A revision on the standard ΛCDM CMB temperature power spectrum

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Abstract. The Cosmic Microwave Background (CMB) is one of the most powerful cosmological probes to confront the plethora of models that have arisen to solve the understood shortcomings of modern Cosmology. Even though the standard cosmological model ΛCDM is currently the preferred model by several observations, there are entities that we have not yet fully understood: as for example the dark sector. It is crucial to understand the necessity of these entities in the standard ΛCDM, even thought we do not know its Nature. In this revision, we perform a revision on that the CMB temperature angular power spectrum is based on ΛCDM. We have focused on the physics behind to calculate the theoretical CMB temperature power spectrum and determine the cosmological parameters using the new Boltzmann code called CLASS with the purpose to compare the ΛCDM model with alternative models e.g. a Universe without dark matter, open and closed Universes, and variation of the Hubble parameter to show the necessity of these unknown components and how the shape of the CMB theoretical curve depends on the cosmological parameters: Ω_m, Ω_k, H, etc.

1. Introduction

In 1965, A. Penzias and R. Wilson, who were investigating the origin of radio interference, reported a signal coming from all the directions of the sky [1]. R. Dicke proposed the explanation that the sky signal detected by Penzias and Wilson was emanating from a Hot Big Bang [2]. Their prediction was based on considering the conditions required for successful nucleosynthesis in an expanding Universe [3]. This signal, which was called as the Cosmic Microwave Background (CMB), has an spectrum comparable with a black-body with a temperature around 3K (K= Kelvin) and this signal detected was isotropic, unpolarized, and free from seasonal variations as well [3]. However, in the early 90s, small fluctuations in the temperature, called temperature anisotropies, was detected by the Cosmic Background Explorer (COBE) satellite [4]. Better and more accurate measurements of the CMB anisotropies were up to this day, first the Wilkinson Microwave Anisotropy Probe (WMAP) [5] and, the most current, Planck Collaboration [10]. The temperature anisotropies detected owe their existence to primeval density inhomogeneities, there is growing evidence that the origin of these perturbations is microscopic quantum fluctuations during an inflationary era in the early Universe [12]. According to Quantum Field Theory, the vacuum is not entirely empty. It is filled with quantum fluctuations of all types of physical fields, e.g. inflaton. These fluctuations create and annihilate each other and the averaged values of these fields [7], vanish, then the space filled with these fields seems to us empty. The wavelengths of all vacuum fluctuations of the inflaton field grow exponentially, and these wavelengths expand faster than the Hubble horizon, who expands as H^{-1}. When the wavelength of any particular fluctuation becomes greater than H^{-1}, this fluctuation is frozen since it is out of the Hubble horizon, and it does not interact with the fluctuations that are inside. Once the inflationary era has ended, the Hubble radius increases faster than the scale factor, so the fluctuations, that were out of the Hubble volume, eventually will reenter the Hubble radius during the radiation or matter dominated eras and these fluctuations will not annihilate. Therefore, the appearance of such frozen fluctuation is equivalent to the appearance of a perturbation in a classical field $\delta\Phi$ that does not vanish after having averaged over some macroscopic interval of time [7]. Therefore, being the inflaton fluctuations to be connected to the metric perturbations through Einstein's field equations, this caused the temperature fluctuations of the CMB and the matter perturbations $\delta\rho$. We are entering the era of precision cosmology, and with this precision of the observational data comes also the need for better precision of theoretical calculations of CMB power spectra[6]. There are some software that compute the CMB anisotropies spectrum that are standard in cosmology. For instance, the Cosmic Microwave Background EASY (CMBEASY) [14][15], the Code for Anisotropies in the Microwave Background (CAMB) [16] and the Cosmic Linear Anisotropies Solving System (CLASS) [17], [18], [19], [20]. In this revision, we have computed the CMB power spectrum for the ΛCDM model and other alternative models e.g. a Universe without dark matter, open and closed Universe and variation of the Hubble parameter, using the new Boltzmann code: CLASS. This paper is structure as follows: in section 2, we describe the dynamics of the standard cosmology, we first must understand deeply what each background quantities mean before perturbing the theory. In section 3, we describe the perturbed universe: we perturb the FRW metric and compute the equation of motion of the perturbations. The derivation of the perturbed Boltzmann equations for photons is given in section 4. In section 5, we compute the theoretical curve of CMB Power Spectrum for ΛCDM and other exotic models using CLASS.

2. Elements of background cosmology

Einstein's General Relativity is a classical theory of gravity that describes the relation between geometry and matter. The equation of motion of the theory are the Einstein's Field Equations (EFE) that describe the connection between the metric and the matter/energy; and can be derived from the Hilbert-Einstein action. According to the covariance principle, we have to write an integration measure in the action that is invariant under coordinates transformations,

$$\sqrt{-g}d^4x \longrightarrow \sqrt{-g'}d^4x', \tag{1}$$

where $g = \det(g_{\mu\nu})$ and the minus sign is because of $\det(g_{\mu\nu}) < 0$. Then, we have to propose a scalar to be proportional to the Lagrangian density and construct the action for the geometry of space-time.

$$S_G = \frac{1}{2k}\int R\sqrt{-g}d^4x\ , \tag{2}$$

where $R = g^{\mu\nu}R_{\mu\nu}$ and $\kappa = 8\pi G$ for consistency with the Newton's law of gravitation. Besides, we can derive the equations of motion of this action with the stationary action principle. Applying a variation in 2 we have,

$$\delta S_G = \frac{1}{2k}\int [g^{\mu\nu}\sqrt{-g}\delta R_{\mu\nu} + R_{\mu\nu}\delta(g^{\mu\nu}\sqrt{-g})]d^4x\ . \tag{3}$$

