Backtracking (1)

By: Aminul Islam

Based on Chapter 5 of Foundations of Algorithms

Objectives

Objectives

- Describe the backtrack programming technique
- Determine when the backtracking technique is an appropriate approach to solving a problem
- Define a state space tree for a given problem
- Define when a node in a state space tree for a given problem is promising/non-promising
- Create an algorithm to prune a state space tree
- Create an algorithm to apply the backtracking technique to solve a given problem

Depth-First Search (DFS)

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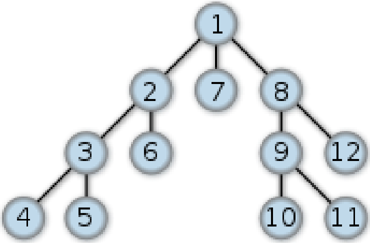

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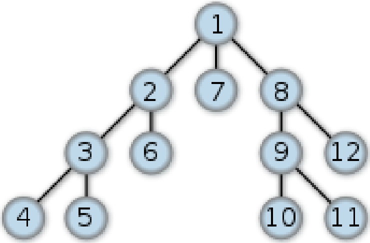

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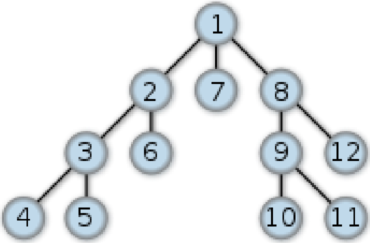

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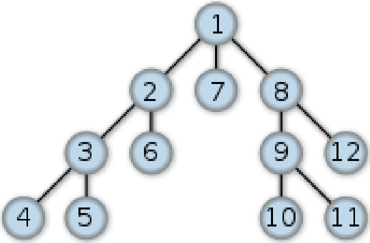

- The root is visited first
- followed by its descendants
- Assume the children of a node is visited from left to right

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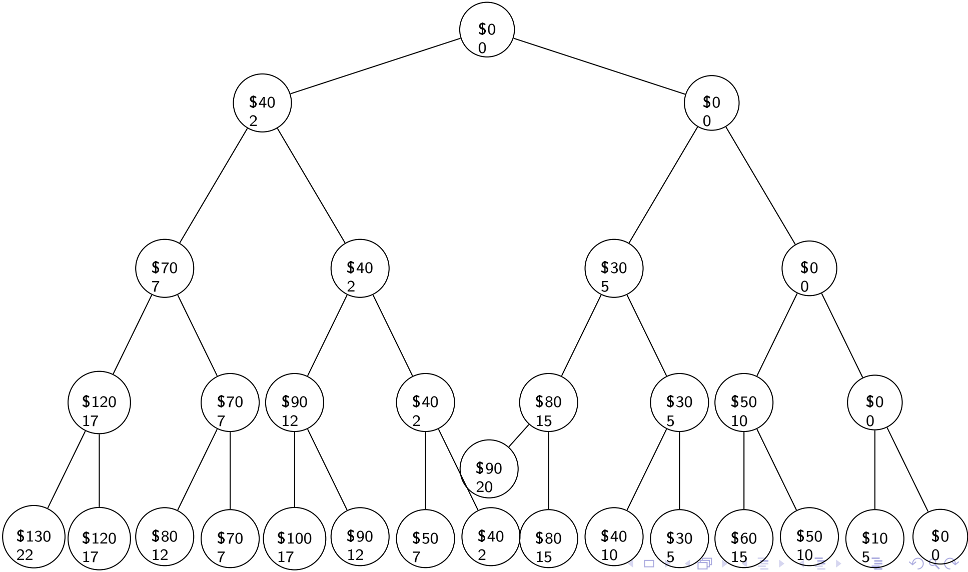

