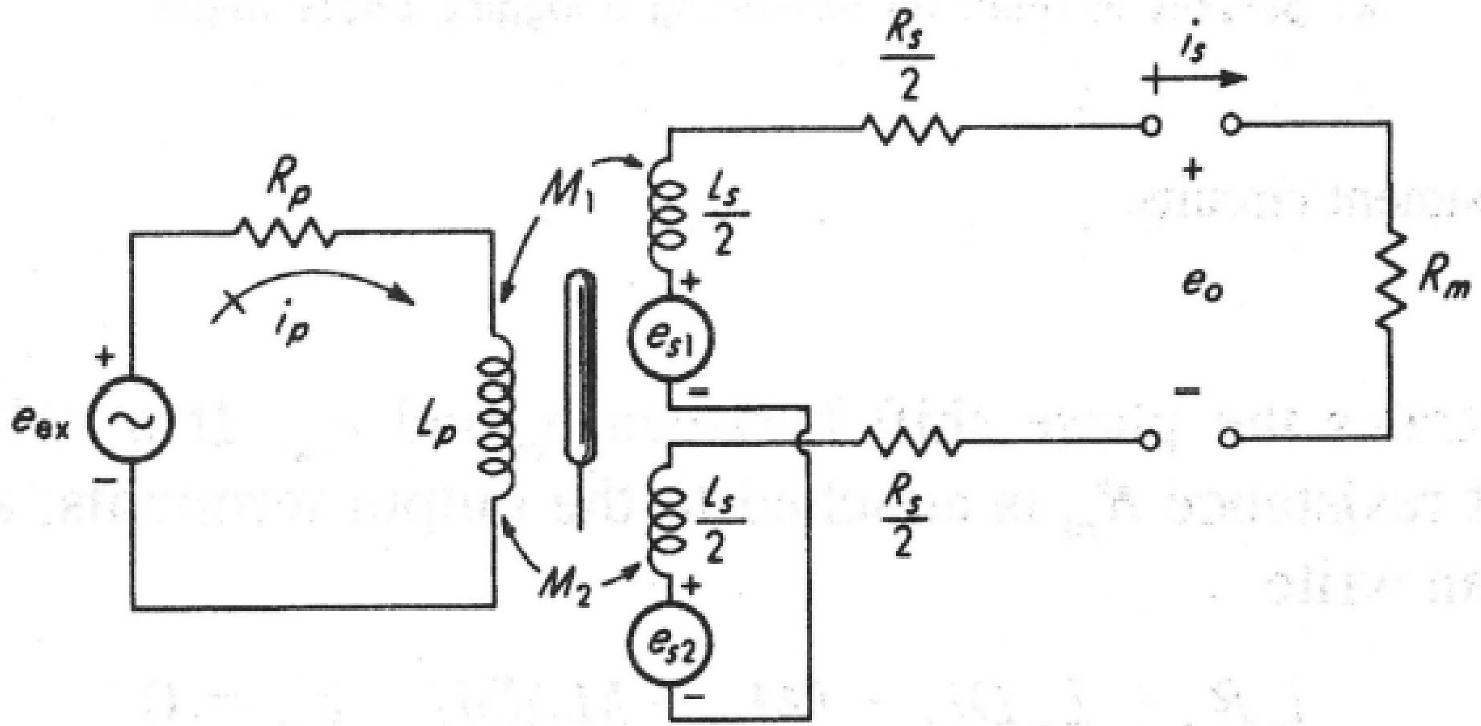


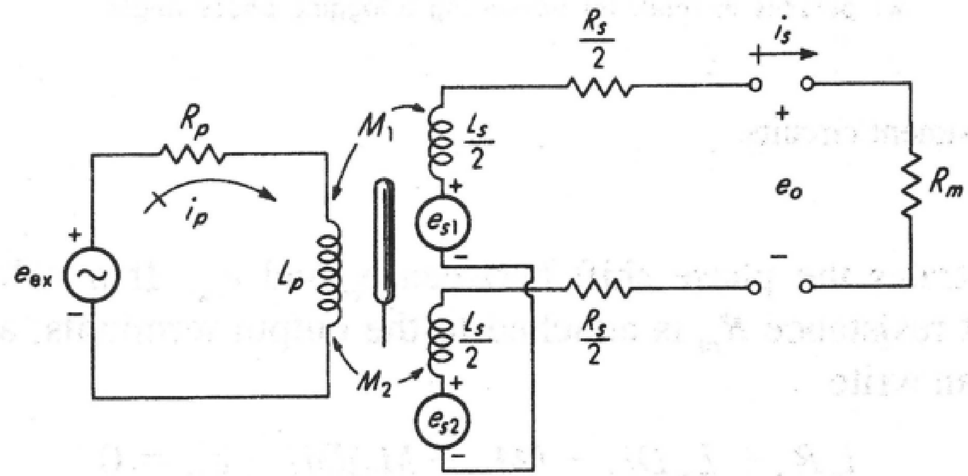
e_{ex}
e_o
+
−
e_{ex}
t
e_o
Core above null
e_o
Core at null
e_o
Core below null
Series-opposing secondaries



Output Vs Input of LVDT

Circuit Analysis of LVDT



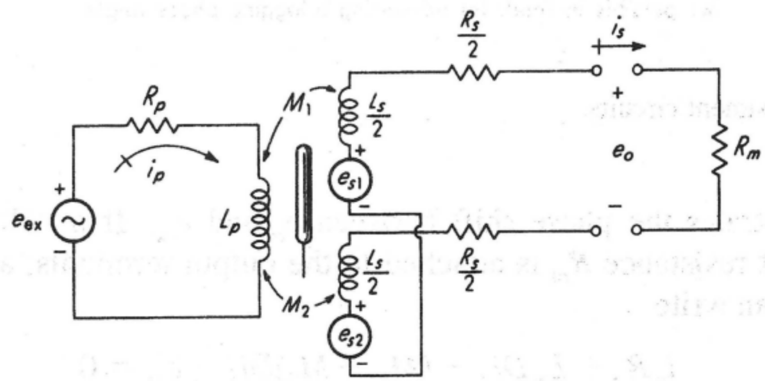


Applying Kirchhoff's voltage-loop law, if the output is an open circuit (no voltage-measuring device attached), we get

$$i_p R_p + L_p \frac{di_p}{dt} - e_{ex} = 0 \qquad (1)$$

Now the voltage induced in the secondary coils is given by

$$\begin{aligned} e_{s1} &= M_1 \frac{di_p}{dt} \\ e_{s2} &= M_2 \frac{di_p}{dt} \end{aligned} \qquad (2)$$



M_1 and M_2 are the respective mutual inductances. The net secondary voltage e_s is then given by

$$e_s = e_{s1} - e_{s2} = (M_1 - M_2)\frac{di_p}{dt} \tag{3}$$

The net mutual inductance $M_1 - M_2$ is the quantity that varies linearly with core motion. We have for a fixed core position

$$e_o = e_s = (M_1 - M_2)\frac{D}{L_pD + R_p}e_{ex} \tag{4}$$

and thus

$$\frac{e_o}{e_{ex}}(D) = \frac{[(M_1 - M_2)/R_p]D}{\tau_pD + 1} \qquad \tau_p \triangleq \frac{L_p}{R_p} \tag{5}$$

In terms of frequency response,

$$\frac{e_o}{e_{ex}}(i\omega) = \frac{\omega(M_1 - M_2)/R_p}{\sqrt{(\omega\tau_p)^2 + 1}}\angle\phi \qquad \phi = 90^\circ - \tan^{-1}\omega\tau_p \tag{6}$$

If a voltage-measuring device of input resistance R_m is attached to the output terminals, a current i_s will flow, and we can write