To simplify the derivation, we can work in the local coordinate system when the Christoffel symbols vanish, but its first derivatives do not. It can be shown that the first term in 3 is a total derivative in this coordinate system [9].

$$g^{\mu\nu}\delta R_{\mu\nu} = \partial_\lambda(g^{\mu\nu}\delta\Gamma^\lambda{}_{\mu\nu} - g^{\mu\lambda}\delta\Gamma^\lambda{}_{\mu\lambda})\,. \tag{4}$$

Therefore, it does not contribute to the equations of motion. Let us calculate the second term in 2,

$$\delta[g^{\mu\nu}\sqrt{-g}] = \sqrt{-g}\delta g^{\mu\nu} + g^{\mu\nu}\delta\sqrt{-g}\,. \tag{5}$$

Let us calculate the variation in the second term: $\delta\sqrt{-g}$

$$\delta\sqrt{-g} = \frac{\partial\sqrt{-g}}{\partial g_{\alpha\beta}}\delta g_{\alpha\beta} = -\frac{1}{2\sqrt{-g}}\frac{\partial g}{\partial g_{\alpha\beta}}\delta g_{\alpha\beta}\,, \tag{6}$$

to calculate $\frac{\partial g}{\partial g_{\alpha\beta}}$ we have to use the following expression,

$$g = g_{\alpha\beta}\mathrm{Cof}^{\;\alpha\beta} = \frac{\mathrm{Cof}^{\;\alpha\beta}}{g^{\alpha\beta}}\,, \tag{7}$$

where Cof $^{\alpha\beta}$ is the determinant of the cofactor's matrix of the element $g_{\alpha\beta}$.

$$\frac{\partial g}{\partial g_{\alpha\beta}} = \mathrm{Cof}^{\;\alpha\beta} = g g^{\alpha\beta}\,. \tag{8}$$

The other variation in the second term in 5 to be calculated is $\delta g_{\alpha\beta}$,

$$g^{\mu\alpha}g_{\alpha\beta} = \delta^\mu_\beta \Rightarrow \delta(g^{\mu\alpha}g_{\alpha\beta}) = 0 \Rightarrow g^{\mu\alpha}\delta g_{\alpha\beta} = -\delta g^{\mu\alpha}g_{\alpha\beta}\,, \tag{9}$$

replacing the expressions 8 and 9 in 6 we have,

$$\delta\sqrt{-g} = -\frac{1}{2\sqrt{-g}}g g^{\alpha\beta}\delta g_{\alpha\beta} = \frac{1}{2}\sqrt{-g}\left[-g_{\alpha\beta}\delta g^{\alpha\beta}\right] \tag{10}$$

Then, replacing the last expression in total variation in 5 we have

$$\delta[g^{\mu\nu}\sqrt{-g}] = \sqrt{-g}\delta g^{\mu\nu} + g^{\mu\nu}\delta\sqrt{-g} = \sqrt{-g}(\delta g^{\mu\nu} + \frac{1}{2}g^{\mu\nu}[-g_{\alpha\beta}\delta g^{\alpha\beta}]) \tag{11}$$

$$= \sqrt{-g}(\delta g^{\mu\nu} - \frac{1}{2}g^{\mu\nu}g_{\alpha\beta}\delta g^{\alpha\beta})\,, \tag{12}$$

let us introduce this expression in the second term of the action 3,

$$R_{\mu\nu}\delta[g^{\mu\nu}\sqrt{-g}] = R_{\mu\nu}\sqrt{-g}(\delta g^{\mu\nu} - \frac{1}{2}g^{\mu\nu}g_{\alpha\beta}\delta g^{\alpha\beta}) = \sqrt{-g}(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})\delta g^{\mu\nu}\,. \tag{13}$$

Finally we have the variation of the Einstein-Hilbert action 3,

$$\delta S_G = \frac{1}{2k}\int\sqrt{-g}(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})\delta g^{\mu\nu}d^4x\,. \tag{14}$$

Before applying the stationary action principle, we must consider the matter/energy contribution in the whole action.

$$S_M = \int\mathcal{L}_{\mathcal{M}}\sqrt{-g}d^4x\,, \tag{15}$$

where $\mathcal{L}_{\mathcal{M}}$ is the Lagrangian density of the matter and energy. When we apply the variation in the action of matter, we are going to find the expression of the energy-momentum tensor inside. It is straightforward shown in [9]

$$\delta S_M = \int \delta \left(\mathcal{L}_{\mathcal{M}} \sqrt{-g} \right) d^4x = -\frac{1}{2} \int T_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4x \,, \tag{16}$$

considering the both variations: in geometry and in matter, we can use the stationary action principle.

$$\delta(S_M + S_G) = \frac{1}{2k} \int \sqrt{-g} \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - k T_{\mu\nu} \right) \delta g^{\mu\nu} d^4x = 0, \tag{17}$$

the only way to this variation vanishes for an arbitrary variation $\delta g_{\mu\nu}$ is when EFE are satisfied,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{18}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor. EFE are clearly non-linear, so it's not straightforward to find analytic solutions. However, the presence of generic symmetries exhibit simple analytical solutions. The Friedmann-Robertson-Walker (FRW) metric is based on the assumption of homogeneity and isotropy of the Universe, which is approximately true on large scales as the cosmological principle states. There are three geometries only that satisfy the cosmological principle: open, close and flat universe. They are resumed in the FRW metric and is given by [21],