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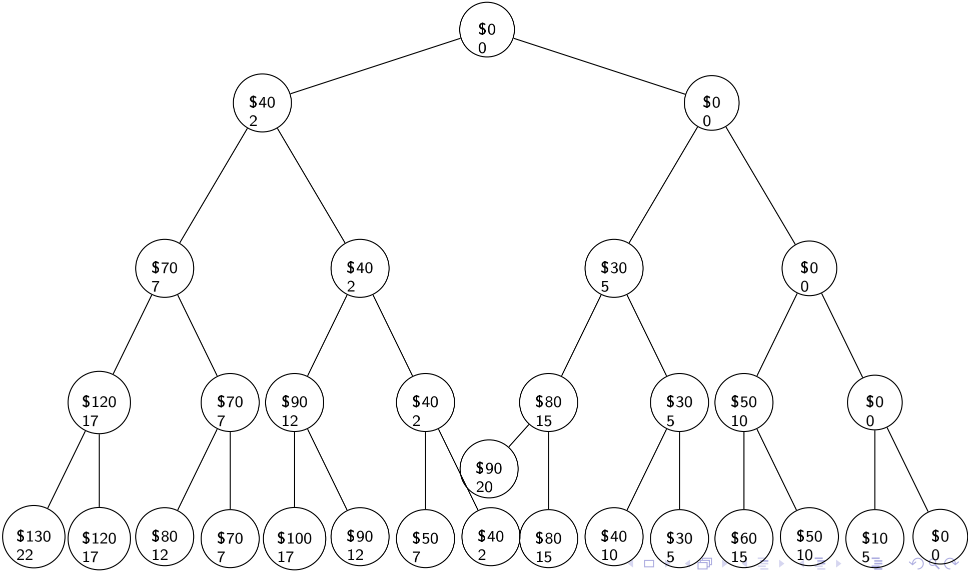

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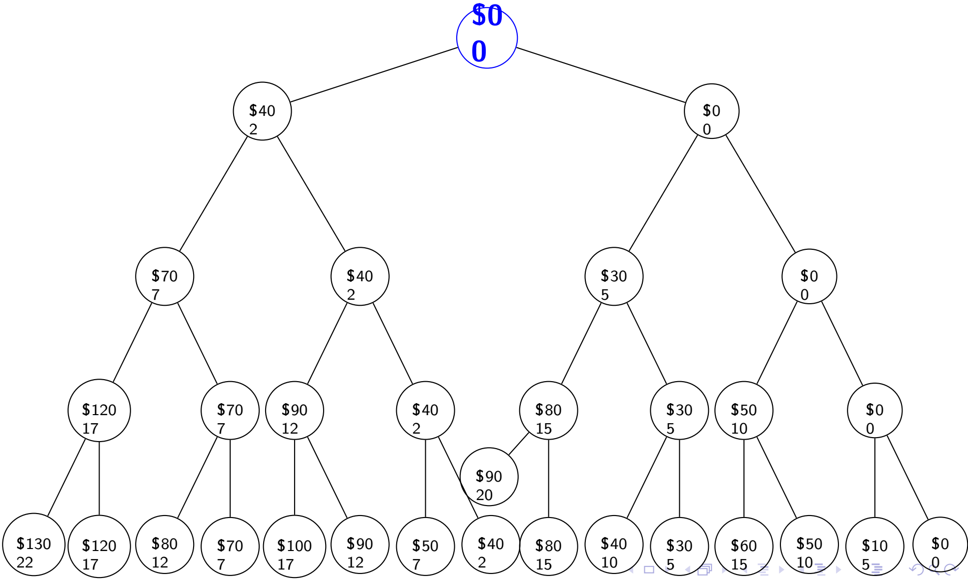

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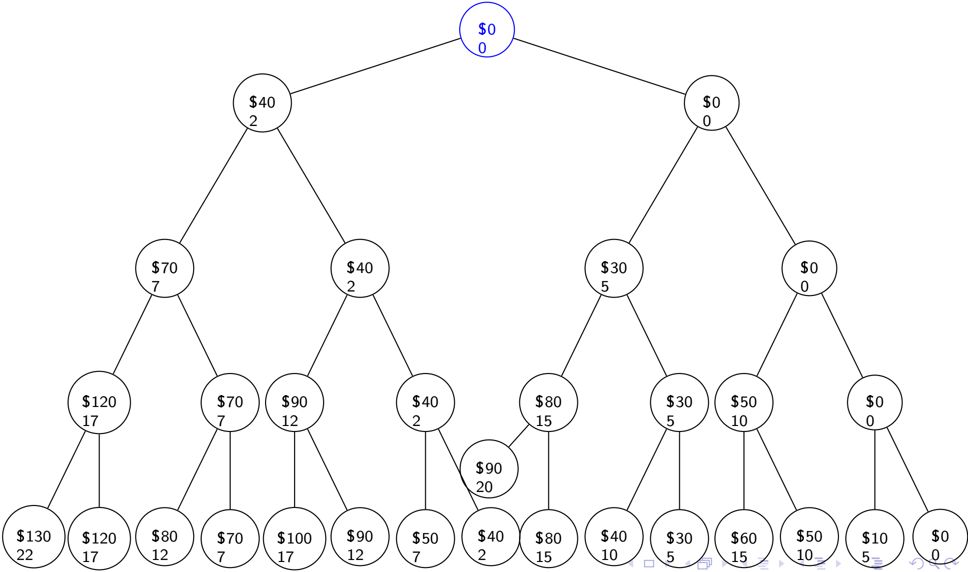

