Branch and Bound (2)

By: Aminul Islam

Based on Chapter 6 of Foundations of Algorithms

Objectives

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- Describe the branch-and-bound technique for solving optimization problems
- Contrast the branch-and-bound technique with the backtracking
- Apply the branch-and-bound technique to solve the 0-1 Knapsack Problem, the Travelling Salesman Problem and Assignment Problem

Travelling salesman problem (TSP)

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- Given a list of cities and the distances between each pair of cities, the task is to find the shortest possible route that visits each city exactly once and returns to the origin city.

Travelling salesman problem (TSP) (2)

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- Goal: find an optimal tour
  - Starts at a given city
  - Visits every city exactly once
  - Returns to the starting city
- Such that the total distanced traveled is minimal

Example of TSP using Best-First Search with Branch and Bound

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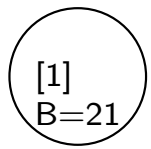

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- Determine promising, unexpanded node with the smallest bound
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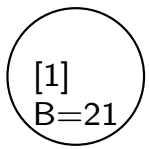

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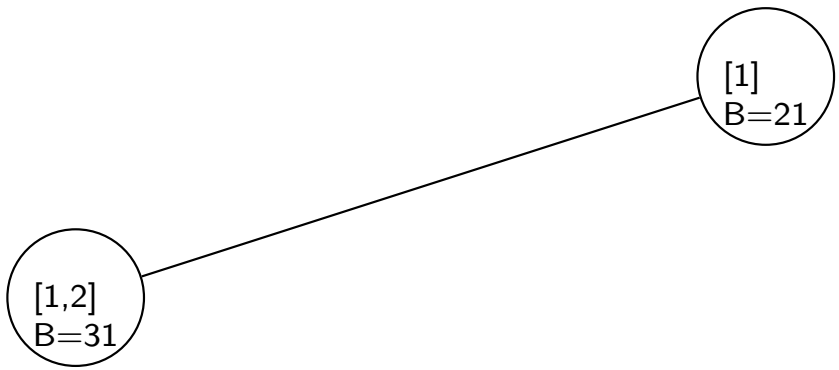

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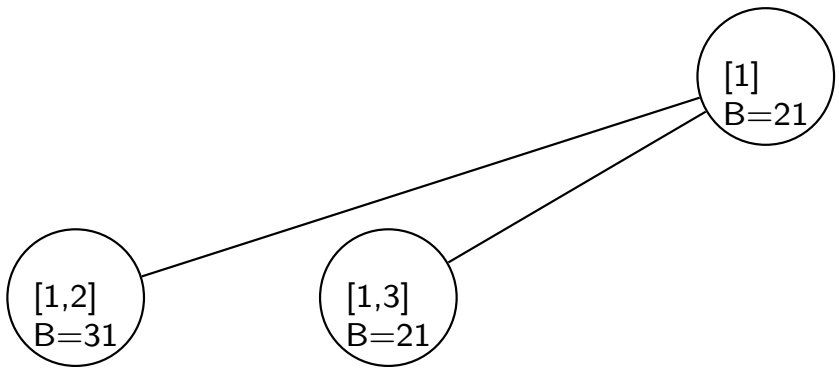

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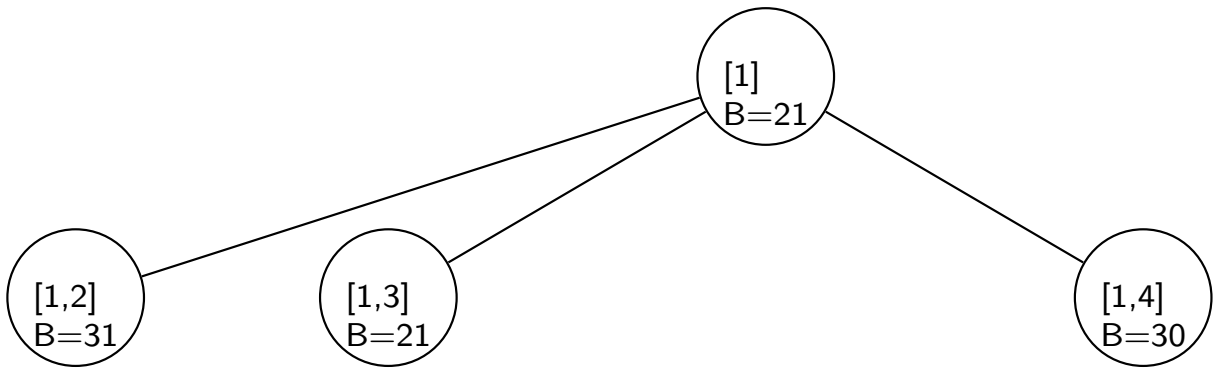

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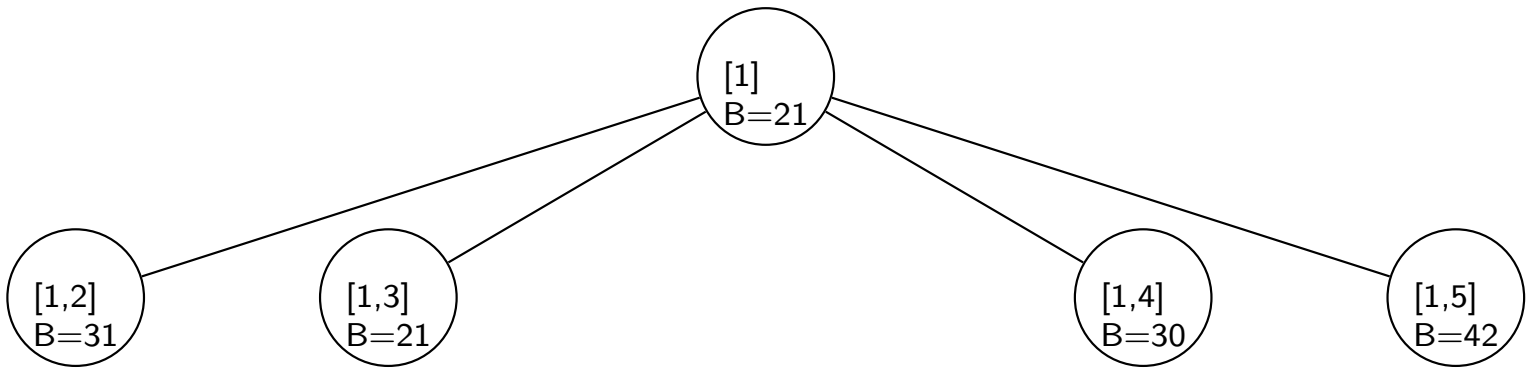

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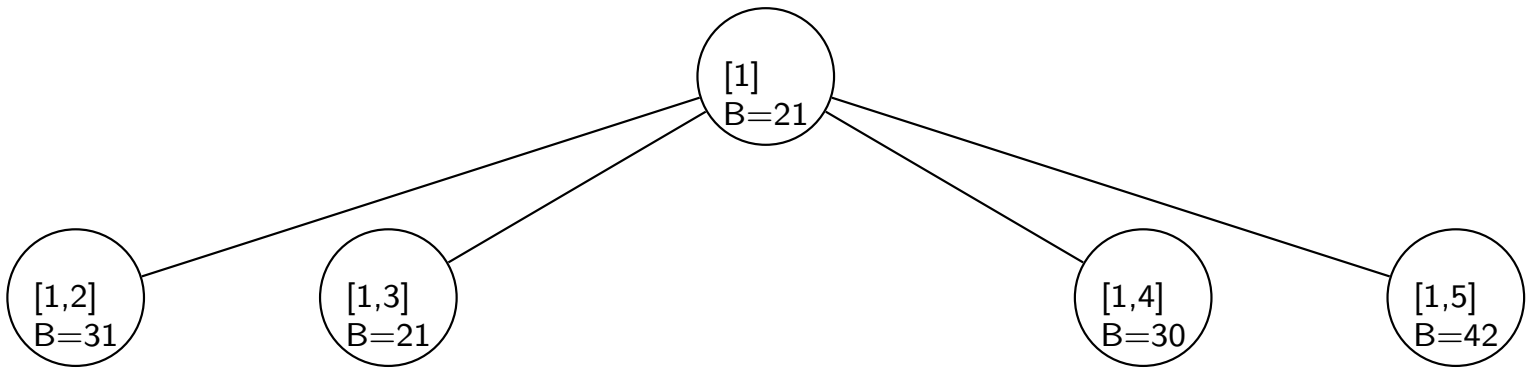

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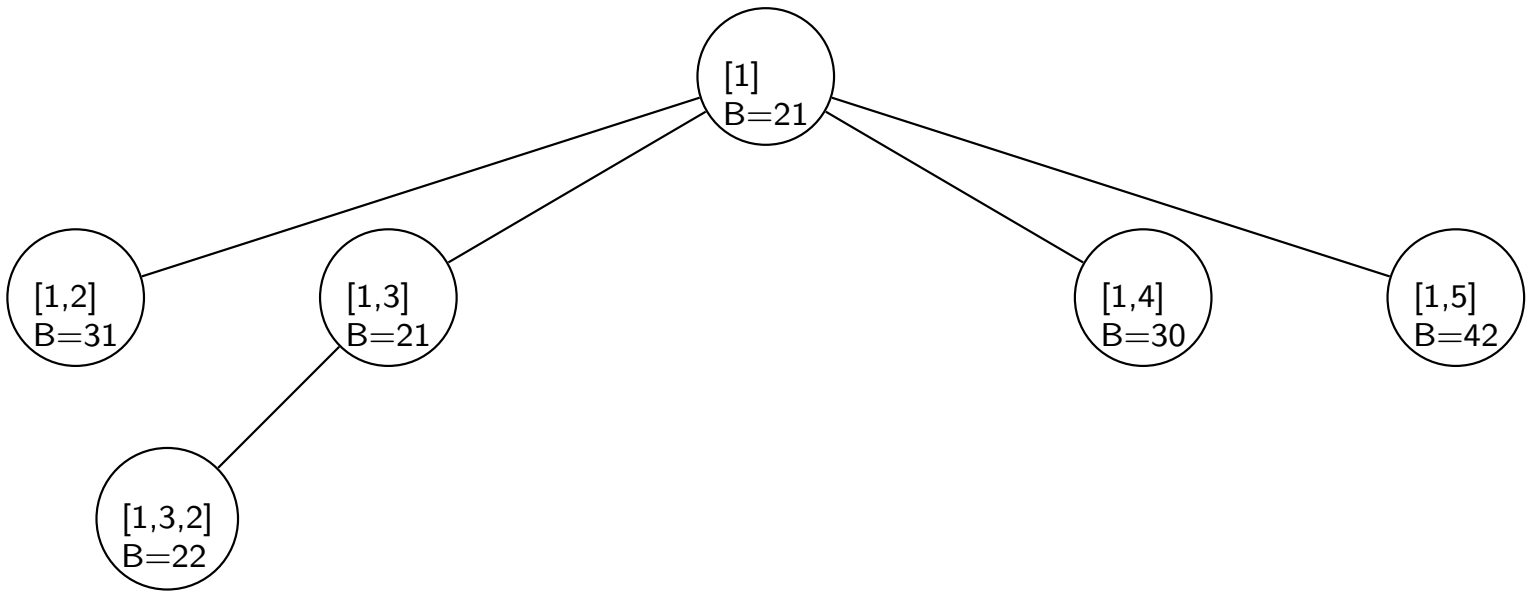

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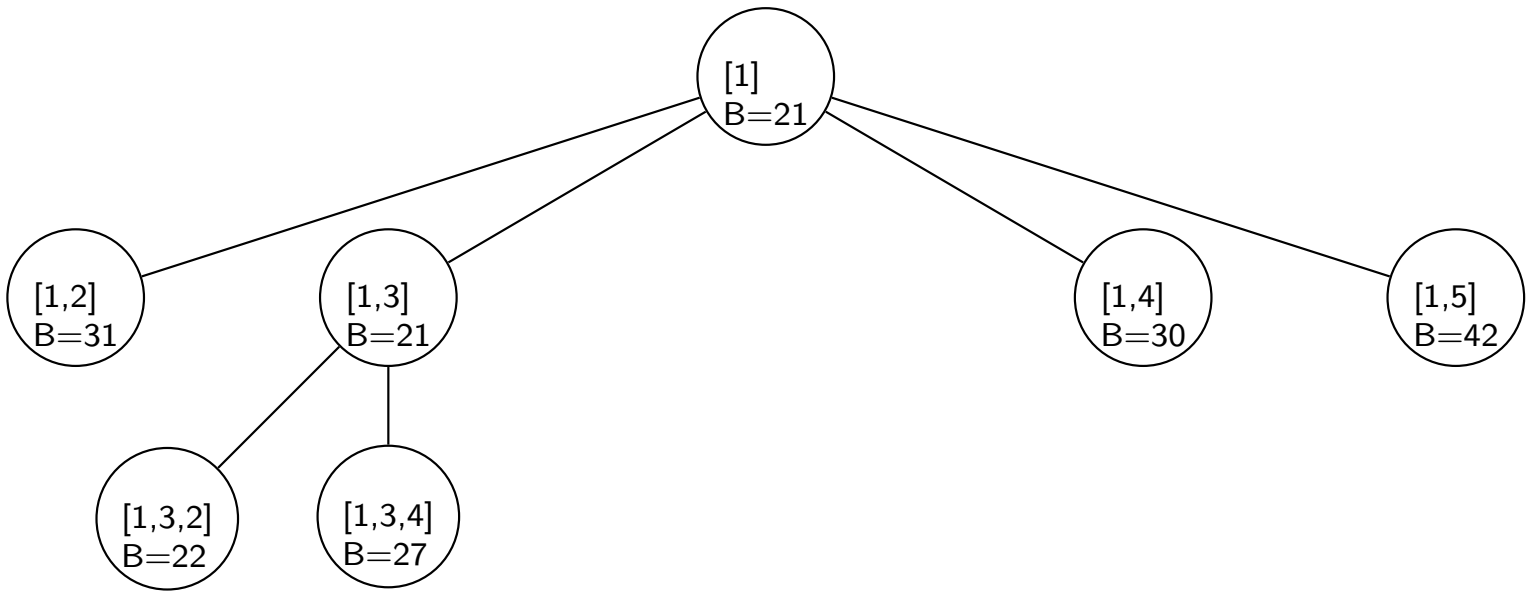

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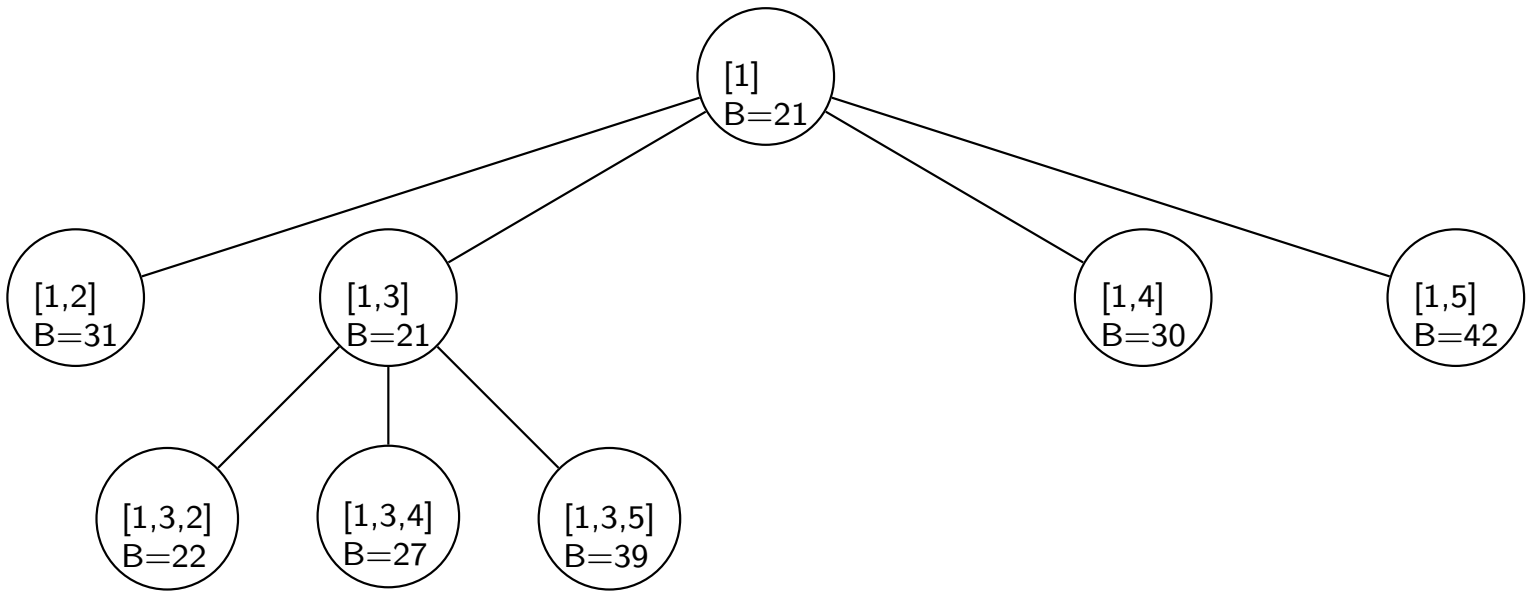

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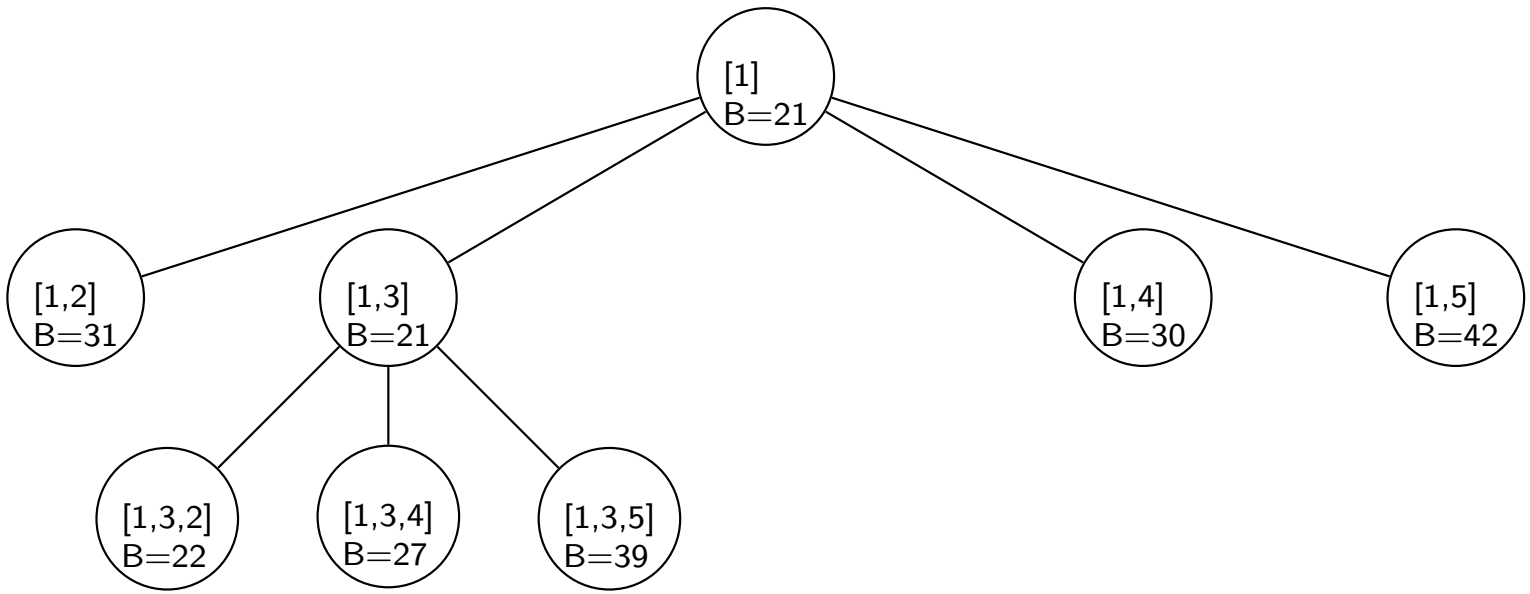