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t) \left[\frac{\mathrm{d}r^2}{1 - kr^2} + r^2 (\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \right], \tag{19}$$

where $a(t)$ is the scale factor with cosmic time t , the constant k in the metric (19) describes the geometry of the spatial curvature, which could be closed, flat or open universes corresponding to $k = +1, 0, -1$, respectively and the coordinates r, θ and ϕ are known as *commoving* coordinates[1]. The dynamics is determined by the scale factor $a(t)$. At this time, the scale factor is any function of the time and EFE allow us to determine the scale factor provided the equation of state. FRW metric has to satisfy EFE. That is how we obtain Friedmann equations. These equations are differential equations of the scale factor. Solving these two equations, we can know the dynamics of our universe.

$$H^2 \equiv \left(\tfrac{\dot{a}}{a}\right)^2 = \tfrac{8\pi G}{3} \textstyle\sum_i \rho_i - \tfrac{K}{a^2}, \tag{20}$$

$$\dot{H} = -4\pi G \textstyle\sum_i (\rho_i + P_i), \tag{21}$$

Where ρ and P are the energy density and pressure respectively, and the index "i" represents the matter contents, as we describe shortly, the matter content is made up of ordinary matter, dark matter, radiation and dark energy. We can rewrite the first Friedmann equations in terms of density parameter $\Omega(t)$.

$$\textstyle\sum_i \Omega_{M_i} + \Omega_K = 1 \,, \tag{22}$$

$$\Omega_{M_i} = \tfrac{\rho_i}{\rho_c} \qquad \Omega_K = -\tfrac{k}{(a.H)^2} \,. \tag{23}$$

In the frame of ΛCDM [2] we have that matter content in the universe can be modelled as a perfect fluid, it means the equation of state is constant $w = P/\rho = \text{cte}$. The Universe is filled with a mixture of different components of matter:

[1] A freely moving particle comes to rest in these coordinates

[2] It is named like this for the cosmological constant Λ and the cold dark matter (CDM) content [12, 22]

(i) Matter: We will refer to matter as any form of matter such that its pressure is very small compared to its energy density, i.e $P << \rho$. This is the case of particles not relativistic ($\omega = 0$). Then, we can solve the second Friedmann equation, 21 for this equation of state,

$$\rho \propto a^{-3}, \tag{24}$$
$$a \propto t^{2/3}. \tag{25}$$

- Baryons: In cosmology, ordinary matter (nucleons and electrons) are called baryons.
- Dark matter: It is a hypothetical kind of matter that has been proposed to explain the Galaxy rotation curve.

(ii) Radiation: In the case of relativistic particle, there is a relationship between pressure and density energy ($\rho = 3P$), that is why its equation of state is $w = \rho/P = 1/3$. Then, we can solve the second Friedmann equation, 21 for this equation of state,

$$\rho \propto a^{-4}, \tag{26}$$
$$a \propto t^{1/2}. \tag{27}$$

(iii) Dark energy: Recently, we have realized that the Universe is expanding. To reproduce this expansion, we need a fluid that satisfies the following equation of state condition: $\omega < -1/3$ [21], [22]. We can introduce the cosmological constant in the Lagrangian density, this will modify the EFE where Einstein tensor will still being divergenceless. Then, we have to derive one more time the Friedmann equation considering this cosmological constant,

$$H^2 = \tfrac{8\pi G}{3}\rho - \tfrac{k}{a^2} + \tfrac{\Lambda}{3}. \tag{28}$$
$$\tfrac{\ddot{a}}{a} = \tfrac{-4\pi G}{3}(\rho + 3P) + \tfrac{\Lambda}{3}. \tag{29}$$

These expressions are consistency with the old Friedmann equations: Eq. 20 and Eq. 21 because if $\Lambda = 0$ we recover the old expressions. We can recover the old expressions without making $\Lambda = 0$.

$$H^2 = \tfrac{8\pi G}{3}\tilde{\rho} - \tfrac{k}{a^2}, \tag{30}$$
$$\tfrac{\ddot{a}}{a} = \tfrac{-4\pi G}{3}(\tilde{\rho} + 3\tilde{P}), \tag{31}$$

where the energy density and pressure has been redefined: $\tilde{\rho} = \rho + \Lambda/8\pi G$ and $\tilde{P} = P - \Lambda/8\pi G$. Therefore, the equation of state of the cosmological constant is $\omega = -1$. We can solve the redefined Friedmann equations for the equation of state $\omega = -1$.

$$\rho_\Lambda = \tfrac{\Lambda}{8\pi G} = cte \tag{32}$$
$$a \propto e^{H_0 t}. \tag{33}$$

Therefore, introducing the cosmological constant, we have an exponentially expansion of the Universe due to a perfect fluid with constant energy density. Once we know the evolution of the energy density of each perfect fluid, we can plot the evolution of the density parameter in function of the scale factor as we can see in Fig. 1.