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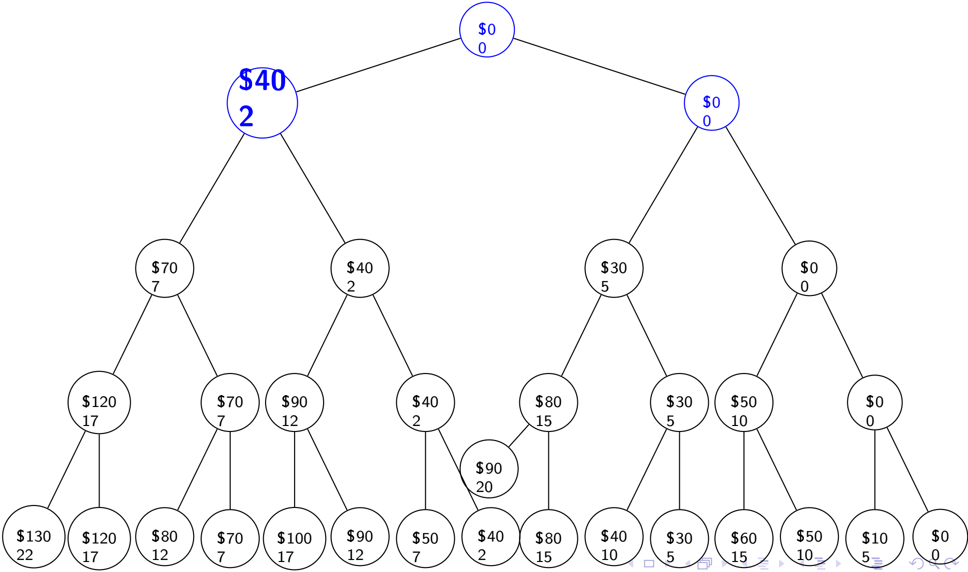

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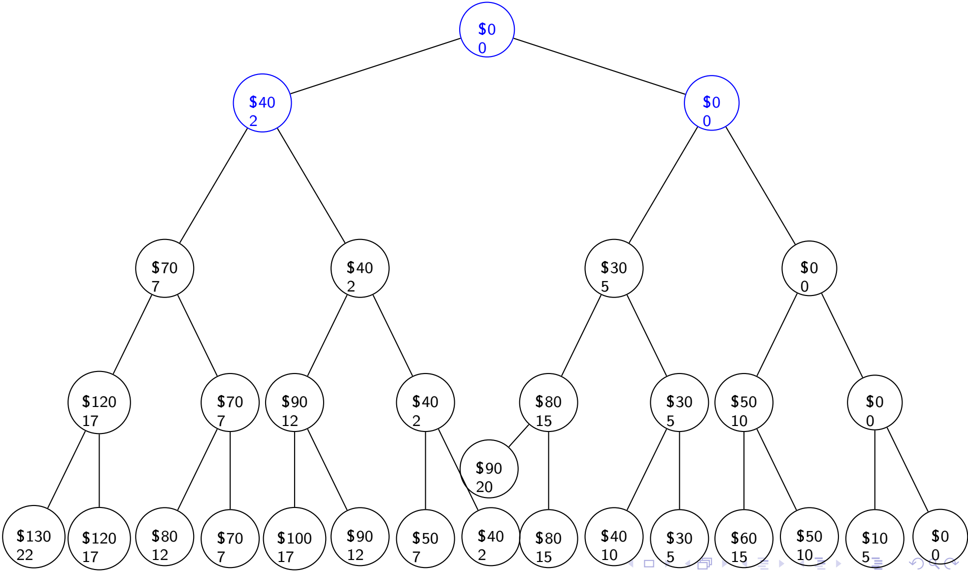