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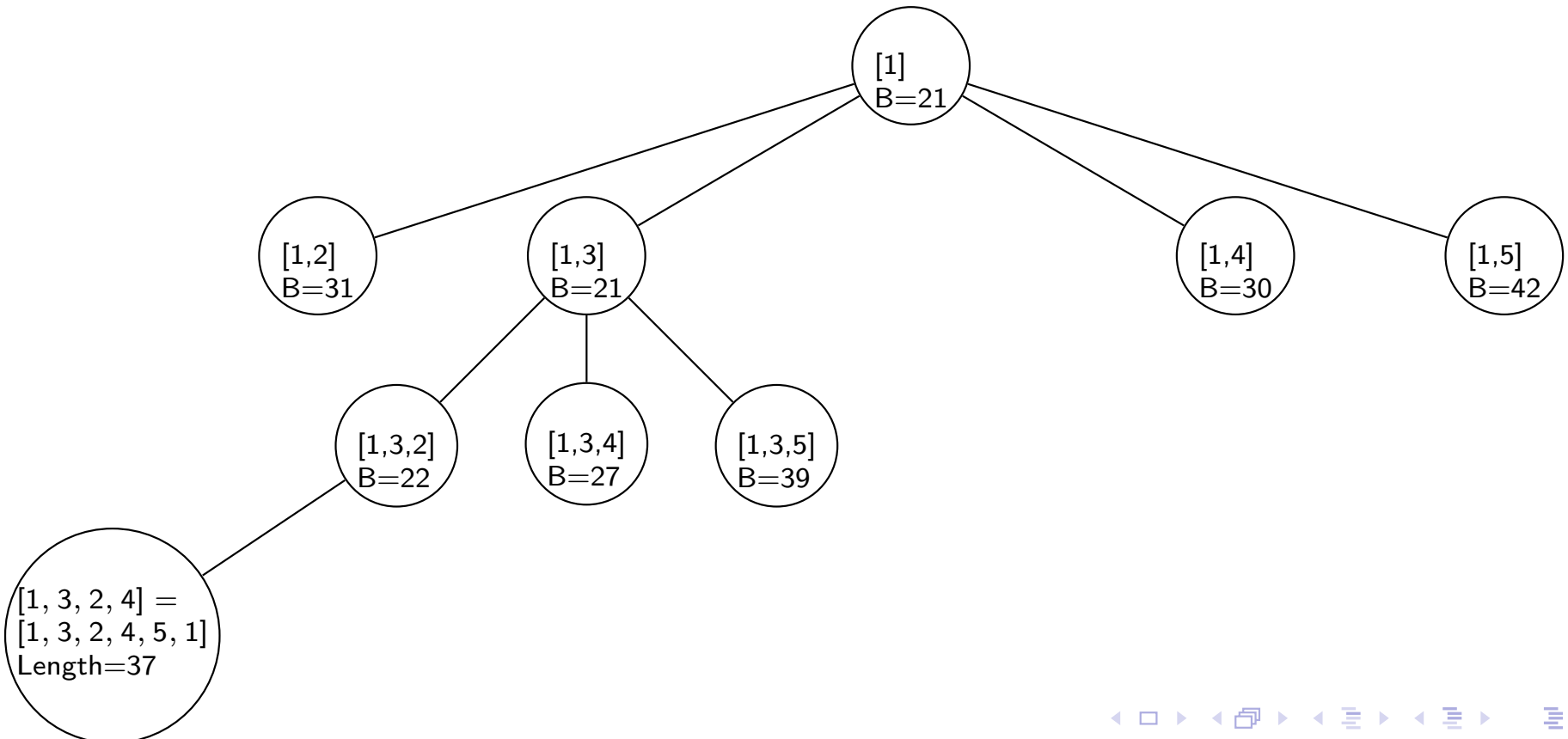

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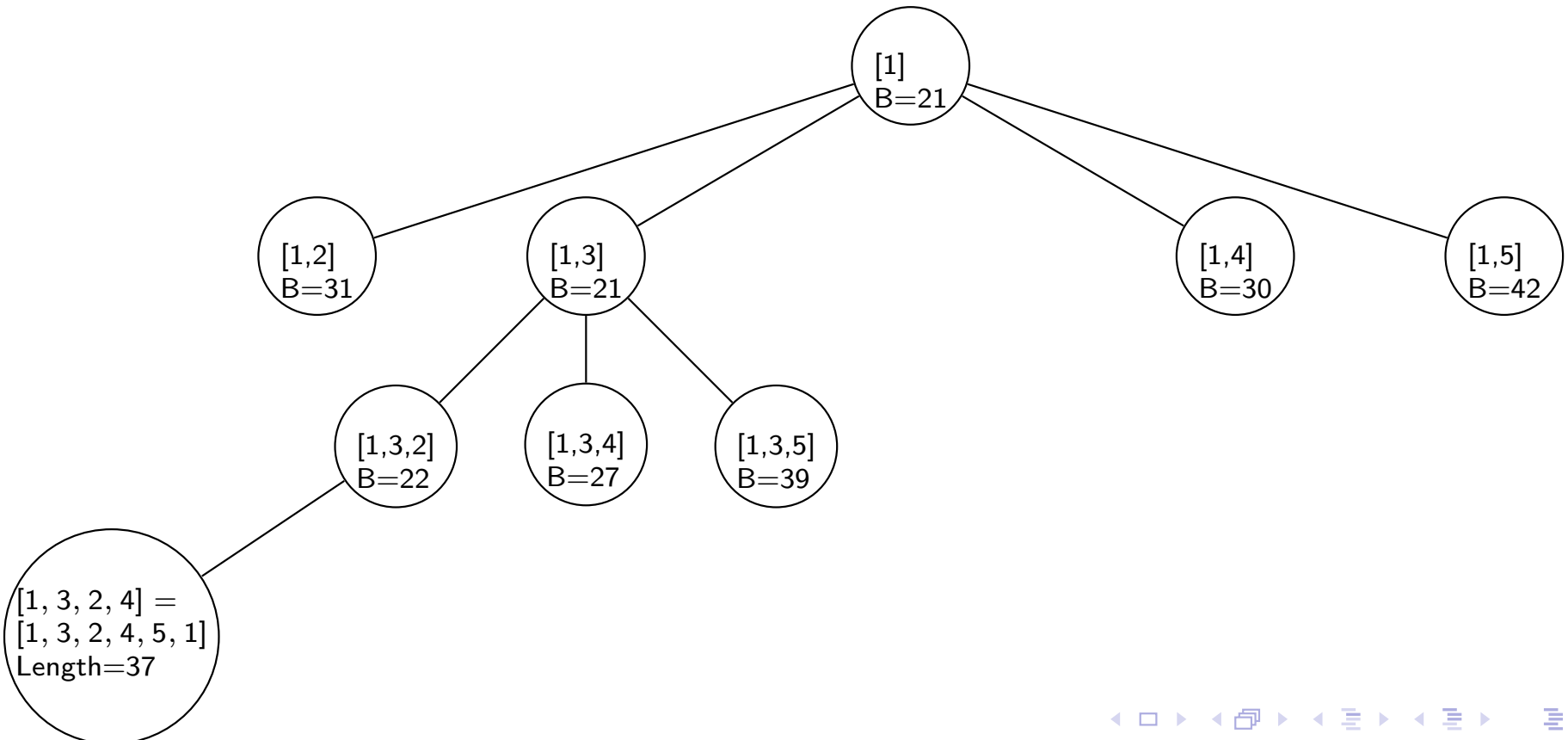

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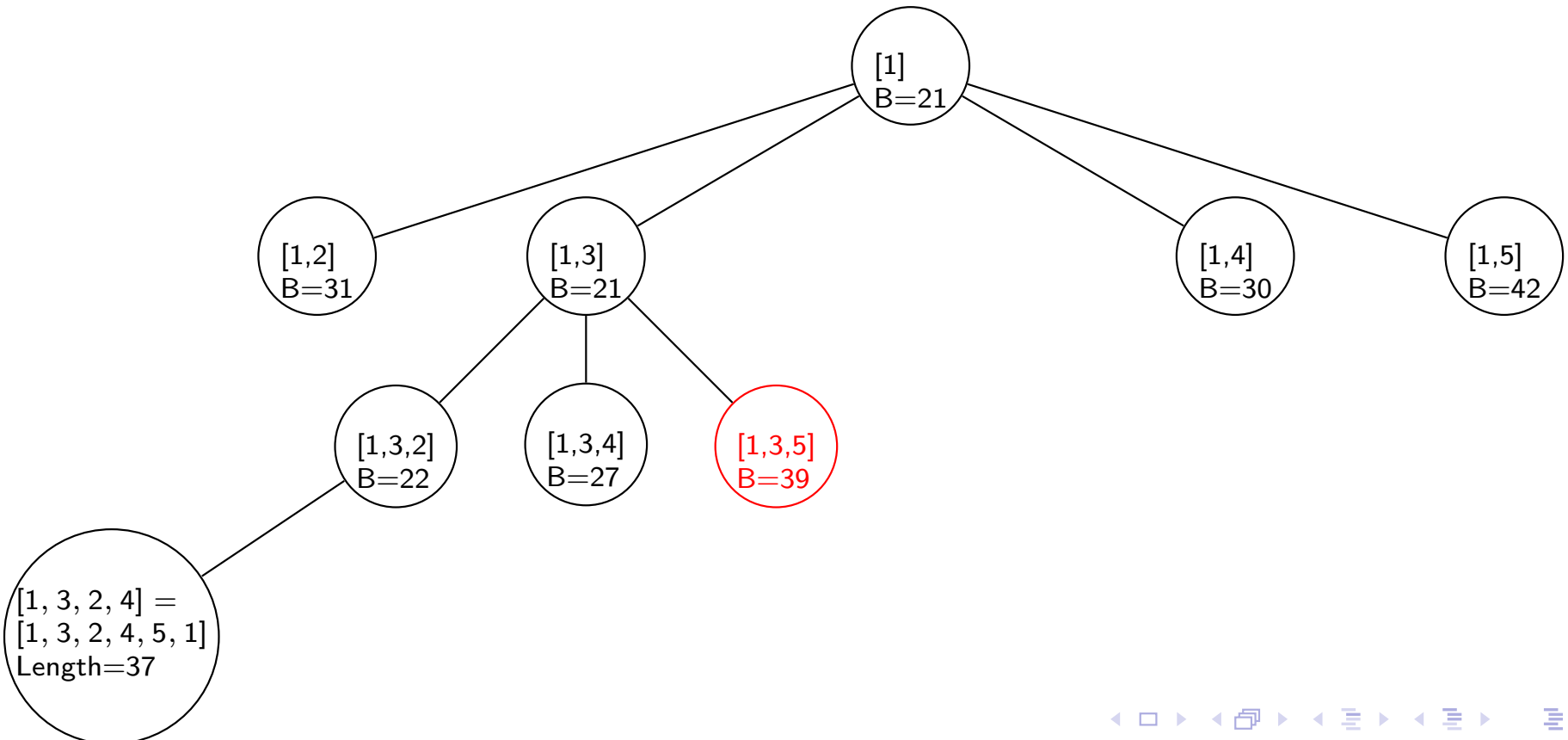

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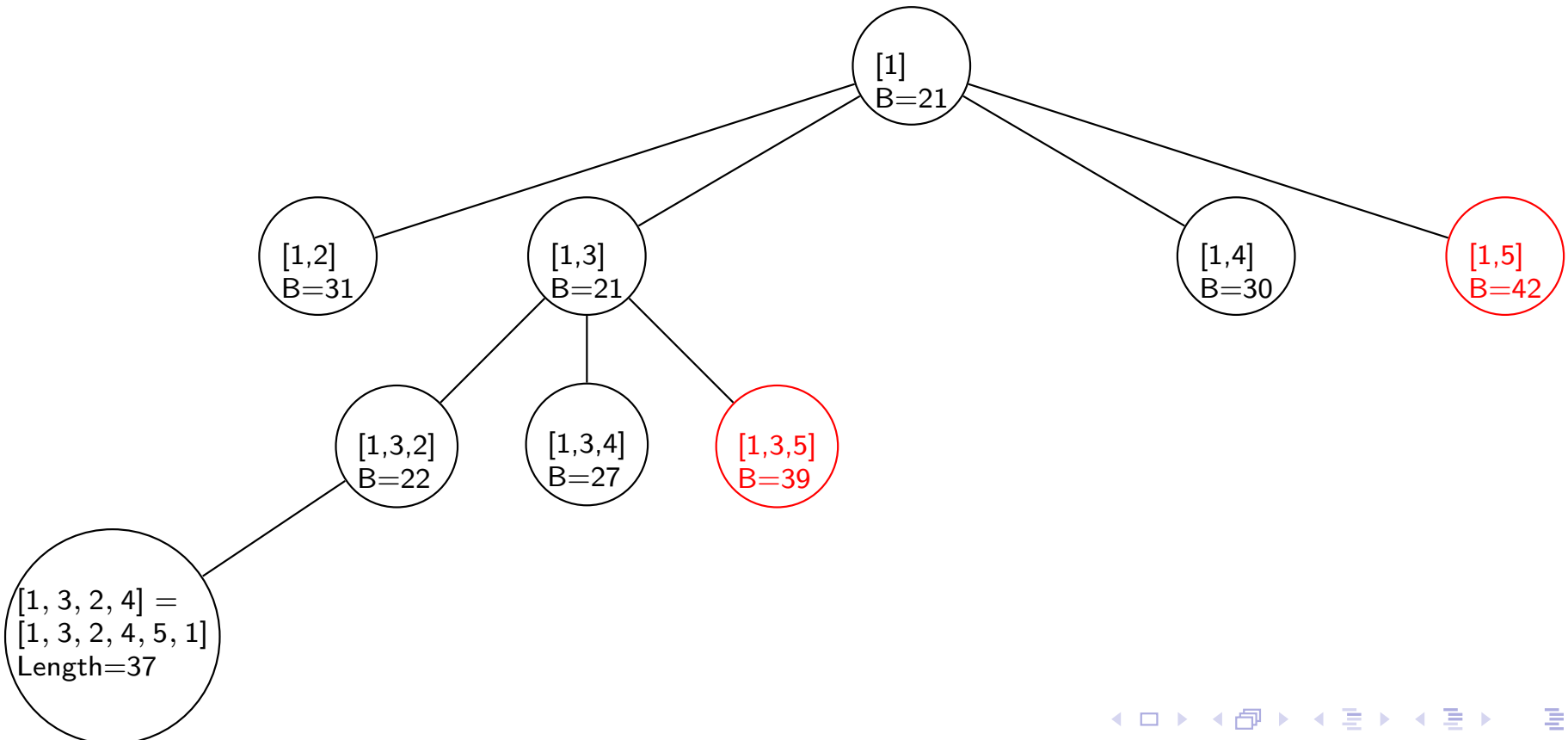

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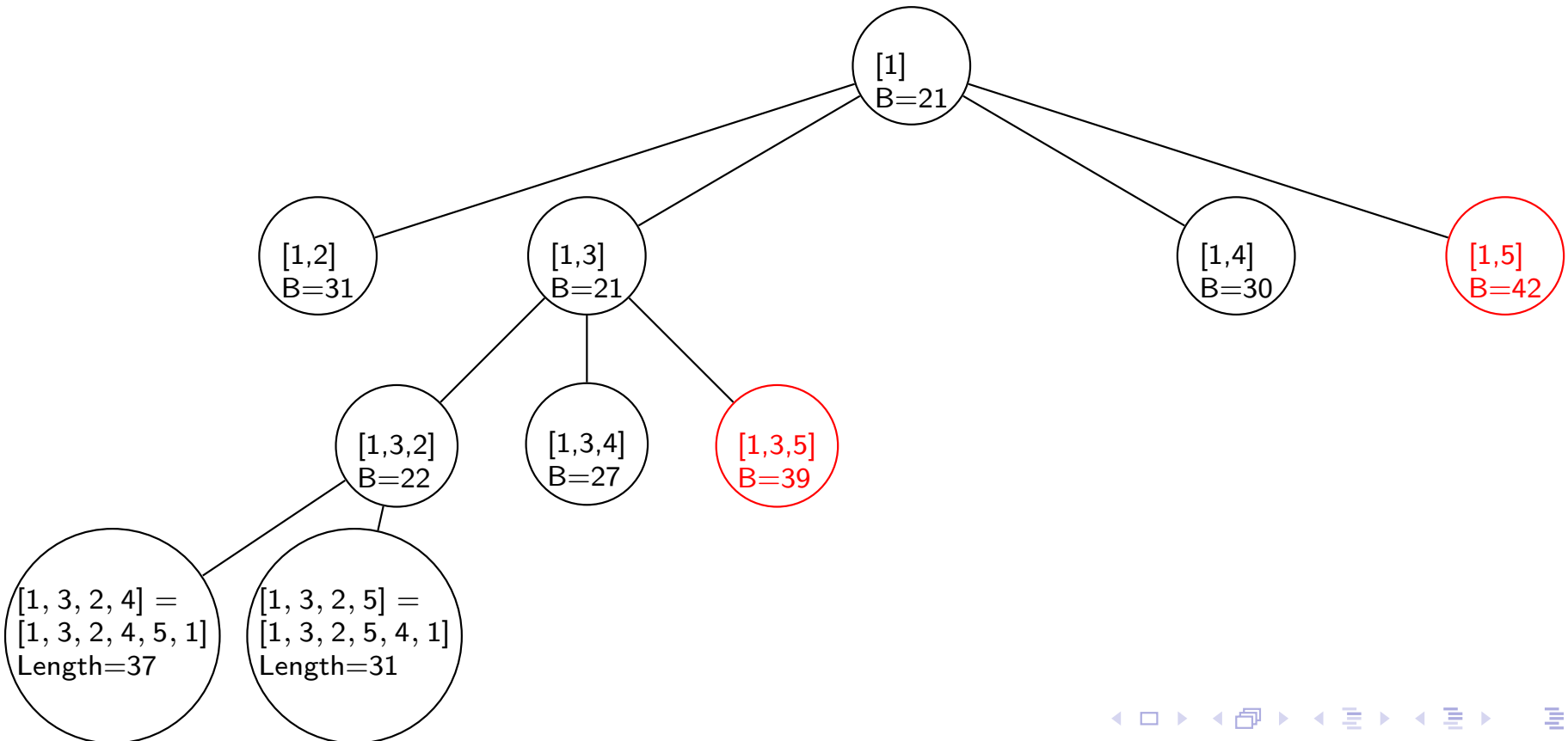

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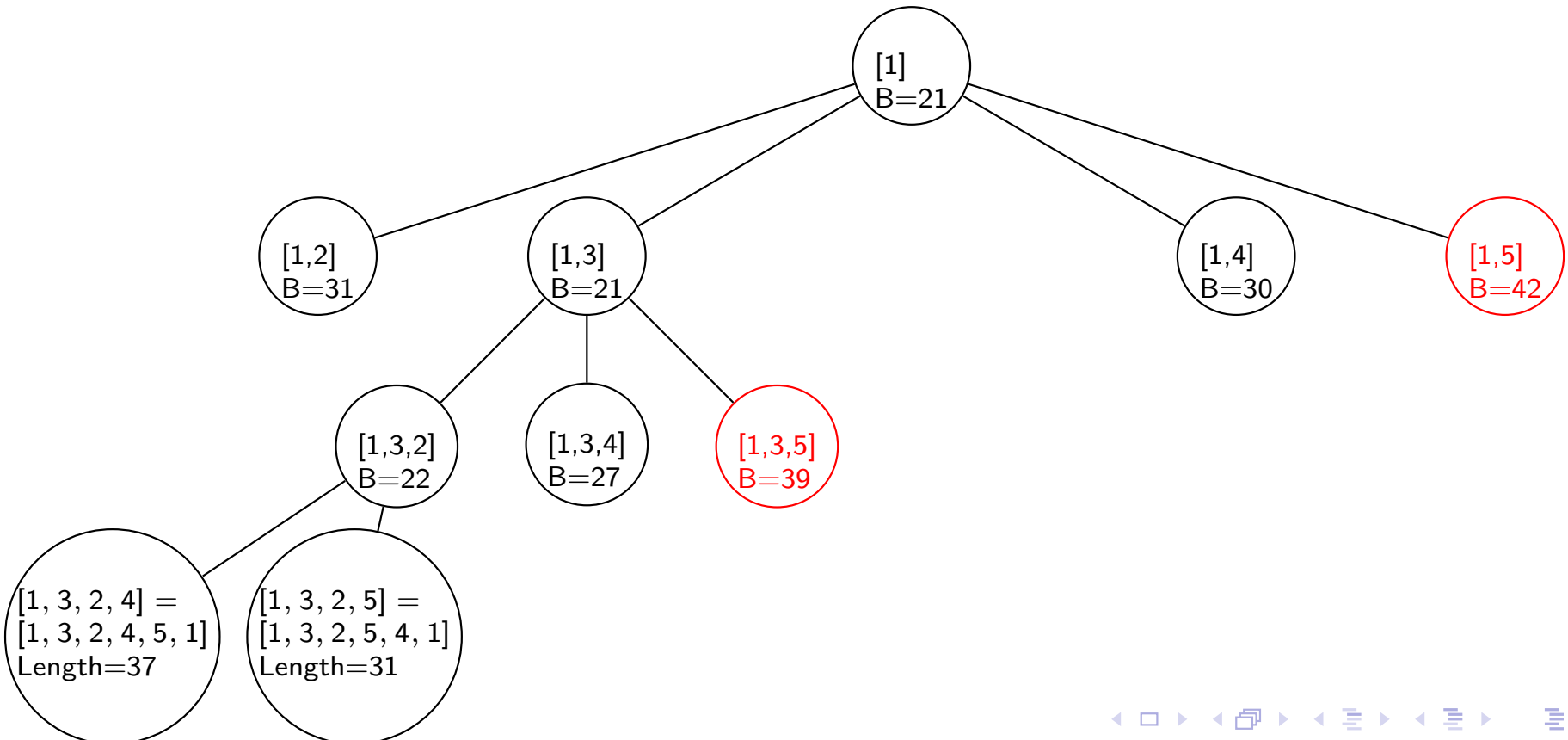

