Pricing and Hedging

Master thesis
Mathematical Modelling, Simulation and Optimization

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Declaration of authorship

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Abstract

We create a financial product that utilises targeted delta hedging on Bitcoin. It aims to provide competitive returns with consistent growth. Bitcoin is postulated to have long-term growth because of deflation. Our financial product should benefit from the positive development of Bitcoin and at the same time minimize risk and reduce the impact of market crashes. Based on the Black-Scholes model, we simulate a marketplace to test different hedging strategies. Specifically, we explore three approaches to optimize the switch between hedging with call and put options. All three models attempt to predict market movements based on historical data. The first model focuses on prior short-term market conditions, the second focuses on prior long-term market conditions, whereas the final model combines data from both timeframes with a self-learned weighting mechanism. We demonstrate that the final model has the potential to serve as a financial product with the required properties.

Contents

1 Theoretical Framework

The entire 'Theoretical Framework' section refers to this source.[2]

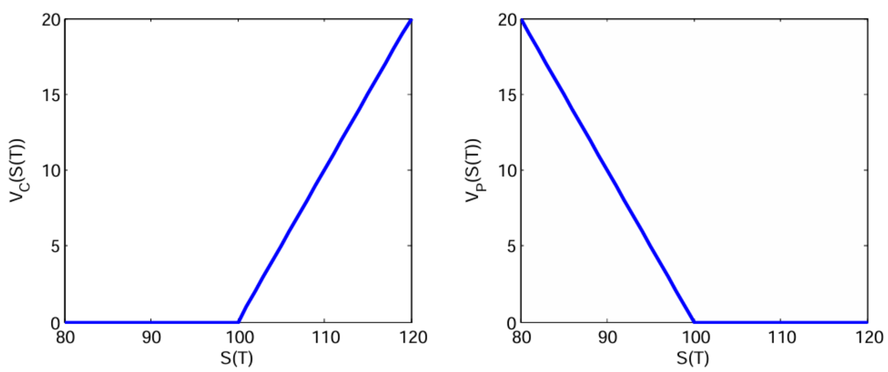
1.1 Fundamentals of the Option Stock Market

An option is a financial product that grants the right to buy or sell a specific quantity of an asset at a predetermined strike price K at a specific point in time T. There are European and American Options. European options can only be exercised after their maturity, whereas American options can be exercised at any time during their validity. The call option provides you the opportunity to buy and the put option to sell under the agreed conditions. During a market uptrend, a call option can experience a disproportionately high increase in value, just as a put option does during a market downturn. This disproportionate increase is due to the ability to realize gains from the stock market under favorable market conditions, while the right to do so can be purchased at a lower cost. Since options grant the right to buy or sell at a fixed price, they also provide the opportunity to hedge against price fluctuations of the underlying asset. The value of an option warrant depending on the current stock price S at expiration day T is as follows:

$$V(T, S(T)) = \Lambda(S(T)) = \begin{cases} (S(T) - K)^+, & \text{for a Call Option} \\ (K - S(T))^+, & \text{for a Put Option} \end{cases} \tag{1.1}$$

whereby $(S(T) - K)^+ := \max\{S(T) - K, 0\}$ resp. $(K - S(T))^+ := \max\{K - S(T), 0\}$.

This relationship is represented in following figure:



Option Value of a European Call Option at t=T Option Value of a European Put Option at t=T

Figure 1.1: Call-Put

Similar to the stock price itself, options can be traded before their expiration date. This implies that options have a market value during their validity. In addition to the stock price, this market value depends on other factors, as shown in the following table:

values	Europe Call	Europe Put	American Call	American Put
Current stock price (S)	$+$	-	$+$	-
Strike Price (K)	$-$	$+$	-	$+$
remaining time ($T-t$)	?	?	$+$	$+$
volatility (σ)	$+$	$+$	$+$	$+$
risk-free interest rate (r)	$+$	-	$+$	-
dividends (d)	-	$+$	-	$+$

[1] $+$: increase in the value of the variable
[2] $-$: decrease in the value of the variable
[3] ? : means that the connection is not clear

Table 1.1: Factors influencing Option Prices

There are different approaches to calculate the *fair* price for an option taking all the mentioned dependancies into account. *Fair* in this context means that these models value the opportunity of disproportionate gains as well as the insurance the options provide against stock price fluctuations correctly. These models are further examined in the following section.

1.2 Basic Option Pricing Models

Two mathematical models have become established for determining option prices: the Black-Scholes model and price estimation via a Monte Carlo simulation. The Black-Scholes model can only be used for European options as it does not account for the early exercise of the option. Another limitation of the Black-Scholes model is that the volatility of the stock price has to be set to a constant value a priori. As shown, the following formulas to calculate the corresponding value of an European option at a point t in time, the volatility remains the only unkown variable.

$$\text{Let } a = \frac{\ln\left(\frac{S(t)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad \text{Let } b = a - \sigma\sqrt{T-t} \tag{1.2}$$

Price of a Call Option:

$$V_C(t) = S(t)\Phi(a) - Ke^{-r(T-t)}\Phi(b) \tag{1.3}$$

Price of a Put Option:

$$V_P(t) = S(t)(\Phi(a) - 1) - Ke^{-r(T-t)}(\Phi(b) - 1) \tag{1.4}$$

Using a Monte Carlo simulation proves to be more flexible in terms of estimating the $fair$ option price. The Monte Carlo method can be used to estimate the price of American options and can also model trends in volatility development when applying the Mean Reversion method. In comparison to the Black-Scholes model, the value of an European option with constant volatility can be estimated using the Monte Carlo simulation with the following formula:

$$V(t) \approx e^{t-T} \left[\frac{1}{N} \sum_{i=1}^{N} \Lambda \left(S_0 \exp \left(\mu - \frac{1}{2}\sigma^2 T + \sigma\sqrt{T} Z_i \right) \right) \right] \tag{1.5}$$

whereby N is the number of simulations, S_0 is the current stock price and Z_i is a normally distributed random variable.