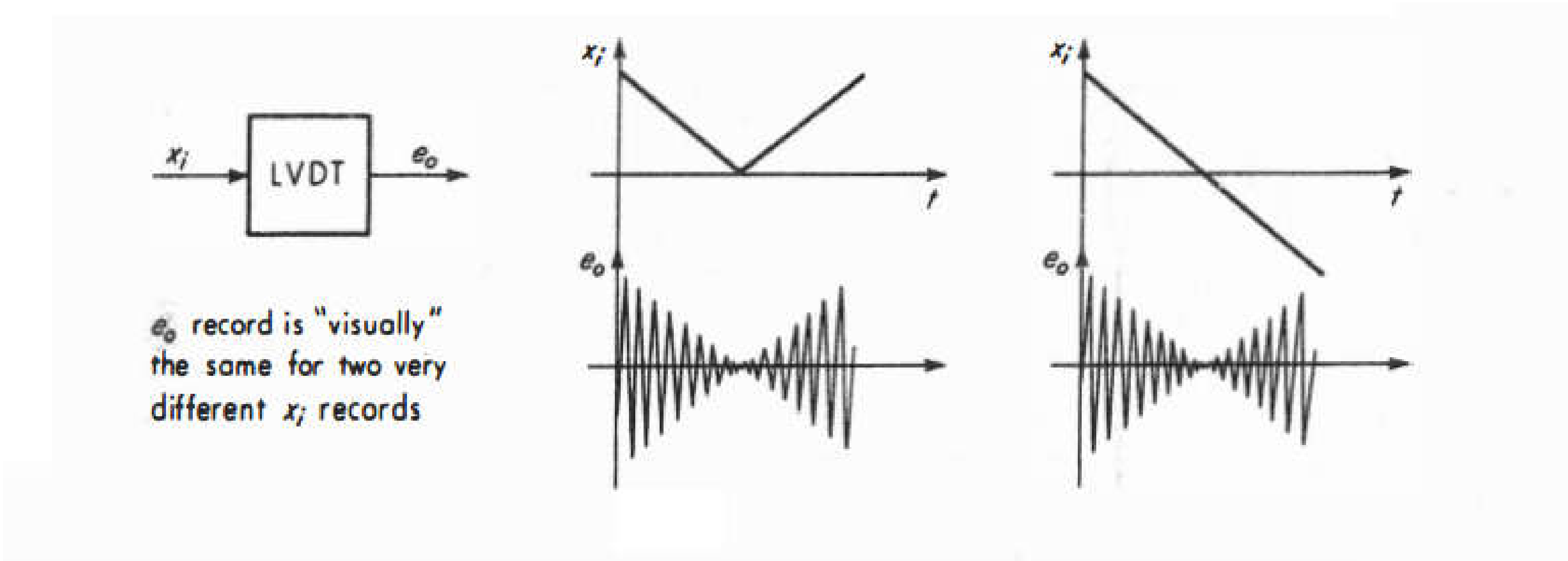
$$i_p R_p + L_p D i_p - (M_1 - M_2) D i_s - e_{ex} = 0 \tag{7}$$

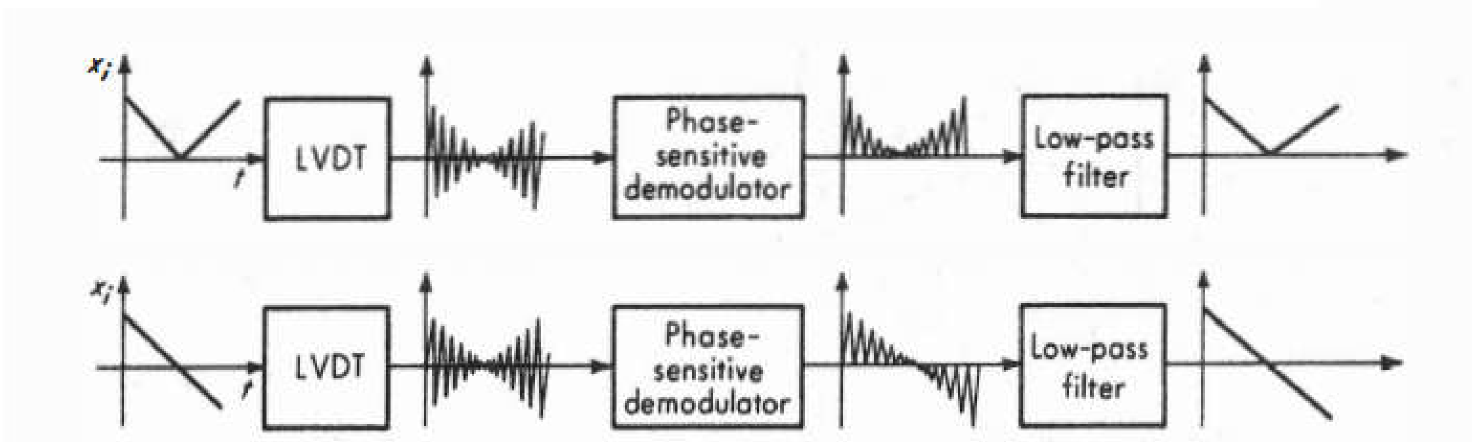
$$(M_1 - M_2) D i_p + (R_s + R_m) i_s + L_s D i_s = 0 \tag{8}$$