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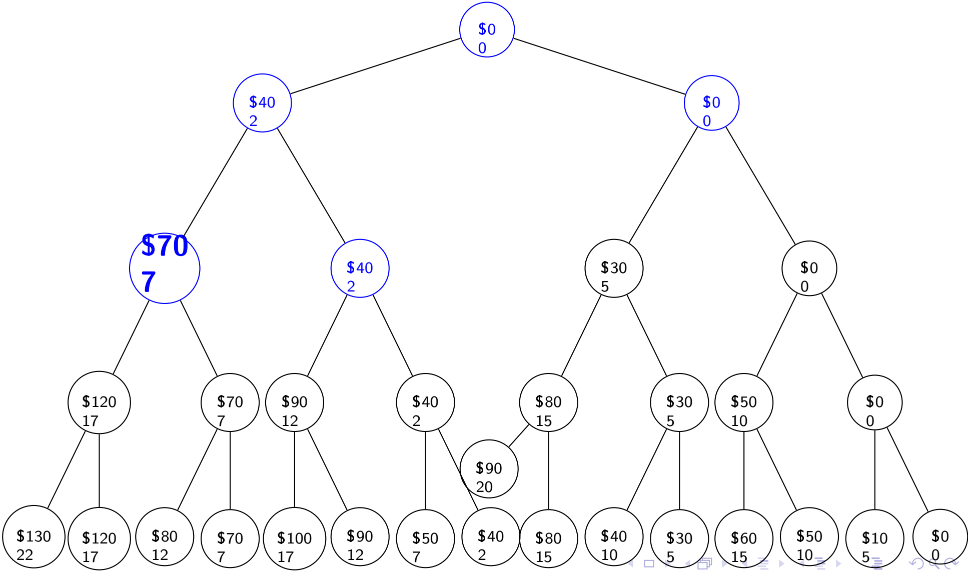

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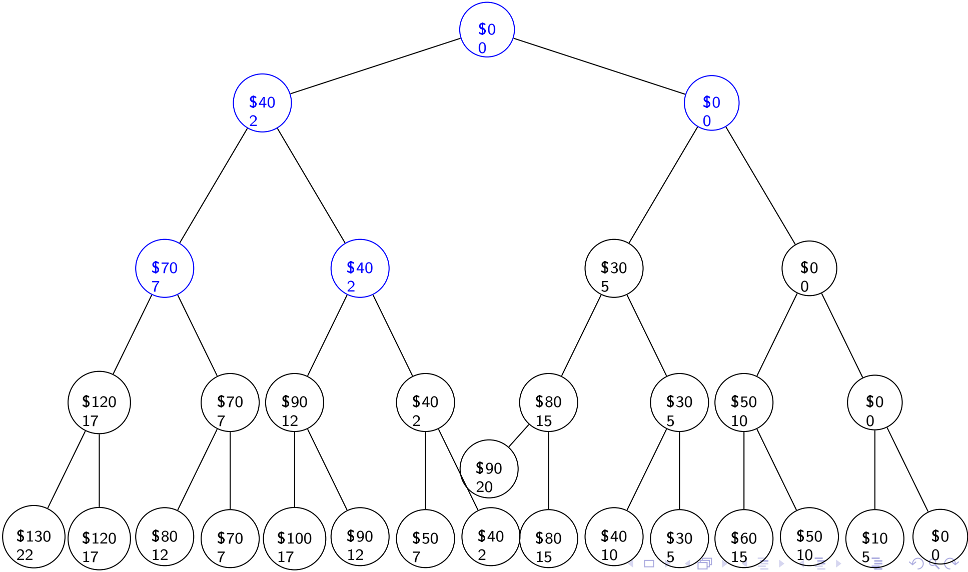