1.3 Historical versus Implied Volatility

As previously mentioned, volatility in the option pricing formulas is an unknown variable. Two fundamental types of volatility can be distinguished: Historical volatility and implied volatility. Historical volatility is calculated based on the price fluctuations of the underlying stock price. The historical volatility can be calculated using the following formula, where n represents the number of past days considered in the calculation. By default, the closing prices of the last 30 days are used. The historical volatility is annualized by the second factor of the equation. Typically, 252 days are used for annualization, as this corresponds to the number of trading days in traditional stock markets. For financial instruments that can be traded around the clock, annualizing with 365.2425 days provides the appropriate equivalent.

$$HV = \sqrt{\frac{1}{n} \sum_{t=1}^{n} \ln \left(\frac{S(t)}{S(t-1)} \right)^2} \cdot \sqrt{365.2425} \tag{1.6}$$

In contrast to historical volatility, implied volatility is not directly calculated from the stock price of the underlying asset. Instead, traders defined this parameter in course of time by using the Black-Scholes model. As mentioned earlier, volatility is a factor in the pricing of options. When expectations of future volatility increase, the option becomes more valuable, while its value decreases when expectations of future volatility decrease. From this motivation, traders have defined implied volatility as follows:

Let V be a given option price in the market, and let $BS(V,\sigma_{implied})$ be the theoretical value based on the Black-Scholes model. Then $\sigma_{implied}$ can be calculated by solving:

$$V = \mathrm{BS}(V, \sigma_{\text{implied}})$$

In this way, traders attempt to identify options for which an unusually low expectation of future volatility must have been assumed, meaning that these options are relatively cheap, since volatility is a key driver of the option price.
In summary, it can be said that both volatilities follow different approaches. Historical volatility uses past price movements of the underlying asset as an estimator for the future, while implied volatility directly reflects the expectations of option traders. The Black-Scholes model is now primarily used to determine the implied volatility of options in the market, rather than in its original sense to calculate a fair option price based on given parameters.
Since the expectations in the options market are largely based on the developments of the underlying asset's market, historical volatility serves as a good estimator for a lower bound of implied volatility. In 1.2, you can see the comparison between the historical volatility of Bitcoin and the DVOL index, which reflects the average implied volatility over the given time span.

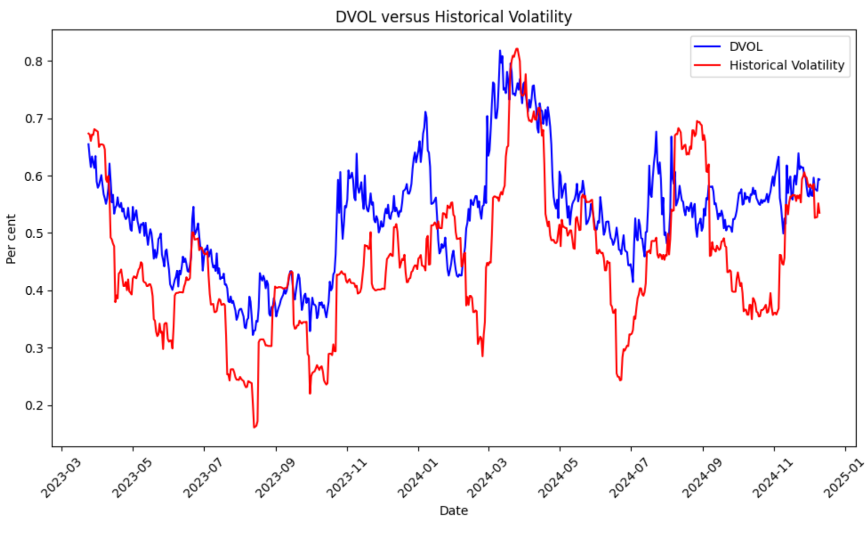


Figure 1.2: Implied Volatility[1] vs Historical Volatility

The difference between historical volatility and implied volatility is called the volatility premium. 1.3 shows that implied volatility is lowest when the strike price K equals the current stock price S. This suggests that in this specific case, the volatility premium is expected to reach a minimum.

