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Synchronization of thermostatically controlled first-order systems

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Abstract

We analyze the collective dynamics of a number of individually heated rooms arranged in the form of a ring. The temperature of each room is kept within a given band by a thermostat which makes it behave as a self-sustained oscillator. Each room interacts thermally only with its immediate neighbors and with the exterior. The temperature in each room is modeled by a first-order differential equation. Numerical results of the complete system show the presence of a rich array of synchronization dynamics in frequency and phase as well as clustering and coupling-induced amplitude death.

1. Introduction

Synchronization of a large number of weakly coupled, self-sustained oscillators has recently been shown to exist in many natural as well as engineering fields. Examples of these, which include biological, chemical, optical, mechanical, and electrical systems, are discussed in several books [1–3]. Thermal systems also exhibit such behavior, as has recently been experimentally demonstrated by the present authors in a thermal-hydraulic network with secondary loops [4] in which the behavior of the secondaries had previously been shown to be coupled [5,6]. The phenomenon is of importance in practice from the perspective of design and energy usage, particularly for temperature control systems. Even though it is common practice in thermal engineering to design a complex system as if the sub-systems that constitute it were functioning independently, coupling could lead to complex or undesired dynamic behavior. From a theoretical point of view, an understanding of the dynamics of thermal systems that could lead to synchronization is therefore essential to their design. Our goal here is to study the simplest thermal system with coupling that could exhibit such behavior. Simple mathematical models have been proposed and explored for some systems in synchrony [7–10]. Local coupling, which considers coupling only between neighboring sub-systems, such as that assumed here, has been used for the analysis of lasers [11], Josephson junction arrays [12], electronic circuits [13,14], biological [15,16] and chemical oscillators [17,18].

To analyze the possibility of synchronization we consider a collection of sub-systems that, if left to themselves, exhibit self-sustained oscillations. There should be some interaction between these sub-systems; it is usually found that, if the coupling is too weak, the sub-systems behave independently and, if too strong, the entire system behaves as one [8]. In the present example, self-sustained oscillations in temperature are a natural consequence of using thermostats, while the interaction is provided by heat transfer between sub-systems. The simplest mathematical model is obtained on using a lumped capacitance approximation which gives a first-order differential equation for each sub-system. There is a forcing function that represents heating sources which go on or off as temperatures go below or above predetermined levels, respectively. A large number of such systems arranged in the form of a ring, each of which is locally coupled with its two nearest neighbors through thermal transport, can be viewed as a system of coupled oscillators. From a practical perspective, such a system can represent, for example, an arrangement of heated rooms in a building that are individually controlled by thermostats.

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Nomenclature

a_l	lth element of first row of circulant matrix **A**
A	circulant matrix, defined after Eq. (5)
C	equivalent heat capacity of walls and room air
K	non-dimensional thermal coupling between rooms
n	total number of rooms
N	number of rooms that are on
$\overline{N}$	average number of rooms that are on
q	magnitude of heat source
q_i	heat source for room i
Q_i	non-dimensional heat source for room i
Q	heat source vector
$\mathbf{Q}(\tau_j^+)$	heat source vector for interval $\tau_j < \tau \leqslant \tau_{j+1}$
r	Kuramoto order parameter
R	thermal resistance between adjacent rooms
R_∞	thermal resistance between room and outside
t	time
T	temperature
T_∞	outside temperature

Greek symbols

θ	non-dimensional temperature
$\mathbf{\Theta}$	vector of room temperatures
λ_i	eigenvalues
ξ_j	jth eigenvector
ρ_j	$=\exp(-i2\pi j/n)$
τ	non-dimensional time
τ_{on}	time interval for on
τ_{off}	time interval for off
τ_{p}	period
ϕ_j	phase angles

Subscripts

i	room i
L	lower temperature setting of thermostat
max	maximum possible temperature
min	minimum possible temperature
U	upper temperature setting of thermostat

2. Mathematical modeling

Consider an arrangement of n identical rooms as shown in Fig. 1. The temperature $T_i(t)$ of room i depends on time t. It loses heat to the environment at a constant temperature T_∞, exchanges heat through separator walls with its two neighbors $i-1$ and $i+1$, and gains heat from a source $q_i(t)$ which compensates for the heat loss. Each room also has an individual thermostat, the objective of which is to control q_i so as to maintain T_i within a given band $[T_{\mathrm{L}},T_{\mathrm{U}}]$. If T_i rises above the upper limit T_{U} the heat source for room i is turned off and $q_i(t)=0$, and if it falls below the lower limit T_{L} it is turned on so that $q_i(t)=q$. The temperature of each of the rooms, if isolated, would thus oscillate in time.

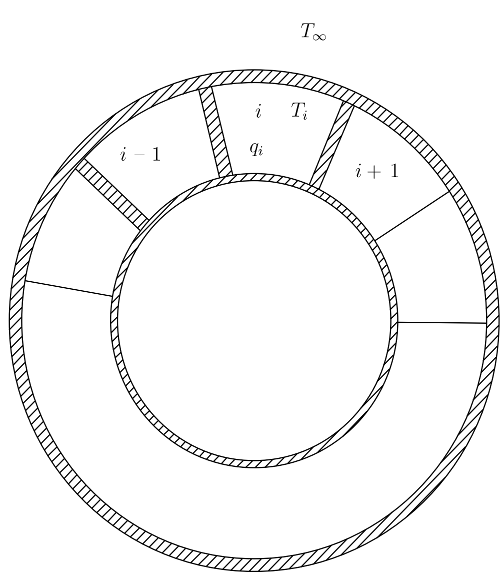


Fig. 1. Multiple rooms in a ring.

Assuming lumped capacitances, heat balance gives

$$C\frac{\mathrm{d}T_i}{\mathrm{d}t}+\frac{1}{R}(T_i-T_{i+1})+\frac{1}{R}(T_i-T_{i-1})+\frac{1}{R_\infty}(T_i-T_\infty)=q_i, \tag{1}$$

for $i=1,\ldots,n$, where C is the equivalent heat capacity of the walls of a room and the air within it, R is the thermal resistance between adjacent rooms, and R_∞ is that between a room and the outside. For simplicity all the rooms are taken to be identical, even though obviously there could be large differences in practice. The subscript i is to be read cyclically in the range $1\leqslant i\leqslant n$, so that room numbers $n+1$ and -1 are $i=1$ and $i=n$, respectively. It should be pointed out that the problem is nonlinear since the value of q_i depends on T_i.

