A revision on understanding the CMB spectrum in a ΛCDM cosmology

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Cosmic Microwave Background (CMB) is one of the most powerful cosmological probes to test the plethora of models that have arisen to solve the shortcomings of modern cosmology. Even though the standard cosmological model ΛCDM is currently the best-fit model by many observations, specially the CMB Power Spectrum, there are entities that we have not yet fully understood: dark energy as a cosmological constant Λ and cold dark matter. It is crucial to understand the necessity of these entities in ΛCDM even thought we do not know their nature. In this article, we show why ΛCDM is the best-fit model according with CMB Temperature Power Spectrum. we have focused on both, all the physics behind to compute the theoretical curve of CMB Temperature Power Spectrum and determine the cosmological parameters using the code CLASS with the intention to compare ΛCDM with other exotic models to show the necessity of these unknown entities. Finally, getting approach of the code CLASS we have computed the theoretical curve of CMB Temperature Power Spectrum with normal, inverted hierarchy of neutrino and a complex quintessence with attractive self-interaction.

PACS numbers: 98.80.-k,04.25.Nx,98.70.Vc

I. INTRODUCTION

In 1965, A. Penzias and R. Wilson, who were investigating the origin of radio interference, reported(first) a 3.5 K signal from the sky[1]. R. Dicke proposed the explanation that the sky signal detected by Penzias and Wilson was emanating from a hot big bang, as had been suggested by Alpher in 1948 [2]. This signal, so-called Cosmic Microwave Background (CMB), has a black-body spectrum with a temperature around 3K and this signal detected was isotropic, unpolarized, and free from seasonal variations as well[3]. However, in the early 90's, small fluctuations in the temperature[1] was detected by the Cosmic Background Explorer (COBE) satellite [4]. Since their discovery, anisotropies in the CMB is one of the most powerful tools to study the early universe. [7]. Better and more accurate measurements of the CMB anisotropies are the Wilkinson Microwave Anisotropy Probe (WMAP) [5],[6] and, the most current, Planck satellite [9].

The temperature anisotropies detected owe their existence to primeval density inhomogeneities, our best guess for the origin of these perturbations is quantum fluctuations during an inflationary era in the early universe. According to quantum field theory, empty space is not entirely empty. It is filled with quantum fluctuations of all types of physical fields. These fluctuations annihilate between them and the averaged values of these fields, over some macroscopically large time, vanish then the space filled with these fields seems to us empty[8]. The wavelenghts of all vacuum fluctuations of the inflaton field grow exponentially in the inflationary era and these wavelengths expand faster than the horizon scale, who expands as H^{-1}. When the wavelength of any particular fluctuation becomes greater than H^{-1},this fluctuation frozen. Once inflation has ended, the Hubble radius increases faster than the scale factor, so the fluctuations eventually reenter the Hubble radius during the radiation or matter dominated eras and these fluctuations will not annihilate. Therefore, the appearance of such frozen fluctuation is equivalent to the appearance of a classical field $\delta\Phi$ that does not vanish after having averaged over some macroscopic interval of time[8]. Therefore, being the inflaton fluctuations to be connected to the metric perturbations through Einstein's field equations (EFE), this caused the anisotropies in the CMB and the matter perturbations $\delta\rho$. These small deviations from homogeneity and isotropy at early times played an important role in the dynamic of our universe. Small initial density perturbations has grew via gravitational instability into the structure we see today in the universe

we are entering the era of precision cosmology and with this precision of the observational data comes also the need for better precision of theoretical calculations of CMB power spectra[7]. There are several software that compute the CMB anisotropies spectrum that are standard in cosmology. For instance, CMB-FAST [11], CMBEASY [12][13], CAMB[14] and CLASS [24],[25],[26],[27],[28]. In this text we have computed the CMB anisotropies spectrum, which is one of the purpose, with the last software. As we will shortly see in section V to compute the CMB spectrum we must determine first the cosmological parameters. According to the last measurements by Planck Satellite [9], the matter

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[1] Henceforth these small temperature fluctuations will be called anisotropies

content [2] accounts of the matter/energy budget of the universe is $\Omega_m = 0.3166 \pm 0.0084$, $\Omega_K = 0.0007 \pm 0.0019$ and $\Omega_\Lambda = 0.6847 \pm 0.0073$. These results stay that dark matter and baryons are 31% of the energy budget, the universe is flattened and the dark energy is around 68% of the energy budget.

This paper is structure as follows: in section II, we describe the dynamics of the standard cosmology, we first must understand deeply what each background quantities mean before perturbing the theory. In section III, we describe the perturbed universe: we perturb the FLRW metric and compute the equation of motion of the perturbations. The derivation of the perturbed Boltzmann equations for photons is given in section IV. In section V, we compute the theoretical curve of CMB Power Spectrum for ΛCDM and other exotic models using CLASS.