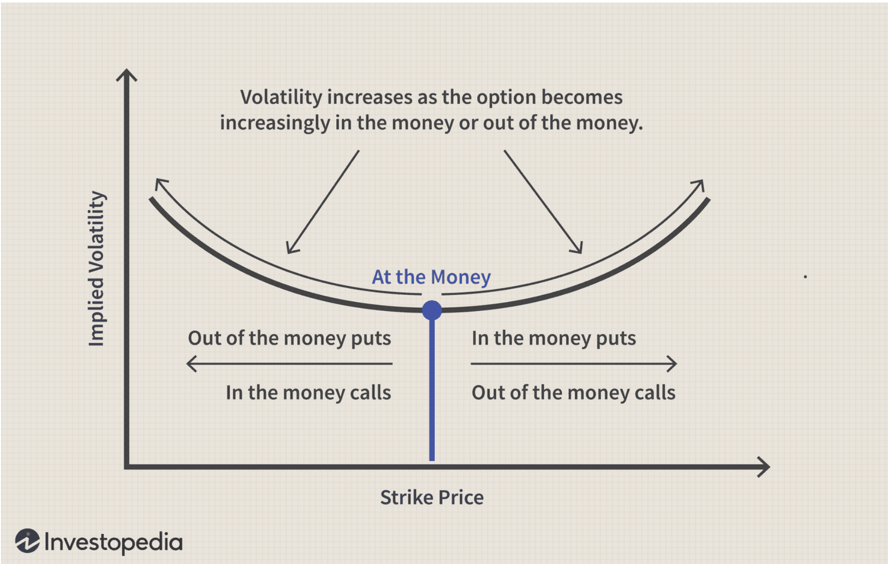


Figure 1.3: Volatility Smile[3]

1.4 Delta Hedging

Delta is an established term in the options market that reflects the partial derivative of the option price with respect to changes in the stock price of the underlying asset. Thus, Delta quantifies the sensitivity of the option price to changes in the stock price. Based on that, delta hedging is a method used to neutralize the price risk of an option. A delta-hedged portfolio consists of an option (either a put or a call) and a position in the underlying asset. For simplicity, let us assume that the option refers to exactly one share of stock, meaning that the hedging unit is one. Then, the goal is to align the quantity of the underlying asset with the current option's delta. This process of keeping the quantity of the underlying asset equal to the option's delta (which changes over time) is called making the portfolio delta neutral. This practice should, in theory, always be carried out (leading to the algorithm in 1.4) to keep the portfolio delta neutral at all times. If the hedging unit is different, the necessary quantity of the underlying asset has to be multiplied by the hedging unit additionally. In this way, traders ensure that small price changes in the asset have a marginal effect on the portfolio's overall value. This makes delta hedging a key tool for risk management in options trading.

An option's delta can be determined either by using the Black-Scholes model or a Monte Carlo simulation to estimate the option's value.

Calculating the Δ for a call option using the Black-Scholes model:

$$\Delta_C(t, S(t)) = \frac{\partial V_{Ce}}{\partial S}(t, S(t)) = \Phi(a) \tag{1.7}$$

Calculating the Δ for a put option using the Black-Scholes model:

$$\Delta_P(t, S(t)) = \frac{\partial V_{Pe}}{\partial S}(t, S(t)) = \Phi(a) - 1 \tag{1.8}$$

Estimating the Δ for an option using a Monte Carlo simulation:

$$\Delta(t) \approx \frac{V(t, S(t) + h) - V(t, S(t))}{h}, \tag{1.9}$$

$$\frac{\partial V}{\partial S}(t, S(t)) = \lim_{h \to 0} \frac{V(t, S(t) + h) - V(t, S(t))}{h}. \tag{1.10}$$