To non-dimensionalize the equation, we will use qR_∞ which is the maximum temperature increment above T_∞ that would be reached if the heat source were to be constantly left on. With $\tau=t/CR_\infty$, $\theta_i=(T_i-T_\infty)/qR_\infty$, and $Q_i=q_i/q$, we get

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}\tau}+K(2\theta_i-\theta_{i+1}-\theta_{i-1})+\theta_i=Q_i, \tag{2}$$

where the thermal coupling parameter is $K=R_\infty/R$. K represents the heat transfer between neighboring rooms as compared to that with the exterior. The non-dimensional heat source is

$$Q_i=\begin{cases}0 & \text{if off,}\\ 1 & \text{if on.}\end{cases} \tag{3}$$

The highest and lowest non-dimensional temperatures possible if the heaters are all continuously left on or off are $\theta_{\max} = 1$ and $\theta_{\min} = 0$, respectively. The lower and upper temperature settings of the thermostats are non-dimensionalized as

$$\theta_{\mathrm{L}} = \frac{T_{\mathrm{L}} - T_{\infty}}{qR_{\infty}}, \tag{4a}$$

$$\theta_{\mathrm{U}} = \frac{T_{\mathrm{U}} - T_{\infty}}{qR_{\infty}}. \tag{4b}$$

The parameters of the problem are the total number of rooms n, the thermal coupling parameter K, the thermostat settings θ_{L} and θ_{U}, and the initial conditions on temperature.

We can rewrite Eq. (2) in matrix form as

$$\frac{\mathrm{d}\boldsymbol{\Theta}}{\mathrm{d}\tau} = \mathbf{A}\boldsymbol{\Theta} + \mathbf{Q}, \tag{5}$$

where $\mathbf{A}$ is an $n \times n$ circulant matrix [19] with the first row being $\{-(2K+1), K, 0, \ldots 0, K\}$, and $\mathbf{Q} = \{Q_1(\tau), \ldots, Q_n(\tau)\}^{\mathrm{T}}$. The eigenvalues of $\mathbf{A}$ are

$$\lambda_j = \sum_{l=0}^{n-1} a_l \mathrm{e}^{-\mathrm{i}2\pi l j/n}, \tag{6}$$

where $j = 0, 1, \ldots, (n-1)$ and a_l is the lth element of the first row of $\mathbf{A}$. The corresponding eigenvectors are

$$\xi_j = \frac{1}{\sqrt{n}} \{1, \rho_j, \rho_j^2, \ldots, \rho_j^{n-1}\}^{\mathrm{T}}, \tag{7}$$

where $\rho_j = \exp(-\mathrm{i}2\pi j/n)$.

The nonlinear set of Eqs. (2) and (3) are differential equations in which the right-hand sides are discontinuous in the dependent variables θ_i [20]; this is often known as a switched system without impulse effects, in this case with hysteresis in the switching [21]. The stability of switched systems, even if the vector field for each switched mode is linear as is the case here, is difficult to reduce to an eigenvalue problem and is often analyzed numerically [22,23].

3. Oscillations without coupling

If there is no coupling between rooms, i.e., $K = 0$, each room temperature can be solved separately. Eq. (2) becomes

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}\tau} + \theta_i = \begin{cases} 0 & \text{if off}, \\ 1 & \text{if on}. \end{cases} \tag{8}$$

The solution to which is

$$\theta_i = \begin{cases} c_1 \mathrm{e}^{-\tau} & \text{if off}, \\ 1 + c_2 \mathrm{e}^{-\tau} & \text{if on}, \end{cases} \tag{9}$$

where c_1, c_2 are constants that can be determined from the initial condition. It can be shown that the on and off time intervals are

$$\tau_{\mathrm{on}} = \ln \frac{1 - \theta_{\mathrm{L}}}{1 - \theta_{\mathrm{U}}}, \tag{10a}$$

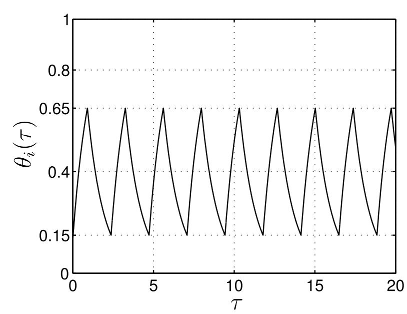
$$\tau_{\mathrm{off}} = \ln \frac{\theta_{\mathrm{U}}}{\theta_{\mathrm{L}}}, \tag{10b}$$

respectively. The total period of the oscillation is

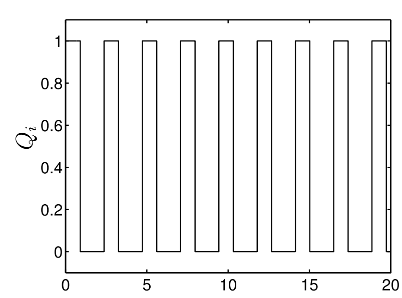
$$\tau_{\mathrm{p}} = \ln \frac{\theta_{\mathrm{U}}(1 - \theta_{\mathrm{L}})}{\theta_{\mathrm{L}}(1 - \theta_{\mathrm{U}})} \tag{11}$$

and the frequency is $1/\tau_{\mathrm{p}}$. The oscillation of temperature is self-sustained and, on the average, the energy generated by the heater is lost to the environment.

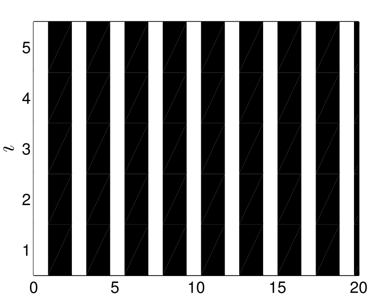
The oscillations of the temperatures θ_i and heating source Q_i are shown in Fig. 2 using $n = 5$ as an example; this value of n will also be used in the rest of the paper. The period and amplitude of the temperature oscillation in Fig. 2a depends on the temperature limits; here we have chosen $\theta_{\mathrm{L}} = 0.15$ and $\theta_{\mathrm{U}} = 0.65$. The initial conditions are assumed identical for all rooms so that the $\theta_i(\tau)$ traces are all the same. The heating function $Q_i(\tau)$ is shown in Fig. 2b, and is again the same for all rooms. This representation is



(a) Variation of temperature.