which lead to

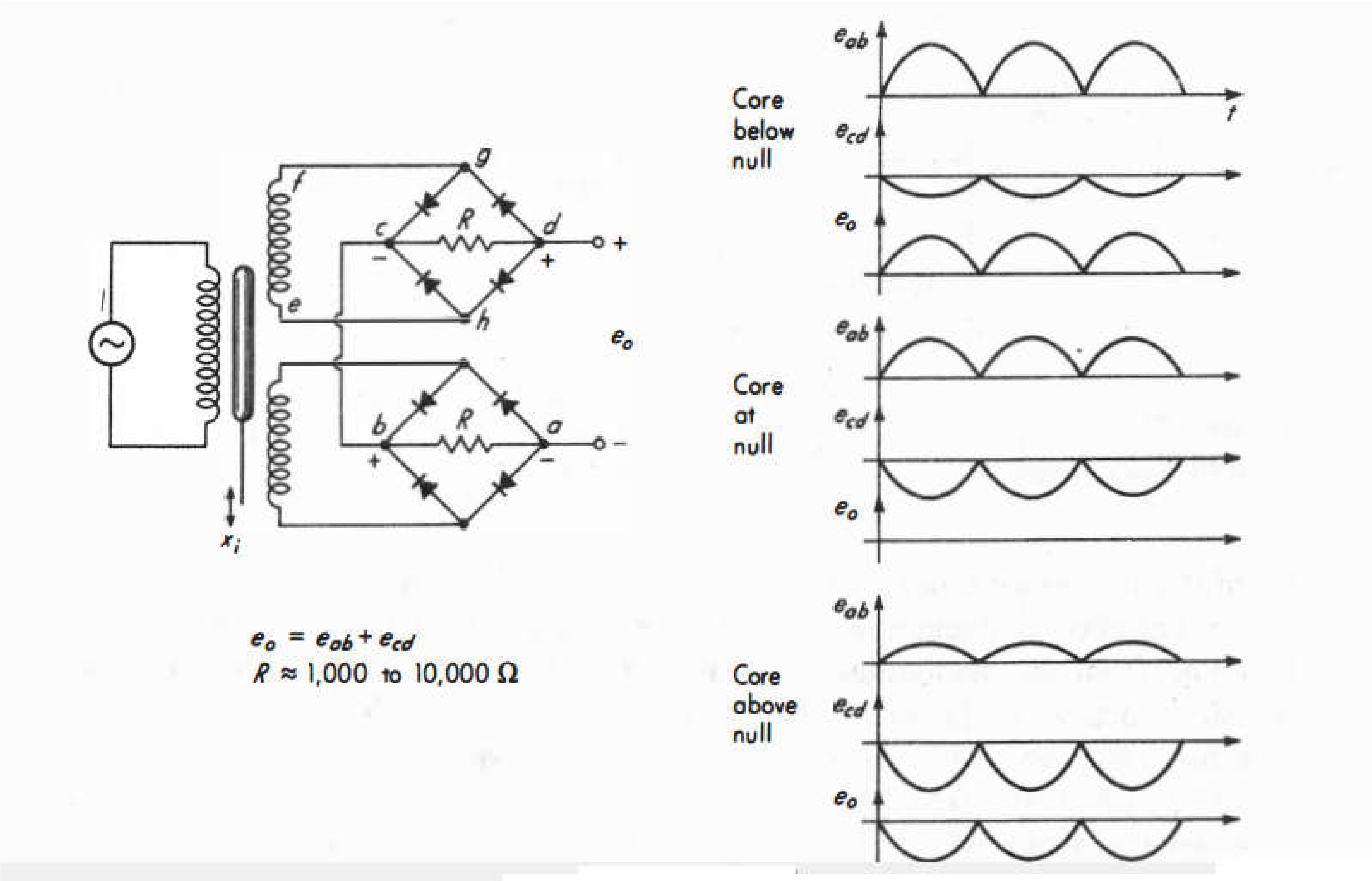
$$\frac{e_o}{e_{ex}}(D) = \frac{R_m (M_2 - M_1) D}{[(M_1 - M_2)^2 + L_p L_s] D^2 + [L_p (R_s + R_m) + L_s R_p] D + (R_s + R_m) R_p} \tag{9}$$