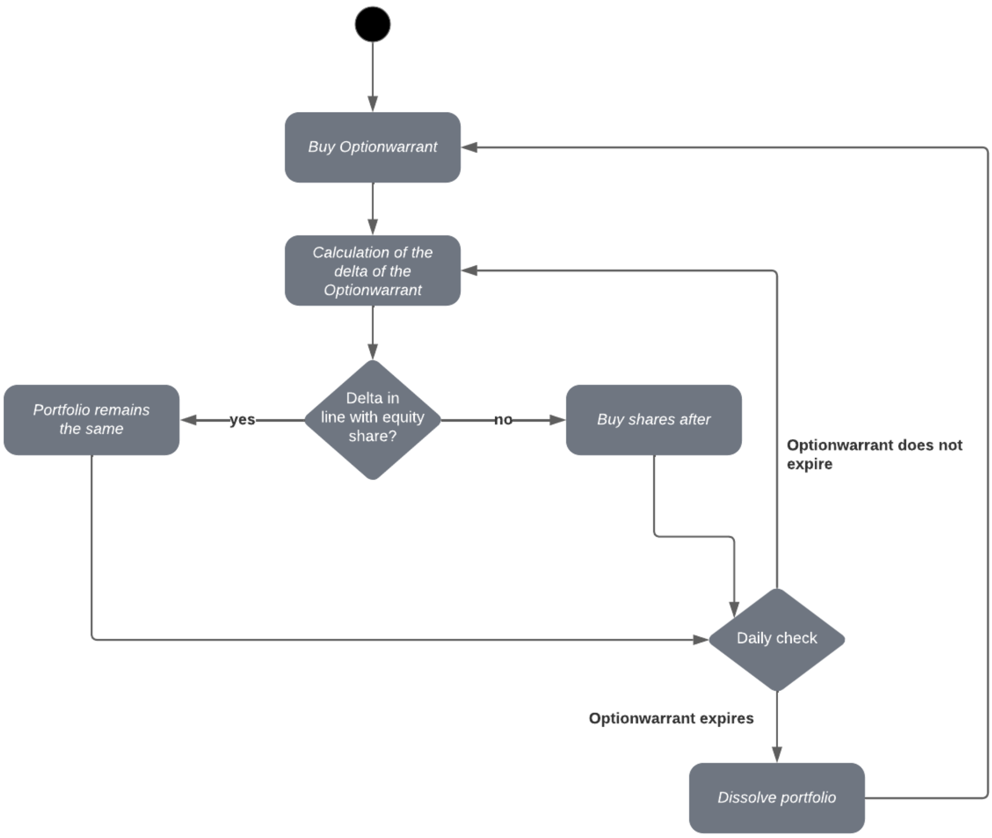


Figure 1.4: Programme schedule of the delta hedging

1.5 Hedging with Put Options versus Hedging with Call Options

Based on the described characteristics of put and call options, as well as the delta hedging method, obvious conclusions can be drawn regarding the differences between hedging with calls and hedging with puts.

Let us first consider hedging with put options. If the underlying asset price increases, the value of your put option declines, but the value of the underlying asset you own increases by the same amount, provided your portfolio is delta-neutral. It works the other way around if the underlying asset price declines. That said, one can notice that the value of your portfolio stays the same regardless of the underlying asset price. Moreover, you are also insured against price fluctuations of the underlying asset, meaning that you will neither realize losses nor gains in your portfolio's value due to changes in the underlying asset's price.

In contrast, hedging with calls does not offer this insurance protection. If the underlying asset increases in value, the value of the call option also rises. Conversely, if the underlying asset decreases in value, the value of the call option also falls. This means that gains or losses are realized simultaneously in both parts of the portfolio. However, potential gains can be disproportionate, while losses from the option are limited to its purchase price. Hedging with calls, therefore, allows for a riskier position and enables disproportionate gains. If you make your portfolio delta-neutral in this case, it merely ensures that the portfolio value remains constant.

1.6 Characteristics of the Bitcoin

Bitcoin is a cryptocurrency traded in an unregulated market availability at all trading times. Transactions are validated by the *Proof of Work* model. For the argumentation required, the following abstract representation of this concept suffices: *Miners* provide computer power to validate transactions in a certain timespan. In return for their investment they are rewarded with Bitcoin and the transactions are stored on a *blockchain*. A blockchain is a database which stores data in a peer-to-peer network by forcing every participant to store a copy of the data on their local device. Thus, the *Proof of Work* model addresses the oracle problem by providing a reliable way to verify external data in a decentralized manner and it becomes possible to transparently track the maximum number of Bitcoin in circulation at any time. In comparison to stocks of a company, these stocks are threatened by the possibility of bankruptcy which leads to a total loss of value. Furthermore, Bitcoin is merely a currency that does not produce any intrinsic value. While its value is also driven by speculation, unlike the stocks of a company, investors' expectations are not influenced by quarterly reports, management decisions, or similar factors. As a result, its value directly reflects supply and demand and the volatility of the Bitcoin price is comparably high.

Moreover Bitcoin is limited to around 20.999.999,97690000 coins which will be distributed till the year 2141 as the amount of rewarded Bitcoin increases in course of time[6]. Additionally, coins that are on a wallet which can no longer be accessed for whatever reason are lost. This fact will increasingly lead to deflation of Bitcoin.

Finally, it should be stated that Bitcoin is the biggest cryptocurrency with a trading volume of 55 billion € per day (January 15th, 2025). Therefore, Bitcoin's network value is correspondingly high. This supports the assumption that Bitcoin, as the primary cryptocurrency used for investment, is unlikely to be replaced by another in the near future. A bear market induced by this scenario, based on this argument, can be considered unlikely.

1.7 Research Questions

Based on the information given and the traits of Bitcoin, we want to examine following questions in the subsequent paragraphs:

Is it possible to hedge Bitcoin with the goal of making the portfolio self-financing?
Is it possible to leverage the properties of Bitcoin to generate long-term gains?
Can we build a financial product with a competitive and constant growth rate?

2 Modelling

2.1 Structure

For our further analysis, we will focus on European options for hedging purposes, as they sufficiently serve our objectives. We will calculate the theoretical option price as well as the option's delta using the Black-Scholes model, either by utilizing the average implied volatility or by applying the historical volatility with a volatility premium of 10 percentage points. As the risk-free interest rate, we will use the ECB's key interest rate[4]. The additional properties of the generated options are based on our attempt to keep the implied volatility low, ensuring that modeling with historical volatility for periods where no data on implied volatility is available remains meaningful under prior conventions. That said, the maturity of the options is set to 90 days and the strike price K is set to the current stock price S when generating the option. The decision to set the option's term to 90 days is based on the consideration that longer terms provide more opportunities for price fluctuations in the underlying asset. Consequently, it is reasonable to assume that implied volatility increases with longer terms. Since the portfolio can be protected against price drops by delta hedging with puts, this is an argument for setting the term as long as possible to achieve the best insurance coverage. Based on these considerations, we decided to set the term to 90 days as a compromise between the two arguments. The decision to set the option's strike price to the current price of the underlying asset is motivated by the fact that, in this case, implied volatility tends to be the lowest (see 1.3 for reference). We assume that there are no transaction costs and that we can buy and sell options with the determined properties at the theoretically $fair$ option price at any point in time. The last assumption can be considered reasonable, as there are so-called synthetic options. These are financial instruments created by combining various positions in underlying assets and/or other derivatives to replicate the same payoff profile as a regular option. We make the portfolio delta-neutral once a day, using the average Bitcoin value of the respective day. In addition to our portfolio, we maintain a clearing account. This clearing account finances the rebalancing of the portfolio and receives any potential returns after a rebalancing. The clearing account has no credit limit, ensuring that the delta-hedging can always be financed. The balance on the clearing account is, however, daily interest-bearing at the ECB's key interest rate. In the case of a positive balance, it serves as a normal investment. In the case of a negative balance, it reflects the opportunity costs, as the capital used for financing the portfolio could have instead earned the ECB's key interest rate. Based on this, *equity* $E(t)$ can be defined:

