Applied Thermal Engineering 65 (2014) 337–342

Contents lists available at ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng

Synchronization of temperature oscillations in heated plates with hysteretic on–off control

Miguel A. Barron [a,*], Mihir Sen [b]

[a] Departamento de Materiales, Universidad Autonoma Metropolitana-Azcapotzalco, Av. San Pablo 180, Mexico D.F., 02200, Mexico
[b] Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame, IN 46556, USA

HIGHLIGHTS

- We model the thermal synchronization of two coupled plates with on–off control.
- We determine the self-oscillation frequency of an uncoupled controlled plate.
- Frequency and phase locking arise when coupled plates share the deadband width.
- Differences in the deadband width between plates cause frequency detuning.
- A detuning window for locked synchronization is found for small values of detuning.

ARTICLE INFO

Article history:
Received 20 September 2013
Accepted 17 January 2014
Available online 28 January 2014

Keywords:
Coupled behavior
Coupled plates
Frequency detuning
Heated plate
Self-oscillation frequency
On–off control
Temperature oscillations

ABSTRACT

The synchronized behavior of two coupled, top heated, square plates with on–off control based on plate temperature is analyzed in this work. Each plate is represented by a two dimensional heat equation, and thermal communication between the plates is modeled by a thermal resistance; each plate also has an internal point at which the temperature is monitored for control. The problem is nonlinear, and numerical simulations are used to determine the long-time dynamic response of the system. As a first step the self-oscillation frequency of a single uncoupled controlled plate is determined as function of the deadband width of the controller. Then the dynamics of the coupled plates is analyzed, and the effect of the thermal resistance and the deadband width on synchronization of temperature oscillations is studied. Like in other complex systems with synchronization, a detuning window is found over which there is synchronization and beyond which the plates have different oscillation frequencies.

1. Introduction

Among the large variety of available control schemes, on–off control with hysteresis (also called thermostatic, bang–bang, or two-position control) is the simplest tool to regulate the dynamic performance of domestic and industrial thermal-fluid systems such as air-conditioners, water heaters, furnaces, and level controls [1–3]. In open-loop operation, i.e. in the absence of control, these systems behave in a non-oscillatory manner, but exhibit oscillatory dynamics under control. The amplitude and frequency of closed-loop temperature oscillations depend on the width of the deadband. When these systems are coupled, closed-loop oscillations interact and propagate through the ensemble.

There have been few publications on the coupled behavior of interconnected systems with self-oscillations induced by controllers. Among them can be cited the synchronization of oscillations in a thermal-hydraulic network [4]. Frequency locking, phase synchronization as well as phase slips are observed to occur due to thermal-hydraulic coupling between the controllers. In a later paper, the collective dynamics of a number of individually heated thermostatically controlled rooms arranged in the form of a ring was analyzed [5]. It was reported that the coupled system shows the presence of a rich array of synchronization dynamics in frequency and phase as well as clustering and coupling-induced amplitude death. The effect of walls on synchronization of thermostatic room-temperature oscillations has also been looked at [6]. Interconnections in the form of simple geometrical configurations, such as rings, can be studied if the self-oscillations are governed by ordinary differential equations [7]. The study of synchronization in coupled self-oscillation of systems governed by partial differential

* Corresponding author.
E-mail address: bmma@correo.azc.uam.mx (M.A. Barron).

1359-4311/$ – see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.applthermaleng.2014.01.026

equations is less common, one exception being that of coupled vibrating beams [8].

Unlike the lumped parameter assumption made in a previous study [5], temperature distributions are usually governed by PDEs. One example of that is that of temperature self-oscillations in heated plates with on–off control. For simplicity, two square plates are chosen for this study; the plates are coupled through a finite thermal resistance. When the temperature at a selected point (chosen here to be the center) reaches an upper limit, the heater shuts off, and when it reaches a lower bound, it comes on again. The energy equation, which models the thermal response of the plates, was numerically solved using a finite-differences technique. Self-oscillation frequencies of a single uncoupled controlled plate are determined as function of the deadband width of the on–off controller. The effect of the thermal resistance and the deadband width on the coupled behavior of two plates is studied.