II. ELEMENTS OF BACKGROUND COSMOLOGY

Einstein's General Relativity is a classical theory of gravity that describes the relation between geometry and matter. EFE are the equations of motion of General Relativity (GR) that describe the connection between the metric and the matter/energy; and can be derived by the Hilbert-Einstein action by the minimal action principle.

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor. EFE are clearly non-linear so it's not straightforward to find analytic solutions. However the presence of generic symmetries exhibit simple analytical solutions. The Friedmann-Robertson-Walker (FRW) metric is based on the assumption of homogeneity and isotropy of the universe which is approximately true on large scales as the cosmological principle states. There are three geometries only that satisfy the cosmological principle: open, close and flat universe. They are resumed in the FRW metric and is given by[Cris: [21]],

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-Kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)\right], \tag{2}$$

where $a(t)$ is the scale factor with cosmic time t, The constant K in the metric (2) describes the geometry of the spatial curvature, which could be closed, flat or open universes corresponding to $K = +1, 0, -1$, respectively and the coordinates r, θ and ϕ are known as *comoving* coordinates[3]. The dynamics is determined by the scale factor $a(t)$. At this time, the scale factor is any function of the time and EFE allow us to determine the scale factor provided the equation of state. FLRW metric has to satisfy EFE. That is how we got Friedmann equations. These equations are differential equations of the scale factor. Solving these two equations we can know the dynamics of our universe.

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{K}{a^2}, \tag{3}$$

$$\dot{H} = -4\pi G \sum_i (\rho_i + P_i), \tag{4}$$

Where ρ and P are the energy density and pressure respectively, and the indice "i" represents the matter contents, as we describe shortly, the matter content is made up of ordinary matter, dark matter, radiation and dark energy. In the frame of ΛCDM we have that matter content in the universe can be modeled as a perfect fluid. It means the equation of state is constant $w = P/\rho$ =cte. For instance, we have Cold Dark Matter(CDM) and baryons who are non relativistic particles and since its pressure is too small in comparison with its density energy, its equation of state is $w = \rho/P = 0$. In the case of relativistic particle, there is a relationship between pressure and density energy ($\rho = 3P$), that's why its equation of state is $w = \rho/P = 1/3$ and finally we have dark energy as a cosmological constant. First, we can infer from Friedmann equations that the equation of state for a perfect fluid to reproduce an expansion rapidly should be $w < -1/3$ [15][21]. Thus, we can introduce the cosmological constant in the Lagrangian density, modify the EFE where Einstein tensor still being divergenceless. In the same way, Friedmann equations get modified but redefining the energy density and pressure: $\tilde{\rho} = \rho + \Lambda/8\pi G$ and $\tilde{P} = P - \Lambda/8\pi G$. We can come back to last form of the friedmann equations. When you redefined the energy density and pressure a new fluid with a equation of state $w = -1$ is added. We can rewrite the first Friedmann equation in terms of density paremeter $\Omega(t)$.

$$\sum_i \Omega_{M_i} + \Omega_K = 1\,, \tag{5}$$

$$\Omega_{M_i} = \frac{\rho_i}{\rho_c} \qquad \Omega_K = -\frac{k}{(a.H)^2}\,. \tag{6}$$

where, $\rho_c = \frac{3H^2}{8\pi G}$ and Ω_M represents the density parameter of matter content[4] and Ω_K represents the density parameter of curvature and we can see how these parameters are related,

$$\Omega_M > 1(\rho > \rho_c) \leftrightarrow k = 1, \tag{7}$$
$$\Omega_M = 1(\rho = \rho_c) \leftrightarrow k = 0, \tag{8}$$
$$\Omega_M < 1(\rho < \rho_c) \leftrightarrow k = -1. \tag{9}$$

[2] It includes dark matter and baryons(ordinary matter)
[3] A freely moving particle comes to rest in these coordinates
[4] dark matter, dark energy, dust, radiation.

III. PERTURBED UNIVERSE

Early universe is far from homogeneous and isotropic and FLRW metric is no longer useful. we need to construct a perturbed universe. we can construct a linearly perturbed metric from the background metric as the following way,

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + \delta g_{\mu\nu}. \tag{10}$$

Where $\overline{g}_{\mu\nu}$ is the background metric, in our cause is the FLRW metric, and $g_{\mu\nu}$ is the perturbed metric at first order which we will use to describe the early universe. The most general perturbed metric considering scalar, vector and tensor perturbation is a bit long and messy but is given in the following lectures [19][10].