(b) Variation of heat source (as a function).



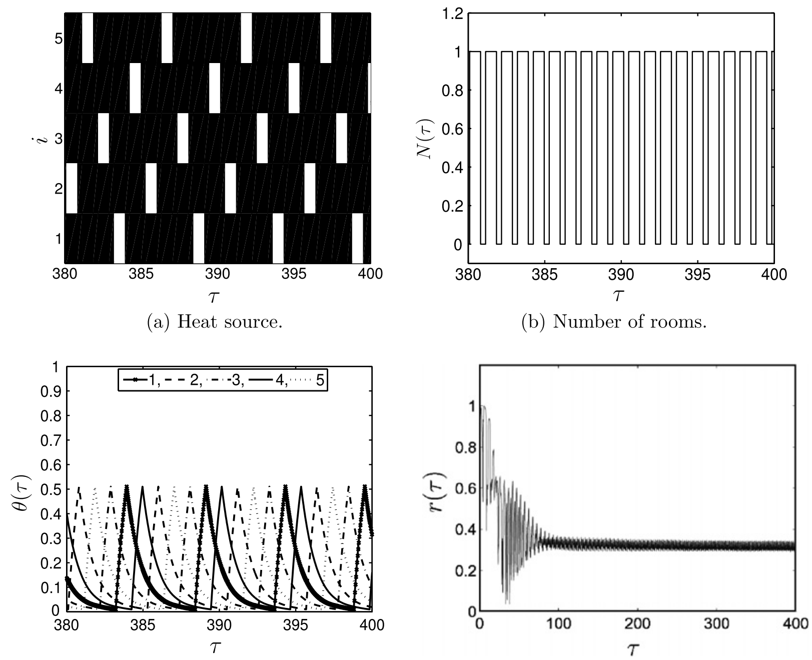
(c) Variation of heat source (symbolic).

Fig. 2. Temperature and heat source oscillations; $K = 0$, $\theta_{\mathrm{U}} = 0.65$, $\theta_{\mathrm{L}} = 0.15$.

not very convenient to show the distribution of $Q_i(\tau)$ for $i = 1,\ldots,n$ if they were all different, which will be the case in general. An alternative is in Fig. 2c: the room number is indicated in the ordinate and time in the abscissa; white indicates that the heating is on in that room at that instant in time, and black that it is off.

4. Oscillations with coupling

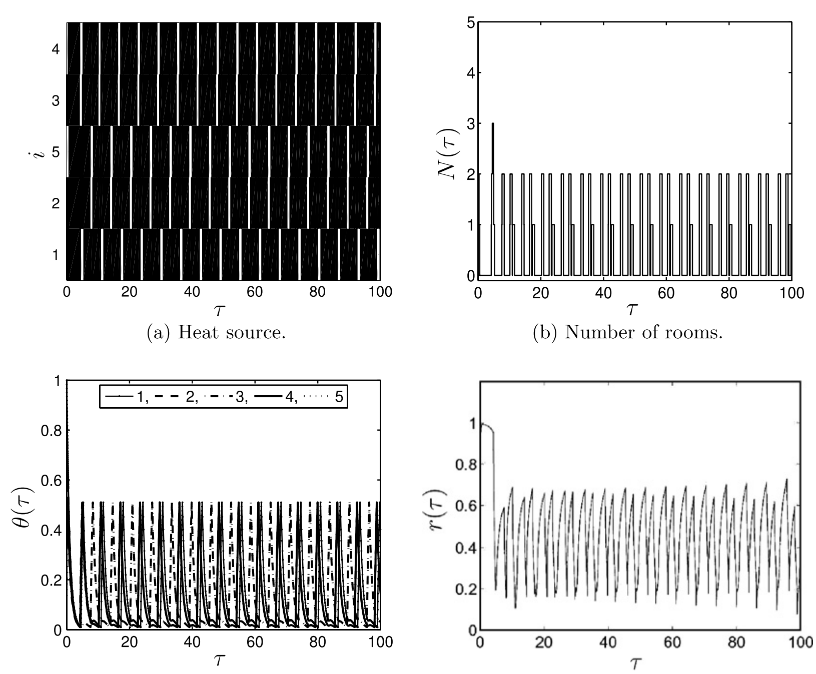
Now we take $K \neq 0$ in Eq. (2). Even though the problem is nonlinear, we can take advantage of the fact that $\mathbf{Q}$ in Eq. (5) is piecewise constant to devise a numerical method. Let $\tau_1, \tau_2, \tau_3, \ldots$ be the successive times when any one of the



(a) Heat source. (b) Number of rooms.

(c) Temperature (with room numbers). (d) Order parameter.

Fig. 3. System behavior for $\theta_L = 0.01$ and $K = 0.015$.



(a) Heat source. (b) Number of rooms.

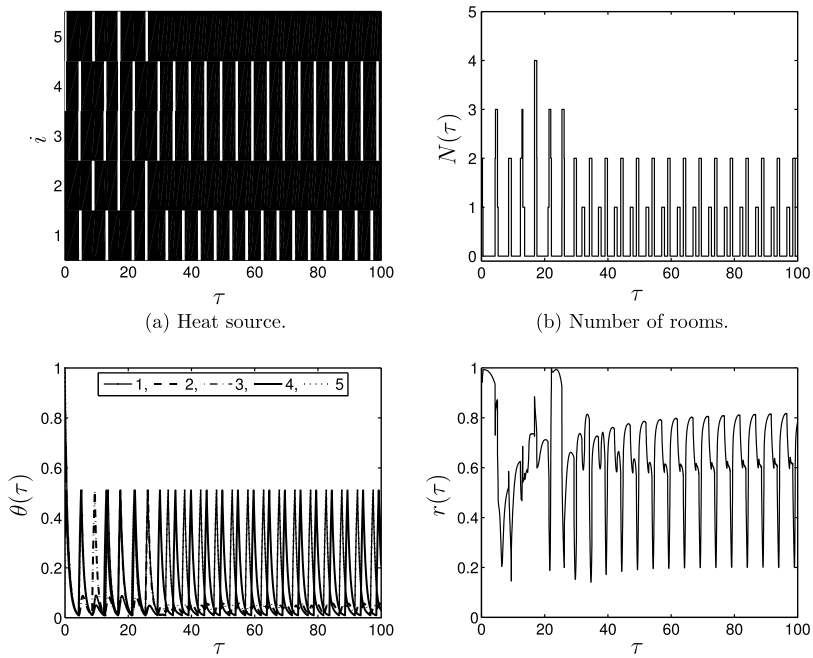
(c) Temperature (with room numbers). (d) Order parameter.