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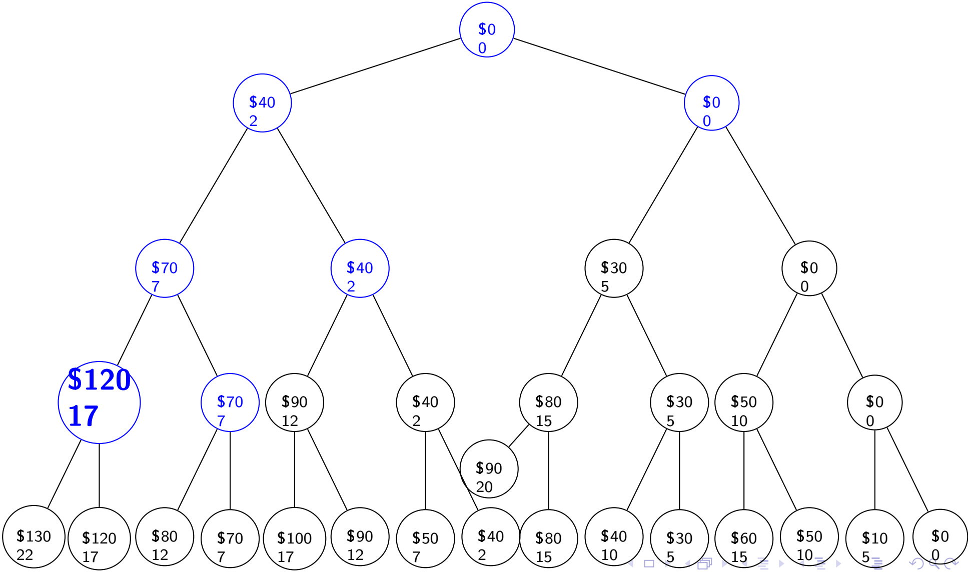

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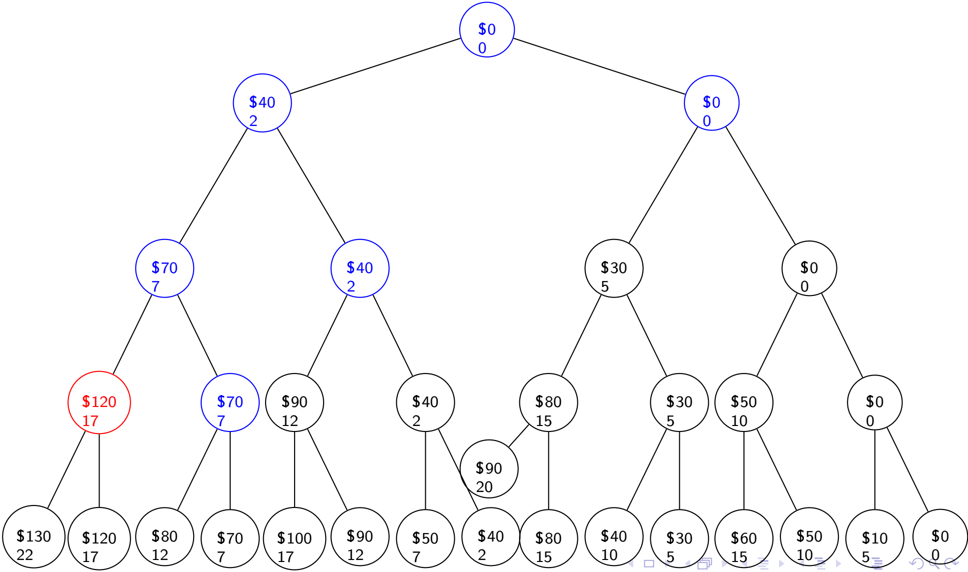