Let ψ be the hedging unit defined in the option.
Let $\Pi(t) = V(t) + \psi \cdot \Delta \cdot B(t)$, where V is the option price and B is the Bitcoin price[5].
Let $\alpha(t)$ be the balance on the clearing account.
$E(t) := \alpha(t) + \Pi(t)$

2.2 Assumptions of Price Movements

The further development of our hedging algorithms is based on the following assumptions. Since Bitcoin is particularly influenced by supply and demand dynamics, we postulate that Bitcoin prices are subject to short-term fluctuations, which transition into medium-term trend movements. Based on the deflationary argument, we assume that the value of Bitcoin will increase in the long term. In the algorithms we are developing, we switch between hedging with put options and call options at specific points in time. In this way, we aim to eliminate short-term movements through hedging, ensuring that they do not impact the equity. The medium-term trends are to be identified intentionally, and in the case of a positive price trend, hedging will be done with call options, whereas in the event of a price decline, hedging will be done with put options to provide insurance coverage. This approach aims to allow the equity value to grow in a bull market while being protected in a bear market. The challenge lies in accurately identifying these trends and leveraging them accordingly, and it is intended to be addressed as effectively as possible through the development of a suitable algorithm.

2.3 Introduction of Different Models

Three different models are to be considered: the naive model, the conservative model, and the flexible model. In the following, we will develop a function Π and a forecast function Φ for each model. In all models, they have an equivalent meaning and will be used later to make the models comparable. The Π function provides a value that does not necessarily have to be 0 or 1. The forecast function Φ acts as an indicator function, mapping to either 0 or 1. Here, 0 represents an expected price decline for the respective day, and 1 represents an expected price increase. Accordingly, hedging is done either with a put or a call option.

The naive model assumes that the price development for each day will be the same as the previous day. The forecast function Φ for this model is given as follows:

$$\text{Let } \Pi(t) = \begin{cases} 1, & \text{if the Bitcoin price on the previous day was lower} \\ 0, & \text{else.} \end{cases} \tag{2.1}$$

$$\phi(t) := \begin{cases} 1, & \text{if } \Pi(t) > 0.5, \\ 0, & \text{else.} \end{cases} \tag{2.2}$$

The conservative model is not meant to react directly to price fluctuations. For this, a 5-day cache is introduced, which stores the developments of the last 5 days. If there are 3 or more price increases in this cache, a further price increase is assumed; otherwise, a price decline is expected.

$$\text{Let } K(t) = \begin{cases} 1, & \text{if the Bitcoin price on the previous day was lower} \\ 0, & \text{else.} \end{cases} \tag{2.3}$$

$$\text{Let } \Pi(t) = \frac{K(t) + K(t-1) + ... + K(t-4)}{5} \tag{2.4}$$

$$\phi(t) := \begin{cases} 1, & \text{if } \Pi(t) > 0.5, \\ 0, & \text{else.} \end{cases} \tag{2.5}$$

The flexible model is meant to combine the properties of both models. The flexible model is designed to combine the characteristics of both models. While the naive model is very sensitive to price fluctuations, the conservative model requires some time to recognize a price uptrend or downturn. It is therefore less prone to frequent changes in the forecast during sideways markets, but it cannot capture market dynamics as quickly and may not immediately recognize a bull market, which can be extremely profitable with correct hedging. Instead of a 5-day cache, the flexible model should use a short-term and a long-term cache, which operate according to the same scheme. In this way, a short-term and a long-term perspective on the market should be used for the forecast. While the long-term cache is set to a constant 10 days, the short-term cache can vary between 2 and 5 days.

$$\text{Let } K(t) = \begin{cases} 1, & \text{if the Bitcoin price on the previous day was lower} \\ 0, & \text{else.} \end{cases} \tag{2.6}$$

$$\text{Let } \Sigma_{short,k}(t) = \frac{K(t) + K(t-1) + ... + K(t-k+1)}{k}, \; k \in [2,5] \tag{2.7}$$

$$\text{Let } \Sigma_{long}(t) = \frac{K(t) + K(t-1) + ... + K(t-9)}{10} \tag{2.8}$$

For each possible short-term cache, a separate forecast Φ_k should now be developed. The weighting between the short-term and long-term caches should be learned from the past three days using linear regression.

$$\text{Let } \Pi_k(t) = \mu_k \cdot \Sigma_k(t) + \mu_{10} \cdot \Sigma_{10}(t) \tag{2.9}$$

$$\text{Solve for } \begin{pmatrix} \mu_k \\ \mu_{10} \end{pmatrix} : \quad \begin{cases} \Pi_k(t) = K(t) \\ \Pi_k(t-1) = K(t-1) \\ \Pi_k(t-2) = K(t-2) \end{cases} \tag{2.10}$$

