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Chaos in a Double Pendulum

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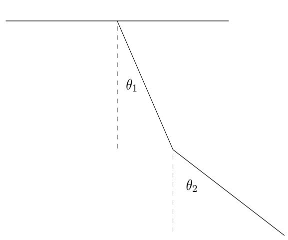
ABSTRACT

The double pendulum is a very simple and useful example of a chaotic system. Small differences in initial conditions yield exponentially diverging outcomes. In this project, I made a simulation of the trajectory of the double pendulum to see the behavior. Also, the Poincare maps to near identical initial conditions are found to see the transition of the chaos. Finally, the flipper time fractal for the secondary pendulum was approximately found.

INTRODUCTION

There is not a universal definition about chaos but there are three criteria that scientists agree. It's not an oscillatory movement. Its behavior is irregular, it arises from non-linearity but is deterministic and the most important is sensitive to the initial conditions. The double pendulum is one of the simplest systems to exhibit chaotic motion. It consists of two rods coupled. The system is governed by a set of coupled non-linear ordinary differential equations. Poincare Maps and Flipper time fractal are solved by The Gauss-Legendre numerical method(accurate up to 8^{th} order) and they are used to explain the chaos.

THEORETICAL FRAMEWORK



$$x_1 = \frac{l}{2}sin\theta_1$$
$$y_1 = -\frac{l}{2}cos\theta_1$$
$$x_2 = l(sin\theta_1 + \frac{1}{2}sin\theta_2)$$
$$y_2 = -l(cos\theta_1 + \frac{1}{2}cos\theta_2)$$

The Lagrangian is given by

$$L = \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2 + \dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}I(\dot{\theta_1}^2 + \dot{\theta_2}^2) - mg(y_1 + y_2)$$

$$\frac{\partial L}{\partial t} = 0 \Rightarrow H = E \text{ is conserved}$$

$$H = \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2 + \dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}I(\dot{\theta_1}^2 + \dot{\theta_2}^2) + mg(y_1 + y_2)$$

Using the Euler-Lagrange equation we have the following equations of motion

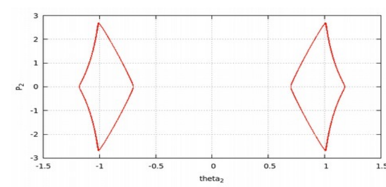
$$8\ddot{\theta}_1 + 3\ddot{\theta}_2 cos(\theta_1 - \theta_2) + 3\dot{\theta_2}^2 sin(\theta_1 - \theta_2) + \frac{9g}{l}sin(\theta_1) = 0$$

$$2\ddot{\theta}_2 + 3\ddot{\theta}_1 cos(\theta_1 - \theta_2) - 3\dot{\theta_1}^2 sin(\theta_1 - \theta_2) + \frac{3g}{l}sin(\theta_2) = 0$$

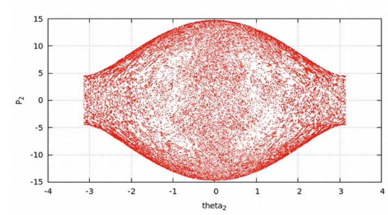
These equations of motion cannot be integrated analytically but they can be solved numerically. Here, they are integrated by 8^{th} order Gauss-Legendre method.

RESULTS AND DISCUSSIONS

POINCARE MAPS: A plane in the four-dimensional phase space is fixed (theta1 = 0 rad, P1 = 0 rad/s).

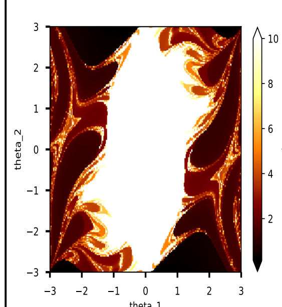


$\theta_1 = 1.1\,rad$
$\theta_2 = 1.1\,rad$
$\dot{\theta}_1 = 0.56\,rad/s$
$\dot{\theta}_2 = 0.56\,rad/s$



$\theta_1 = 1.1\,rad$
$\theta_2 = 1.1\,rad$
$\dot{\theta}_1 = 0.57\,rad/s$
$\dot{\theta}_2 = 0.56\,rad/s$

FLIP OVER TIME FRACTAL: Chaotic motion is intimately connected with fractals.



Energetically is impossible to flip in

$$3cos\theta_1 + cos\theta_2 > 2$$

This region is called Forbidden Zone. Outside this region, the second pendulum can flip but in this case, energy cannot determine whether the second pendulum will flip within a set period of time.

CONCLUSIONS

- The transition of the chaos was shown with the Poincare maps and the flip over time fractal was found approximately.

BIBLIOGRAPHY

- Jeremy S. Heyl. The Double Pendulum Fractal. 2008. University of British Columbia.

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