We have to be careful with fake perturbations and recognize which perturbations are real. To illustrate, if we make a translation, that depends on spacetime, in spacetime coordinate to the FLRW metric II. we are going to get a metric that seems to be perturbed. We have to realize that these are fake perturbations because they appear in this new coordinate system if we go back to the old one this 'perturbation' will vanish. However, physics does not depend on coordinate systems and that is how we state that real perturbations should be invariant under this coordinate system change[5] and that is how we can distinguish between fake and real perturbations.

Therefore, it does not matter which coordinate system we have chosen, the form of the perturbation is not going to change Now is better explained in the last paragraph why this does not change. We can use this freedom and choose a coordinate system[6] where the perturbed metric is written in the simplest way. Fixing the Newtonian gauge[7], we got the following expression for the perturbed metric[10],

$$\mathrm{d}s^2 = a^2(\tau)\left[-(1+2\Psi)d\tau^2 + (1+2\Phi)\delta_{ij}dx^i dx^j\right]. \tag{11}$$

Thus, the perturbation metric is given by,

$$\delta g_{\mu\nu} = \begin{pmatrix} -2\Psi & 0 \\ 0 & 2\Phi I_{3\times 3} \end{pmatrix}.$$

Where Φ and Ψ are the scalar perturbations that perturb the background metric. From EFE we know that matter tells spacetime how to curve and geometry tells matter how to move. Thus, if metric is perturbed, the momentum-energy tensor should be perturbed as well.

$$T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}\,. \tag{12}$$

[5] This coordinate system change is also called gauge transformation.
[6] Choosing a coordinate system
[7] We can find a coordinate system where $B = E = 0$.

Where $\bar{T}_{\mu\nu}$ is the energy-momentum tensor from background universe, and $T_{\mu\nu}$ is the energy-momentum tensor for perturbed universe. From the section II, we have modeled the matter content as a perfect fluid, so the background energy-momentum tensor is,

$$\overline{T}_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}\,, \tag{13}$$

where, $u_\mu = -a\delta^0_\mu$ is the fourth velocity. Then, we can apply a variation to the background energy-momentum tensor 13 and get the most general perturbed energy-momentum tensor,

$$\delta T_{\mu\nu} = (\delta\rho + \delta p)\bar{u}_\mu \bar{u}_\nu + (\bar{\rho} + \bar{p})(\bar{u_\nu}\delta u_\mu + \bar{u_\mu}\delta u_\nu) \\ + \delta p\, \bar{g}_{\mu\nu} + \bar{p}\, \delta g_{\mu\nu}\,. \tag{14}$$

Where all the upper lines live in the background universe and the rest are the perturbations of each physical magnitude from the background universe. Then, since the perturbed metric refperturbed metric and the background metric 2 both satisfy EFE 1, we can infer that the perturbation of the metric must satisfy the EFE as well,

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \Rightarrow \delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}\,, \tag{15}$$

Thus, background, perturbed metric and perturbation of the metric have to satisfy EFE. Then, the equation of motion of the scalar perturbations Φ and Ψ are determined when the perturbation of the metric III satisfy the perturbation of EFE 15. Analogously to the derivation to Friedmann equations, we must compute the Christoffel symbols for the perturbation of the metric and then compute the Ricci tensor and do the whole crossroads again. That is how we got, after a tedious calculation, the equation of evolution of these perturbations.

$$\Phi_k = -\Psi_k \tag{16}$$

$$4\pi G a^2 \delta\rho = k^2\Phi_k + 3\mathcal{H}(\Phi'_k - \mathcal{H}\Psi_k)\,, \tag{17}$$

$$-4\pi G a^2 \delta P = \Phi''_k + \mathcal{H}(2\Phi'_k - \Psi'_k) - (\mathcal{H}^2 + 2\mathcal{H}')\Psi_k \tag{18}$$

Where $\mathcal{H} = a'/a$ is the hubble parameter but the derivate is in function of conformal time [10] . We have expanded the perturbations Φ and Ψ in their Fourier modes, $\Phi = \int e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\Phi_k d^3k$, thus equations 17 and 18 are the equation of evolution for each wavenumber k. The equation 16 comes from the assumption the energy-momentum tensor is still diagonal in the perturbed universe, this implies that Einstein's tensor is diagonal as well and the only way to make it possible is if equation 16 is satisfied. Then, we can find an equation of evolution for Φ_k for each wavenumber k from 17 and 18 Φ_k,

$$\Phi''_k + 3\mathcal{H}(1+\omega)\Phi'_k + wk^2\Phi_k = 0. \tag{19}$$

Where $\omega = \frac{\rho}{P}$ is the equation of state of the background universe. The equation 19 is the equation of evolution of the perturbation Φ for each wavenumber k.