2. Mathematical model

Heat conduction in a single two-dimensional square plate with constant properties and no heat sources is governed by

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{1}$$

where $T(x,y,t)$ is temperature, t is time, α is the thermal diffusivity of the plate material, and x and y are Cartesian coordinates, respectively. Two temperatures, one hot T_h, and another cold T_c, are selected. Defining the non-dimensional variables $\theta = (T - T_c)/(T_h - T_c)$, $\tau = \alpha t/L^2$, $\xi = x/L$, $\psi = y/L$, where L is the plate length, we get

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial \theta^2}{\partial \xi^2} + \frac{\partial \theta^2}{\partial \psi^2}. \tag{2}$$

The effects of thermal diffusion and size of the plate have been absorbed into the new variables.

Eq. (2) and its boundary conditions, to be described later, were numerically solved using finite-differences. Central differences with equal spacing in the two directions were used for the spatial derivatives, and the boundary conditions were approximated by first-order differences. A forward difference was used for the first-order time derivative so that the temperature field was calculated explicitly at each time step. The code was verified in a series of tests using the method of manufactured solutions [9]. Care was taken in the different tests to exercise all the terms in the governing equations and boundary conditions. For a mesh of 100 × 100 and a time step of 10^{-5}, the error was found to be negligible.

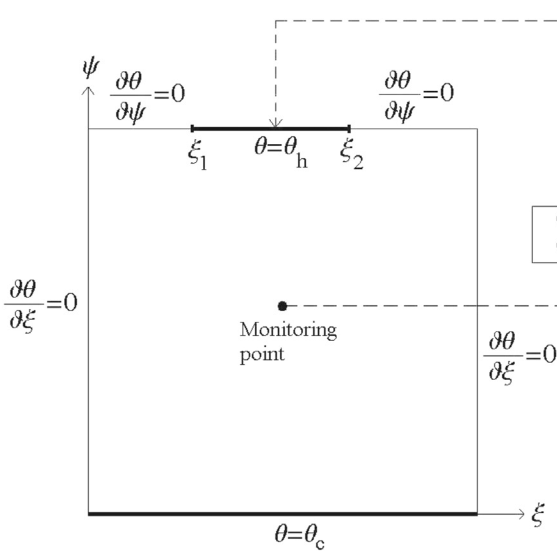

Fig. 1. Single plate with on–off controller.

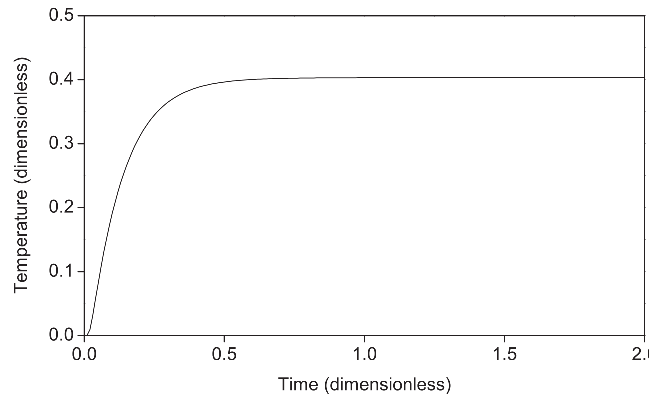

Fig. 2. Open-loop behavior of single uncoupled plate.

3. Behavior of a single plate

A single plate that will first be analyzed is shown in Fig. 1. The temperature in the region $\xi_1 \le \xi \le \xi_2$ of the top wall is θ_h or θ_c, whereas bottom wall is always maintained at θ_c (the values $\xi_1 = 0.25$ and $\xi_2 = 0.75$ are assumed here). The left and right walls of the plate are adiabatic. The input to the controller is the