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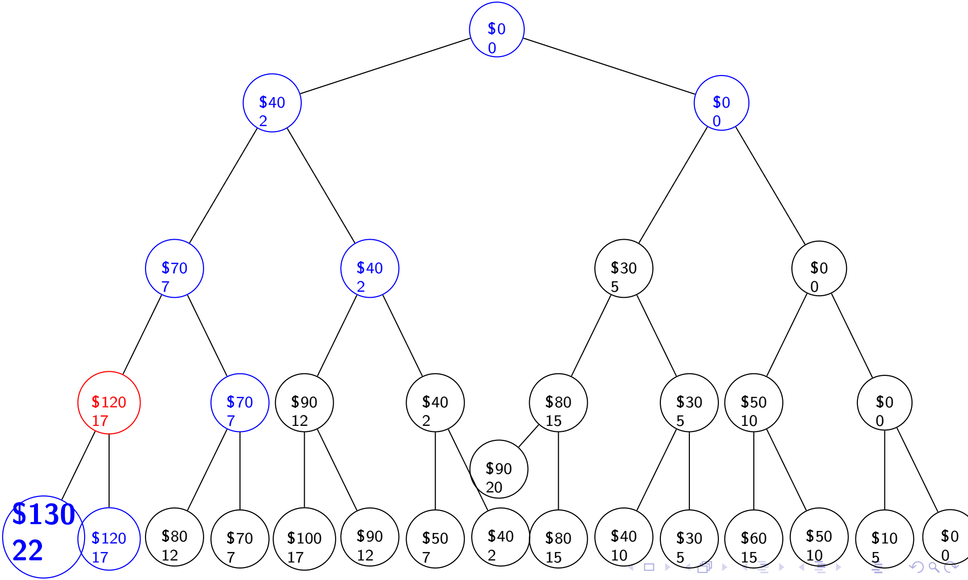

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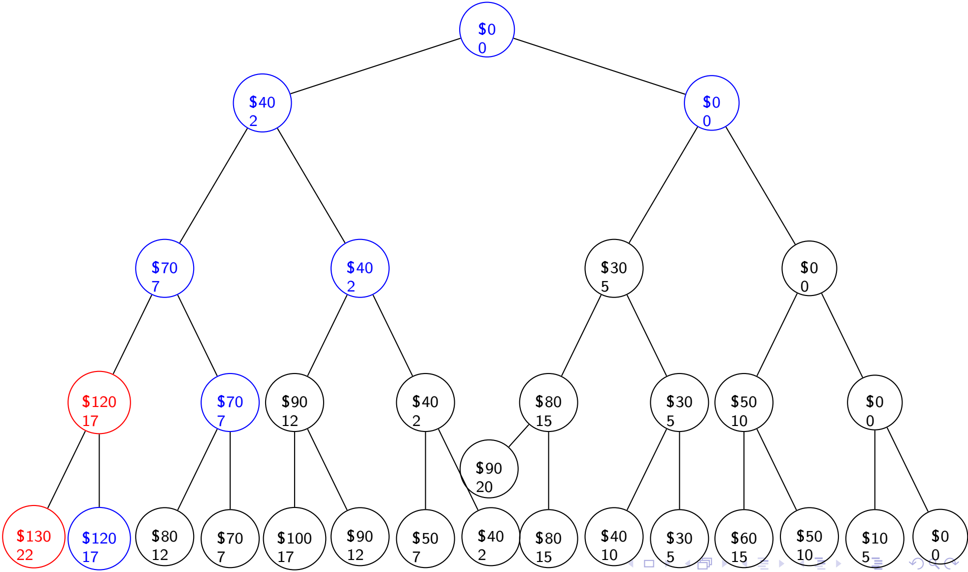

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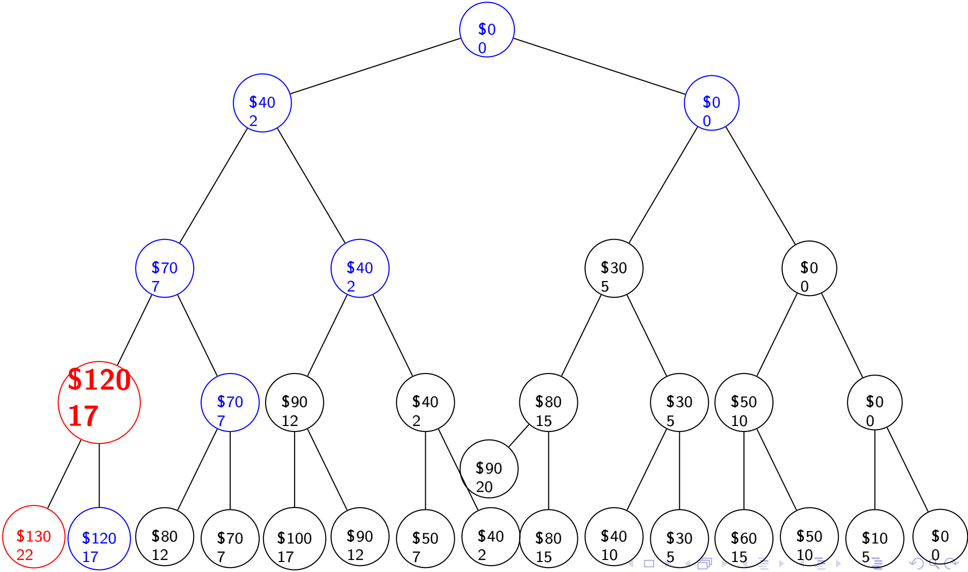