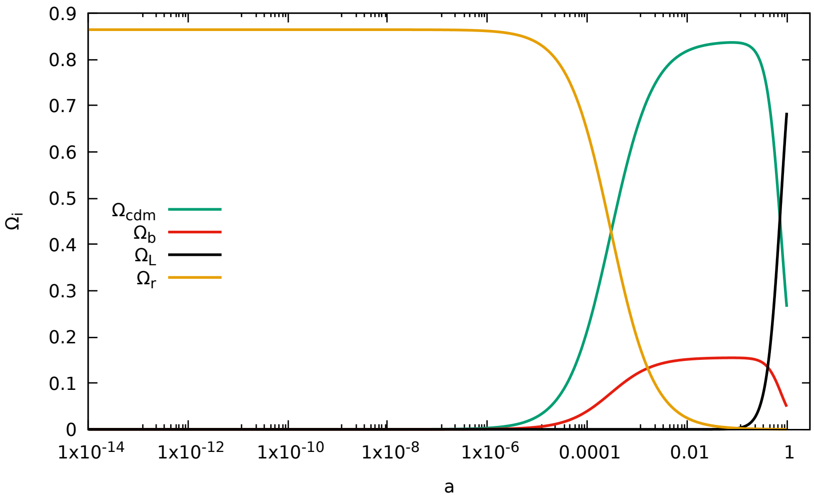

Figure 1. Evolution of density parameter in function of scale factor. The green line corresponds to the density parameter of cold dark matter. The red line corresponds to the density parameter of ordinary matter. The black line corresponds to the density parameter of dark energy and the yellow one corresponds to the density parameter of radiation.

As we can see in Fig. 1 the scale factor goes from zero to one, where close to zero represents the origin of the universe and one represents today. According to the Fig. 1 the radiation dominated when the Universe started, it means that the Universe was born and the first steps it takes are in a radiation era. Then, radiation becomes less prominent and matter started to be predominated. As we can see in Fig. 1, radiation and matter, currently, became less prominent and dark energy started to be predominated.

3. The Perturbed Universe

The Universe is isotropic and homogeneous on large scales, and it can be described very well by the FRW metric, 19. If we want to go to the early Universe, the FRW metric is not useful any more because the early Universe is no longer homogeneous and isotropic on these scales. That is why we need to construct a metric that can be useful on small scales. We can construct a perturbed metric from the FRW metric and compute the perturbations at linear order as,

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + \delta g_{\mu\nu} \,, \tag{34}$$

where $\overline{g}_{\mu\nu}$ is the background metric, in our cause is the FRW metric, and $g_{\mu\nu}$ is the perturbed metric at first order which will describe, approximately, the early Universe. We can consider more orders, e.g. second or third order; but a first order is a good approximation. The most general perturbed metric considering scalar, vector and tensor components are straightforward developed in [12] and references there in. We have to be careful to identify the perturbations. To illustrate, if we do a space-time translation, that depends on spacetime coordinates, to the FRW metric 19, we are going to obtain a metric that seems to be perturbed because the metric written in the new coordinates is going to be the metric written in the old coordinates plus extra terms. We have to realize that these extra terms are not perturbations because they appear in this new coordinate system, but they will vanish when we come back to the old coordinate system. However, physics does not depend on coordinate systems and that is how we state that a real perturbation should be invariant under this coordinate system change[3]. The perturbations

[3] This coordinate system change is also called gauge transformation.

are defined to be invariant under this coordinate system change. Therefore, we do not have to worry about which coordinate system is chosen, the form of the perturbation is not going to change. We can use this freedom to fix a gauge, that is, choosing a coordinate system where the perturbed metric is written in the simplest way. Fixing the Newtonian gauge, we obtain the following expression for the perturbed metric[12],

$$\mathrm{d}s^2 = a^2(\tau)\left[-(1+2\Psi)\mathrm{d}\tau^2 + (1+2\Phi)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right] , \tag{35}$$

Thus, the perturbation of the metric is given by,

$$\delta g_{\mu\nu} = \begin{bmatrix} -2\Psi & 0 \\ 0 & 2\Phi I_{3\times3} \end{bmatrix} \tag{36}$$

where Φ and Ψ are the scalar perturbations that perturb the background metric. From EFE we know that matter tells space-time how to curve and geometry tells matter how to move, it means that geometry and matter are intimately related. Therefore, if the metric is perturbed, the momentum-energy tensor should be perturbed as well

$$T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu} , \tag{37}$$

where $\bar{T}_{\mu\nu}$ is the energy-momentum tensor from background Universe, and $T_{\mu\nu}$ is the energy-momentum tensor for perturbed Universe. The energy-momentum tensor of the background Universe is the tensor of a perfect fluid according to the section 2. Therefore, we have to apply a variation to the energy-momentum tensor for a perfect fluid and obtain the most general perturbed energy-momentum tensor

$$\delta T_{\mu\nu} = (\delta\rho + \delta p)\bar{u}_\mu\bar{u}_\nu + (\bar{\rho} + \bar{p})(\bar{u}_\nu\delta u_\mu + \bar{u}_\mu\delta u_\nu) + \delta p\,\bar{g}_{\mu\nu} + \bar{p}\,\delta g_{\mu\nu} , \tag{38}$$

where, $\bar{u}_\mu = -a\delta^0_\mu$ is the fourth velocity and where all the upper lines live in the background Universe and the rest are the perturbations of each physical magnitude from the background Universe. Then, since we want to describe the early Universe with the perturbed metric, this metric has to satisfy EFE. Analogously to the derivation of the Friedmann equations, we must do the whole crossroads to solve the EFE. That is how we obtain, after calculations, the evolution equation of these perturbations.