Fig. 4. System behavior for $\theta_L = 0.01$ and $K = 0.05$.

rooms (generally speaking, different rooms) hits either the lower or upper prescribed temperature bounds. $\mathbf{Q}$ changes at these instants, and we will use the notation $\mathbf{Q}(\tau_j^+)$ to indicate $\mathbf{Q}$ immediately *after* the discontinuity at $\tau = \tau_j$. Thus $\mathbf{Q}$ is constant for $\tau_{j-1} < \tau < \tau_j$, and Eq. (5) can be easily solved in that time interval using Eqs. (6) and (7).

Knowing $\mathbf{Q}(\tau_{j-1}^+)$ and $\mathbf{\Theta}(\tau_{j-1})$, we would like to integrate the equation to find the next τ_j, $\mathbf{Q}(\tau_j^+)$ and $\mathbf{\Theta}(\tau_j)$ numerically. Using $\mathbf{\Theta}(\tau_{j-1})$ as initial condition, we find that

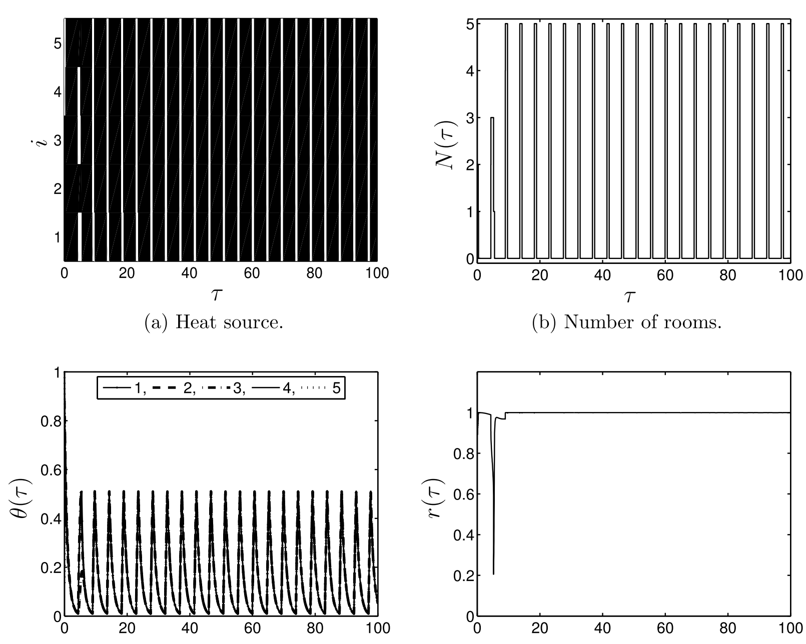
$$\mathbf{\Theta}(\tau) = \exp\{\mathbf{A}(\tau - \tau_{j-1})\}[\mathbf{\Theta}(\tau_{j-1}) + \mathbf{A}^{-1}\mathbf{Q}(\tau_{j-1}^+)] - \mathbf{A}^{-1}\mathbf{Q}(\tau_{j-1}^+). \tag{12}$$



(a) Heat source. (b) Number of rooms.

(c) Temperature (with room numbers). (d) Order parameter.

Fig. 5. System behavior for $\theta_L = 0.01$ and $K = 0.2$.



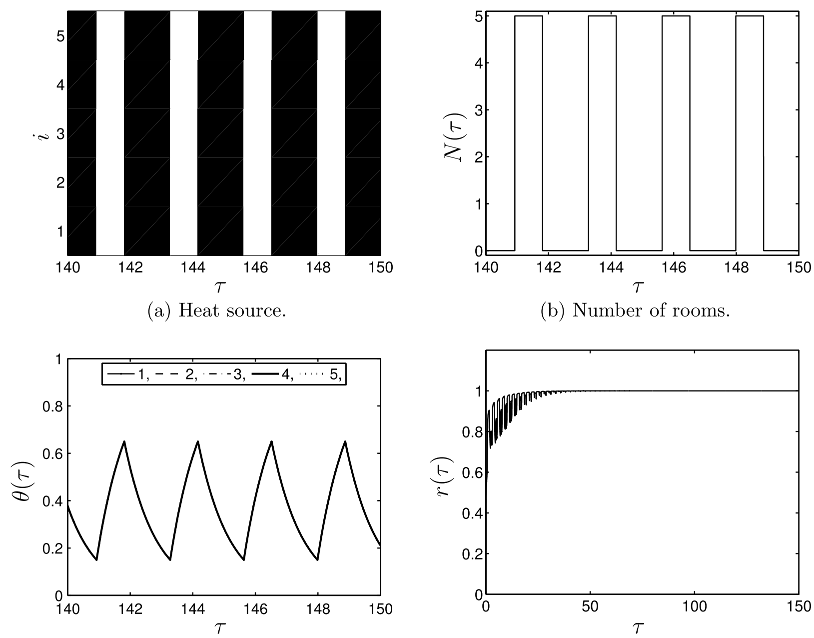
(a) Heat source. (b) Number of rooms.

(c) Temperature (with room numbers). (d) Order parameter.

Fig. 6. System behavior for $\theta_L = 0.01$ and $K = 0.5$.