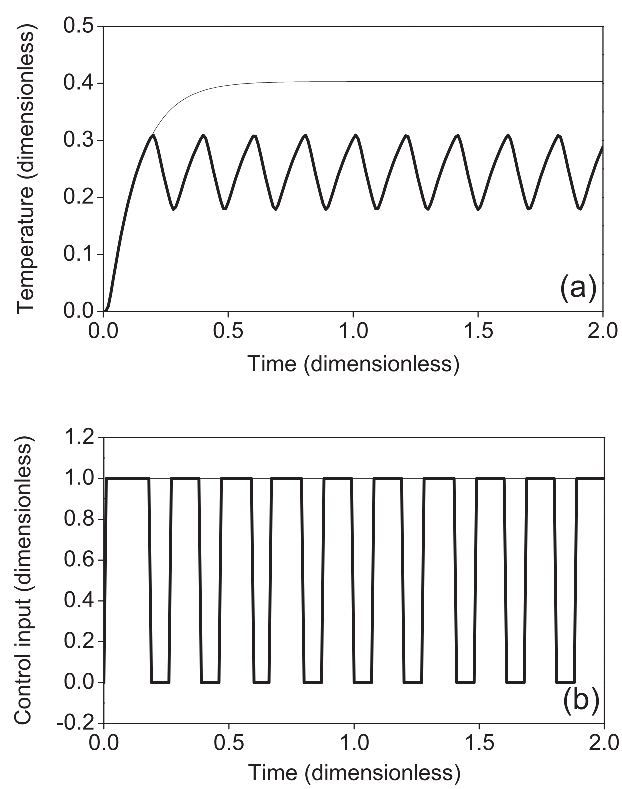

Fig. 3. Closed-loop behavior of single uncoupled plate for two set points: $\theta_{sp} = 0.25$ (thick) and $\theta_{sp} = 0.40$ (thin); $W = 0.1$. (a) Monitored temperature, (b) control input.

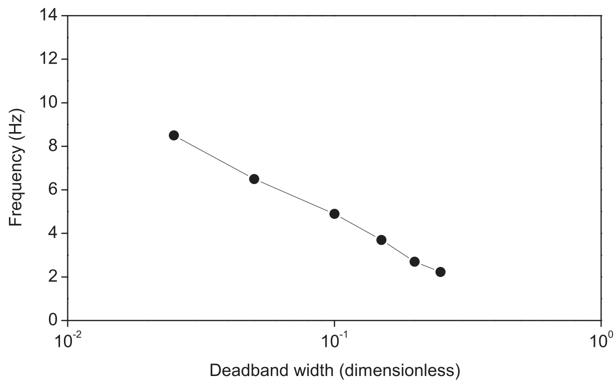

Fig. 4. Self-oscillation closed-loop frequency of single plate as function of W.

temperature at a monitoring point M in the plate (which is taken to be the center of the plate). The output of the controller is the temperature at the top wall. A set-point θ_{sp} and deadband W are chosen. The controller reads the temperature θ_m at the monitoring point and takes a control action: if the reading θ_m rises to $\theta_U = \theta_{sp} + W/2$ (upper limit), the top wall becomes θ_c; if the reading θ_m falls to $\theta_L = \theta_{sp} - W/2$ (lower limit), the top wall becomes θ_h. A non-zero deadband width $W = \theta_U - \theta_L$ is thus required to maintain θ_m within a desired range and to avoid excessive oscillations.

Fig. 2 shows the open-loop behavior of a single uncoupled plate with the temperature at the top wall unchanging at θ_h. The temperature at the monitoring point increases until a steady state value of $\overline{\theta}_m = 0.4032$ is reached after a dimensionless time around 1.13. $\overline{\theta}_m$ represents the equilibrium point of the single uncoupled open-loop plate, and is the maximum possible temperature that the on–off controlled plate can achieve any time at the monitoring point.

The closed-loop evolution of the monitored temperature and the control input is shown in Fig. 3(a) and (b), for $\theta_{sp} = 0.25$ and 0.40, respectively, with $W = 0.1$. One can observe in Fig. 3(a) that for $\theta_{sp} = 0.40$, temperature oscillation death (OD) arises as a result of saturation of the control input, which is depicted in Fig. 3(b). During OD the monitored temperature is frozen at $\theta = \overline{\theta}_m$. The emergence of OD takes place whenever $\theta_U \geq \overline{\theta}_m$.