$$\Phi_k = -\Psi_k , \tag{39}$$

$$4\pi G a^2\delta\rho = k^2\Phi_k + 3\mathcal{H}(\Phi'_k - \mathcal{H}\Psi_k) , \tag{40}$$

$$-4\pi G a^2\delta P = \Phi''_k + \mathcal{H}(2\Phi'_k - \Psi'_k) - (\mathcal{H}^2 + 2\mathcal{H}')\Psi_k , \tag{41}$$

where $\mathcal{H} = a'/a$ is the Hubble parameter, but the derivative is in function of conformal time [12]. We have expanded the perturbations Φ and Ψ in their Fourier modes, $\Phi = \int e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\Phi_k d^3k$, thus Eq. 40 and Eq. 41 are the equation of evolution for each wave number k. The Eq. 39 comes from that assumption the energy-momentum tensor is still diagonal in the perturbed Universe, this implies that Einstein's tensor is diagonal as well and the only way to make it possible is if Eq. 39 is satisfied. Then, replacing 40 in 41 and after a basic algebraic calculation, we find an equation of evolution for Φ_k for each wave number k.

$$\Phi''_k + 3\mathcal{H}(1+\omega)\Phi'_k + wk^2\Phi_k = 0 , \tag{42}$$

where $\omega = \rho/P$ is the equation of state of the fluid in the background Universe, 2. The Eq. 42 is the evolution equation of the perturbation Φ for each wave number k. In this section, we have seen that when the metric is perturbed, then the EFE are perturbed as well. EFE are not the only equations that are affected when metric is perturbed, in the next section we are going to derive it the perturbed Boltzmann equation for the photons.

4. Perturbed Boltzmann Equation for Photons

The equations of evolution for the perturbations come from the perturbed Universe as well as the Boltzmann equation for photons, Cold Dark Matter, baryons and neutrinos [6].These particles interact between them in the early Universe according to the Fig. 2 and these interactions can be described by Boltzmann equation.

$$\frac{df}{dt} = C[f] \,, \tag{43}$$

where f is the distribution function of the photon and the r.h.s of the equation is the collision term. To find the equation of evolution of temperature anisotropies, we have to solve the Boltzmann equation for photons. First, we need to calculate the *r.h.s*, this term depends on the interaction between the particles. In this case, the interaction is determined by Compton scattering: $e^-(\boldsymbol{q}) + \gamma(\boldsymbol{p}) \longleftrightarrow e^-(\boldsymbol{q'}) + \gamma(\boldsymbol{p'})$. The collision term determines the influence that Compton scattering has on the photon distribution. We are interested in the change of distribution of photons with initial momentum $\mathbf{p}$. Therefore, we must sum over all other momenta $\mathbf{q}, \mathbf{q}', \mathbf{p}'$ which affect $f(\mathbf{p})$. Then, the collision term is given by,

$$C[f(\mathbf{p})] = \sum_{\mathbf{q},\mathbf{q}',\mathbf{p}'} |A|^2 [f_e(\mathbf{q}') \cdot f(\mathbf{p}') - f_e(\mathbf{q}) \cdot f(\mathbf{p})] \,. \tag{44}$$

where $|A|^2$ is determined by the Fermi's golden rule and implicitly by the Feynman's diagrams [28]. Unfortunately, this term becomes messy if we put it properly for the sums over phase space, but it can be found it in [25]. The other term in the Eq. 44 is the subtraction of the product. The product of distribution function counts the number of particle that have a given momentum; therefore, the subtraction of the product of the distribution functions counts the number of particle have changed their momenta. After the extensive calculation of $|A|^2$ that can be found in many references [22], [25], [28] and references there in. Thus, the collision term is given by,

$$C[f] = -p\frac{\partial f^{(0)}}{dp} n_e \sigma_T [\Theta_0 - \Theta(\hat{p}) + \hat{\boldsymbol{p}} \cdot \boldsymbol{v}_b], \tag{45}$$

where $\Theta \equiv \delta T/T$ is the perturbation in photon temperature, n_e is the number density of free electrons and σ_T is the cross-section and $f^{(0)}$ is the distribution function in the background, remember that as the metric is perturbed, this will perturb the photons temperature as well due to Fig. 2 and this perturbs the Bose-Einstein distribution. Then, we have to compute the *l.h.s* of the Boltzmann equation, and it is given by [25][28].

$$\frac{df}{dt} = -p\frac{\partial f^{(0)}}{\partial p}\left[\frac{\partial \Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Theta}{\partial x^i} + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i}\right] \,. \tag{46}$$

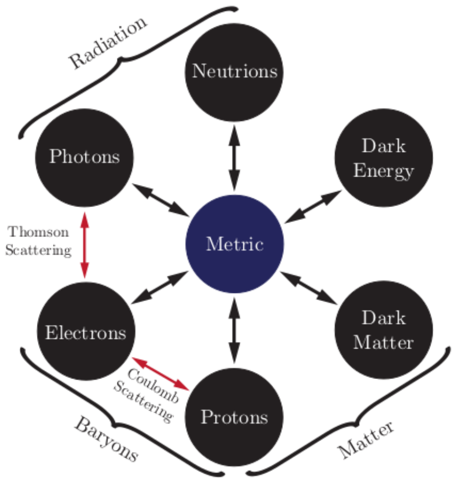

Figure 2. This is the way that metric is related to matter. Neutrinos and dark matter are unsociable and are affected by gravity only. Protons and electrons interact through Coulomb scattering and photons interact with electrons between Compton scattering, and of course all of they are affected by gravity (metric) with no exception.