IV. BOLTZMANN EQUATION FOR PHOTONS

The equations of evolution for the perturbations come from the Boltzmann equations for photons, Cold Dark Matter, baryons, and neutrinos [7]. Photons are affected by gravity and by Compton scattering due to free electrons. Electrons couple lightly to protons by Thomson Scattering and, of course, both are affected by gravity.

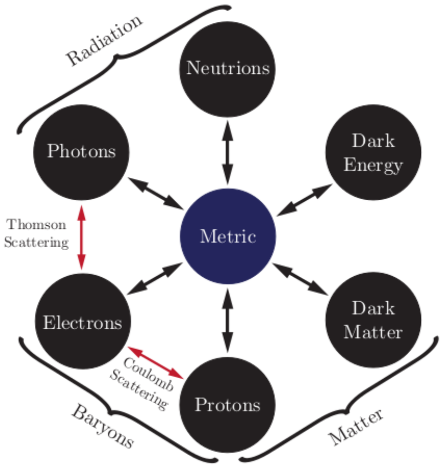

FIG. 1: This is the way that matter interacts with matter. Neutrinos and dark matter are unsociable and are affected by gravity only. Protons and electrons interact through Coulomb scattering and photons interact with electrons through Compton scattering, and of course, all they are affected by gravity (metric) with no exception. This picture is from [10]

The metric is influenced by all the mentioned components plus neutrinos and dark matter. These interactions can be visualized better in FIG. 1. Therefore, we need to describe these interactions between the particles in the early universe. They are described by the Boltzmann equation.

$$\frac{df}{dt} = C[f]\,, \tag{20}$$

where f is the distribution function of the photon and the right side of the equation is the collision term. In general, this term depends on the interaction between the particles. In this case, the interaction is determined by Compton scattering: $e^-(\boldsymbol{q}) + \gamma(\boldsymbol{p}) \longleftrightarrow e^-(\boldsymbol{q}') + \gamma(\boldsymbol{p}')$. The collision term determines the influence Compton scattering has on the photon distribution. We are interested in the change of distribution of photons with momentum $\mathbf{p}$. Therefore, we must sum over all other momenta $\mathbf{q}, \mathbf{q}', \mathbf{p}'$ which affect $f(\mathbf{p})$. Then, the collision term is given by [20],

$$C[f(\mathbf{p})] = \sum_{\mathbf{q},\mathbf{q}',\mathbf{p}'} |A|^2 [f_e(\mathbf{q}') \cdot f(\mathbf{p}') - f_e(\mathbf{q}) \cdot f(\mathbf{p})]\,. \tag{21}$$

where $|A|^2$ is determined by Fermi's golden rule and implicitly by Feynman's diagrams [20]. Unfortunately, this term becomes messy if we put it properly for the sums over phase space, but it can be found in [20]. The other term on the right side of the equation is the subtraction of the product. The product of the distribution function counts the number of particles that have a given momentum; therefore, the subtraction of the product of the distribution function counts the number of particles that have changed their momenta. The calculation of $|A|^2 = 8\pi\sigma_T m_e^2$ is a bit extensive and the outcome is in many references [15][19][20]. Thus, the collision term is given by, **[Cris:**

$$C[f] = -p\frac{\partial f^{(0)}}{dp} n_e \sigma_T [\Theta_0 - \Theta(\hat{p}) + \hat{\boldsymbol{p}} \cdot \boldsymbol{v}_b], \tag{22}$$

] where $\Theta \equiv \delta T/T$ is the perturbation in photon temperature, n_e is the number density of free electrons σ_T is the cross-section, and $f^{(0)}$ is the distribution function in the background, remember that as the metric is perturbed, so the temperature of the photons would be perturbed as well due to Fig. 1 and this perturb the Bose-Einstein distribution. Then, we have to compute the left-handed side of the Boltzmann equation and it is given by [19][20].

$$\frac{df}{dt} = -p\frac{\partial f^{(0)}}{\partial p}\left[\frac{\partial \Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Theta}{\partial x^i} + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i}\right]. \tag{23}$$

where, $\tau_{op} = \int_\tau^{\tau_0} n_e \sigma_T a d\tau$ is the optical deep. At late times, $\tau \ll 1$, while at early times, it is very large [20]. Note that the derivative in 24 is in function of conformal time. We are knocking the doors on finding the Boltzmann equation for photons, so now we only need to replace the distribution function that photons obey: Bose-Einstein distribution and then we have to match the RHS 22 with the LHS of Boltzmann equation 23 and we will get,

$$\Theta_k' + ik\mu\Theta_k + \Phi_k' - ik\mu\Phi_k = -\tau_{op}'[\Theta_0 - \Theta_k + \mu v_b] \tag{24}$$