Requirement of Phase Sensitive Demodulation in LVDT

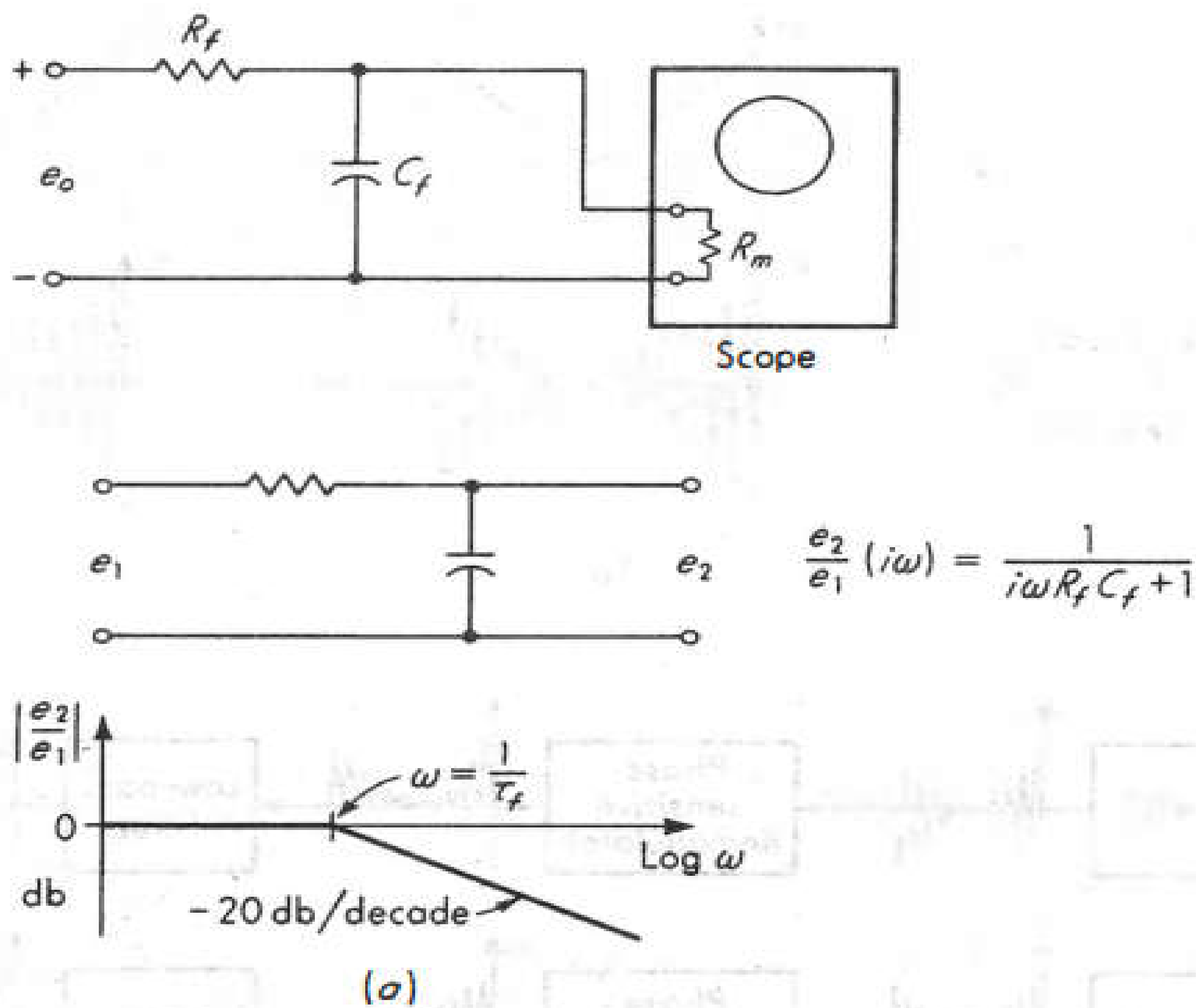




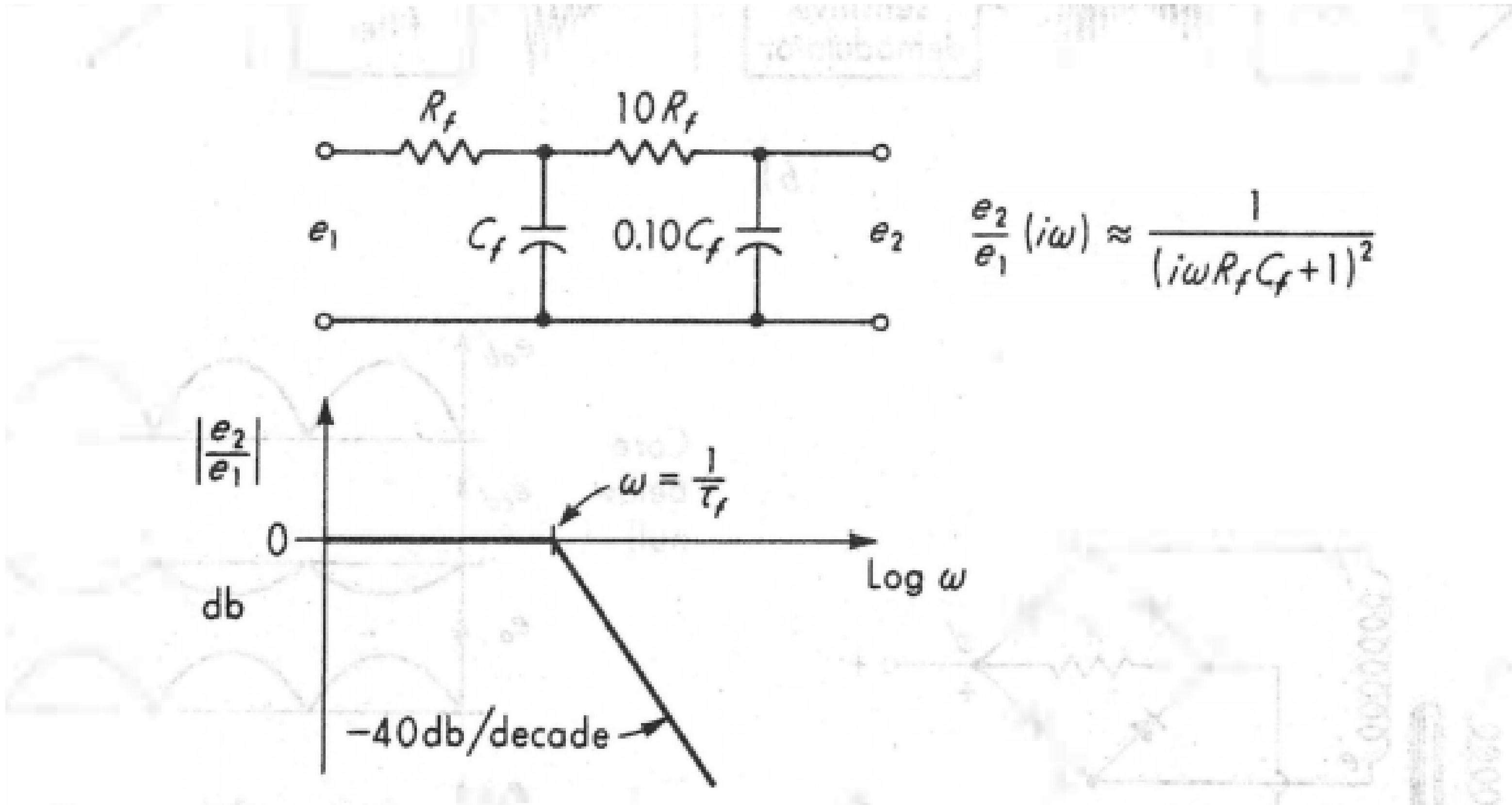
Phase Sensitive Demodulator Circuit of LVDT



Low pass Filtering of Demodulator output



1st Order LPF response



2nd Order LPF response

Application of LVDT

- Transduction of respiration through changes in chest volume

- Measuring dimensional change of internal organ