where, $\tau_{op} = \int_{\tau}^{\tau_0} n_e \sigma_T a d\tau$ is the optical deep. At late times, $\tau \ll 1$, while at early times, it is very large [28]. Note that the derivative in 47 is in function of conformal time. We are knocking the doors on finding the Boltzmann equation for photons, so now we only need to replace the distribution function that photons obey: Bose-Einstein distribution [22] and then we have to match the RHS 45 with the $l.h.s$ of Boltzmann equation 46, and we will obtain,

$$\Theta'_k + ik\mu\Theta_k + \Phi'_k - ik\mu\Phi_k = -\tau'_{op}[\Theta_0 - \Theta_k + \mu v_b] \tag{47}$$

where $\Theta \equiv \delta T/T$ are the CMB temperature fluctuations, $\tau_{op} \equiv \int_{\tau}^{\tau_0} n_e \sigma_T a d\tau$ is the optical deep. The Eq. 47 is the equation of evolution of Θ for each wave number k. Then, we can expand it in multipoles, and obtain the evolution of Θ_l: $\Theta = \sum_{l=0}^{\infty} \frac{2l+1}{i^l} \Theta_l \mathcal{P}_l$, where $\mathcal{P}_l$ are the Legendre Polynomials. Replacing this expansion in Eq. 47 we obtain the equation of evolution for each multipole,

$$-\Phi = \dot{\Theta}_0 + k\Theta_1 \tag{48}$$

$$-\frac{k}{3}\Phi + \dot{\tau}\left[\Theta_1 + \frac{1}{3}v_b\right] = \dot{\Theta}_1 - \frac{k}{3}\Theta_0 + \frac{2k}{3}\Theta_2 \tag{49}$$

$$\dot{\tau}\left[\Theta_l - \frac{1}{10}\Theta_2\delta_{l,2}\right] = \dot{\Theta}_l - \frac{lk}{2l+1}\Theta_{l-1} + \frac{(l+1)k}{2l+1}\Theta_{l+1}, l > 2$$

Where we have omitted the wave number, k, subscript. Observational data from Planck Collaboration [11] is in function of $C_l = \langle a_{lm}\, a^*_{l',m'} \rangle$ where a_{lm} are the Fourier coefficients of the expansion in spherical harmonics. Thus, we have to relate the observables, a_{lm}'s to the Θ_l's. The mean value of all the a_{lm}'s is zero, but they will have some non-zero variance. The variance of the a_{lm}'s is precisely C_l [28]

$$C_l = \frac{2}{\pi}\int_0^{\infty} dk k^2 P(k) \left|\frac{\Theta_l}{\delta(k)}\right|^2, \tag{50}$$

where we have written the photon distribution Θ as $\delta \cdot (\Theta/\delta)$, and the dark matter overdensity[4] δ does not depend on any direction [28] (here it is assumed the statistical isotropy of the Universe; for other results regarding this issue see, e.g., [29]). Resuming this section, to plot the power spectrum of the temperature anisotropies (Eq. 50) we need to solve first the Boltzmann equations for photons (Eq. 47). But, to solve the Boltzmann equation, we need to solve first the equation of evolution of Φ (Eq. 42). Also, this equation of Φ is in function of the Hubble parameter, so we must solve first the Friedmann equations (Eq. 20 and Eq. 21). Thus, to compute the CMB temperature anisotropies spectrum, we have to solve a set of equations that are not trivial to solve it analytically [24].

5. CMB Temperature Anisotropies

In this section, we are going to compute the expected CMB Temperature anisotropies (CMB TT) for ΛCDM and other 'exotic models'. To plot these CMB TT, one needs to solve a set of equations that are coupled between them. You can try to solve it analytically, as you can see in the reference [24]. Otherwise, we can try to solve it numerically with a Boltzmann code, e.g. CLASS. In this revision, we prefer to do it numerically with the help of CLASS. One of the most important things in the CMB TT is its shape, as we can see in Fig. 3. Its shape depends on the cosmological parameters (Ω_m, Ω_Λ, Ω_K, ...) and initial conditions determined by the inflation epoch [12]. The characteristic peaks structure of the CMB TT (see Fig. 3) are due to the acoustic oscillations in the primordial plasma before recombination [12]. The temperature fluctuations in the CMB TT are sourced by scalar fluctuations[5].

We aim to plot the CMB TT for the standard cosmological model ΛCDM and other alternative models e.g. a Universe without dark matter, open and closed Universes, and variation of the Hubble parameter. We try to show how the expected CMB TT depends on the cosmological parameters that is why we consider different values of Ω_{cdm}, Ω_k and H to see how CMB TT shape changes. We first show the CMB TT shape of the ΛCDM standard cosmological model. Then, we show the CMB TT for a Universe without dark matter to show the necessity of this component in the cosmological model. There are two cases that we can have: the first one is when the missing of dark matter is the increasing in baryons, it means all the matter is the ordinary matter. The second case is when the missing of dark matter is the increasing of dark energy to satisfy the density parameter equation (Eq. 22), in this case the ordinary matter is the same that in ΛCDM. In addition, we show the CMB TT of an Universe with different curvatures, in this case we aim to show how the shape of CMB TT depends on the curvature of the Universe. Finally, the last case we show Universes with different Hubble values, we aim to show the dependency of the CMB TT on the Hubble parameter value.

