Computational Complexity and Intractability

By: Aminul Islam

Based on Chapter 9 of Foundations of Algorithms

Objectives

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- Classify problems as tractable or intractable
- Define decision problems
- Define the class P
- Define the class NP
- Define the class of NP-Complete

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- It is the property of the problem not the algorithm

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3. Problems that have not been proven to be intractable, but for which polynomial-time algorithms have never been found

Not proven to be intractable no existing polynomial time algorithm

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- Decision problems
- The class P
- Nondeterministic algorithms
- The class NP
- Polynomial transformations
- The class of NP-Complete

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- Optimization problems are at least as hard as the associated decision problem
- If polynomial-time algorithm for the optimization problem is found, we would have a polynomial-time algorithm for the corresponding decision problem

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  - For a given profit P, is it possible to load the knapsack such that total weight $<= W$?

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    - No one knows
    - To know a decision problem is not in P, it <u>must be proven</u> it is not possible to develop a polynomial-time algorithm to solve it

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- For instance Traveling salesperson decision problem belongs to NP

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- To show P = NP, find polynomial-time algorithm for each problem in NP

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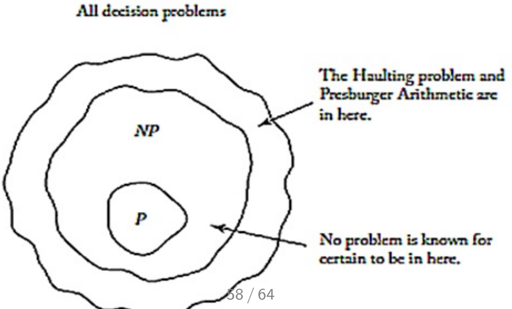

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- How to prove that a problem is NP-complete?
  - Suppose that you want to know if problem B is NP-complete or not?
  - You know problem A as an NP-complete problem
  - If you can reduce problem A to B, you have proved that B is also NP-complete