The influence of W on the self-oscillation frequency f of the closed-loop plate is shown in Fig. 4 for $\theta_{sp} = 0.25$ and several values of W. The initial temperature is θ_c. The values of the self-oscillation frequencies were determined from the power spectrum of the temperature time series at M. The larger the value of W, the higher the amplitude and lower the frequency of the self-oscillations. A logarithmic dependence of f on W is observed. These results are qualitatively similar to those found experimentally in a thermal-hydraulic network [4].

4. Dynamics of coupled plates

Here we consider two coupled heated plates with independent on–off controllers, and with a thermal resistance between them, as shown in Fig. 5. The coupled system is governed by

$$\frac{\partial \theta_i}{\partial \tau} = \frac{\partial \theta_i^2}{\partial \xi^2} + \frac{\partial \theta_i^2}{\partial \psi^2}, \tag{3}$$

for $i = 1,2$. The plates are coupled through a thermal resistance R given by

$$\frac{\partial \theta_1}{\partial \xi} = \frac{\partial \theta_2}{\partial \xi} = -\frac{1}{R}\left(\theta_{1,\xi=1} - \theta_{2,\xi=0}\right). \tag{4}$$

The same parameter values of Section 3 were employed in the numerical simulations. Initial conditions for the left and right plates were θ_c and θ_h, respectively. The effect of R on the coupled behavior is seen in Fig. 6. Fig. 6(a) and (b) shows initial time series of the monitoring points for $R = 0.01$ and $R = 1$, respectively, using $W = 0.1$. A transient phase shift is observed in both cases. However, long time computer runs show complete phase and frequency synchronization in both plates, irrespective of the value of R, as illustrated in Fig. 7 for $R = 1$.

The self-oscillations of the individual plates interact with each other. Since the deadbands determine the self-oscillation frequency of each plate, frequency detuning is obtained by assigning a different deadband width to each plate. Frequency detuning quantifies the difference between uncoupled oscillators [10]. The response is also affected by the thermal resistance that affects the level of coupling between them. Fig. 8(a) shows the time series of the monitoring points using $W_1 = 0.1$ and $W_2 = 0.2$ for the left and the right plates, respectively, for $R = 0.01$. These deadband widths correspond, in accordance to Fig. 4, to self-oscillation frequencies of

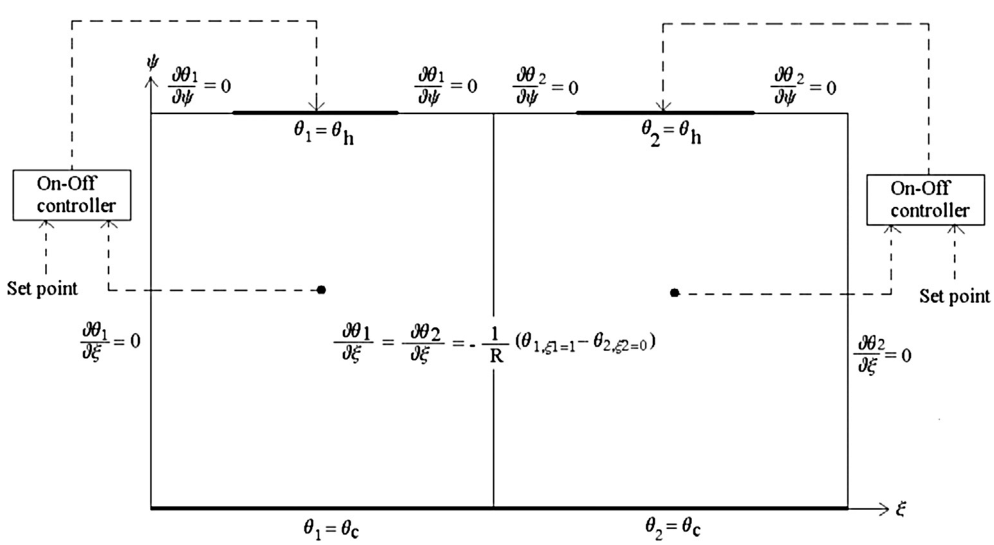

Fig. 5. Two coupled plates with on–off controller.