We can evaluate $\mathbf{\Theta}(\tau)$ forward in time to find τ_j until one of the room temperatures hits either a lower or an upper bound. This is done iteratively using the bisection method. We can then calculate the temperatures and the new heating distribution, $\mathbf{\Theta}(\tau_j)$ and $\mathbf{Q}(\tau_j^+)$, respectively. From the period τ_p it is possible to get an idea of how long it would take for a certain number of periods to occur, and hence for the response of the system to settle down.

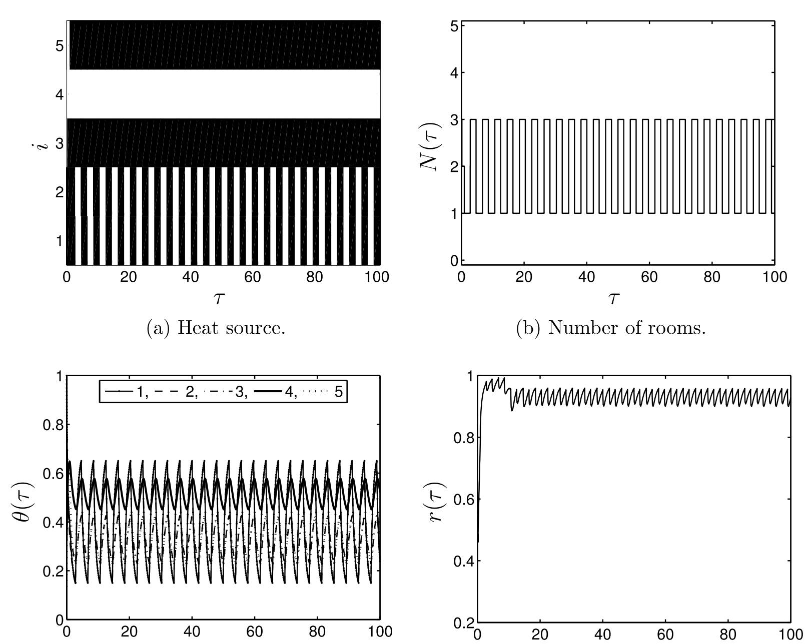
As is commonly done in phase synchronization studies, the Hilbert transform described in [4,24,25] is used to determine the instantaneous phases, $\phi_j(\tau)$, $j = 1,\ldots,n$ for the temperatures. In order to analyze the global behavior of



(a) Heat source. (b) Number of rooms.

(c) Temperature (with room numbers). (d) Order parameter.

Fig. 7. System behavior for $\theta_L = 0.15$ and $K = 0.05$.



(a) Heat source. (b) Number of rooms.

(c) Temperature (with room numbers). (d) Order parameter.

Fig. 8. System behavior for $\theta_L = 0.15$ and $K = 1.3$.

the coupled system, a mean-field quantity $r(\tau)$, also known as Kuramoto order parameter [8,12], is defined as

$$r = \frac{1}{n}\left|\sum_{j=1}^{n} e^{i\phi_j}\right|. \tag{13}$$

This parameter measures the degree of phase synchronization of the oscillators. Full phase synchronization is achieved as $r \to 1$.

There is another parameter that can be calculated to indicate the overall performance of the thermal system; in a building one may be interested in the instantaneous energy input which is proportional to the number of rooms that are being heated at that instant in time. This is

$$N(\tau) = \sum_{i=1}^{n} Q_i(\tau). \tag{14}$$

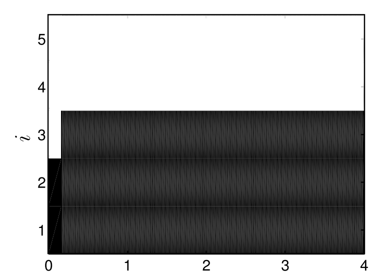
An additional concern could be the cost of running the system that is related to the number of rooms that are on the average on. Thus we can also define $\overline{N}$ as an average of $N(\tau)$ over time. From energy conservation, $\overline{N}$ is in effect related to the average temperature of the rooms.

5. Numerical results

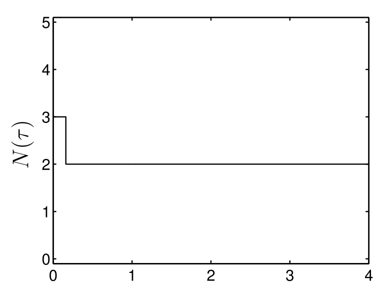
For rooms with the same temperature limits and coupling parameters, if the initial temperatures are identical, there is no difference in temperature between them and, consequently, all temperatures will vary in the same way as described in Section 3. Here we will look at the more interesting case when the initial temperatures are different. We will fix them at $\theta_1(0) = 1$, $\theta_2(0) = \exp(-1/8)$, $\theta_3(0) = \exp(-2^2/8)$, $\theta_4(0) = \exp(-3^2/8)$, $\theta_5(0) = \exp(-4^2/8)$. In addition, the temperature limits will be chosen such that as $\theta_U - \theta_L = 0.5$.

5.1. $\theta_L = 0.01$

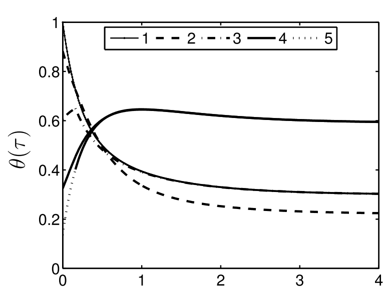
First, we assume θ_L close to θ_{min} for which three different values of K will be studied. The period is $\tau_p = 4.63$. The behavior for $K = 0.015$ is shown in Fig. 3. Frequency synchronization is reached after a while as shown in Fig. 3a, where only the time range from $\tau = 380$ to $\tau = 400$ is shown. The heating sources take turns to be on and off with the same period. The order is 1 on, all off, 4 on, all off, 2 on,



(a) Heat source.

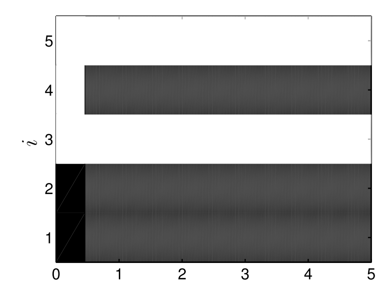


(b) Number of rooms.