Table 1. Cosmological Parameters. We have considered $h = 0.6727 kms^{-1}Mpc^{-1}$ according to the baseline model in [10].

Models	Ω_{cdm}	Ω_Λ	Ω_b	Ω_k
ΛCDM	0.265	0.684	0.049	0
Without CDM (1)	0	0.684	0.314	0
Without CDM (2)	0	0.949	0.049	0
Open Universe	0.265	0.986	0.049	-0.3
Closed Universe	0.265	0.386	0.049	0.3

[4] the equation of overdensity δ is determined by Eq. 40.

[5] a major goal of current efforts in observational cosmology is to detect the tensor component of the primordial fluctuations [12].

(i) **The ΛCDM model**
The standard cosmological model ΛCDM is made of the cosmological constant Λ, cold dark matter (CDM) content, ordinary matter and radiation. The most abundant content of matter in the Universe, according to this model, with $\sim 68\%$ is dark energy that is modelled by a cosmological constant Λ as a perfect fluid with a constant equation of state $w = -1$. Then, we have CDM with $\sim 27\%$ and ordinary matter with, $\sim 6\%$ that both are modelled by a perfect fluid with an equation of state $w = 0$. We have assumed a flat Universe, it means $\Omega_k = 0$. We have assumed three neutrinos species with the normal hierarchy mass: two massless species and a single massive neutrino of mass $m_\nu = 0.06$ eV. We have considered the baseline model ΛCDM from [10] considering only components TT and low E.

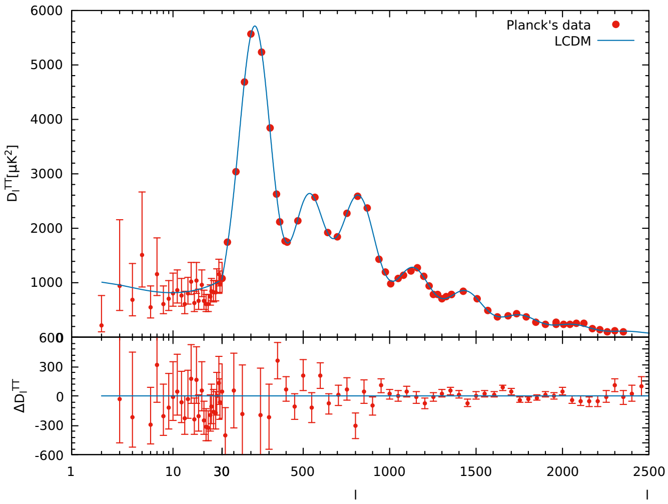

Figure 3. Top: CMB TT for the standard cosmological model ΛCDM. The solid line in blue denotes the ΛCDM model. The dot and error bars in red color denotes the data from the Planck 2018 TT power spectra. Bottom: ΔD_l^{TT} component from the temperature power spectrum of the CMB. The data used was obtained from [11].

As we can see in Fig. 3 the baseline model ΛCDM with the cosmological parameters values according to the Table 1 fits the CMB TT data obtained by Planck Collaboration [10]. Every cosmological parameter for ΛCDM in the Table 1 is relevant in the shape of the CMB TT. If we consider a different value of a single cosmological parameter the expected CMB TT will not fit the data CMB TT data that is how we will show the necessity of these matter components and their values in the standard cosmological model.

(ii) **Cases Without Dark matter:**
In order to highlight the importance of CDM in ΛCDM, we plot the expected CMB TT for a model without CDM. The blue curve is for a Universe without dark matter, and what is missing from dark matter is the increase in baryons. The yellow curve is for a Universe without dark matter as well, and what is missing from dark matter is the increase in dark energy in order to satisfy the density parameter condition 22. In both cases, the first peak is higher than the first peak's data. Clearly, we can see the *l.h.s* in Fig. 4 that these two expected CMB TT's have the same shape of CMB TT for ΛCDM, but the height of their peaks do not coincide with the data. That is how we can state that we can not live in a Universe without dark matter, and it is needed to fit the CMB TT data. Not only that, but dark matter accounts for 26% of the energy budget of the Universe according to what we show in ΛCDM model.

(iii) **Cases with different Hubble parameter values:**
There are some methods to determine the Hubble parameter. One of them is the local measurements from supernovae [30] and other one is from the CMB TT [10]. These values do not coincide between them, and this discrepancy is called the Hubble tension[6]. We know that the Hubble parameter for ΛCDM case is $h = 0.6727 kms^{-1}Mpc^{-1}$ In this case, we aim to show the dependency of CMB TT on the Hubble parameter that is why we chose two values: one less than the usual Hubble parameter (h $= 0.5 kms^{-1}Mpc^{-1}$) and the other one greater than the usual Hubble parameter (h $= 0.9 kms^{-1}Mpc^{-1}$). In the *r.h.s* in Fig. 4, we see that none of these two cases fit the CMB TT data. As we can see, the yellow line that corresponds to the value less than the usual one holds the shape of the CMB TT, but it is shifted to the right. On the other hand, the green line corresponds to values greater than the usual one is shifted to the left as well, and it would hold the shape of the CMB TT if it were not for the second peak.