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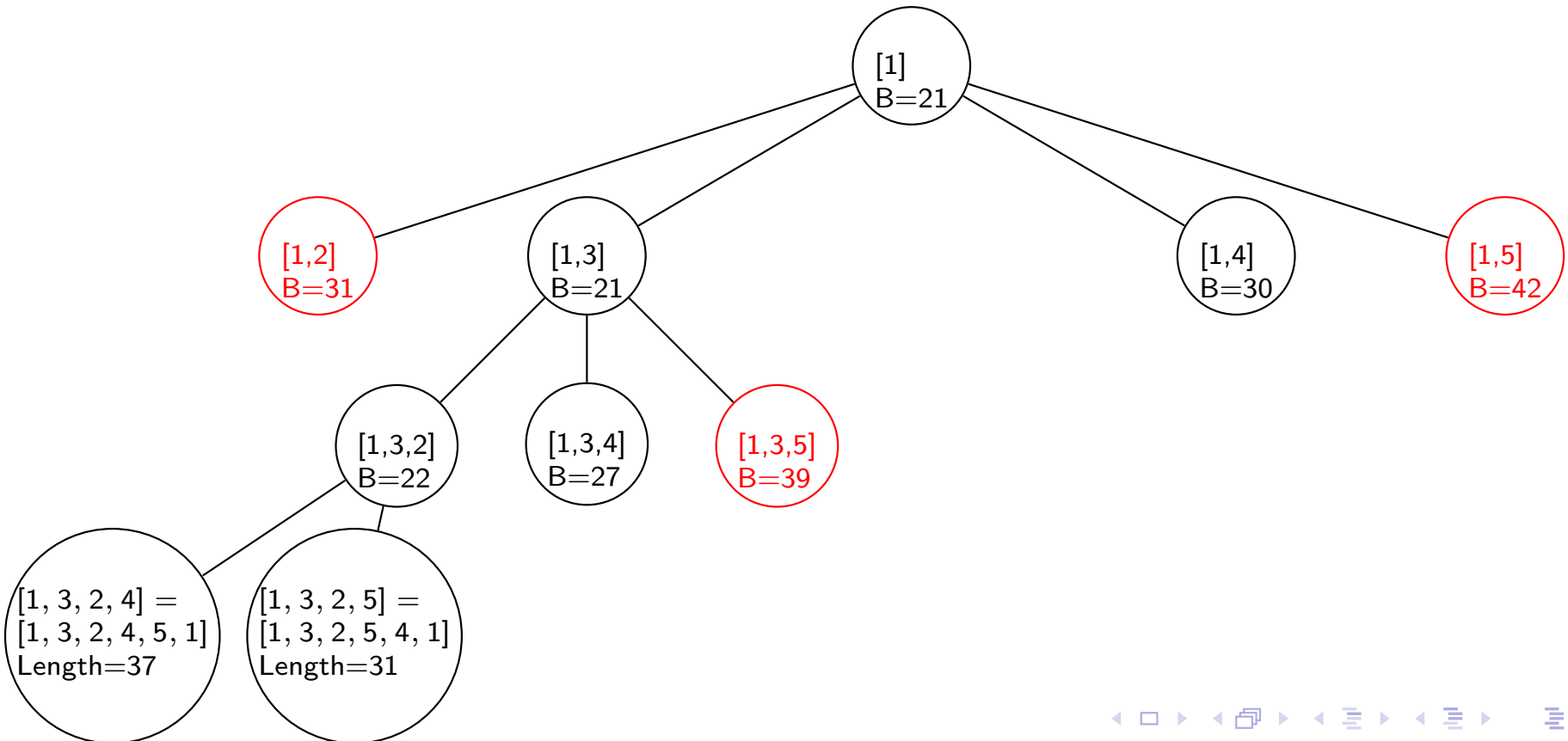

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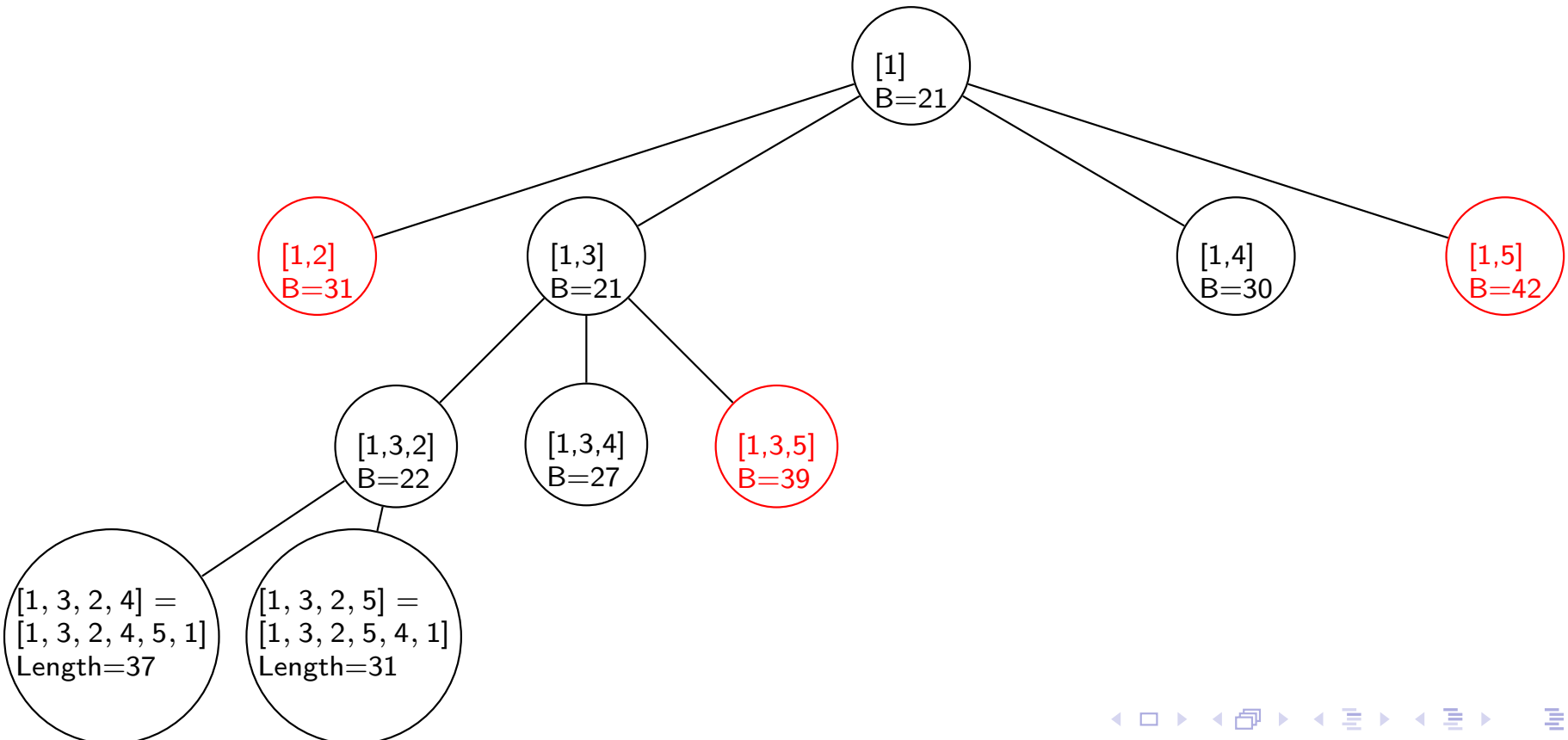

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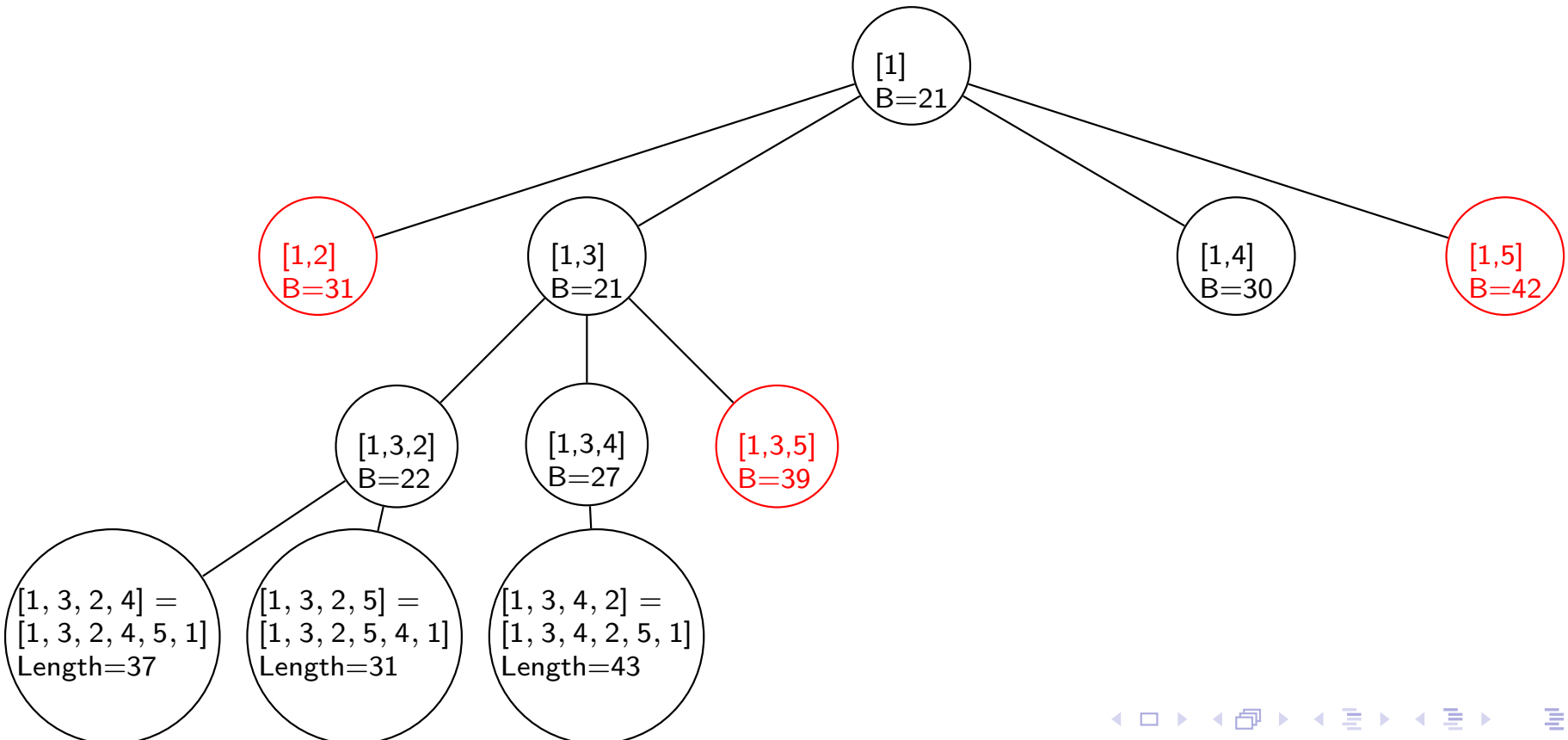

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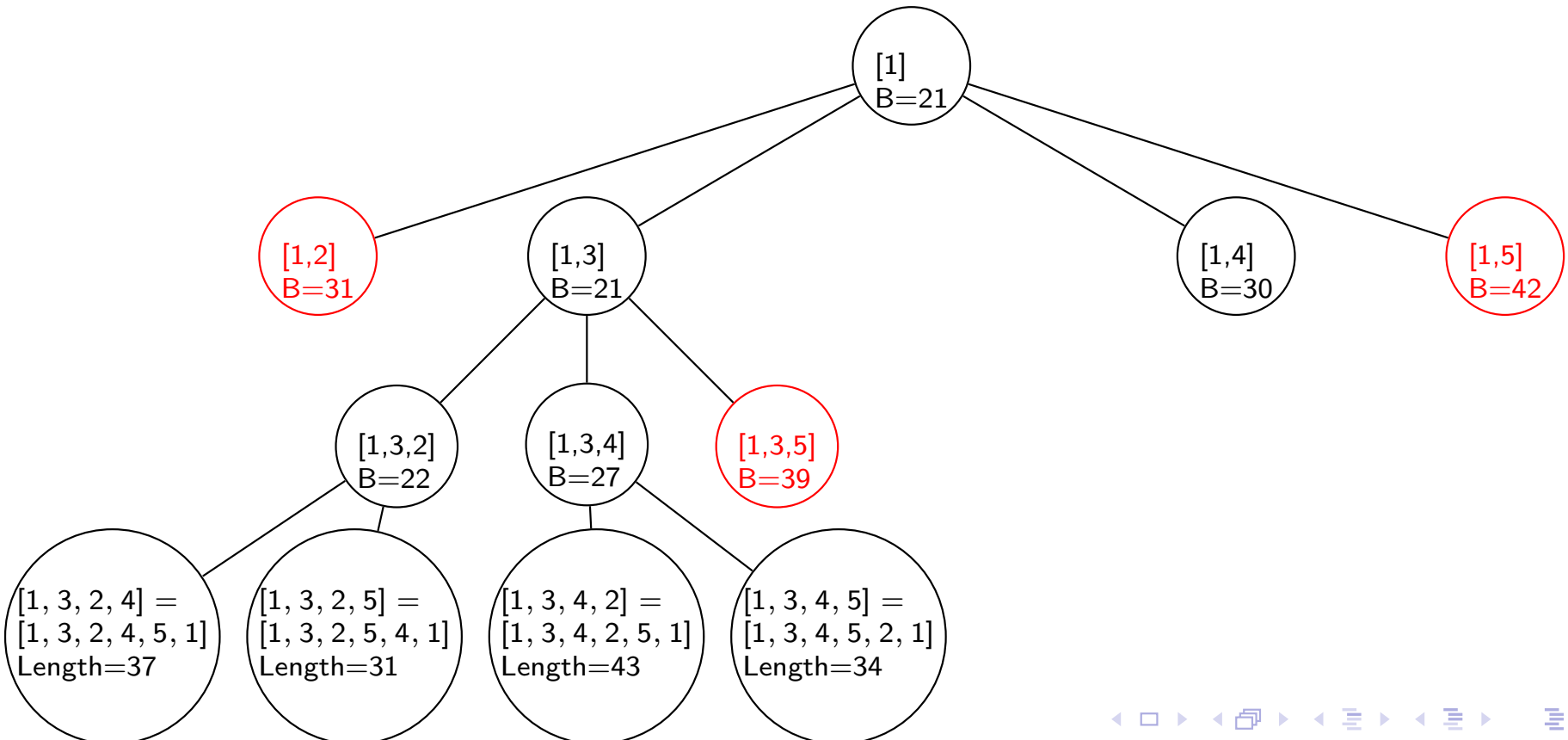

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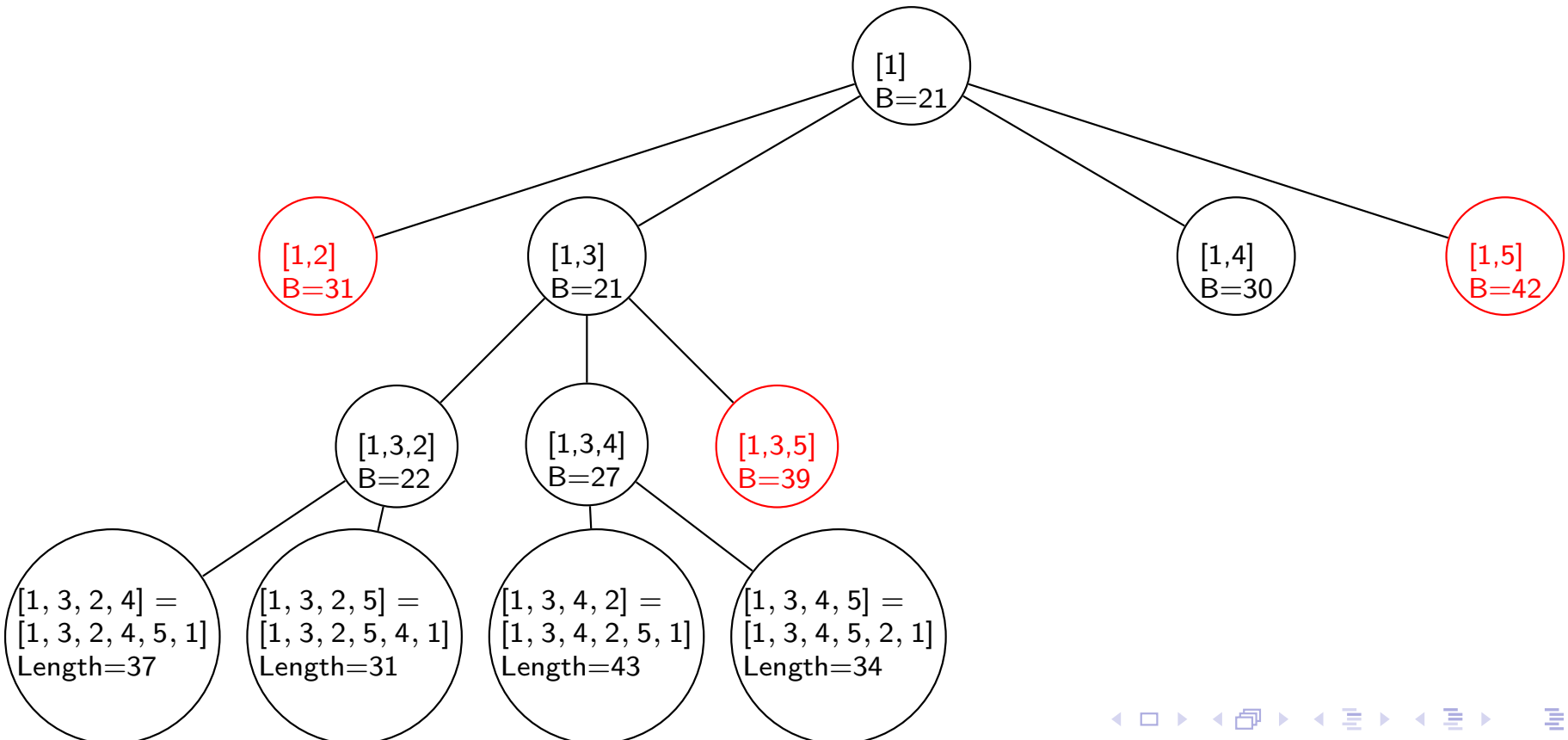