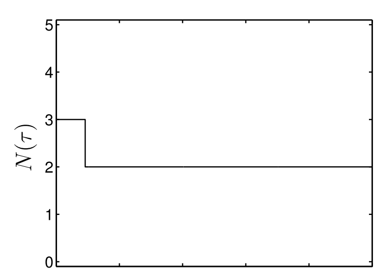


(c) Temperature (with room numbers).

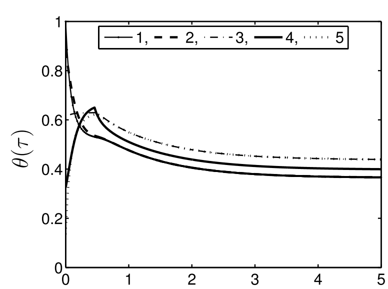
Fig. 9. System behavior for $\theta_L = 0.15$ and $K = 1.4$.



(a) Heat source.



(b) Number of rooms.

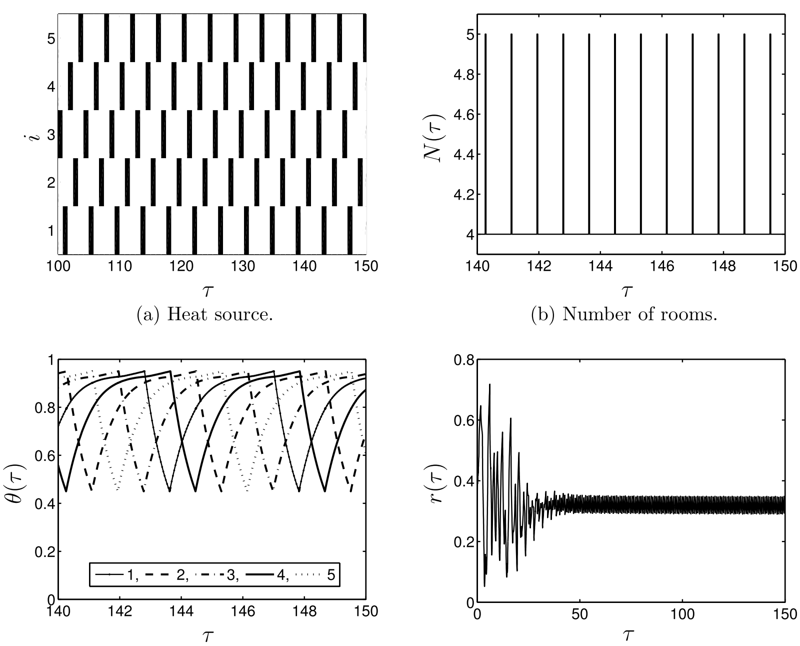


(c) Temperature (with room numbers).

Fig. 10. System behavior for $\theta_L = 0.15$ and $K = 5$.

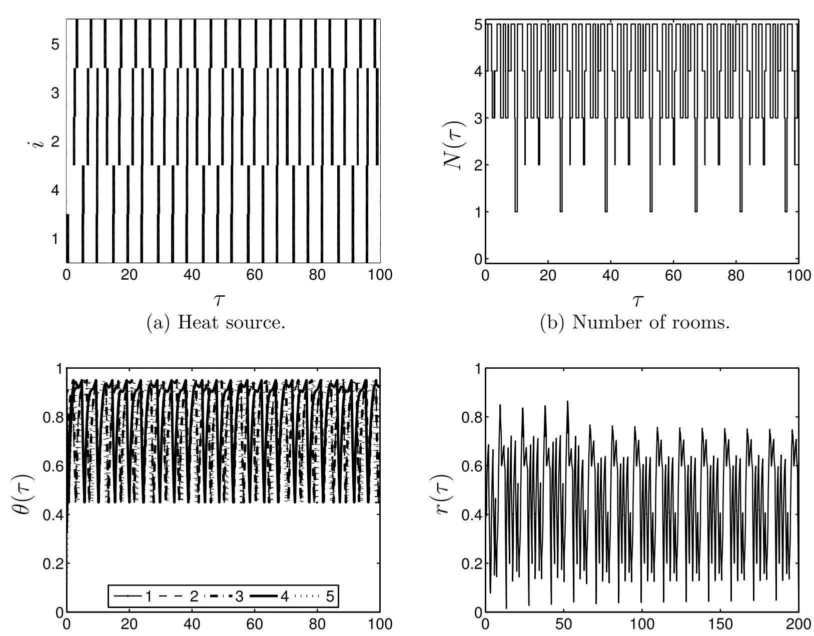
all off, 5 on, all off, 3 on, all off, 1 on, and so on in a pattern that repeats itself forever. The number of rooms being heated at any given instant is shown in Fig. 3b; either no room or only one room is on at any given time. The temperatures oscillations are shown in Fig. 3c; they are all identical except for a phase shift. The order parameter r is shown in Fig. 3d; it is less than unity since phase synchronization is not reached.

Fig. 4 for $K = 0.05$ shows different dynamics. Now, pairs of rooms (3 and 4, 2 and 5) become fully synchronized and are on and off at the same time, as shown in Fig. 4a. This clustering happens not only between neighbors, such as 3 and 4, but also between 2 and 5 which are farther apart. The number of rooms being heated varies between zero and two, as shown in Fig. 4b. There are still phase differences between the temperatures shown in



(a) Heat source. (b) Number of rooms.
(c) Temperature (with room numbers). (d) Order parameter.

Fig. 11. System behavior for $\theta_L = 0.45$ and $K = 0.1$.