The equation 24 is the equation of evolution of anisotropies, Θ. Then, we can expand it in multipoles, $\Theta = \sum_{l=0}^{\infty} \frac{2l+1}{i^l} \Theta_l \mathcal{P}_l$, where $\mathcal{P}_l$ are the Legendre Polynomials. Replacing this expansion in 24 we got the equation of evolution for each multipole[7],[20]

$$-\Phi = \dot{\Theta}_0 + k\Theta_1 \tag{25}$$

$$-\frac{k}{3}\Phi + \dot{\tau}\left[\Theta_1 + \frac{1}{3}v_b\right] = \dot{\Theta}_1 - \frac{k}{3}\Theta_0 + \frac{2k}{3}\Theta_2 \tag{26}$$

$$\dot{\tau}\left[\Theta_l - \frac{1}{10}\Theta_2\delta_{l,2}\right] = \dot{\Theta}_l - \frac{lk}{2l+1}\Theta_{l-1} + \frac{(l+1)k}{2l+1}\Theta_{l+1}, l > 2 \tag{27}$$

Where we have omitted the wavenumber, k, subscript. Resuming this section, to solve the Boltzmann equations for photons 24 we need to solve first the equation of evolution of Φ 19 and in order to solve it we need to solve first the Friedman equations 3,4. Thus, to compute the CMB power spectrum we have to solve a set of equations that are not trivial and we have to ask for help to CLASS.

V. CMB POWER SPECTRUM

In this section we are going to compute the theoretical curve of CMB anisotropies Power Spectrum for ΛCDM and other 'exotic models'. Observational data from Planck Collaboration [23] is in function of $D_l = \langle a_{lm}\, a^*_{l',m'} \rangle$ where a_{lm} is the Fourier coefficient of the expansion in spherical armonics. So, we have to relate the observables, a_{lm}'s to the Θ_l's. The mean value of all the a_{lm}'s is zero, but they will have some nonzero variance. The variance of the a_{lm}'s is precisely[8] D_l.[20]

$$D_l = \frac{2}{\pi} \int_0^\infty dk k^2 P(k) \left| \frac{\Theta_l}{\delta(k)} \right|^2 . \qquad (28)$$

Where we have written the photon distribution Θ as $\delta \cdot (\Theta/\delta)$, and the dark matter overdensity[9] δ does not depend on any direction [20]. Therefore, to plot the theoretical curve of CMB spectrum that is determined by 28 we need to solve a big set of equations that are coupled between them 25, 24, 19, 3,4. You can try to solve it analytically [17] but it is not straightforward or we can solve it numerically with the help of the software CLASS.

The shape of the CMB spectrum depends on the cosmological parameters (Ω_m, Ω_Λ, Ω_K, ...) and initial conditions determined by the inflation epoch [10] that is why we have to set the cosmological parameters when we compute the CMB power spectrum. The characteristic peak structure of the CMB (see FIG. 2) is due to the acoustic oscillation in the primordial plasma before recombination [10]. The temperature fluctuations in the CMB are sourced predominantly by scalar fluctuations[10].

Models	Ω_{cdm}	Ω_Λ	Ω_b	Ω_k
ΛCDM	0.270	0.679	0.049	-0.056
Model B (1)	0	0.679	0.319	-0.056
Model B (2)	0	0.949	0.049	-0,056
Model C (1)	0.270	0.914	0.049	-0.3
Model C (2)	0.270	0.323	0.049	0.3

TABLE I: These are the parameters we have used in the file explanatory.ini in CLASS[24]. We have considered $h = 0.6688\, km\, s^{-1} Mpc^{-1}$ [9].

A. Example 1: The ΛCDM model

The standard cosmological model ΛCDM is named for the cosmological constant Λ and the cold dark matter content [15][10].. The most abundant content matter in the universe, according to this model, with 67% is dark energy that is modeled by a cosmological constant Λ as a perfect fluid with a constant equation of state $w = -1$. Then, we have cold dark matter with 27% that is modeled by a perfect fluid with an equation of state $w = 0$ since we have considered that is not a relativistic particle, that's why the name cold dark matter.