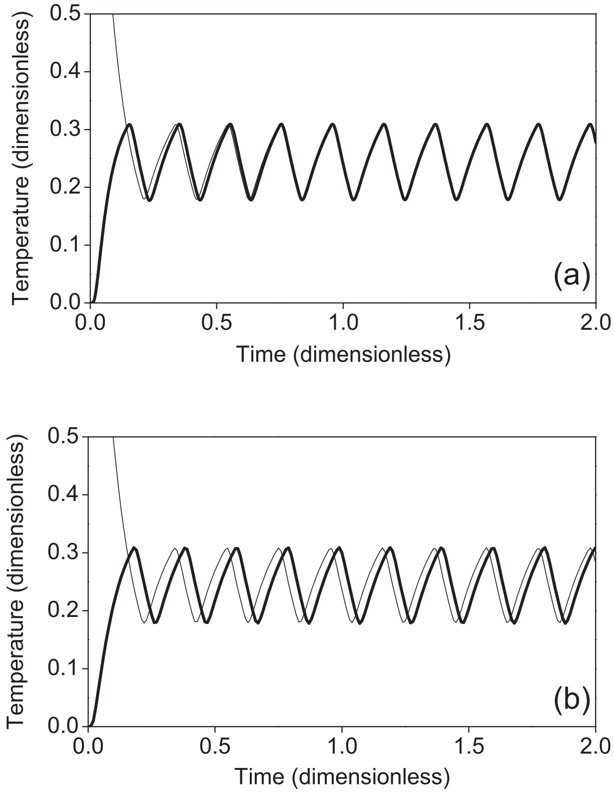

Fig. 6. Initial time series of coupled plates (thick = left plate, thin = right plate) for (a) $R = 0.01$ and (b) $R = 1$. $W_1 = W_2 = 0.1$.

$f_1 = 4.88$ and $f_2 = 2.85$ for the uncoupled left and right plates, respectively, so the detuning is $\Delta f = 2.03$. The power spectrum in Fig. 8(b) shows that the frequency of the left plate increases to 5.07. Fig. 8(c) shows that the frequency of the coupled right plate is composed of a main frequency of 2.54, and a secondary frequency of 5.07, which is the result of the interaction of the right plate with the faster dynamics of the left plate [11]. A phase portrait corresponding to the time series of Fig. 8(a) is in Fig. 9. This phase portrait is a Lissajous figure [12] that indicates that the period of an oscillator corresponds to two periods of the remaining oscillator [10], namely 0.3937 for the right plate and 0.1972 for the left plate.

Fig. 10 shows the power spectrum of the coupled plates for two values of the thermal resistance. For $R = 1$, the power spectra are identical for both plates, with a main frequency of 4.90 and a secondary frequency of 9.80. For $R = 100$ the left plate exhibits a main frequency of 4.89 and small secondary frequency of 9.81, whereas the right plate shows a main frequency of 2.83 and a secondary frequency 5.61.

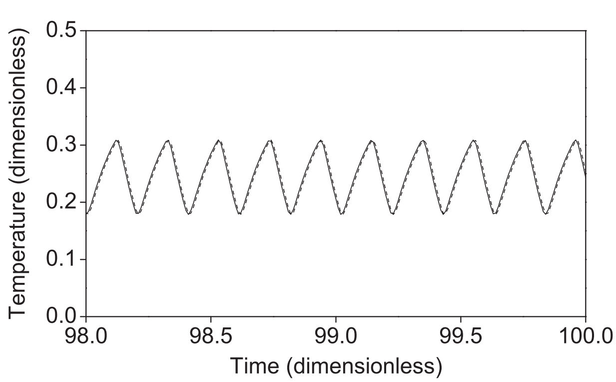

Fig. 7. Long term time series of coupled plates (solid = left plate, dashed = right plate) for $R = 1$, $W_1 = W_2 = 0.1$.