To solve the linear systems of equations, we used the Moore-Penrose inverse of the corresponding matrices. After the coefficients of Π_k have been determined, the forecast function Φ_k of the k^{th} model can now be defined:

$$\phi_k(t) = \begin{cases} 1, & \text{if } \Pi_k(t) > 0.5 + \text{Margin}, \\ 0, & \text{else.} \end{cases} \tag{2.11}$$

The margin is set to 0.05 in this model and is intended to prevent constant switching between hedging with put and call options in sideways markets. Additionally, the margin can be adjusted to control how conservative the investment is.
After calculating for every $k \in [2, 5]$ the corresponding model, these models should evaluate their individual forecast functions for the last 7 days. Based on this comparison, the model that would have provided the most correct predictions over the past 7 days will be selected for forecasting the next day. If two models have provided the same number of correct predictions, the model with the larger k-value should be selected, meaning the one that acts more long-term.

3 Results

3.1 Comparison of Different Models

Figure 3.1 visualizes the theoretical equity development from January 1, 2016, to December 10, 2024, with an initial investment of 1000 € using different hedging algorithms. For this simulation, the historical volatility was assumed with a volatility premium of 0.1 to calculate the option prices. The simulation that generated Figure 3.1, on the other hand, used implied volatility to demonstrate that the approach is comparably profitable even when using implied volatility. Since the motivation was to develop a long-term successful model and only the implied volatility data between March 2023 and December 2024 was available, we used simulations with historical volatility and a volatility premium as an estimator for implied volatility during the model development. Figure 3.2 further breaks down the equity development into the performance of its individual components. It is clearly visible how the portfolio value is regularly reset to 1000 €. This is because 1000 € is the value that is intended to be maintained through hedging. When a new option is purchased (either because the hedging shifts from calls to puts or because the current option expires), the portfolio is reset with 1000 € as the investment. Any potential gains are then transferred to the clearing account. During phases when the value of the clearing account falls below 0 €, the portfolio is not self-financed.