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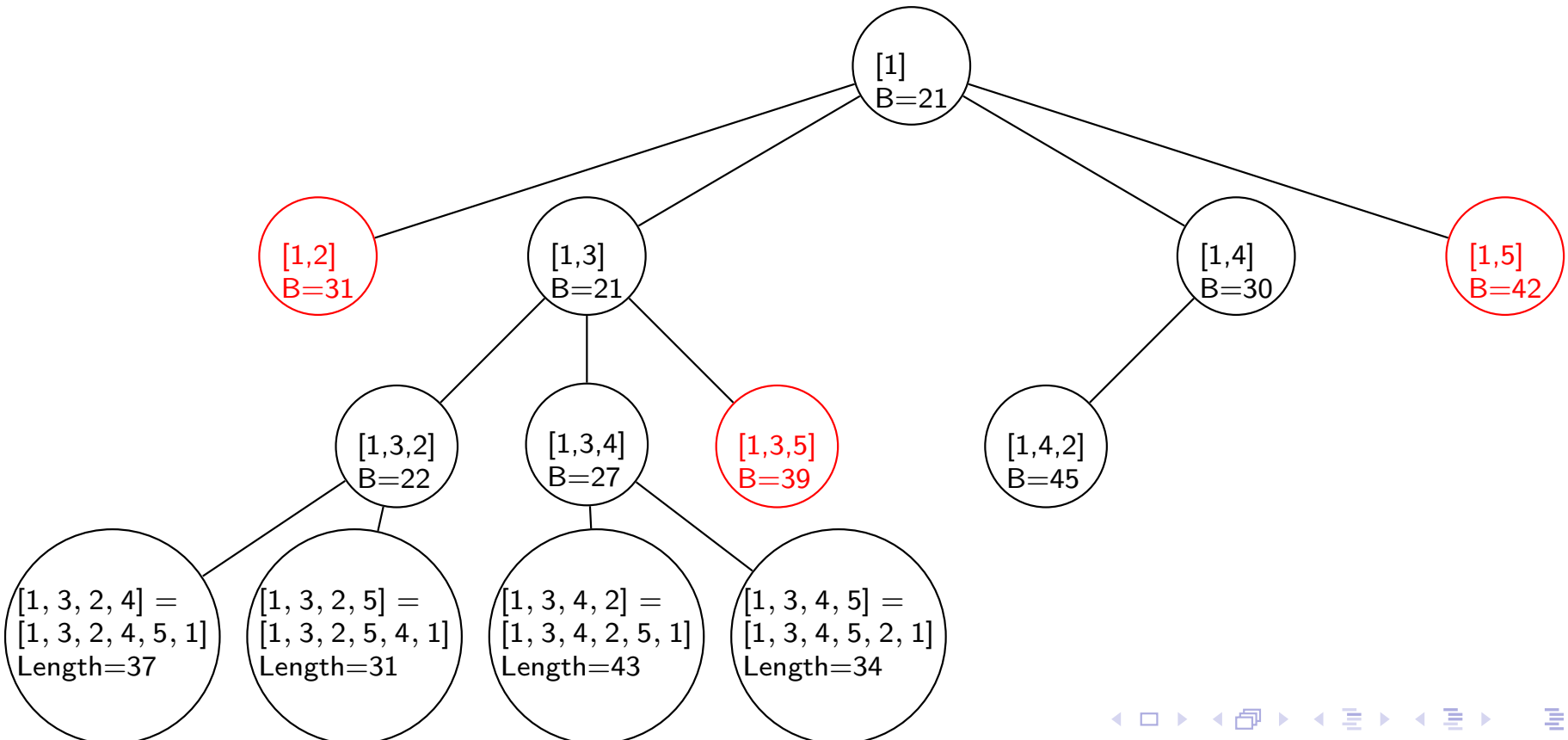

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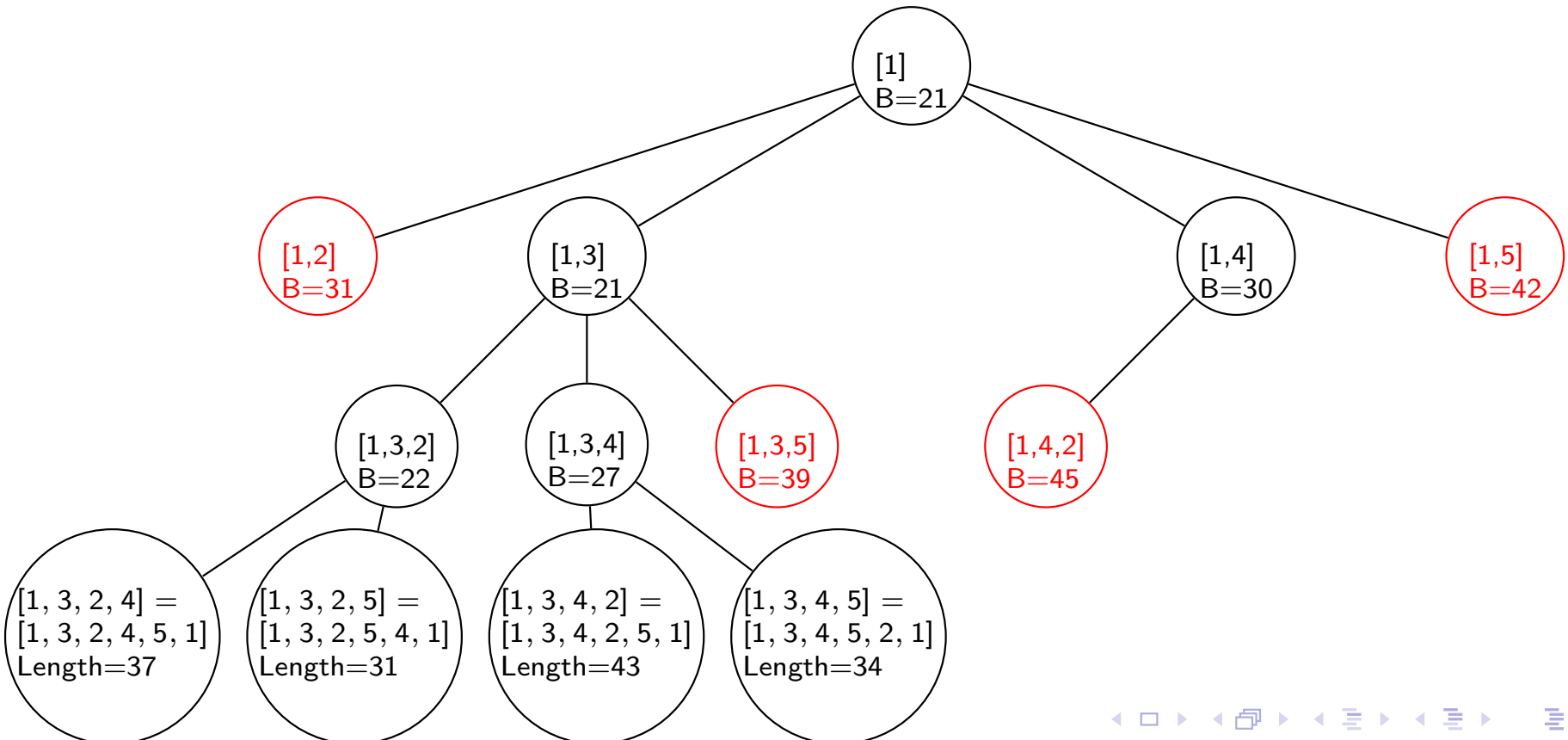

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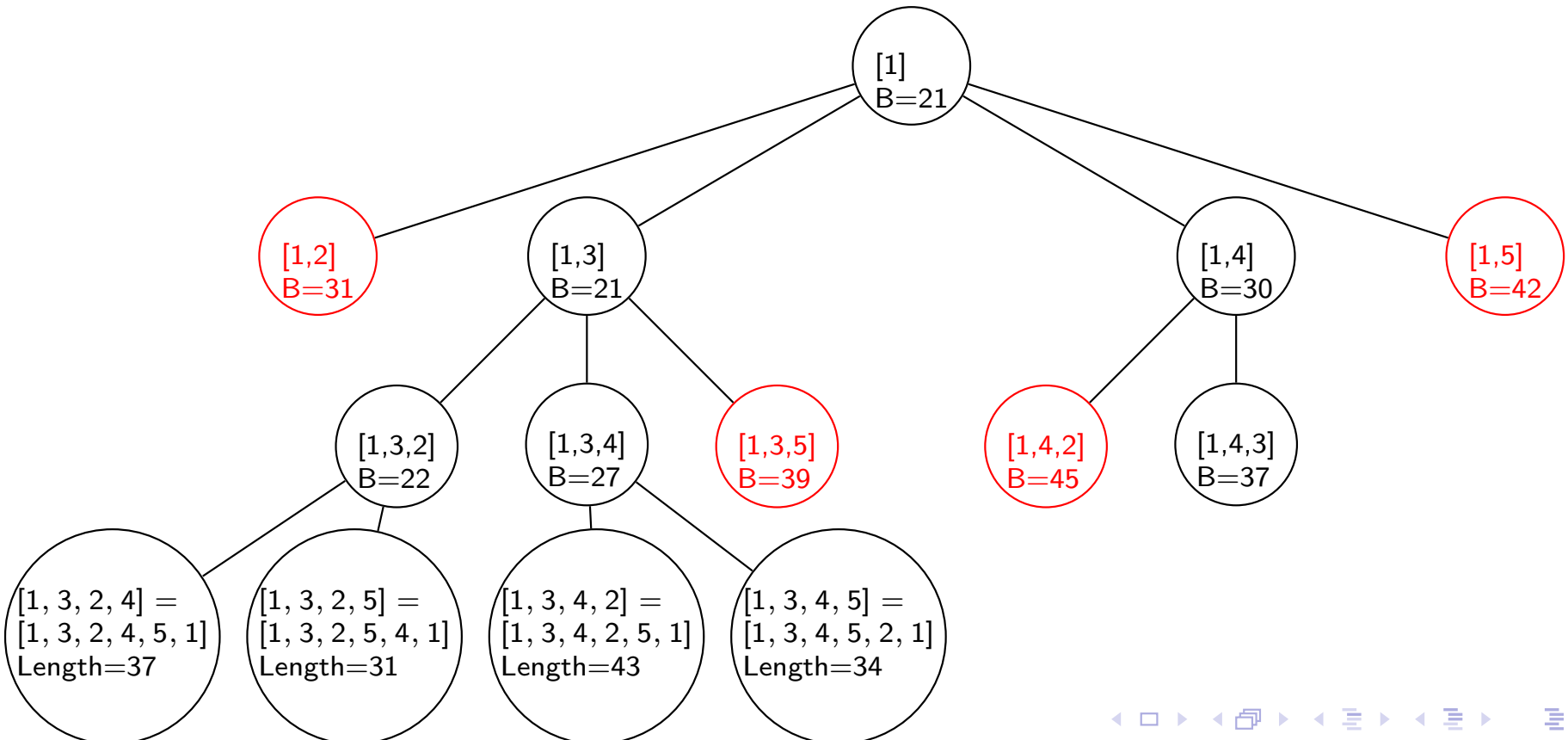

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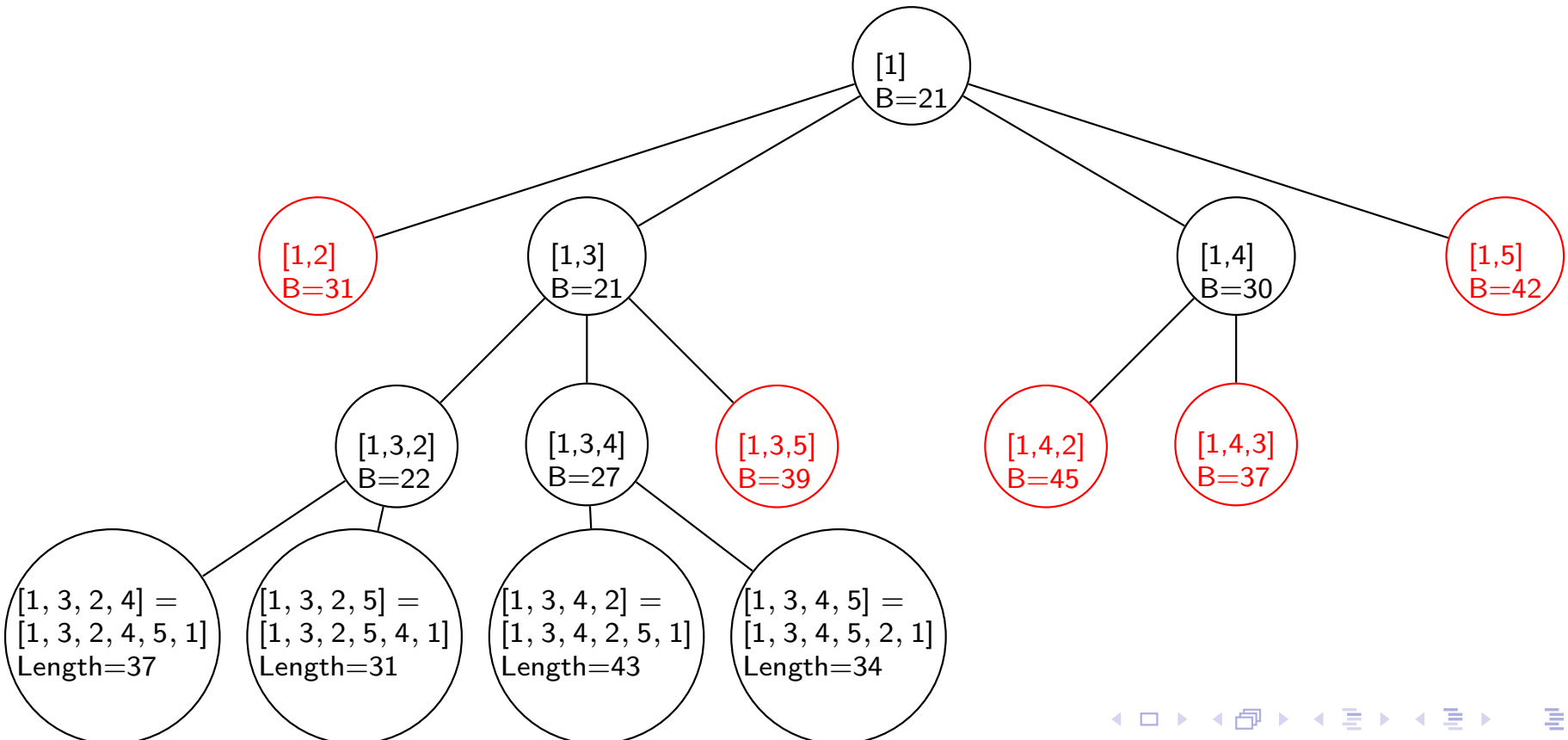

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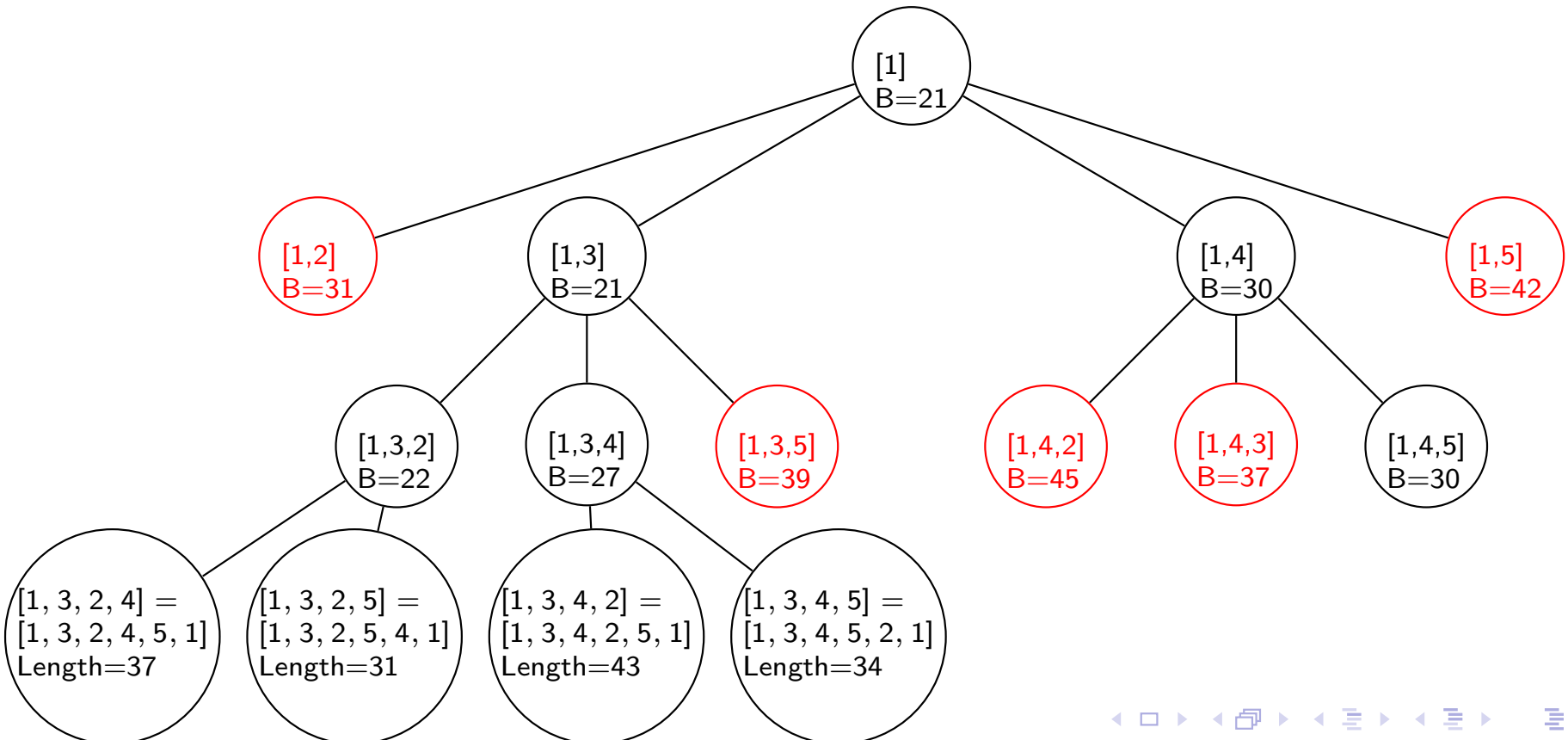

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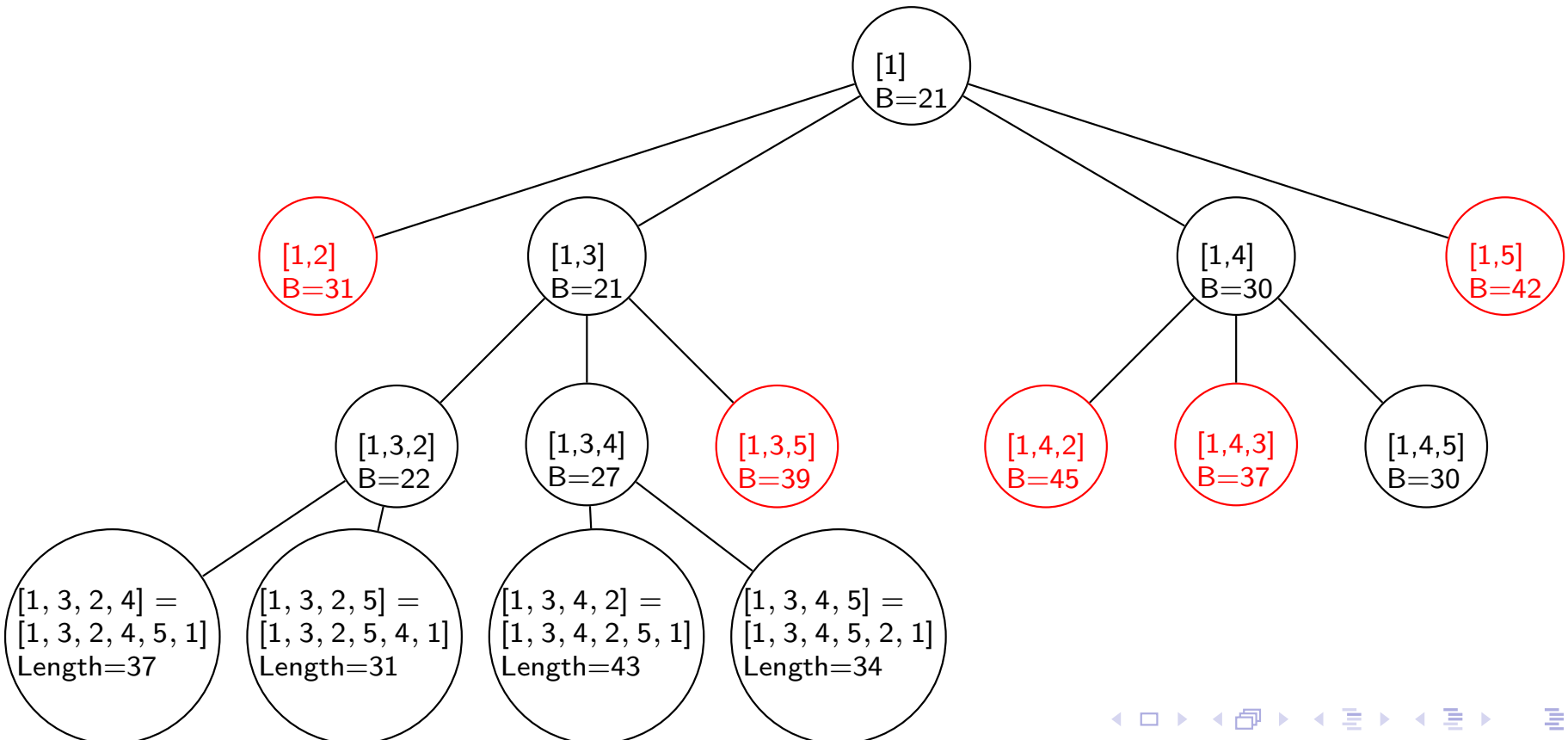