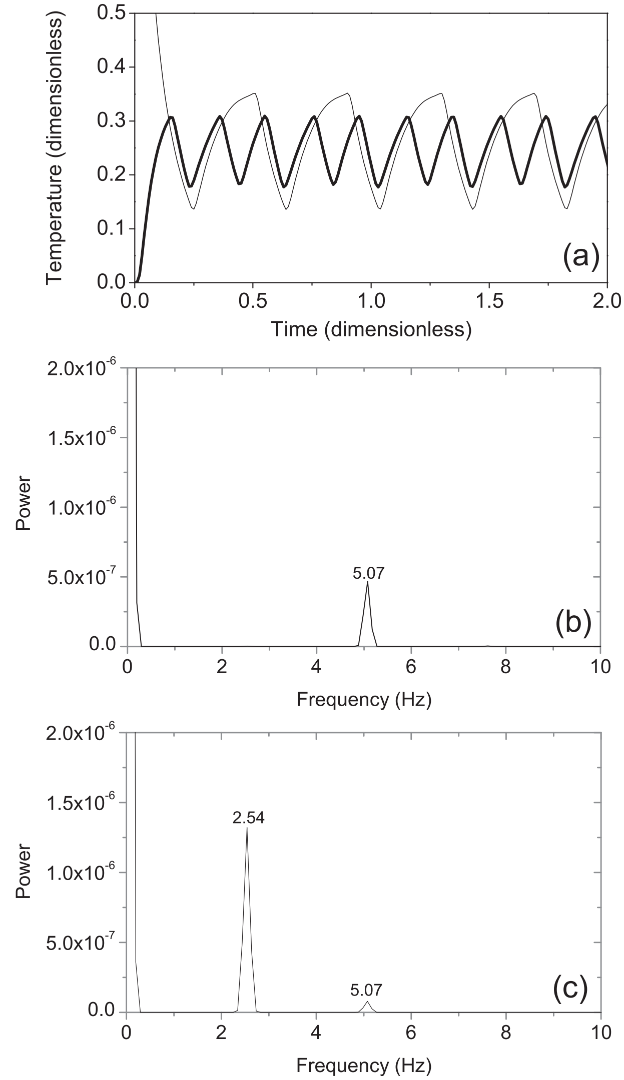

Fig. 8. (a) Time series of coupled plates (thick = left plate, thin = right plate), (b) power spectrum of left plate, and (c) power spectrum of right plate. $R = 0.01$, $W_1 = 0.1$, $W_2 = 0.2$.

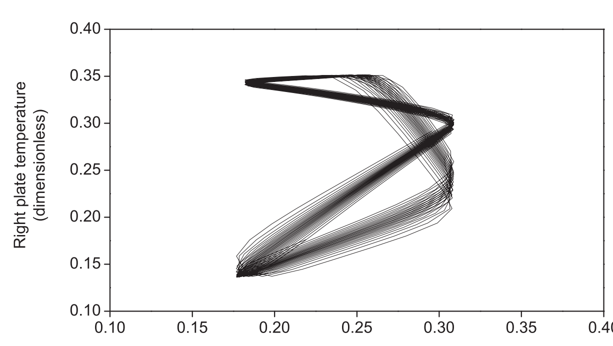

Fig. 9. Phase portrait of coupled plates for time series of Fig. 8(a).