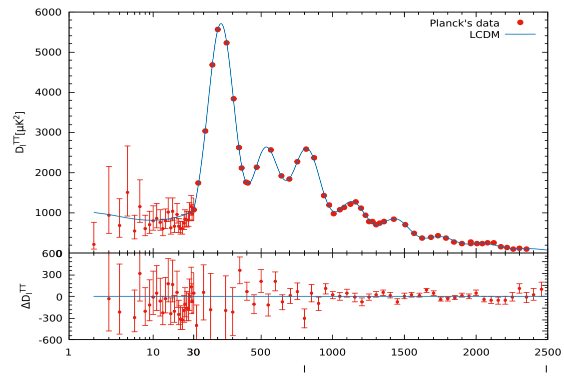

FIG. 2: The red points are the data obtanied by the Planck Collaboration [23] and the blue one is the theoretical curve of CMB Power spectrum for ΛCDM obtanied by CLASS.

we have considered the best fits for ΛCDM from [9] considering only components TT and low E. The theoretical model is the ΛCDM with purely adiabatic scalar primordial perturbations with a power-law spectrum. We have assumed three neutrinos species, approximated as two massless states and a single massive neutrino[11] of mass $m_\nu = 0.06$ eV [9].

B. Example 2: Non Cold Dark matter

From ΛCDM we have found that dark matter accounts for 26% of the energy budget of the Universe. However, we do not know something else about it. For instance, we can wonder why we need dark matter in our cosmological models and why baryons are not enough to fulfill the matter content. [12] In order to highlight the importance of dark matter we compute. the theoretical curve of the CMB of a model without dark matter. The blue curve is for a universe without dark matter and what's missing from dark matter is the increase in baryons, we see that

[8] In many reference this is denoted by C_l
[9] The equation of overdensity δ is determined by 17.
[10] A major goal of current efforts in observational cosmology is to detect the tensor component of the primordial fluctuations[10].

[11] This is the normal hierarchy mass.
[12] This last question can be answered with the galaxy rotation curve, but we can explain with CMB as well.

the second peak in the blue one is missing, this second peak is related to dark matter, moreover, the first peak is very high in comparison with the data. Analogously, the yellow curve is for a universe without dark matter as well and what's missing from dark matter is the increase in dark energy. In this case, the second peak is not missing, but it's higher than the peaks from the data and the first peak so is. Clearly, we can see in Fig. 3, that these two models do not fit the data. Thus, we can explain the necessity of dark matter and why it accounts for 26% of the energy budget of the Universe.

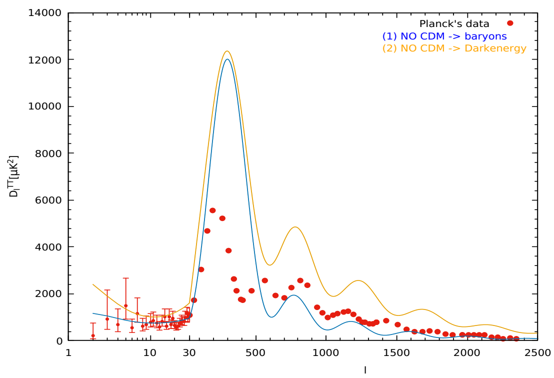

FIG. 3: CMB Temperature Power Spectrum for a universe with no cold dark matter. The red points are the data obtained by the Planck Collaboration [23], the blue one is the theoretical curve of CMB Power spectrum for a universe with no dark matter and an increasing of baryons, and the yellow one is the theoretical curve of CMB Power spectrum for a universe with no dark matter and an increasing of dark energy.

C. Example 3: Cases with different curvature

ΛCDM model assumes a priori that the spatial curvature is flat since is predicted by some inflationary models. This is a prediction that can be tested by CMB. If we do not consider a priori a flat universe and we leave free the density parameter of curvature Ω_K we will find that $\Omega_K = -0.056^{+0.028}_{-0.018}$ considering only components TT and low E [9]. It means that our Universe is closed but flattened. However, if we consider a combination of CMB and Baryonic Acoustic Oscillation (BAO) data, we will find that $\Omega_K = 0.0007 \pm 0.00065$ considering data from components TT, TE, EE + low E+ lensing + BAO. It means that our Universe is open but flattened.

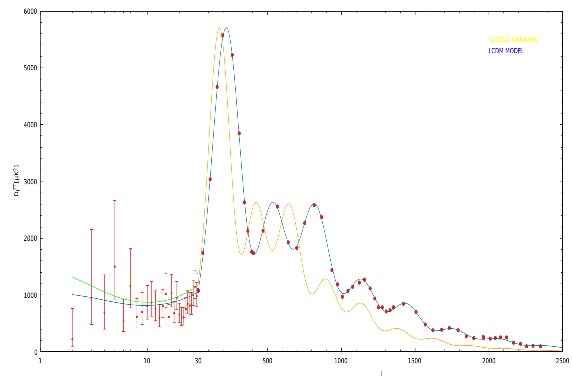

FIG. 4: The red points are the data obtained by the Planck Collaboration [23], the blue one is the theoretical curve of CMB Power spectrum for ΛCDM, and the green one is the theoretical curve of CMB Power spectrum for a closed universe, both obtained by CLASS.