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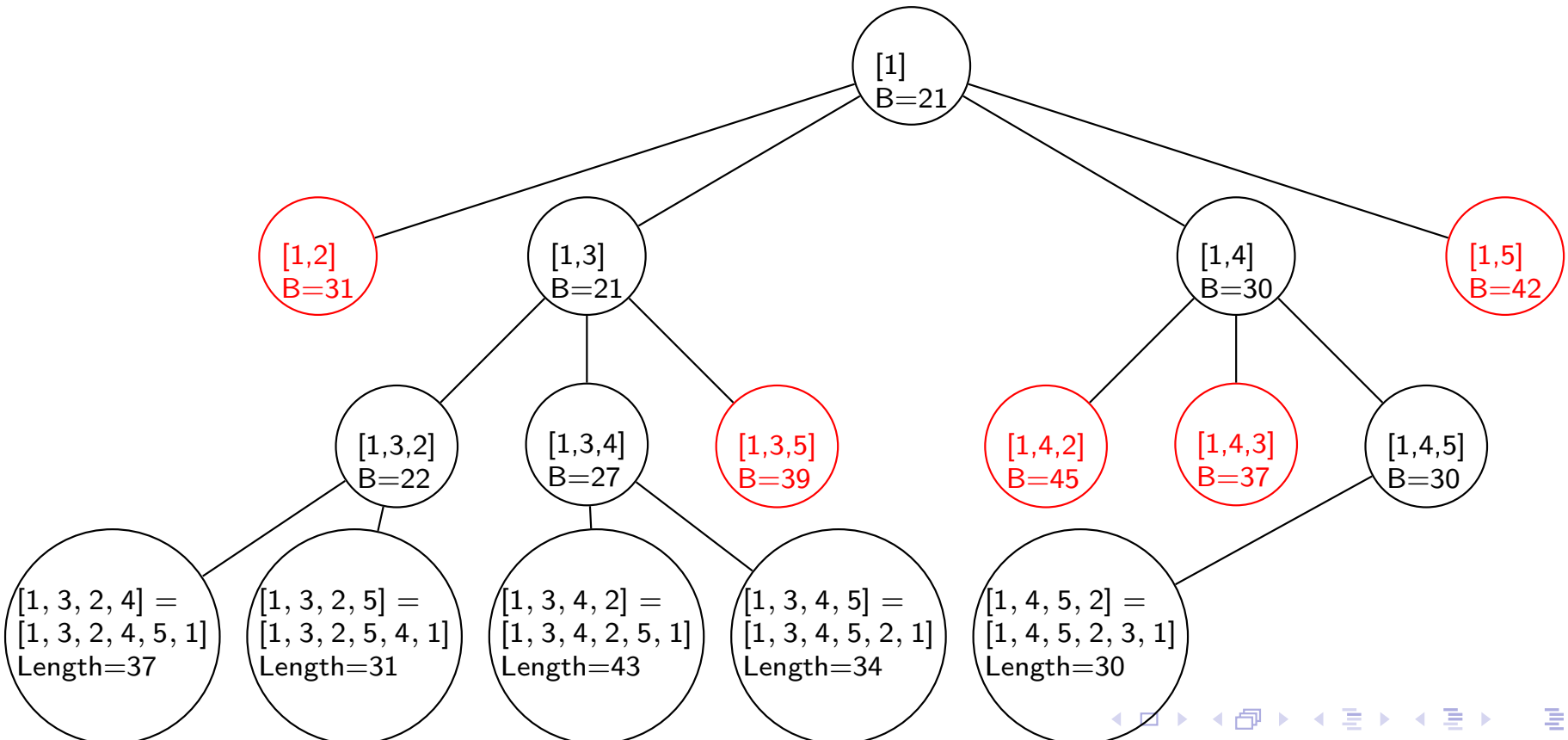

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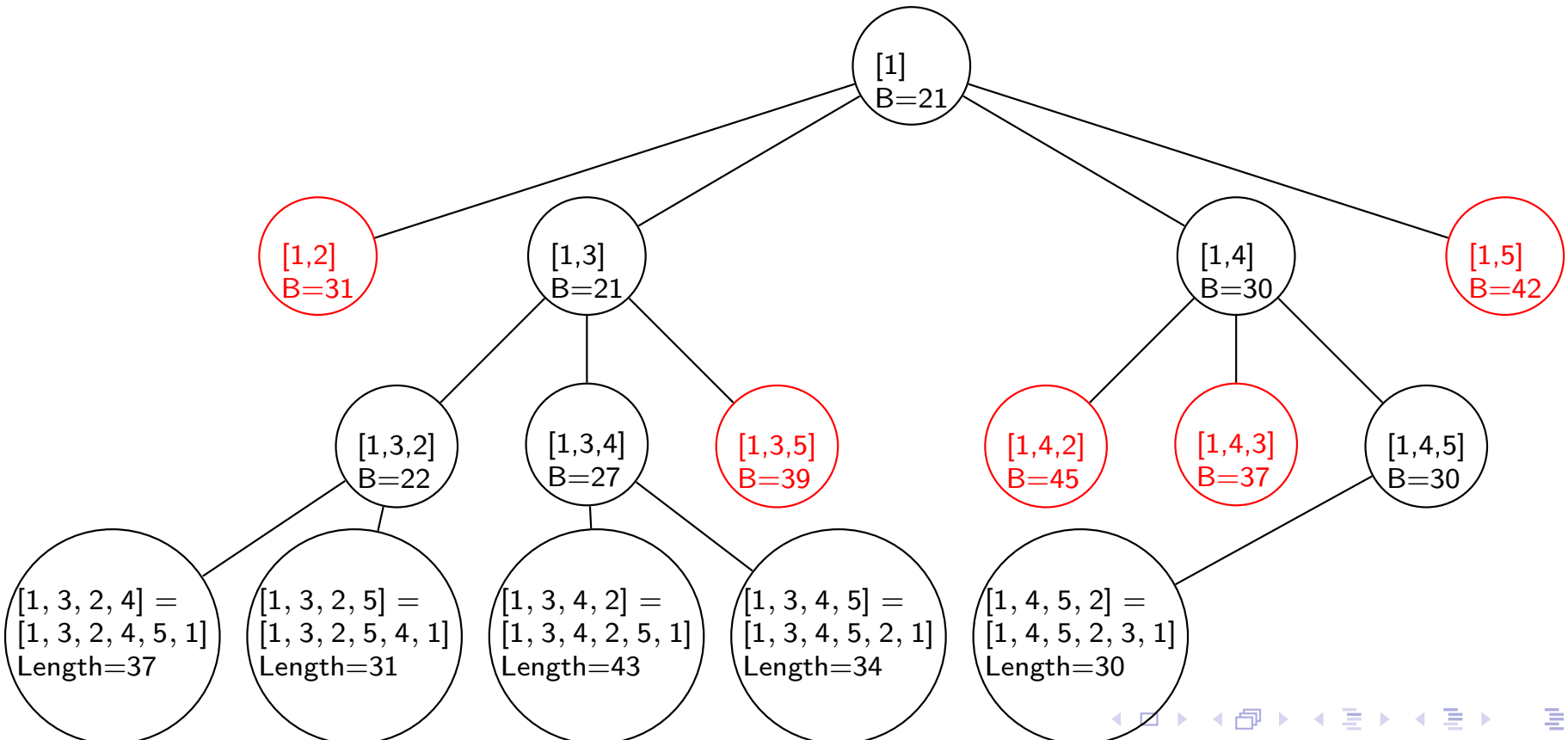

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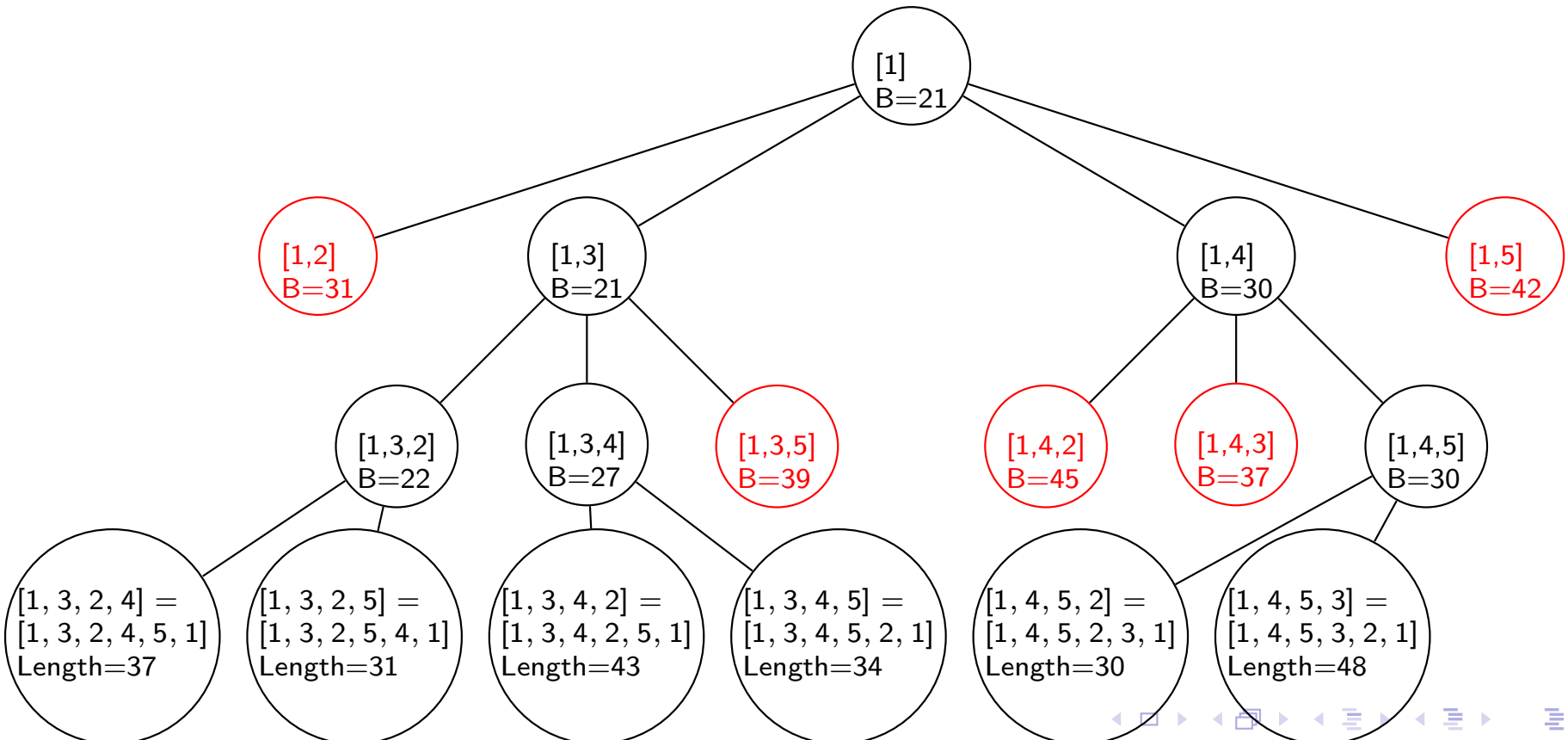

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- Determine promising, unexpanded node with the smallest bound
- Promising: Bound $<$ minlength
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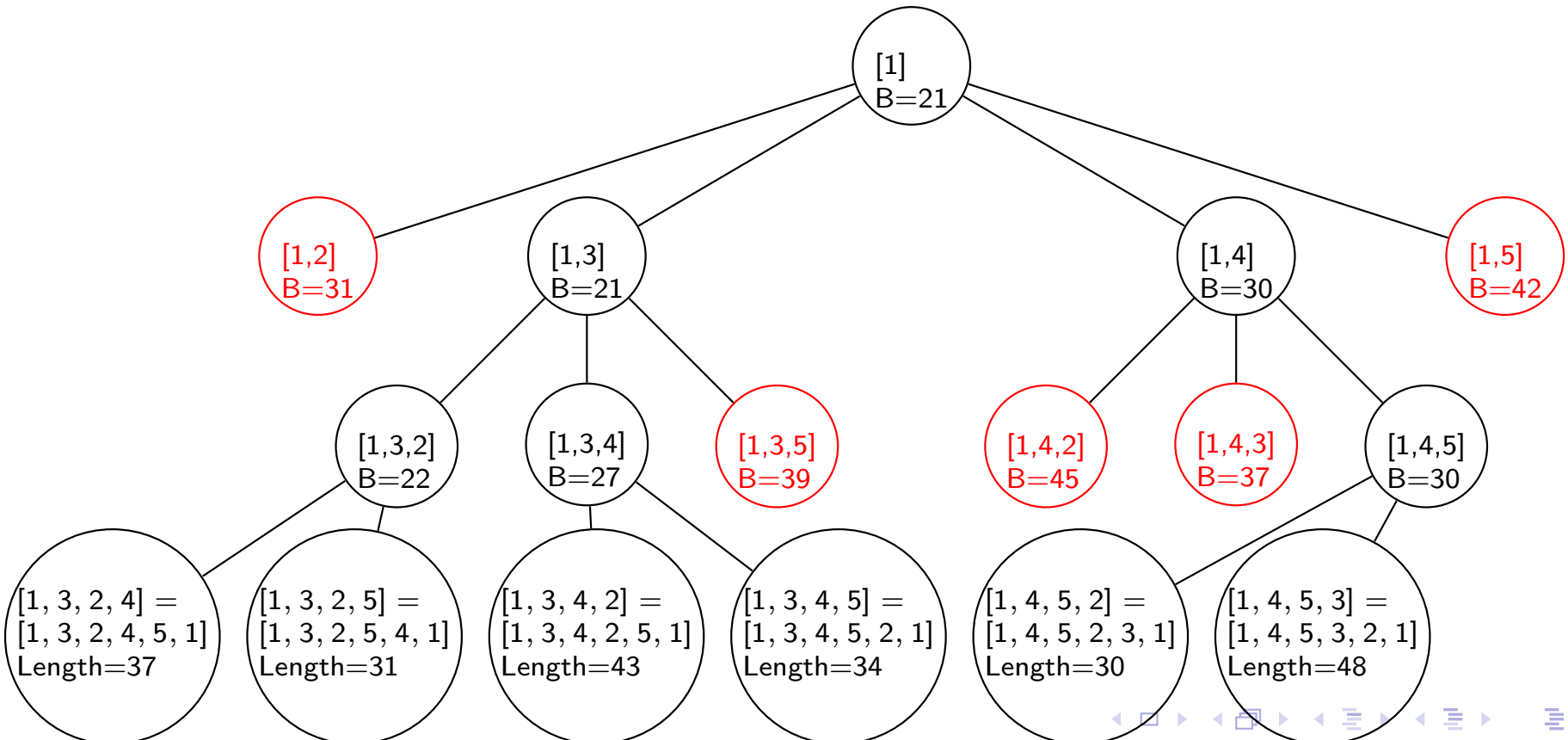

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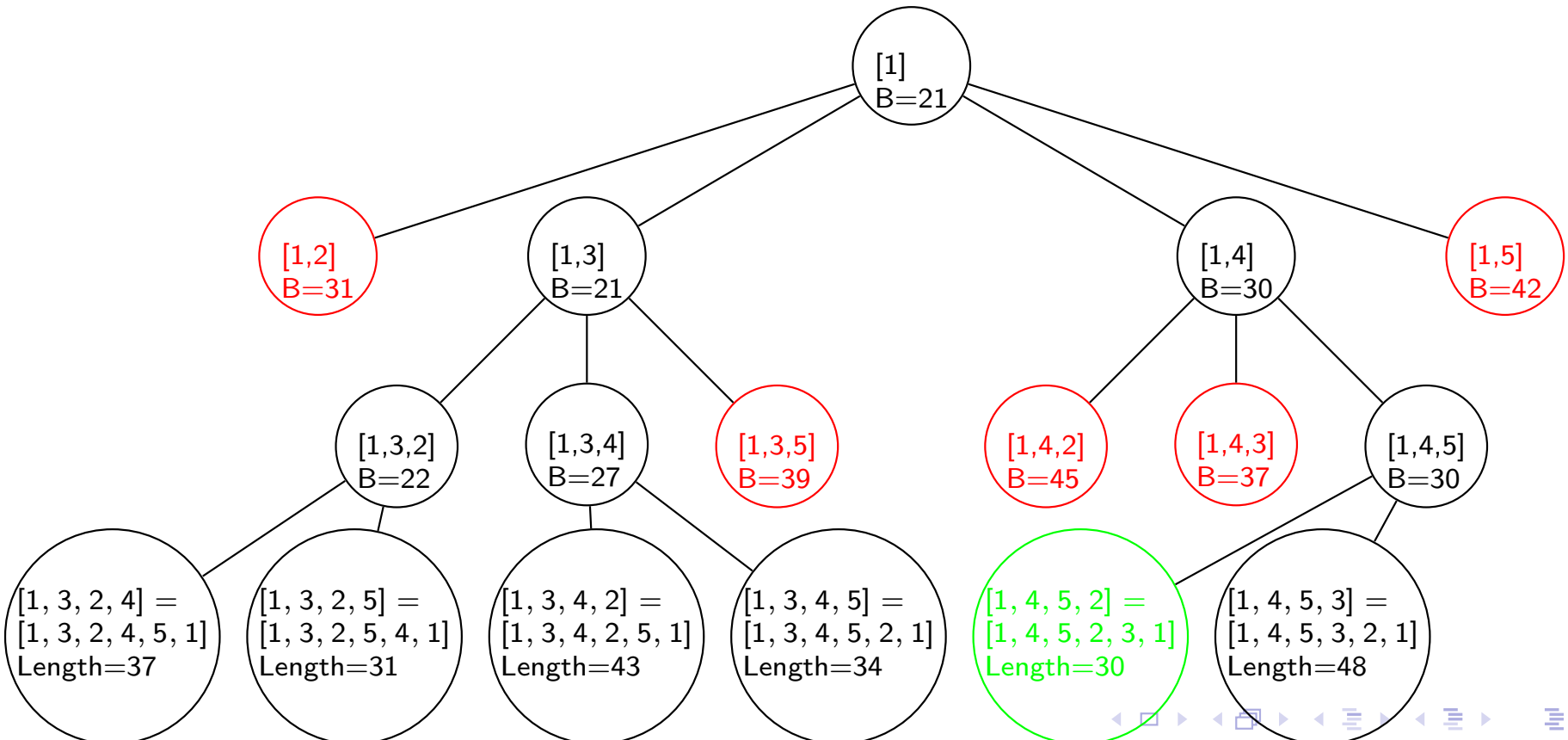

Exercise

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- Use the Best-First Search with Branch-and-Bound Pruning Algorithm for the Travelling Salesperson problem to find an optimal tour and the length of the optimal tour for the graph whose adjacency matrix is given by the following matrix. Show the actions step by step. (Show the pruned state space tree produced by using the above mentioned algorithm. Specify 'city number' and 'bound'/'length' from top to bottom at each node in the pruned state space tree. Mark each nonpromising node with a cross. Mark the node(s) with optimal solution.)

$$\begin{bmatrix} 0 & 6 & 6 & 10 & 8 \\ 3 & 0 & 12 & 7 & 6 \\ 8 & 7 & 0 & 14 & 20 \\ 5 & 13 & 9 & 0 & 8 \\ 9 & 8 & 10 & 6 & 0 \end{bmatrix}$$

Assignment Problem Using B&B

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- What is the best assignment of jobs to different people?