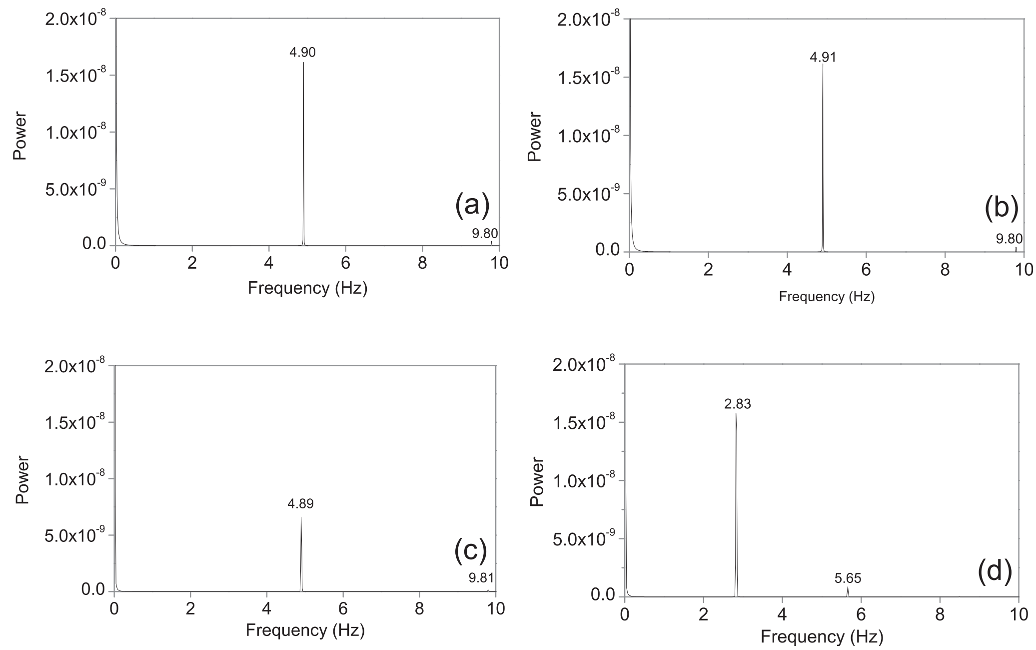

Fig. 10. Power spectra of coupled plates; (a), (b) for $R = 1$, and (c), (d) for $R = 100$. Left figures for left plate, right figures for right plate.

5. Synchronization of coupled plates

One of the fundamental aspects of the synchronization of a complex array of self-excited oscillators with coupling is that they may synchronize even when their parameters and hence their individual, uncoupled frequencies are different. It has been empirically found that in many cases this synchronization happens over a small band of detuning, but when the detuning is large, there is no synchronization and the individual oscillators have different frequencies [4]. Fig. 11 shows that, in spite of being governed by PDEs, the present pair of plates have a window of detuning over which there is synchronization and locking, and beyond which there is not.

6. Effect of thermal diffusivity

Physical properties of the plates determine the rate at which heat is transferred. Given that thermal oscillations depend on delays in heat transfer, it is interesting to analyze the influence of thermal diffusivity on the natural frequency of a controlled plate. In order to take into account physical properties, Eq. (1) was numerically solved. Several values of thermal diffusivity, corresponding to metals such as iron, molybdenum, aluminum and copper were considered. The following values of dimensional parameters and boundary conditions were employed in the numerical simulations: $T_{sp} = 283$ K, $T_c = 273$ K, $T_h = 373$ K, $T_0 = 273$ K, $W = 2$ K, which correspond to temperature set point, cold temperature, hot temperature, initial temperature and temperature deadband width, respectively. Besides, the physical dimensions of the plate were as follows: length, 0.05 m; height, 0.05 m; coordinates of monitoring point, (0.025, 0.025); length of first top adiabatic wall, 0.0125 m; length of second top adiabatic wall, 0.0125 m. Results are depicted in Fig. 12, where is observed that the frequency grows as the thermal diffusivity is increased. Given that thermal diffusivity alters the natural frequency of the controlled plates, coupling plates made of different materials, irrespective of the value of the thermal resistance, causes a frequency detuning in the coupled system.

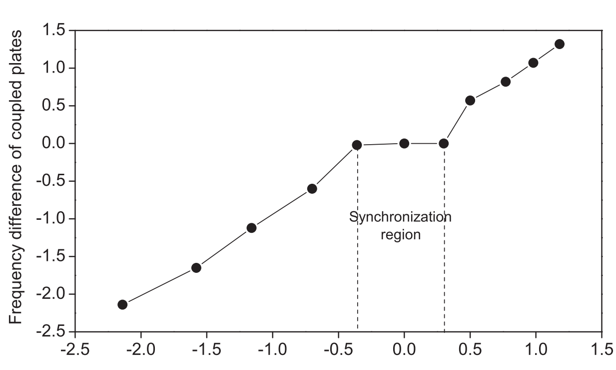

Fig. 11. Detuning window for synchronization.