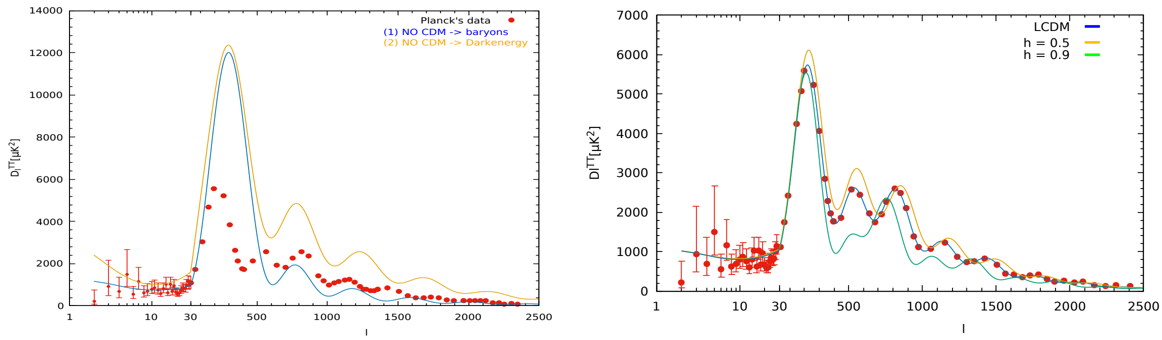

Figure 4. *Left:* CMB TT without dark matter. The solid line in blue denotes the Universe without dark matter and the increasing of baryons. The solid line in blue denotes the Universe without dark matter and the increasing of dark energy. The dot and error bars in red color denotes the data from the Planck 2018 TT power spectra. *Right:* CMB TT with different Hubble's parameter values. The solid line in blue denotes ΛCDM model. The solid line in green denotes a Universe with $h = 0.9 kms^{-1}Mpc^{-1}$. The solid line in green denotes a Universe with $h = 0.5 kms^{-1}Mpc^{-1}$. The dot and error bars in red color denotes the data from the Planck 2018 TT power spectra.

(iv) **Cases with different curvatures**
ΛCDM model assumes a priori that the spatial curvature is flat, since is predicted by some inflationary models [26] and [27]. This is a prediction that can be tested by CMB TT. If we do not consider a priori a flat Universe, and we leave free the density parameter of curvature, Ω_K we will find that $\Omega_K = -0.056^{+0.028}_{-0.018}$ considering only components TT and low E according to [10]. It means that our Universe is closed but close to flat, it means flattened. However, if we consider a combination of CMB TT and Baryonic Acoustic Oscillation (BAO) data, we will find that $\Omega_K = 0.0007 \pm 0.00065$ considering data from components TT, TE, EE + low E+ lensing + BAO [10]. It means that our Universe is open, but flattened. Therefore, it cannot be stated the geometry of the Universe, but, what we can state is that our Universe is flattened whether it is open or closed because in either case the density parameter of the curvature is close to zero, $\Omega_k \sim 0$. An interesting exercise to see how the CMB TT depends on the geometry of the Universe is to assume a geometry and plot the expected CMB TT. We have assumed the values $\Omega_K = -0.3$ for a closed

[6] it is not easily explained by a systematic effect in the measurements.

Universe and $\Omega_K = 0.3$ for an open Universe, you can see in Table 1. We can see in Fig. 5 that both cases: closed and open Universe, have the same form and numbers of peaks as ΛCDM. Curvature of the Universe only shift to the right or to the left the expected CMB TT if it is open or closed, respectively.

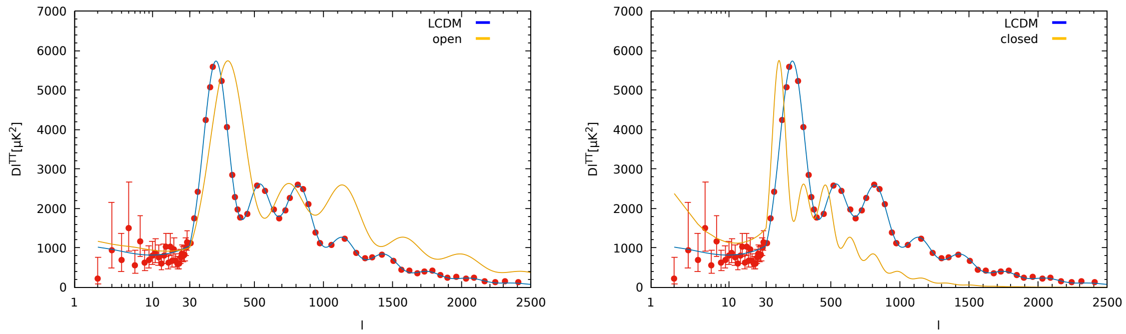

Figure 5. *Left:* CMB TT for an open Universe with a density parameter of curvature negative, $\Omega_k = 0.3$ The solid line in blue denotes ΛCDM model. The solid line in yellow denotes an open Universe with $\Omega_k = 0.3$. The dot and error bars in red color denotes the data from the Planck 2018 TT power spectra. *Right:* CMB TT for a closed Universe with a density parameter of curvature negative, $\Omega_k = -0.3$. The solid line in blue denotes ΛCDM model. The solid line in yellow denotes a closed Universe with $\Omega_k = -0.3$. The dot and error bars in red color denotes the data from the Planck 2018 TT power spectra [10].

6. Conclusions

We have derived the equations of perturbations from the Perturbed Universe and the Perturbed Boltzmann equation for the photons. Besides, we have reviewed the baseline model ΛCDM from the CMB Temperature Anisotropies. In addition, we plotted the expected CMB TT for alternative models e.g. Universes without dark matter, cases with different Hubble parameter values, closed and open Universes. We showed the dependency of the CMB TT on the cosmological parameters.

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