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- Consider the following example:

	job1	job2	job3	job4
A	9	2	7	8
B	6	4	3	7
C	5	8	1	8
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Example of Assignment Problem using Best-First Search with Branch and Bound

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Example of Assignment Problem using Best-First Search with Branch and Bound

mintime $=$

- Promising: Bound $<$ mintime

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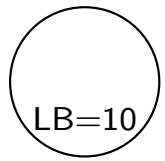

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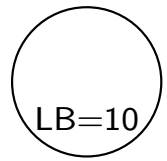
Example of Assignment Problem using Best-First Search with Branch and Bound

mintime $= \infty$

- Determine promising, unexpanded node with the smallest bound
- Promising: Bound $<$ mintime



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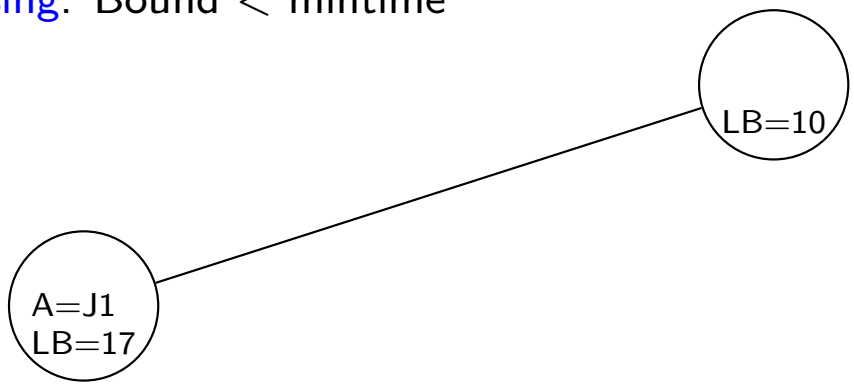

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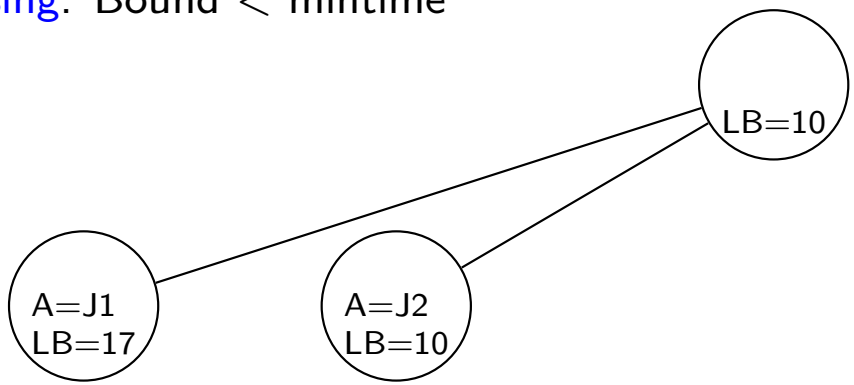

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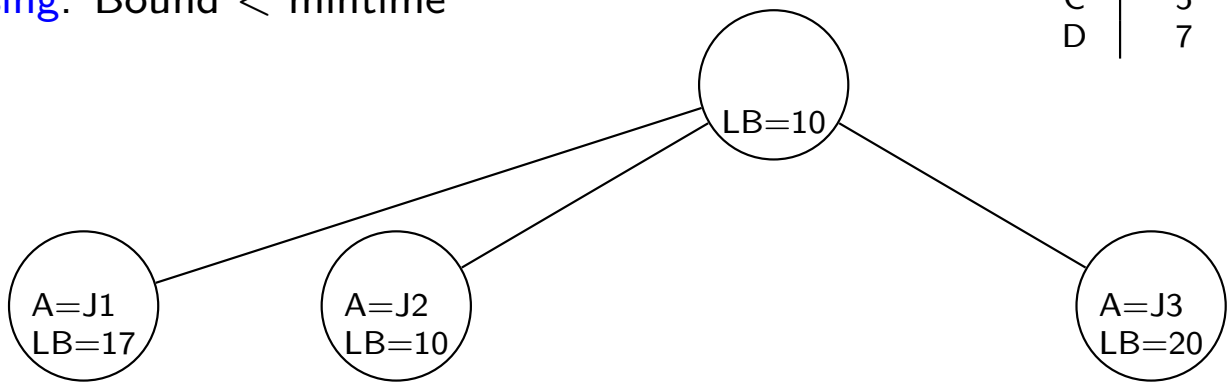

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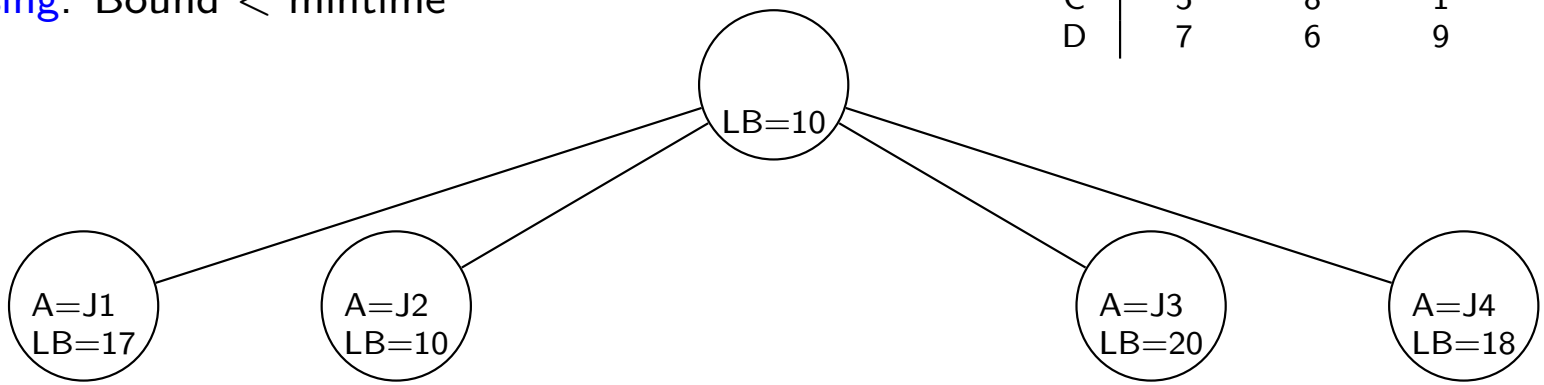

Example of Assignment Problem using Best-First Search with Branch and Bound

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- Determine promising, unexpanded node with the smallest bound
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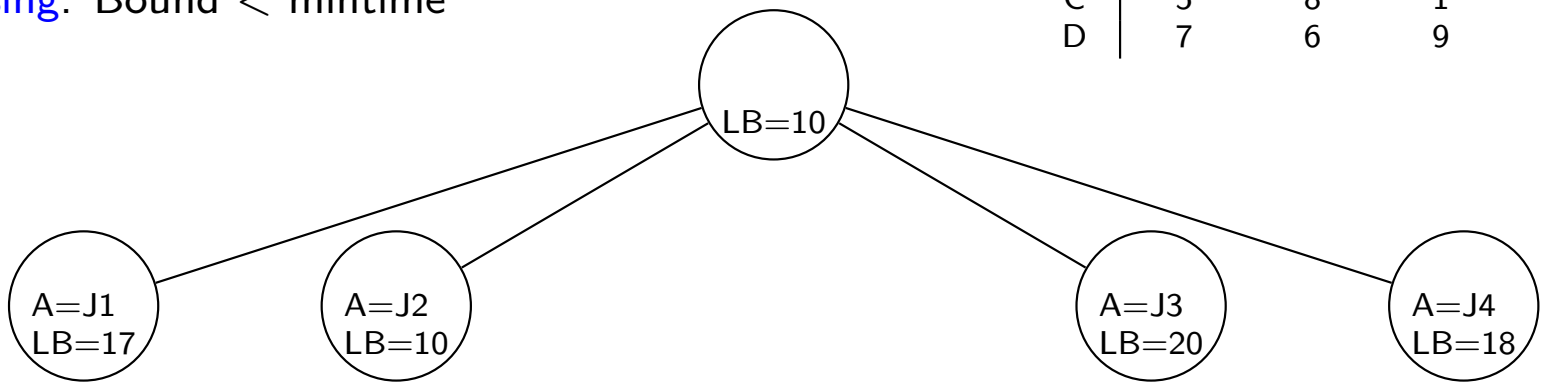

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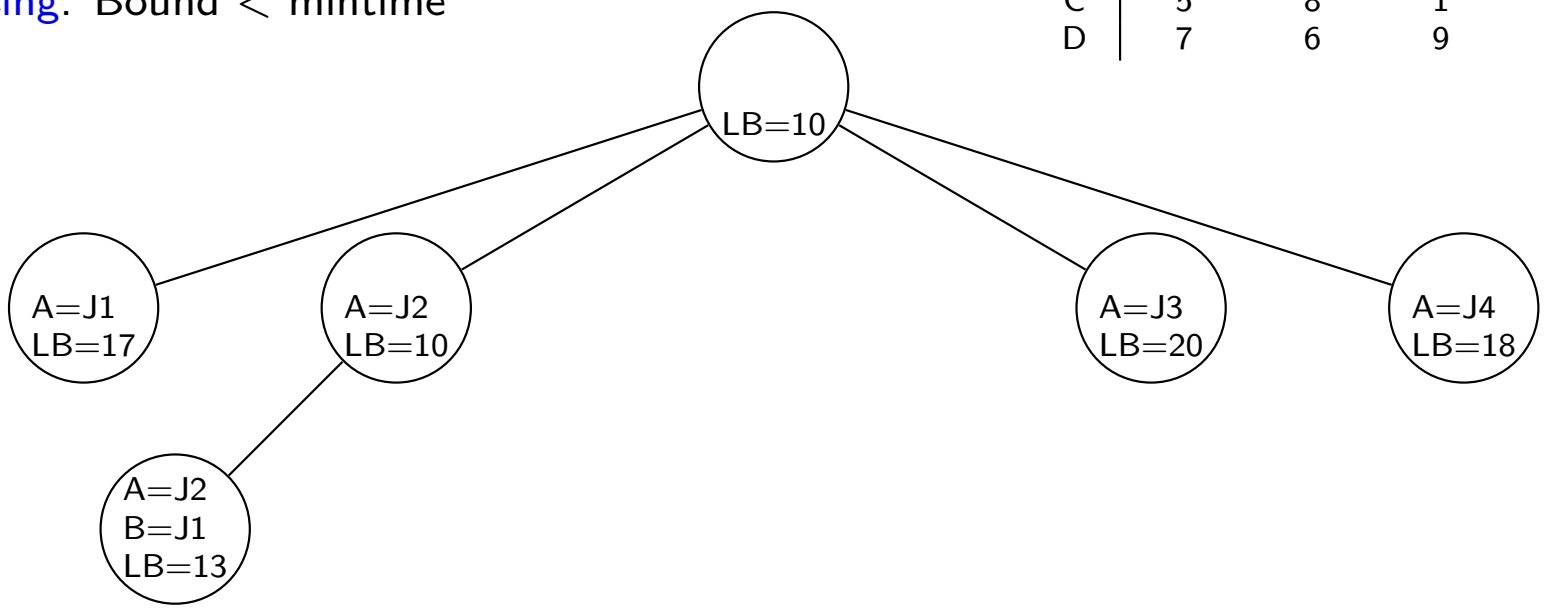

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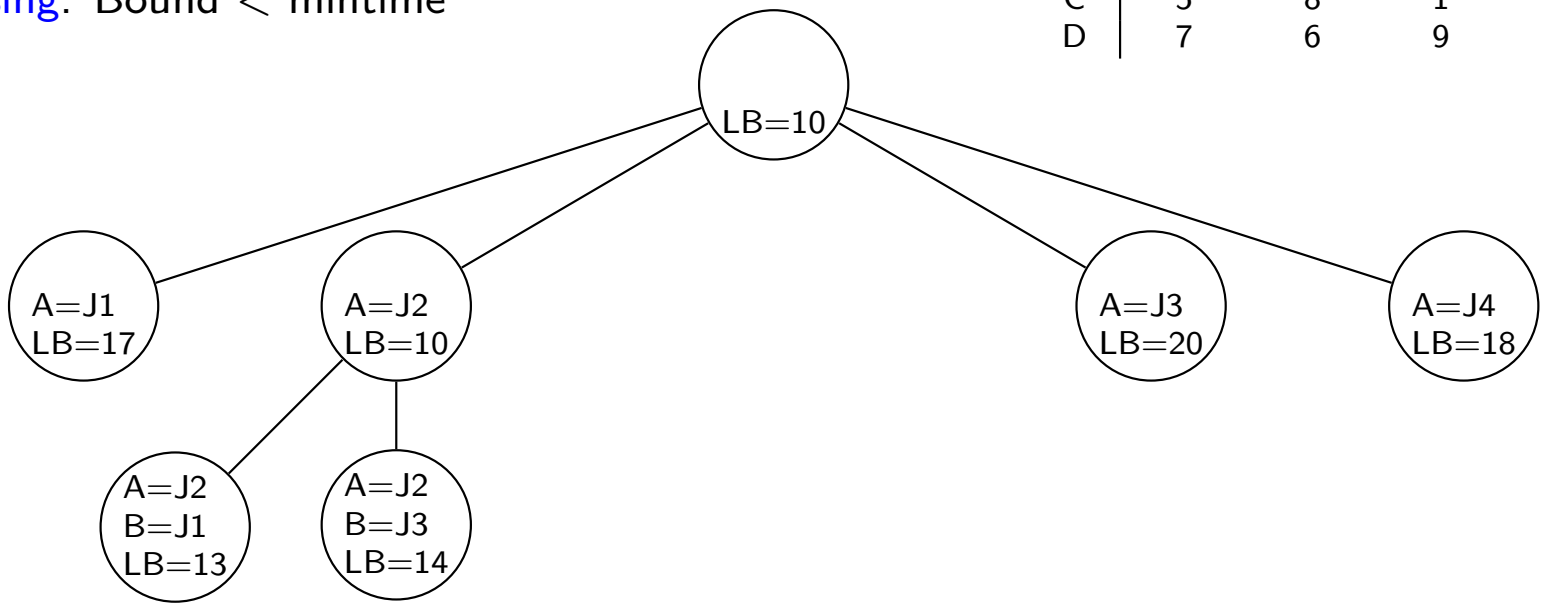

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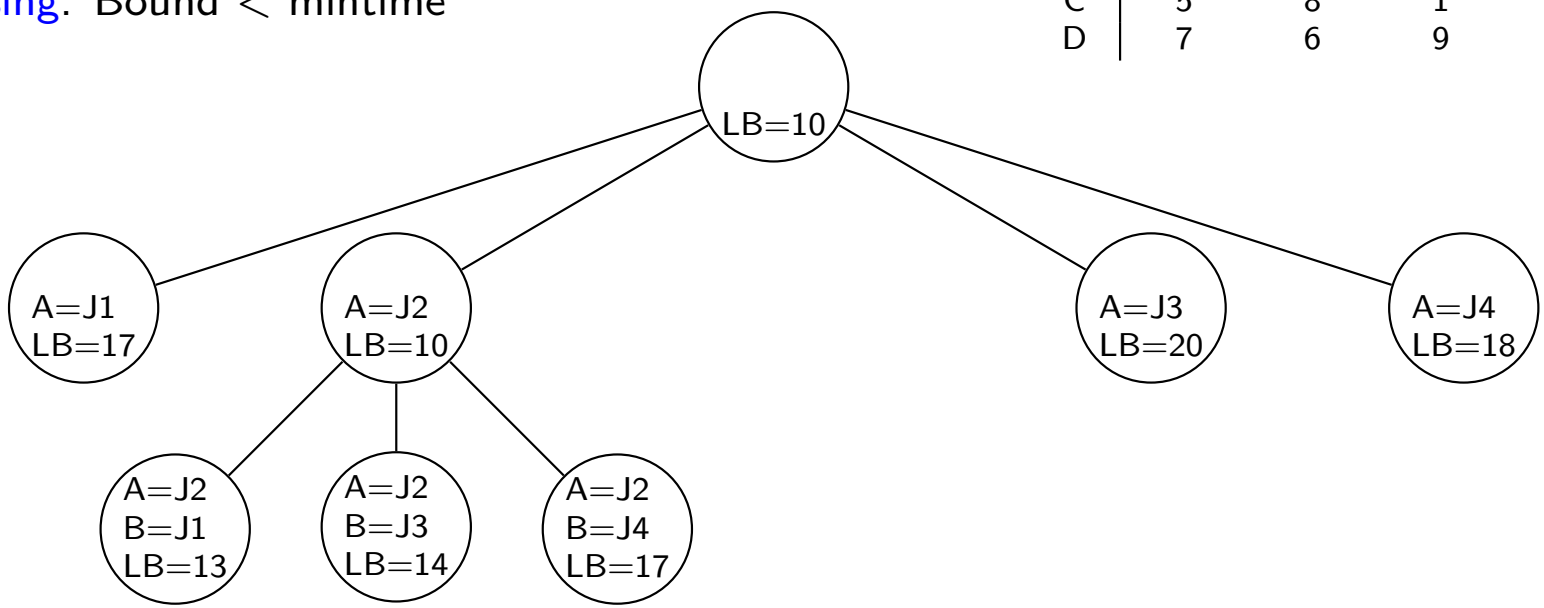

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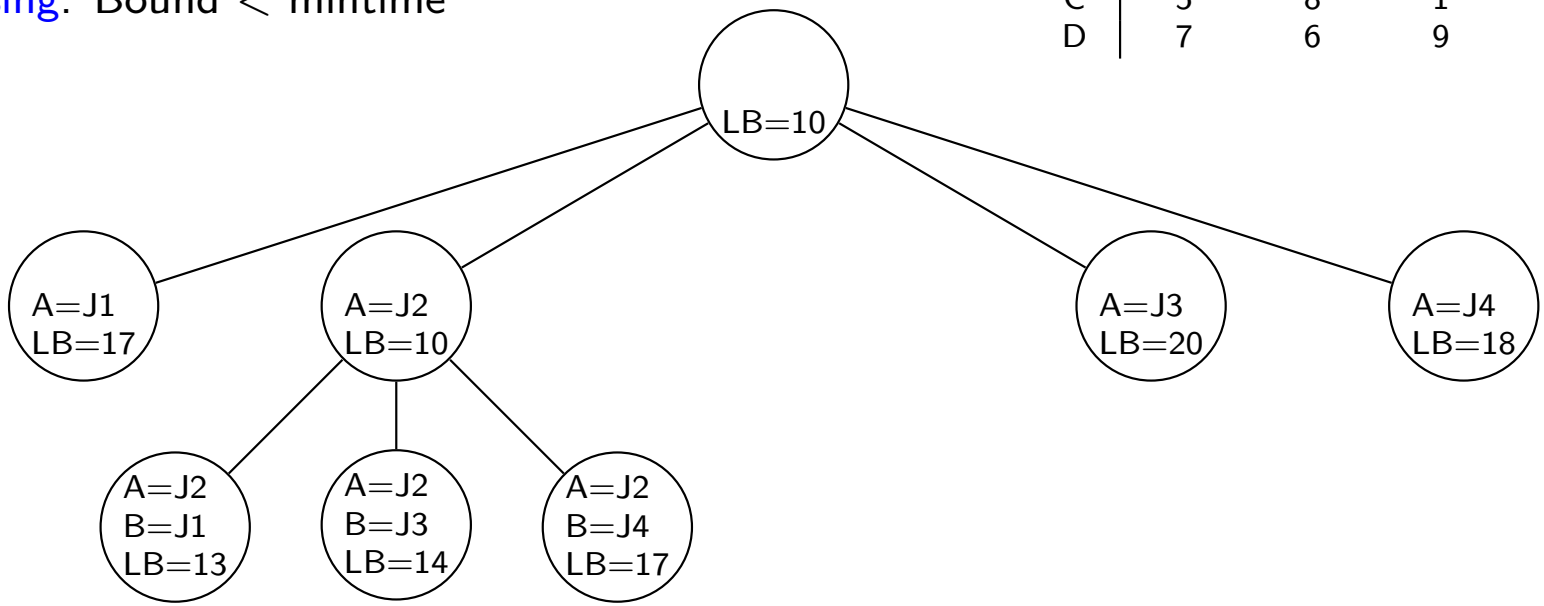