7. Conclusions

The behavior of two heated plates with on–off control and coupled through a thermal resistance was numerically studied. The

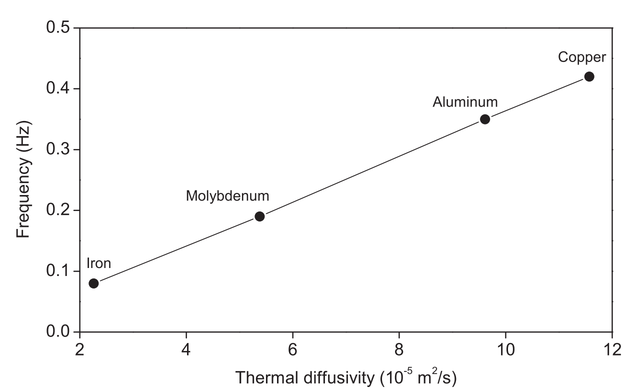

Fig. 12. Natural frequency of an on–off controlled plate as function of the thermal diffusivity.

self-oscillation frequency of thermal oscillations in a single uncoupled plate decreases nonlinearly as the deadband width increases. Frequency and phase locking are exhibited when the coupled plates share the same deadband width, irrespective of the value of the thermal resistance. Differences in the deadband width between closed-loop plates causes frequency detuning, and the power spectra of the time series reveals the presence of two frequencies in the temperature oscillations of the plates due to the interaction of a left plate with fast dynamics. However, a window of detuning for locked synchronization has been found for small values of the detuning.

References

[1] S. de Jonge, Simple thermostat with hysteresis, Electron. World 103 (1739) (1997) 942.
[2] P. Gurevich, W. Jaeger, A. Skubachevskii, On periodicity of solutions for thermocontrol problems with hysteresis-type switches, SIAM J. Math. Anal. 41 (2) (2009) 733–752.
[3] B. Li, R. Otten, V. Chandan, W.F. Mohs, J. Berge, A.G. Alleyne, Optimal on–off control of refrigerated transport systems, Control Eng. Pract. 18 (12) (2010) 1406–1417.
[4] W. Cai, M. Sen, K.T. Yang, R.L. McClain, Synchronization of self-sustained thermostatic oscillations in a thermal-hydraulic network, Int. J. Heat Mass Transfer 49 (2006) 4444–4453.
[5] W. Cai, M. Sen, Synchronization of thermostatically controlled first-order systems, Int. J. Heat Mass Transfer 51 (2008) 3032–3043.
[6] M. Sen, I. Amegashie, E. Cecconi, P. Antsaklis, Dynamics of Air and Wall Temperatures in Multiroom Buildings, ASME IMECE, Houston, TX, Nov. 9–15, 2012. Paper No. IMECE2012-93125.
[7] M.A. Barron, M. Sen, Synchronization of four coupled van der Pol oscillators, Nonlinear Dyn. 56 (4) (2009) 357–367.
[8] M.A. Barron, M. Sen, Synchronization of coupled self-excited elastic beams, J. Sound Vib. 324 (1–2) (2009) 209–220.
[9] K. Salari, P. Kunpp, Code Verification by the Method of Manufactured Solutions, Technical Report 2000-1444, Sandia National Laboratories, June 2000.
[10] A. Pikovsky, M. Rosenblum, J. Kurths, Synchronization: A Universal Concept in Nonlinear Sciences, Cambridge University Press, Cambridge, UK, 2001.
[11] A.H. Nayfeh, Nonlinear Interactions: Analytical, Computational and Experimental Methods, Wiley, New York, NY, 2000.
[12] A. Balanov, N. Janson, D. Postnov, O. Sosnotseva, Synchronization: From Simple to Complex, Springer, Berlin, Germany, 2009.