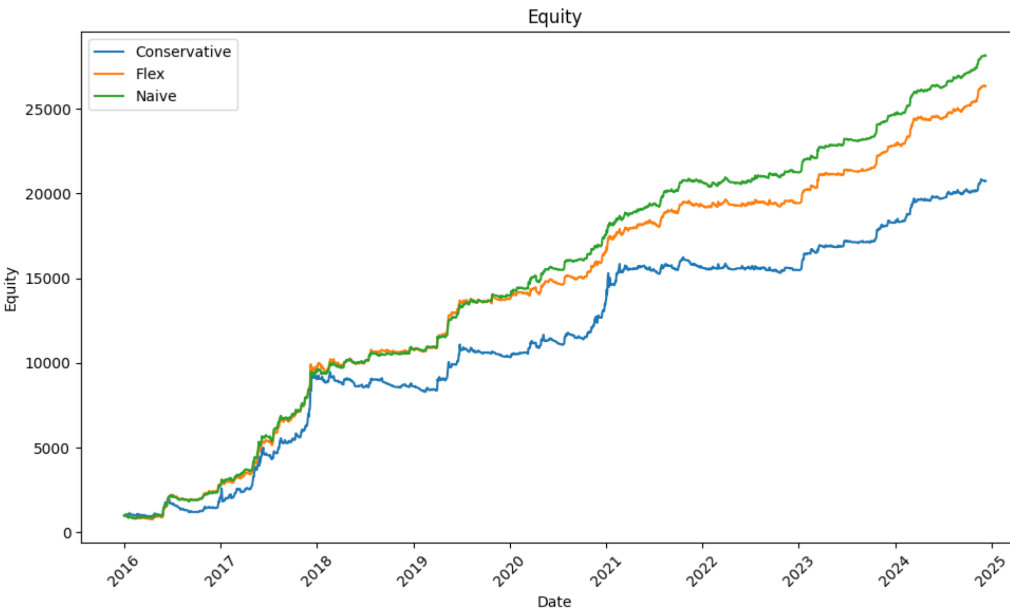


Figure 3.1: Equity development using different models

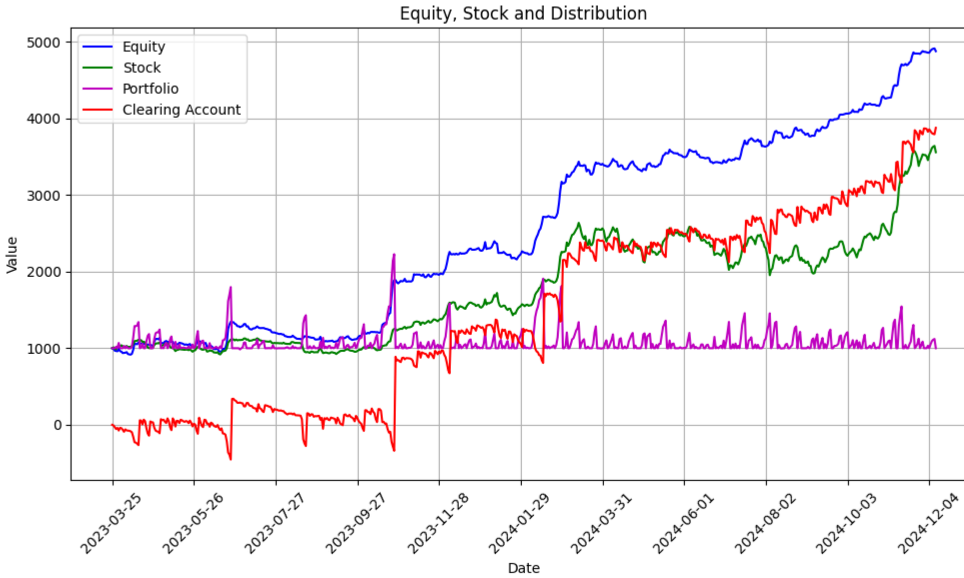


Figure 3.2: Equity development using the naive model and implied volatility

When looking at the price developments in Figure 3.1, one might think that the naive model is the best, as it achieved the highest return in this scenario. In addition to the highest return of the three models, the naive model also shows desirable characteristics in the way the value develops. In comparison, the conservative model shows a more step-like progression. This is because the conservative model tends to focus on secure price increases and, otherwise, hedges its position and value. A look at the evaluations of the Π function of the naive model in Figure 3.3, however, puts this assumption into perspective. The horizontal red line here reflects the critical point at which the Φ function switches between 0 and 1 in the evaluation. However, this switch, which is performed when the graph crosses the red line, means that the hedging strategy must shift from hedging with puts to hedging with calls, and it requires the current option to be replaced with a new one. This results in transaction costs. Since transaction costs are neglected in our simulation, the naive model is the best in this specific setting, but it takes advantage of this assumption. Compared to the naive model, the conservative model uses significantly fewer switches in the hedging strategy, which puts the difference between the returns of the naive model and the conservative model into perspective in terms of transaction costs.
In summary, the naive model has desirable characteristics when transaction costs are neglected, while the conservative model exhibits good properties in terms of the Π function when transaction costs are considered. Based on this insight, the flexible model was developed, with the goal of having a similar price development to the naive model, while requiring significantly fewer switches between hedging with puts and hedging with calls.