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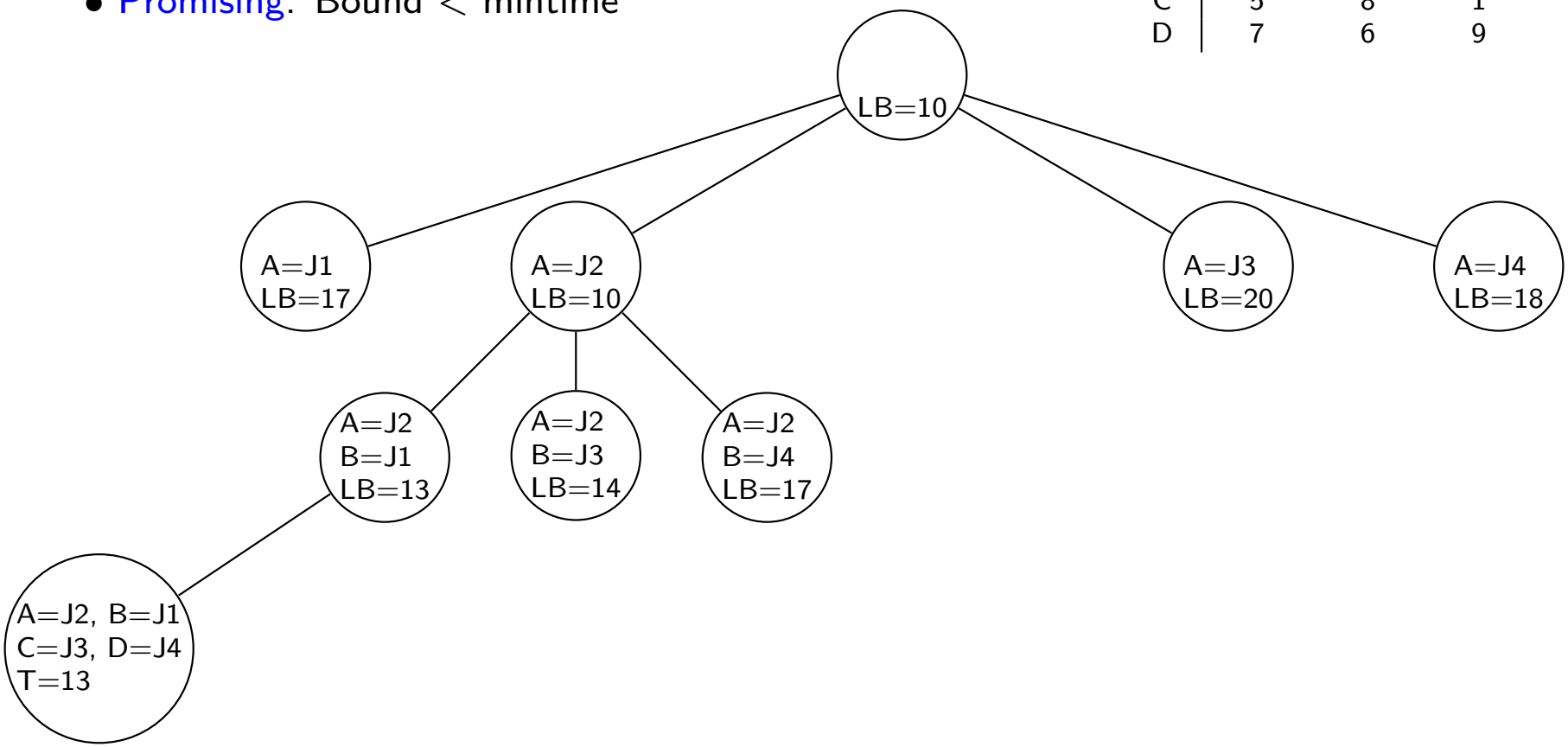

Example of Assignment Problem using Best-First Search with Branch and Bound

mintime $= 13$

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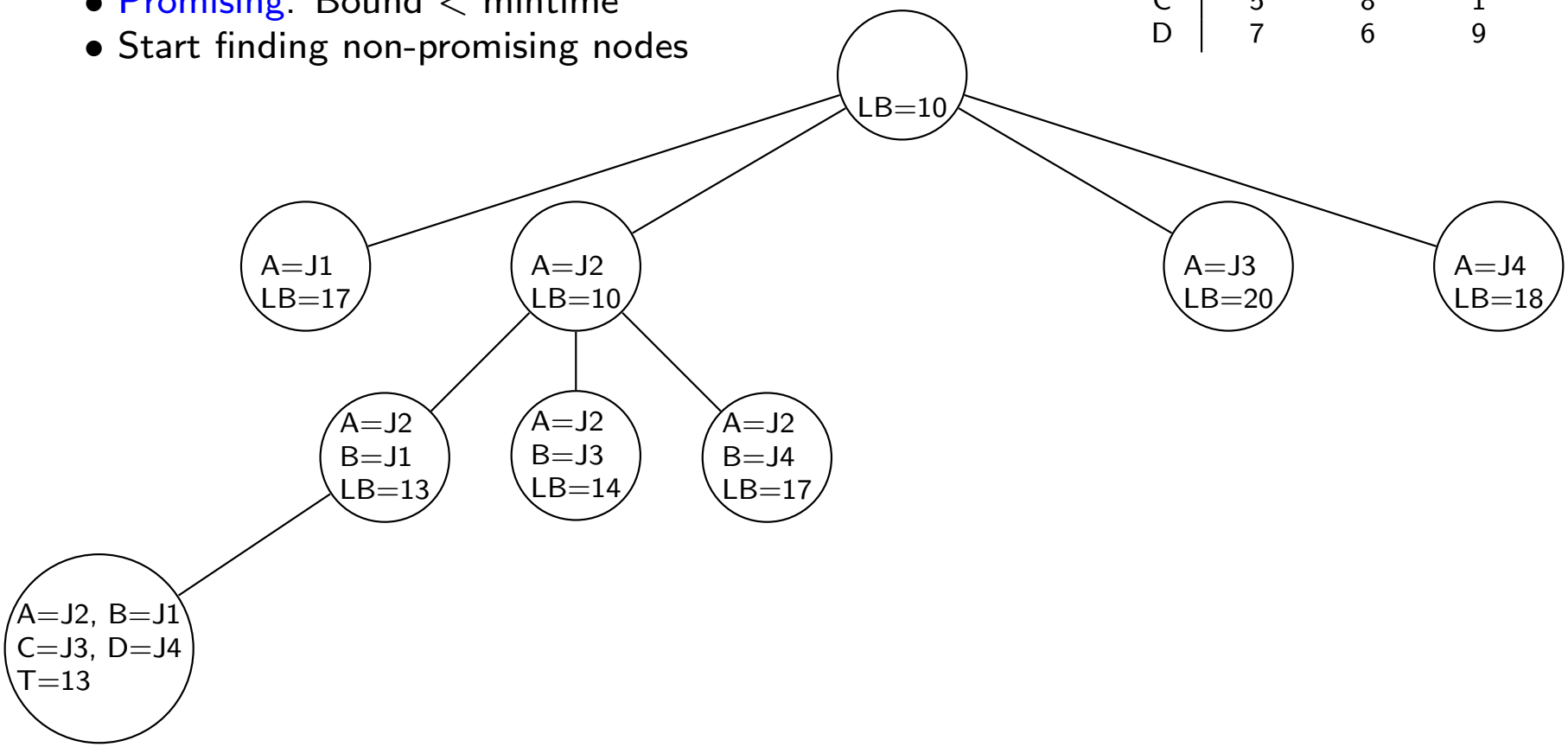

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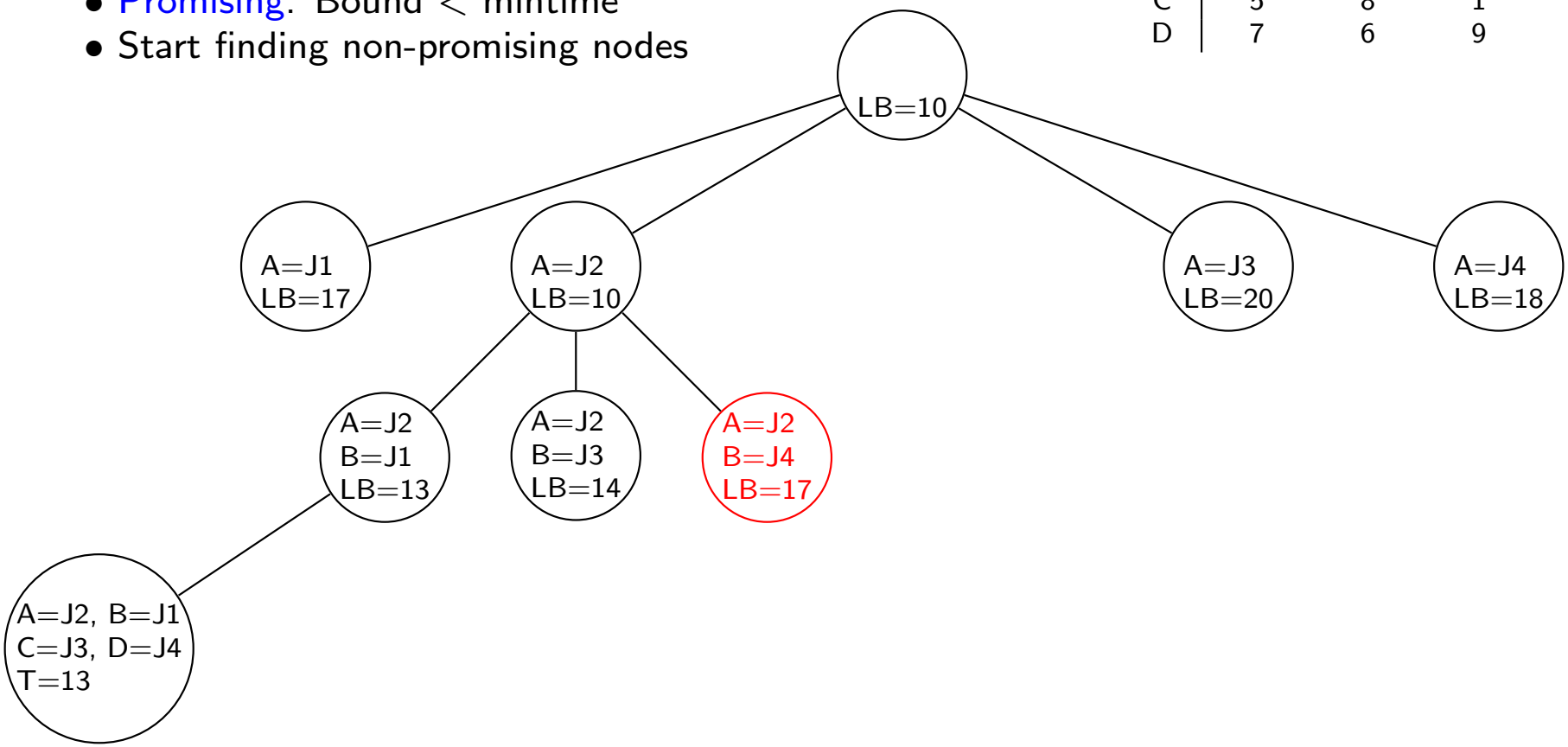

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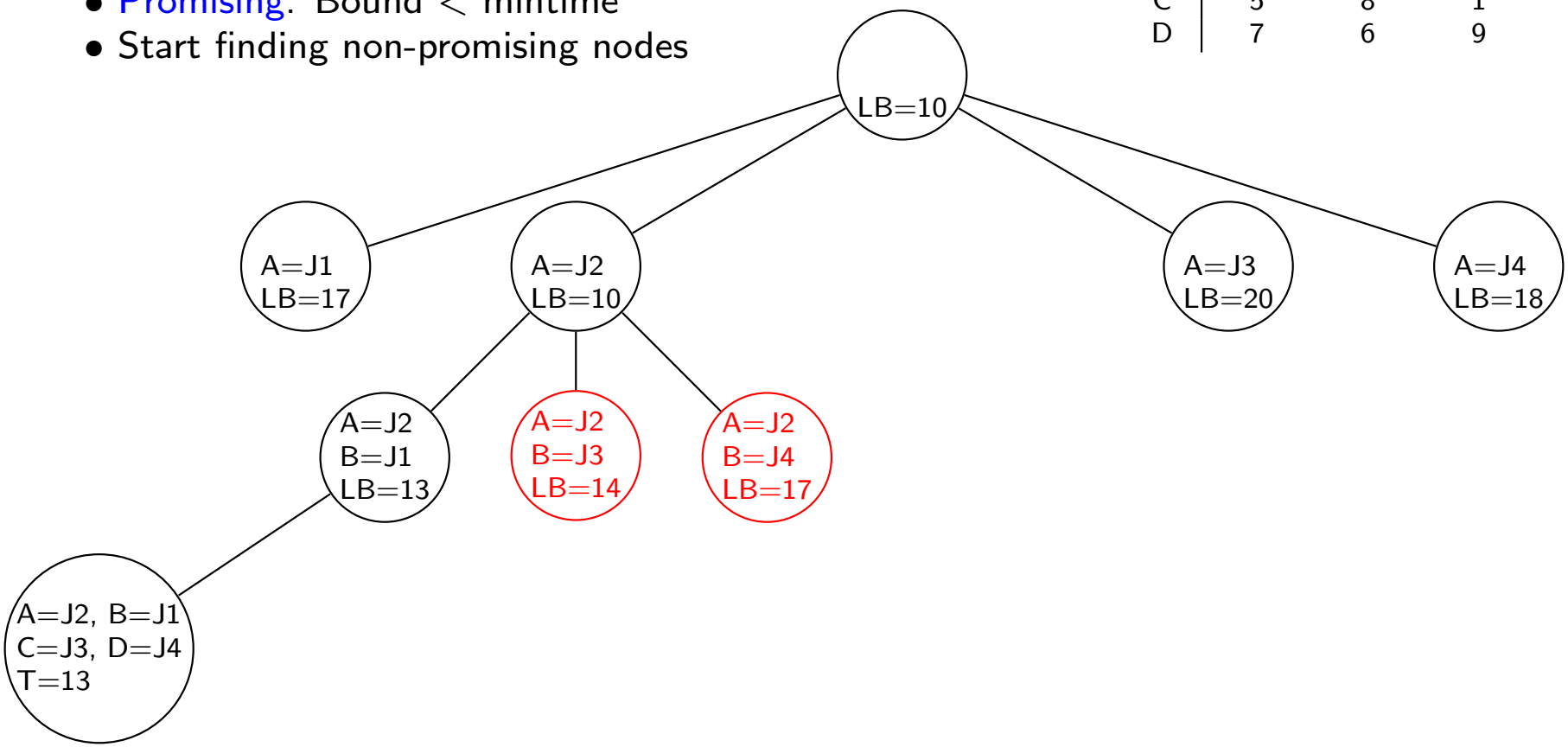

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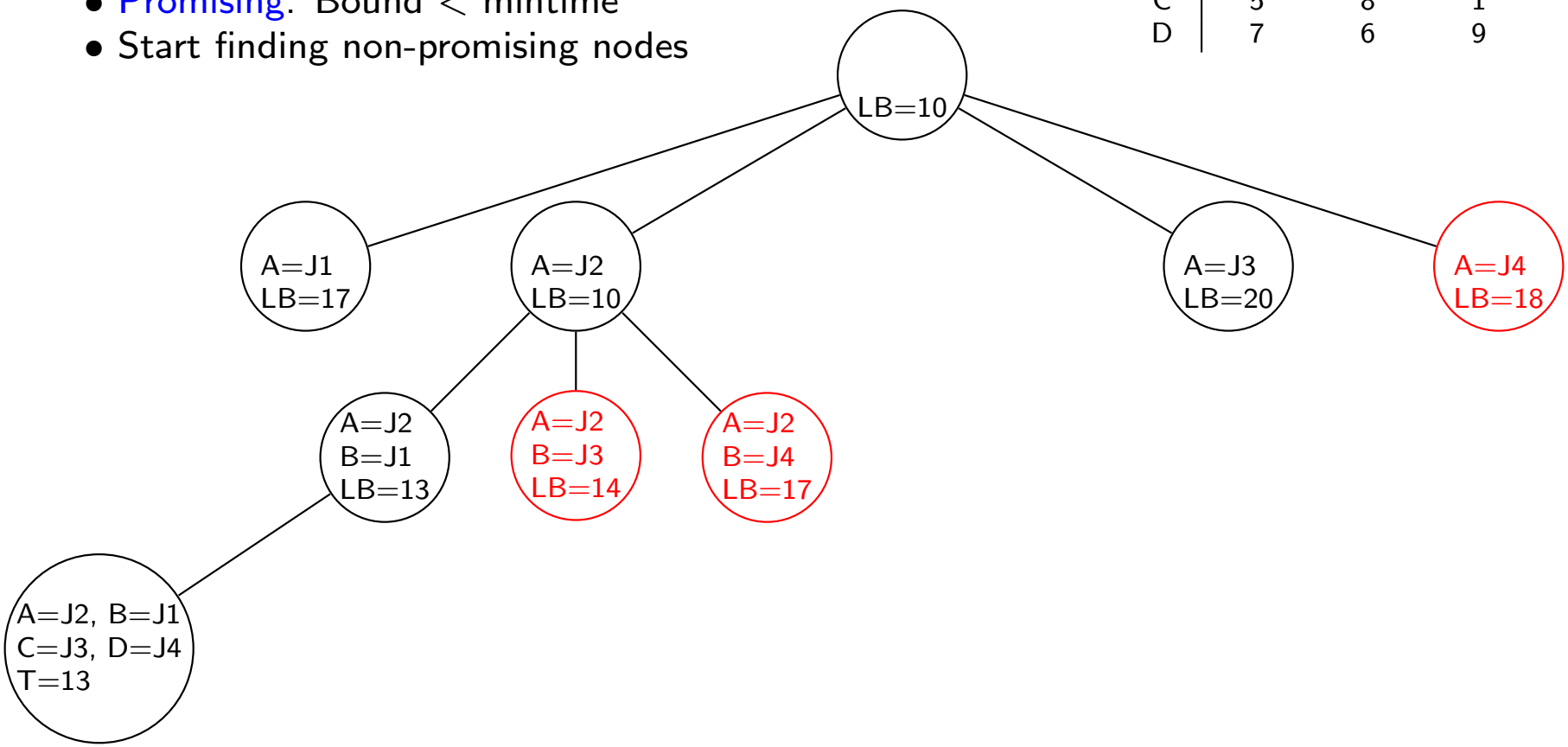