Therefore it cannot be stated the geometry of the universe, but, it can be stated that our Universe is flattened because its geometry is very close to flat independently if it's open or closed.

An interesting exercise to see how the theoretical curve depends on the geometry of the universe is to assume a geometry and graph the theoretical curve of CMB.

We have assumed the values $\Omega_K = -0.3$ for a closed universe and $\Omega_K = 0.3$ for an open universe. You can see Table I. We have taken these values to visualize clearly how geometry affects CMB. We can see that CMB has the number of peaks that ΛCDM and has the same form as well. Geometry only moves to the right or left if it is open or closed respectively.

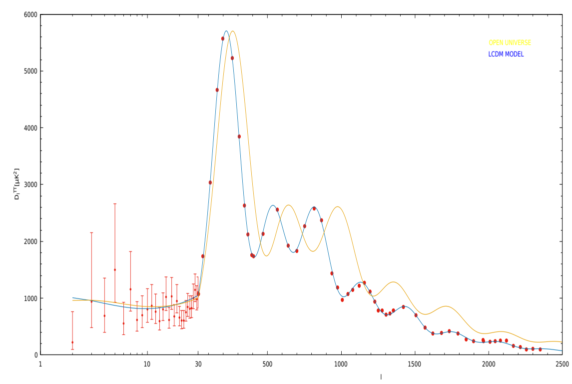

FIG. 5: The red points are the data obtanied by the Planck Collaboration [23], the blue one is the theoretical curve of CMB Power spectrum for ΛCDM, and the green one is the theoretical curve of CMB Power spectrum for an open universe, both obtanied by CLASS.

D. Neutrino mass Hierarchy

Observations from both the sun and our atmosphere strongly suggest that neutrinos have mass [29]. From these observations we know Δm_{12}^2 and Δm_{23}^2 , but we do not know the mass of each one. At least, two of them have masses and one can be massless. We have two scenarios in which is according with the observation. We have normal and inverted hierarchy as we can see in Fig. 6. We do not know which neutrino is the lightest, resulting in two orders.

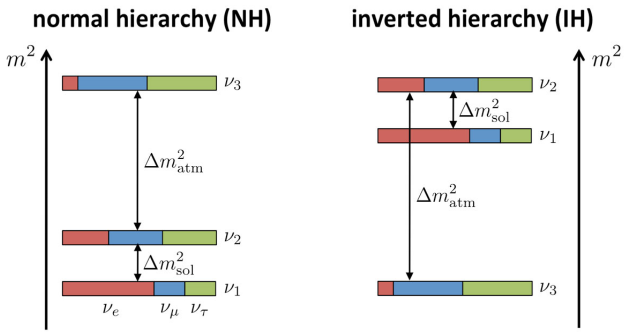

FIG. 6: Normal and Inverted hierarchy. The colors red, blue, and green represent the amount of electron, muon, and tau neutrinos present in the neutrino at the base of mass.

From [9] we have an upper bound for neutrino mass $\sum m_\nu < 0.54eV$ considering components TT and LowE only. Moreover, there are lower bounds for neutrinos masses with these two orders. If neutrinos have normal hierarchy, it means the lowest two mass eigenstates have the smallest mass splitting, can give any $\sum m_\nu \geq 0.06eV$. On the other hand, an inverted hierarchy, in which the two most massive eigenstates have the smallest splitting, requires $\sum m_\nu \geq 0.1eV$ [9]. In this subsection, we have tried to visualize how the CMB changes when we consider normal or inverted hierarchy. As you can see in Fig. 7 we have graphed both cases and they are pretty similar. Analogously to the previous exotic models, when we change one cosmological parameter we must change other one to make up and satisfy $\Omega_M + \Omega_K = 1$. Let's compute the density parameter of massive neutrinos,

$$\Omega_\nu = \frac{\sum m_v}{94h^2eV} \tag{29}$$

For inverted hierarchy we have considered two massive neutrinos with $m_\nu = 0.06eV$ each one, so the variation of the density parameter of massive neutrino is $\propto 10^{-3}$. Therefore, Ω_Λ is increasing in $\propto 10^{-3}$ to make up this change. That's why difference between CMB with normal and inverted hierarchy are not visible in Fig. 7.