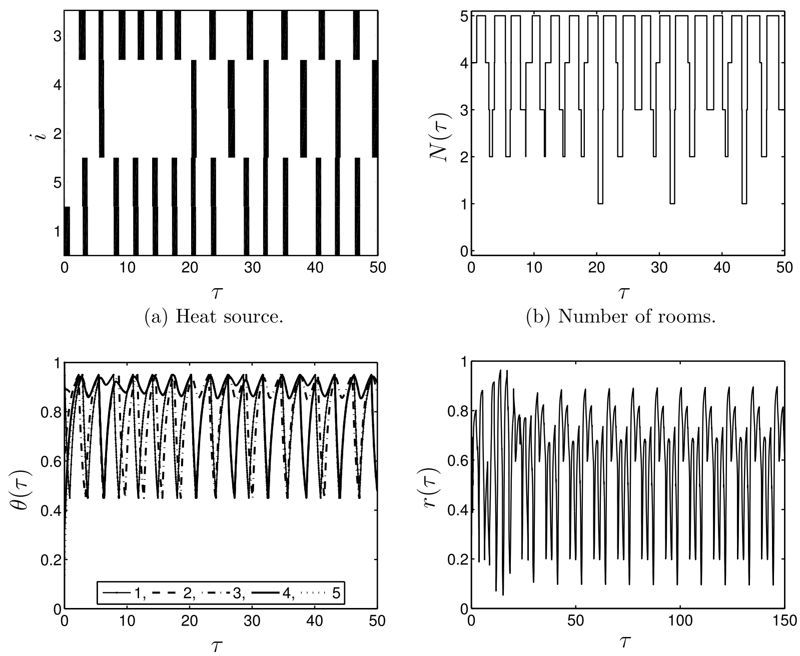
(a) Heat source. (b) Number of rooms.
(c) Temperature (with room numbers). (d) Order parameter.

Fig. 12. System behavior for $\theta_L = 0.45$ and $K = 0.15$.

Fig. 4c, and consequently the order parameter r shown in Fig. 4d is less than unity, though it is larger than in Fig. 3d.

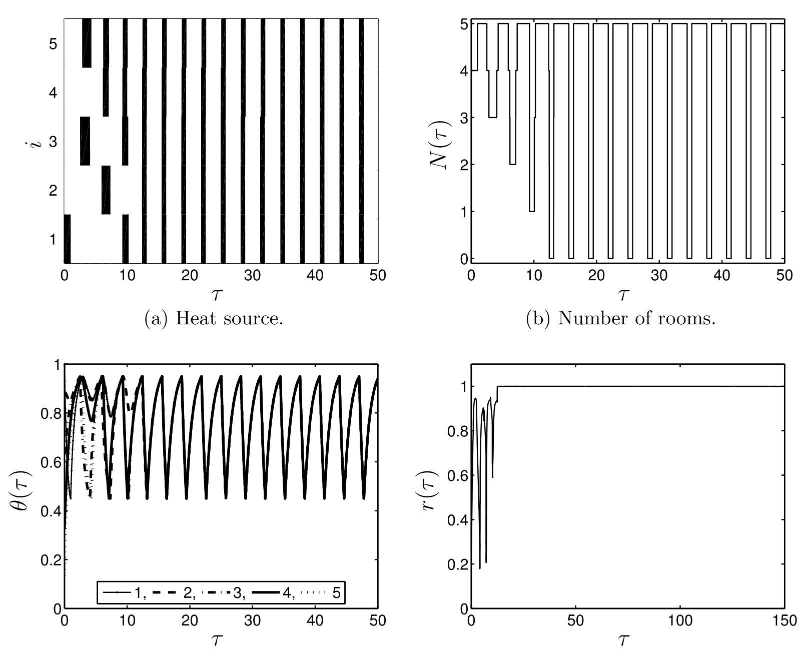
Fig. 5 shows the pattern when K is further increased to 0.2. Fig. 5a shows that the heat sources in 2 and 5 are driven to an off state, while 3 and 4 are still clustered together but out of phase with 1. The number of rooms being heated varies between zero and two. The variation of temperature is shown in Fig. 5c. The order parameter r in Fig. 5d reaches higher values than in Fig. 4d.

At $K = 0.5$, as shown in Fig. 6, phase synchronization occurs around $\tau = 15$; all rooms are then on and off at the same time, as shown in Figs. 6a and 6b, and the room temperatures shown in Fig. 5c are identical. The order parameter at phase synchronization in Fig. 6d quickly



(a) Heat source. (b) Number of rooms. (c) Temperature (with room numbers). (d) Order parameter.

Fig. 13. System behavior for $\theta_L = 0.45$ and $K = 0.3$.



(a) Heat source. (b) Number of rooms. (c) Temperature (with room numbers). (d) Order parameter.

Fig. 14. System behavior for $\theta_L = 0.45$ and $K = 0.5$.

reaches unity. For $K > 0.5$, which will not be shown here, the phases are also synchronized, but much quicker.

5.2. $\theta_L = 0.15$

θ_L is now set slightly higher. Since $\tau_p = 2.35$, the settling-down time is now about half. At $K = 0.05$, phase synchronization is obtained. All rooms are fully synchronized with the same period, as shown in Figs. 7a–c. The order parameter equals unity at full synchronization, as shown in Fig. 7d. For $K = 1.3$ there is on–off synchronization of the temperatures in rooms 1 and 2, as shown in Fig. 8. Heating in 3 and 5 is always off and in 4 always on; the order parameter r is slightly less than unity. On increasing K to 1.4, as shown in Fig. 9, it is seen that temperature oscillations have died out; the neighbors 1, 2 and 3 are always on, and 4 and 5 always off. The temperatures, shown in Fig. 9c reach a time-independent state and do not cross the limits of the thermostat settings. For $K = 5$, Fig. 10 shows that 3 and 5 are on while the others are off. The variation of the steady-state temperatures between the rooms in Fig. 10c is smaller than in Fig. 9c. When $K = 10$ room 4 becomes on again, and the situation is similar to that in Fig. 9, except that three rooms need to be on, instead of only two.