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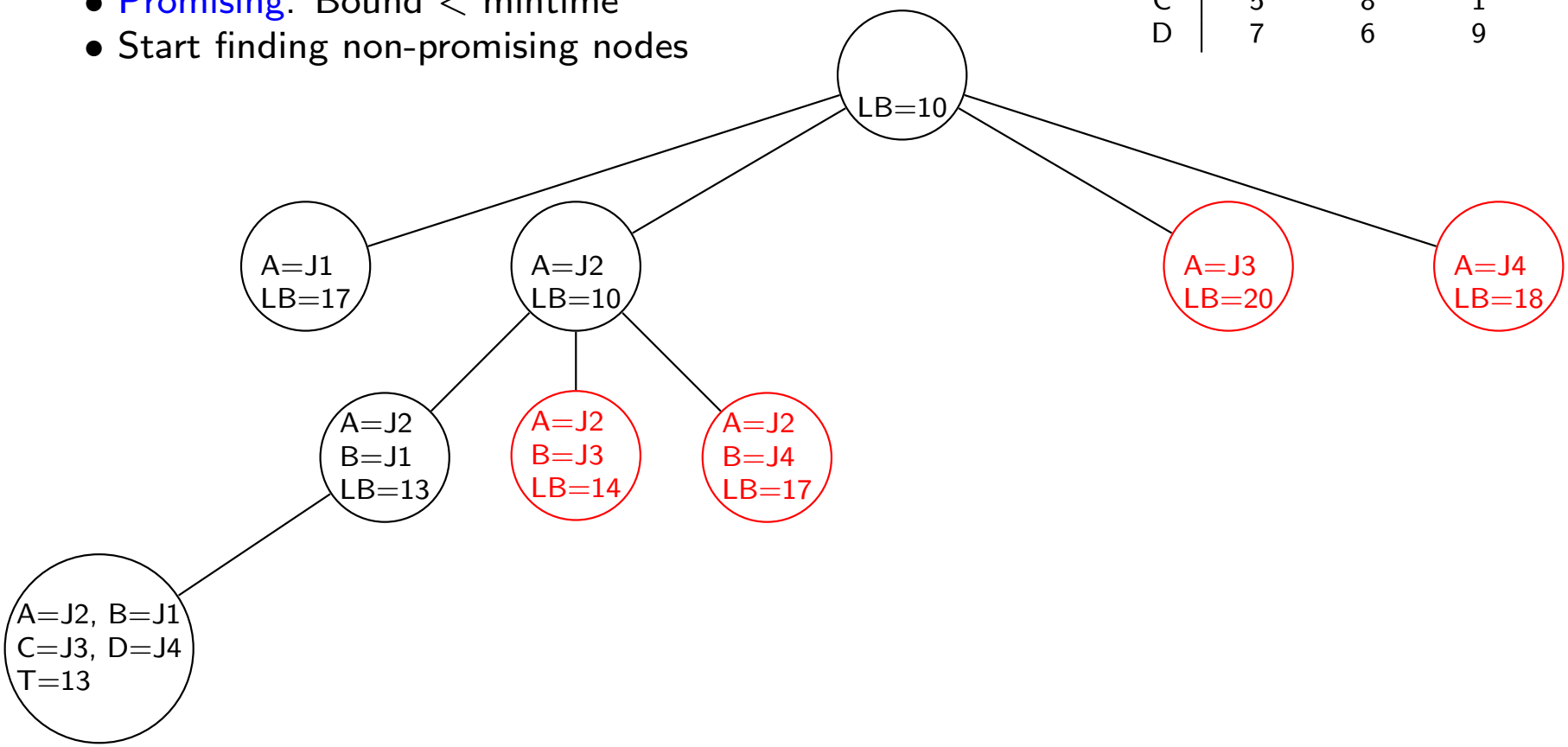

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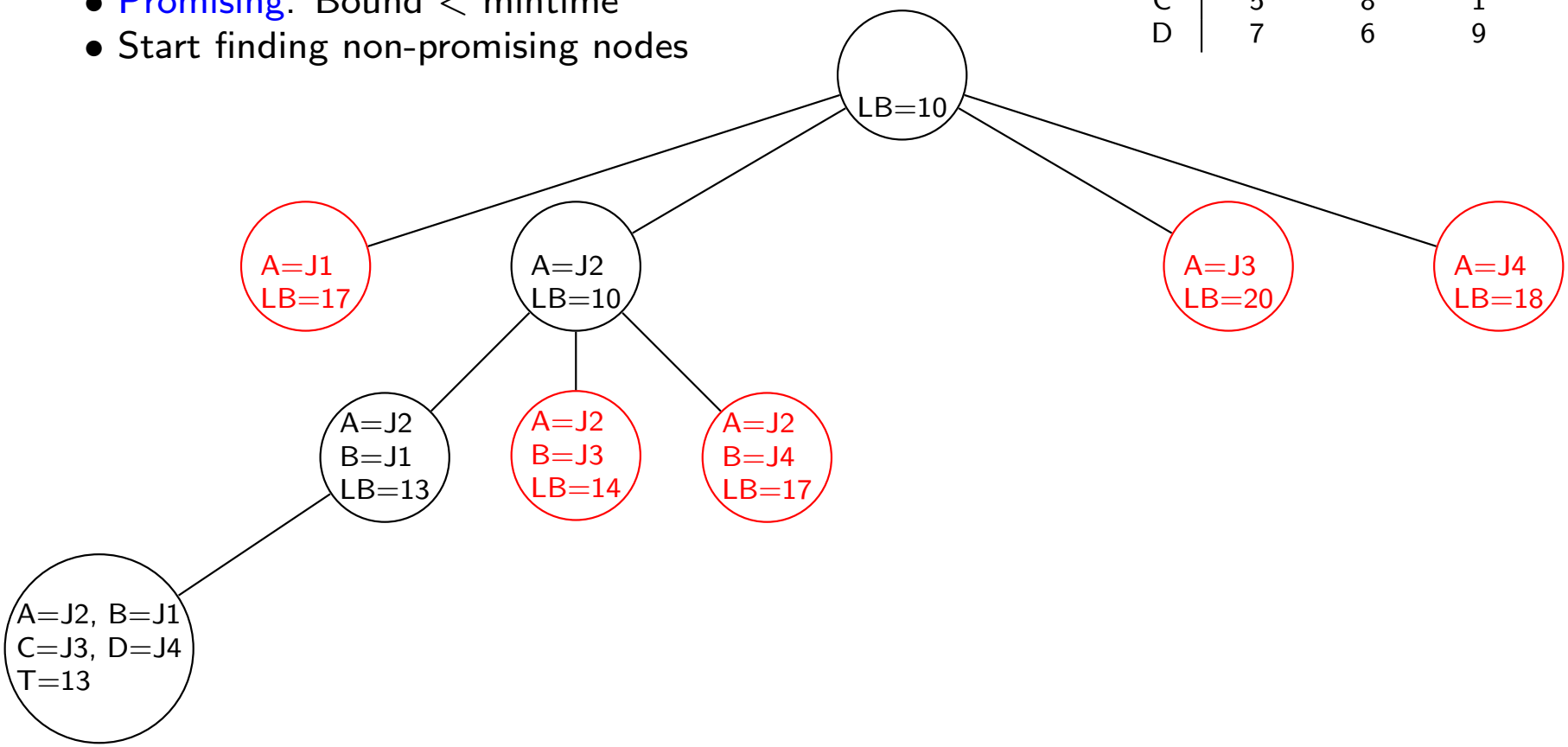

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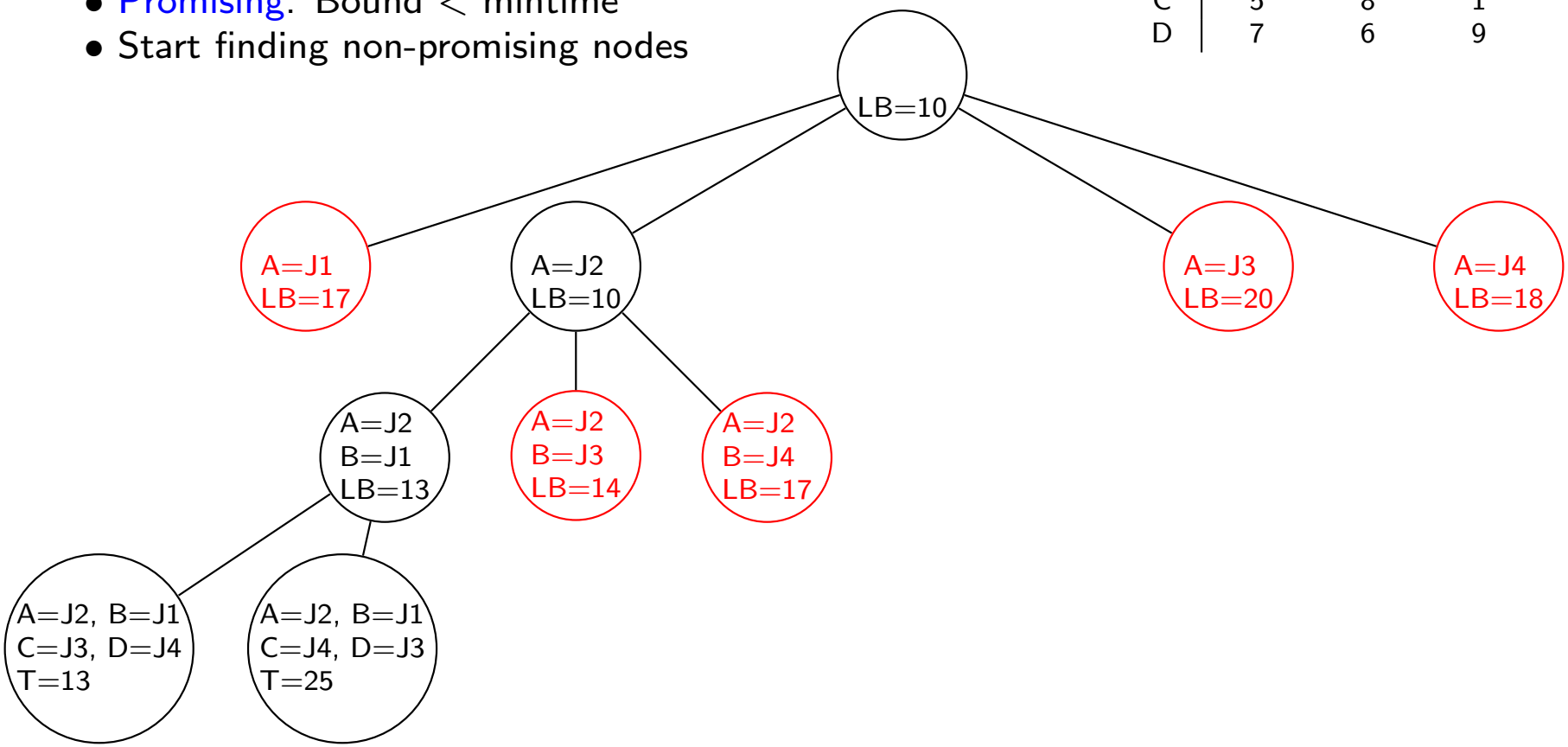

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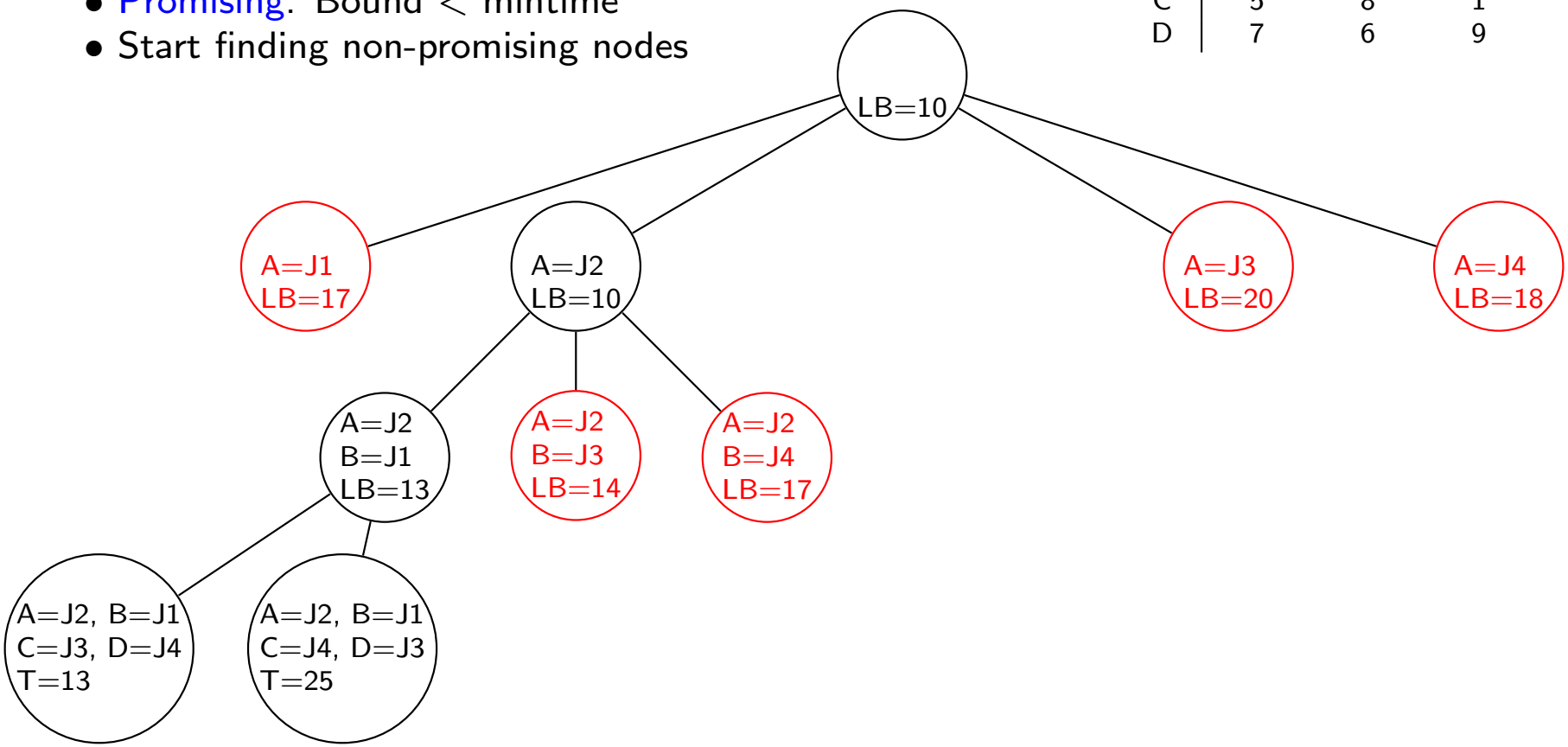

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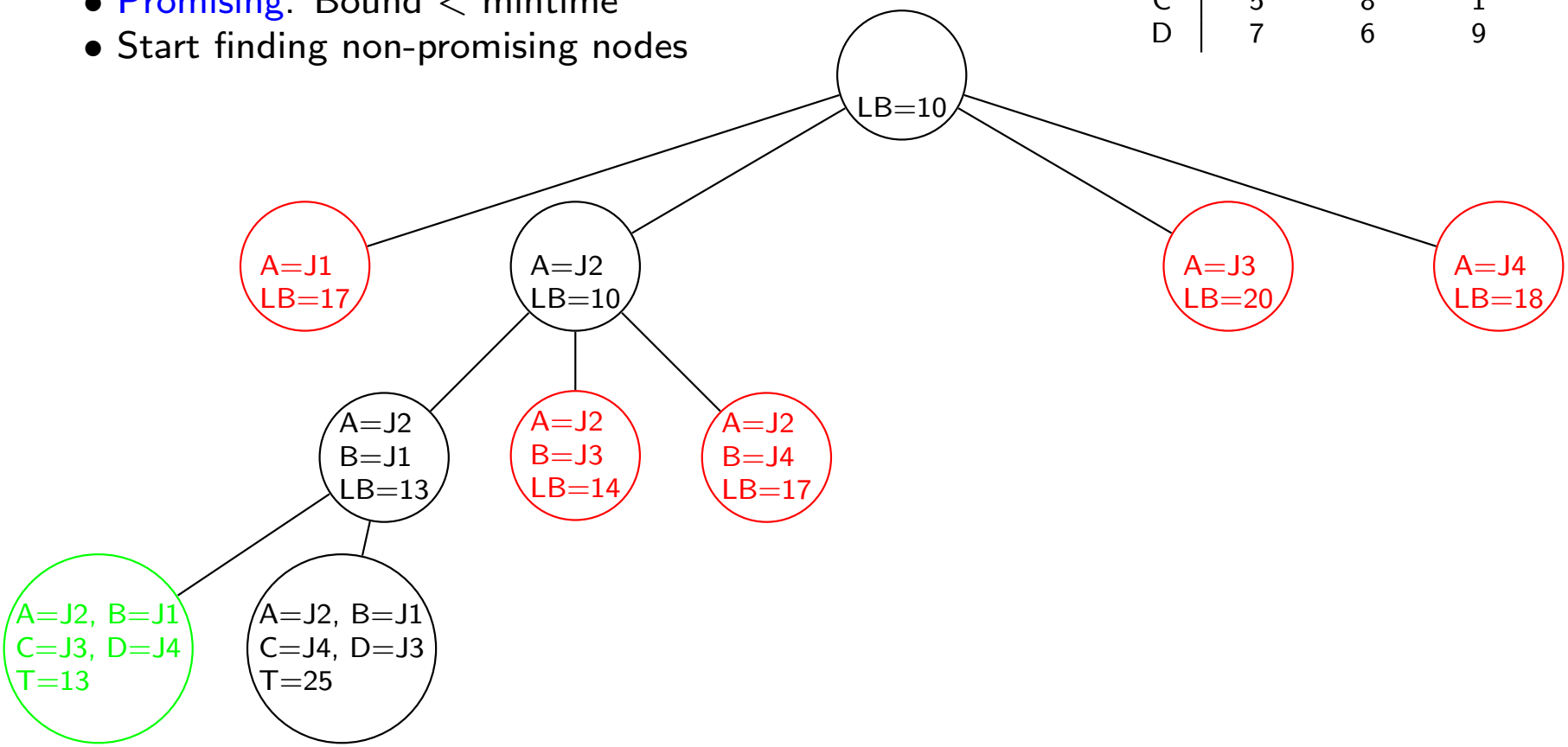

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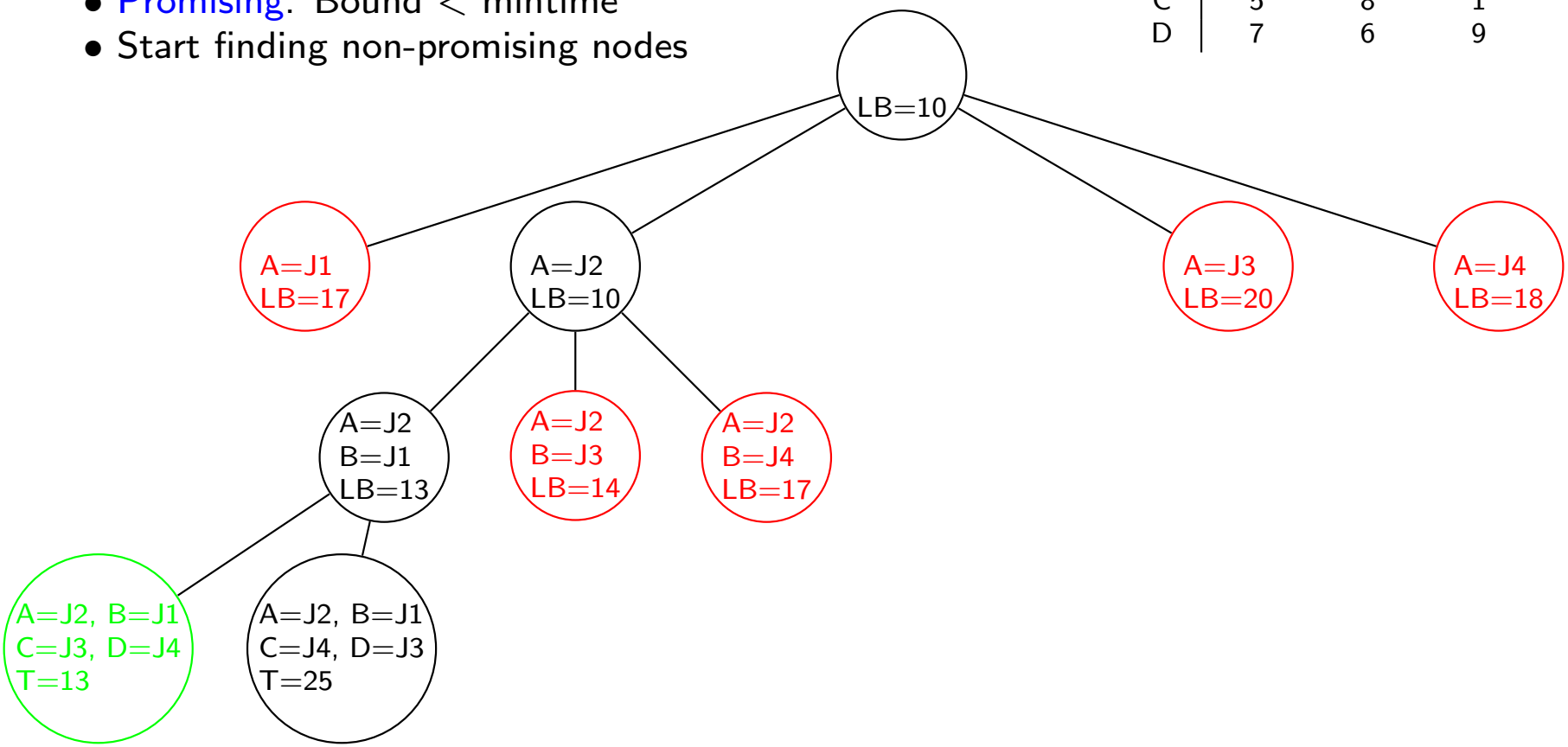

What if C can finish Job1 in 3?