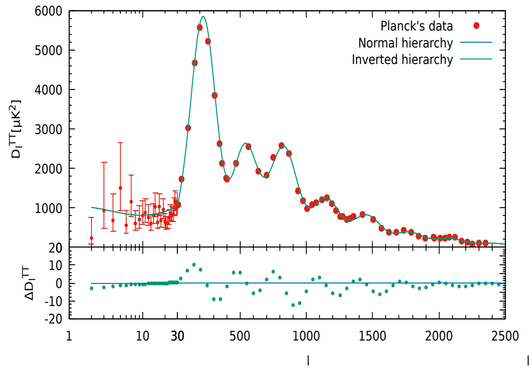

FIG. 7: Hierarchy mass neutrinos. ΛCDM has assumed normal hierarchy, to be specific one massive neutrino $m_\nu = 0.06eV$ and the two massless. Actually, two "massless" neutrinos are not massless, but their masses are very small in comparison with 0.06eV.

Better observations could give better bounds in neutrino masses. For instance, a lower bound $\sum m_\nu < 0.1eV$ could rule out the inverted hierarchy[9]

VI. CONCLUSIONS

We have reviewed the theoretical model behind in order to plot the CMB TT power spectrum. We have verified that ΛCDM fit the data from [23]. We have seen the importance of dark sector and how exotic models are out of data. Finally, there are lower and upper bounds for neutrino masses, but they are not enough to determine their hierarchy.

ACKNOWLEDGEMENTS

C.C would like to thank J. Beltran and A. Bernui for the interesting discussions. Also, to thank Universidad Nacional de Ingenieria for the support to study for a semester at UNAM. CE-R is supported by the Royal Astronomical Society as FRAS 10147 and by DGAPA-PAPIIT-UNAM Grant IA100220. This article is based upon work from CANTATA COST (European Cooperation in Science and Technology) action CA15117, EU Framework Programme Horizon 2020.

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[1] A. A. Penzias and R. W. Wilson, A measurement of excess antenna temperature at 4080 Mc/s, Astrophys. J. 142 (1965) 421.
[2] R. A. Alpher, H. Bethe and G. Gamov, The origin of chemical elements, Phys. Rev. 73 (1948) 803.
[3] Martin Bucher. Physics of the cosmic microwave background anisotropy
[4] G. F. Smoot et al. *Astrophys. J. Lett. 396, L1 (1992)*
[5] C. L. Bennett et al. *Astrophys. J. Suppl. 148, 1 (2003)*
[6] D. N. Spergel et al. *Astrophys. J. Suppl. 148, 175 (2003)*
[7] P. Callin. How to calculate the CMB spectrum.
[8] Antonio Riotto. *Primordial non-Gaussianity in the cosmological perturbations*
[9] Planck Collaboration. *Planck 2018 results. VI. Cosmological parameters*.
[10] D. Baumann. *Cosmology.*
[11] U. Seljak and M. Zaldarriaga. *See its webpage here.*
[12] M. Doran. *CMBEASY:: an Object Oriented Code for the Cosmic Microwave Background*
[13] M. Doran. *See its web page here.*
[14] A. Lewis, A. Challinor and A. Lasenby. *See its web page here.*
[15] L. Amendola, S. Tsujikawa. *Dark Energy* . Cambridge University Press. (2010)
[16] E. Copeland, M. Sami, S. Tsujikawa. *Dynamics of Dark Energy*.
[17] V. Mukhanov. *CMB-slow, or How to Estimate Cosmological Parameters by Hand*.
[18] T. Rindler-Daller. *Understanding CMB physics through the exploration of exotic cosmological models: a classroom study using CLASS*.
[19] Oliver F. Piattella. *Lectures Notes in Cosmology*
[20] Scott Dodelson. *Modern Cosmology.* Academic Express. (2003)
[21] V. Mukhanov. *Physical Foundations of Cosmology.* Cambridge University Press. 2005
[22] Horatiu Nastase. *Classical Field Theory.* Cambridge University Press. (2019)
[23] Planck Satellite. *See the data took by the Planck Mission..*
[24] Julien Lesgourgues. *The Cosmic Linear Anisotropy Solving System (CLASS) I: Overview*
[25] Diego Blas, Julien Lesgourgues, Thomas Tramd. *The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes.*
[26] Julien Lesgourgues. *The Cosmic Linear Anisotropy Solving System (CLASS) III: Comparision with CAMB for LambdaCDM.*
[27] Julien Lesgourgues, Thomas Tram. *The Cosmic Linear Anisotropy Solving System (CLASS) IV: Efficient implementation of non-cold relics.*
[28] Julien Lesgourgues. *See its web page here.*
[29] The Super-Kamiokande Collaboration, Y. Fukuda et al. *Evidence for oscillation of atmospheric neutrinos.*