5.3. $\theta_L = 0.45$

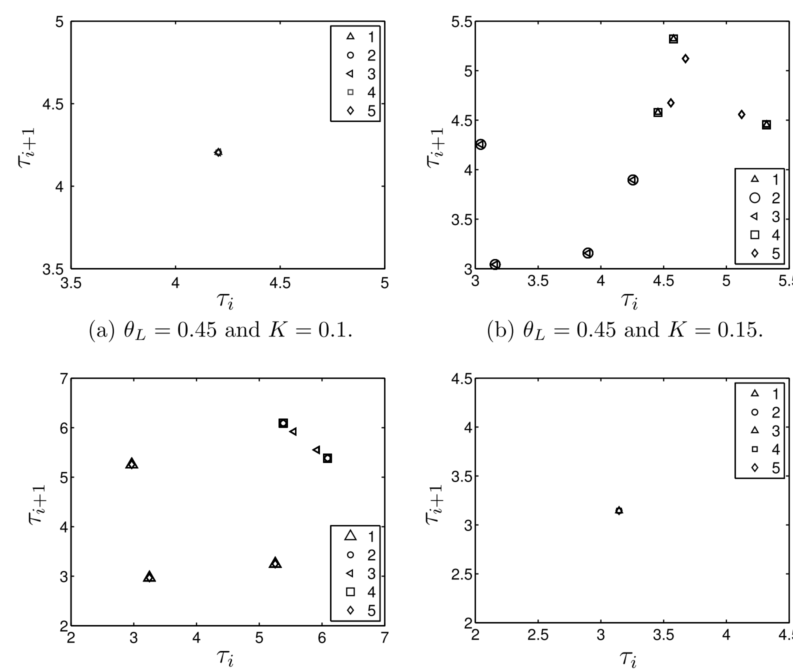
Finally, we choose θ_U near θ_{max} so that $\tau_p = 3.15$. For $K = 0.05$, there are three clusters: 1 and 5; 2 and 4; and 3, but the patterns do not completely repeat. On increasing K to 0.1, the frequency synchronization shown in Fig. 11 occurs in the system after about 50 units of time. For $K = 0.15$, Fig. 12 shows that there are again three clusters, for $K = 0.3$ Fig. 13 is periodic, and for $K = 0.5$ Fig. 14 indicates that there is phase synchronization in all rooms. For $K = 0.6$ rooms 3 and 5 stay on while the others periodically flash. For $K = 0.7$ the oscillations disappear and the temperatures are fixed.

5.4. Return maps

Additional information about the periodicity of temperature oscillations is given by return maps, like for example τ_i vs. τ_{i+1}. All periodic behaviors give a single dot, but the most interesting results are for $\theta_L = 0.45$, as shown in Fig. 15. Frequency synchronization is shown in Fig. 15a. As K increases, periodic behaviors represented by a finite number of dots appear in Figs. 15b and c. Finally there is phase synchronization as shown in Fig. 15d.

5.5. Average fraction of heated rooms

$\overline{N}/n$ is the fraction of rooms that are, on the average, on. This is shown for different θ_L in Fig. 16. When $\theta_L = 0.01$, frequency synchronization gives a least value for $K = 0.2$. When $\theta_L = 0.15$, $\overline{N}/n$ generally increases with the coupling strength and there is no optimum, but for $\theta_L = 0.45$ there may be more than one. It is important to notice that for a given θ_L, i.e., for prescribed thermostat settings, there are different fractions of rooms that are on depending on the thermal coupling K. The energy requirement, and consequently, the average temperature will also be different.



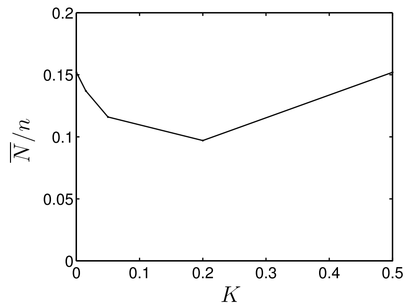
(a) $\theta_L = 0.45$ and $K = 0.1$.

(b) $\theta_L = 0.45$ and $K = 0.15$.

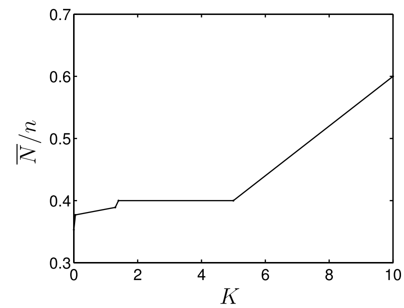
(c) $\theta_L = 0.45$ and $K = 0.3$.

(d) $\theta_L = 0.45$ and $K = 0.5$.

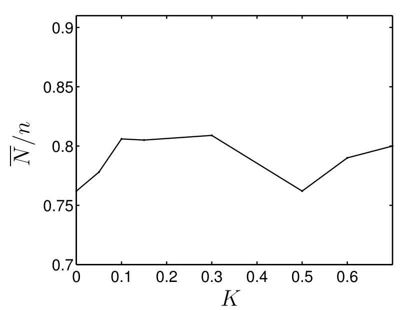
Fig. 15. Return maps (with room numbers).



(a) $\theta_L = 0.01$.



(b) $\theta_L = 0.15$.



(c) $\theta_L = 0.45$.

Fig. 16. Average fraction of rooms that are on.

6. Discussion and conclusions

A simple mathematical model has been used to analyze the dynamic behavior of a system of a number of thermally coupled and thermostatically controlled oscillators. The example of a five-room system is calculated in detail as a function of the thermostat settings and the thermal coupling between neighbors. The results show a rich array of dynamics that has been previously observed for other complex systems. We have found frequency and phase synchronization, clustering, and coupling-induced oscillation death. Other parameters which could have been varied are those that reflect differences between the rooms, the thermal coupling between them, initial conditions, the number of rooms, and the dead-band of the thermostat in each room. All of these will affect the long-time dynamics of the system and their effect should be studied.

The present results are significant for complex thermal systems which consist of a large number of weakly coupled sub-systems, each of which individually acts as a self-sustained oscillator. For building heating, as an example, the temperature dynamics of the rooms are affected by thermal conduction through the wall between adjacent rooms. A fully synchronized situation, if it occurs, will require a demand for maximum heating capacity in the building for an interval of time followed by another of zero demand. For a reduction in installed capacity, it may be desirable to have a smaller variation of the demand over time, in which case the control system will have to be properly designed for the purpose. The dynamics of the collective behavior also affects the average number of rooms that are on, and thus the average temperature of the rooms and the energy usage. By changing the coupling parameter it is possible to increase or decrease the average temperature in the rooms while keeping them within the allowable thermostat limits. In fact, for high thermal coupling, it is even possible to eliminate heating from certain rooms because of heat sharing.

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