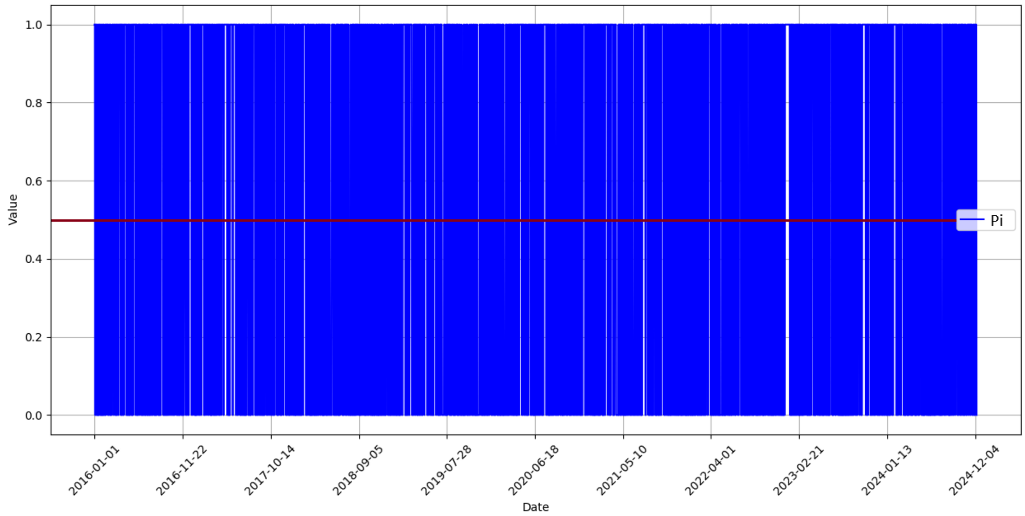


Figure 3.3: Π function of the naive model

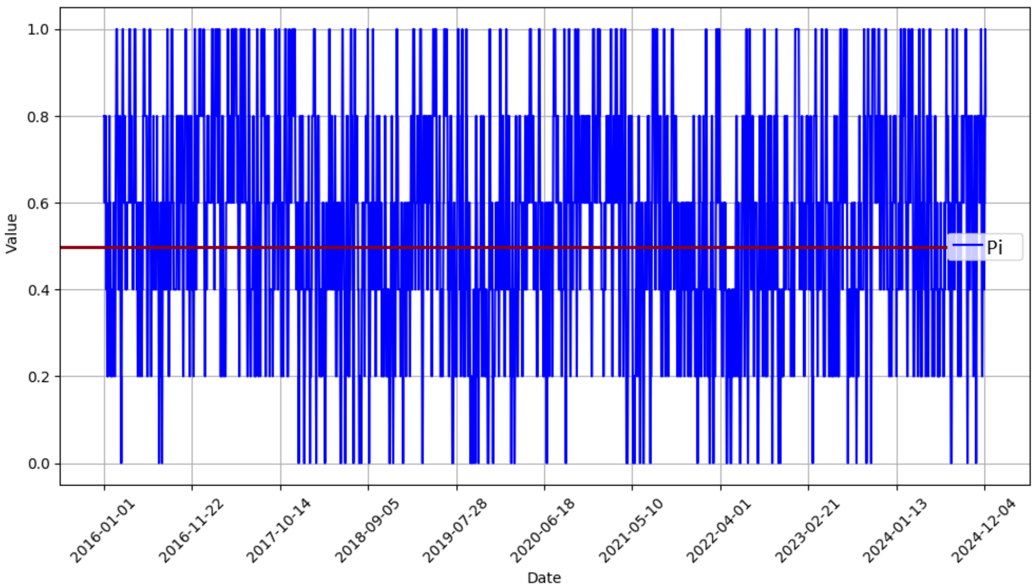


Figure 3.4: Π function of the conservative model

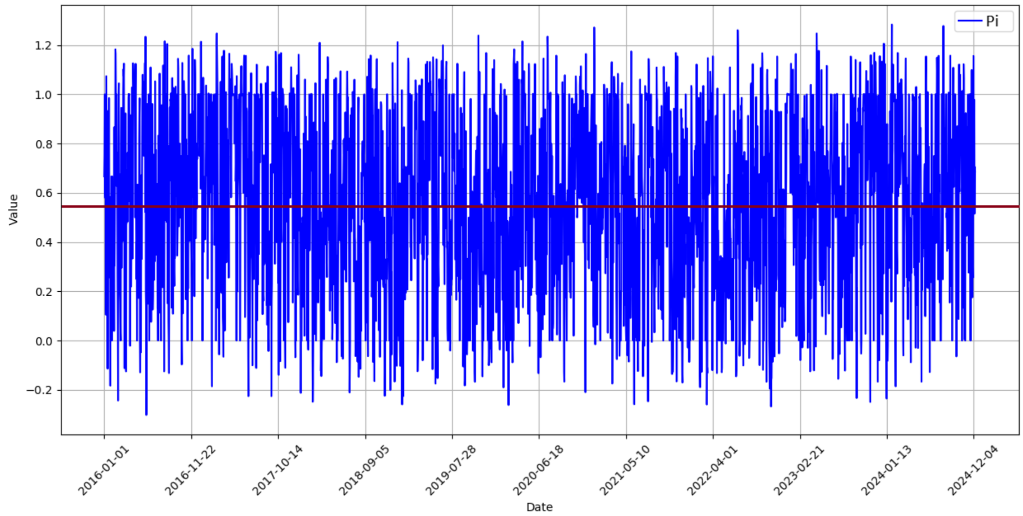


Figure 3.5: Π function of the flexible model

As Table 3.1 quantitatively shows, the geometric mean and the standard deviation of the annual returns of the flexible model are much closer to the values of the naive model than to the conservative model. Additionally, the flexible model manages to deliver a positive return in 2019 and 2023, years in which Bitcoin experienced a significant decline in value.

Year	Bitcoin	Conservativ	Flexibel	Naiv
2017	123.73%	110.74%	184.4%	179.83%
2018	1364.44%	328.92%	240.39%	238.25%
2019	-73.45%	-4.61%	11.5%	14.19%
2020	92.03%	19.85%	27.59%	29.23%
2021	302.98%	33.17%	20.38%	25.85%
2022	59.73%	13.52%	16.35%	17.6%
2023	-64.27%	-0.97%	0.69%	2.73%
2024	155.5%	18.33%	17.62%	16.2%
Geometric Mean	77.42%	42.83%	47.86%	49.29%
Standard Deviation	468.13%	112.64%	92.6%	90.32%

Table 3.1: Model comparison

3.2 Assessment and Outlook

The results obtained in this analysis provide an initial approach for the further development of a financial product, which should be explored in more depth for real-world application. To build upon this, the following limitations of the system used here will be highlighted.
In the model considered here, transaction costs were neglected. This limitation is addressed through the use of the flexible model. A more detailed quantitative analysis should therefore be conducted in future studies, in which transaction costs are explicitly considered and simulated.
It was also assumed that options could be bought at any time at the fair transaction price. This assumption is justified by the potential use of synthetic options. Thus, it is possible to replicate any option using other financial products. However, since the Black-Scholes model calculates the theoretical price of a real option, this may not necessarily align with the price of a possible replication. In fact, it is conceivable that synthetic options could be either more expensive or cheaper, as their pricing is determined by arbitrage-free conditions in their respective markets. A suitable selection of various financial products, which can then specifically replicate the desired position, can be further investigated in future simulations.
In addition to these model errors, the algorithm of this model can also be further developed. Medium-term price movements were determined solely based on past price fluctuations. To identify these trends more reliably, the magnitude of the respective price movements and volatility can be quantitatively considered. A statistical estimation through time series analysis could thus be a feasible approach. Furthermore, instead of using the Black-Scholes model for option pricing, a Monte Carlo simulation with mean-reverting volatility can be employed. This approach eliminates the need to assume constant volatility over the period from option purchase to expiration. In certain cases, this makes it possible to minimize errors in the delta calculation, thereby improving the portfolio's hedging. Additionally, the leverage that a call option has on the development of the underlying asset's price could be adjusted by varying the time to maturity instead of keeping it constant at 90 days.

Overall, however, our analysis shows that despite the simplifications made for the simulation, such a financial product, with further development, potentially possesses the previously required properties, and it makes sense to conduct further investigations based on it.

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