

Dynamic Programming (3)

By: Aminul Islam

Based on Chapter 3 of Foundations of Algorithms

Objectives

- Describe the Dynamic Programming Technique
- Contrast the Divide and Conquer and Dynamic Programming approaches to solving problems
- Identify when dynamic programming should be used to solve a problem

Example of 0-1 Knapsack problem (Brute force sol.)

item (i)	1	2	3	4
value (v)	100	20	60	40
weight (w)	3	2	4	1

- Assume Knapsack's max weight capacity is $W = 5$
- How to fill the knapsack with items such that the value is maximum?

items picked= item 1 and item 4

value= 140

weight= 4

Order of complexity= $\Theta(2^n)$

1	2	3	4	W	V
0	0	0	0	0	0
0	0	0	1	1	40
0	0	1	0	4	60
0	0	1	1	5	100
0	1	0	0	2	20
0	1	0	1	3	60
0	1	1	0	6	80
0	1	1	1	7	120
1	0	0	0	3	100
1	0	0	1	4	140
1	0	1	0	7	160
1	0	1	1	8	200
1	1	0	0	5	120
1	1	0	1	6	160
1	1	1	0	9	180
1	1	1	1	10	220

* 1 means item is picked and 0 means item is not picked

Here, we went through all possible solutions/cases and picked the one that maximizes the value. This is known as **Brute force algorithm**.

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- However, not all optimization problems can be solved using DP
- The “principle of optimality” must apply in the problem!

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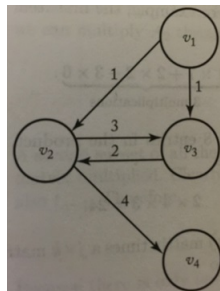
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- The principle of optimality is applied in a problem if an optimal solution for a problem instance includes optimal solutions for all sub-problems
- If the principle holds, we can provide a recursive solution and obtain optimal solutions from smaller ones

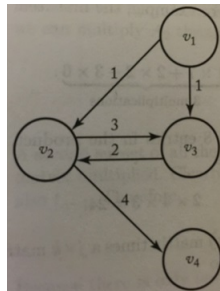
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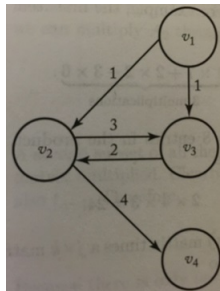
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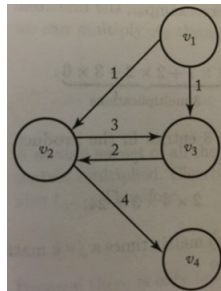
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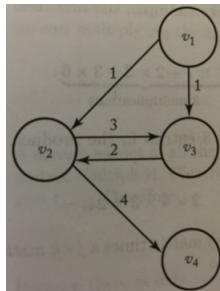
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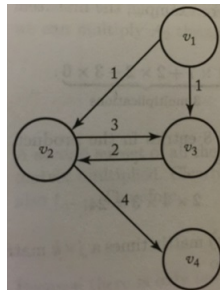
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- Suppose V_k is a vertex in an optimal path from V_i to V_j
- Then the sub-path from V_i to V_k and from V_k to V_j are also optimal
- Therefore, the principle of optimality holds!



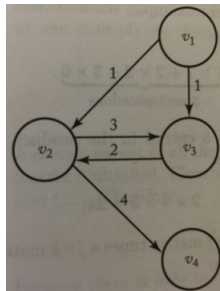
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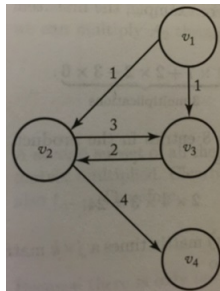
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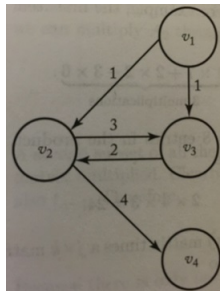
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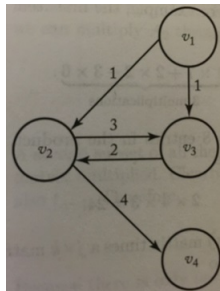
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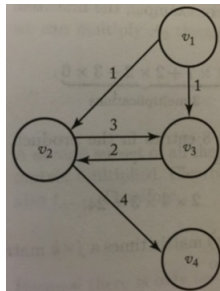
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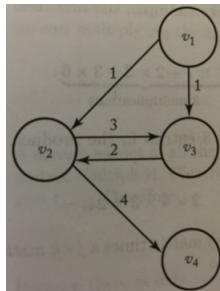
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- However, sub-path $V_1 V_3$ is not the longest possible!
- In fact, $V_1 V_2 V_3$ is the longest simple path from V_1 to V_3
- Principle of optimality does not hold in this case!



Steps in Dynamic Programming

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- Characterize the structure of an optimal solution.
- Recursively define the value of an optimal solution.
- Compute the **value** of an optimal solution, typically in a bottom-up fashion.
- Construct an **optimal solution** from computed values.

Example of 0-1 Knapsack Problem (DP solution)

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Algorithm: DP solution to 0-1 Knapsack Problem

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```
KnapSack (int W, int n, int val[], int wt[] )
{
    int i, w;
    int V[n+1,W+1];
    for (w=0; w<=W; w++){
        V[0,w]=0; }
    for (i=0; i<=n; i++){
        V[i,0]=0; }
    for (i=1; i<=n; i++){
        for (w=1; w<=W; w++){
            if (wt[i]<=w){
                V[i,w] = max(V[i-1,w], val[i]+V[i-1, w-wt[i]]) }
            else {
                V[i,w] = V[i-1,w] }
        }
    }
    return V[n,W];
    // Add the algorithm on the next slide here for optimal sol.
}
```

Constructing the Optimal Solution

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```
w=W;
for (i = n downto 1){
    if (V[i,w] != V[i-1,w]){
        output i ;
        w = w - wt[i] ;
    }
}
```

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    V[i,w] = V[i-1,w]
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$$wt[i] \leq w$$

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i = 0	0	0	0	0	0	0
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item (i)	1	2	3	4
value (val[])	100	20	60	40
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→3	0	0	20			
4	0					

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→3	0	0	20			
4	0					

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3	0	0	20	100	100	120
→4	0	40	40			

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→4	0	40	40			

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→4	0	40	40			

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