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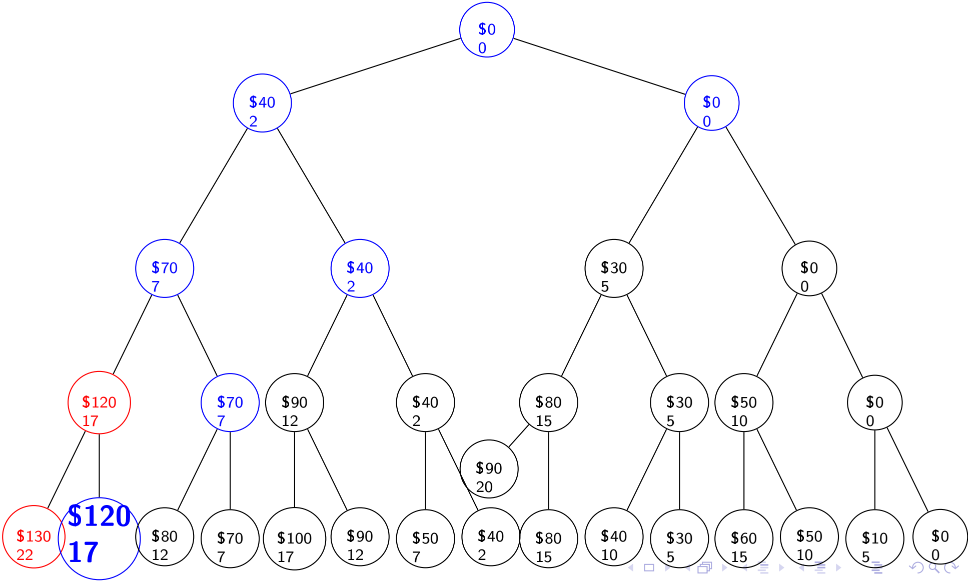

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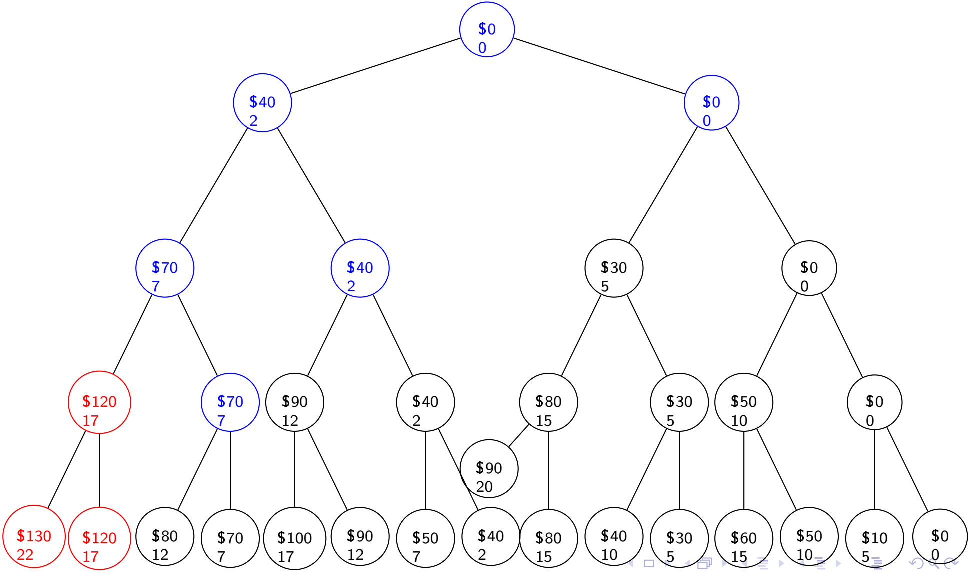

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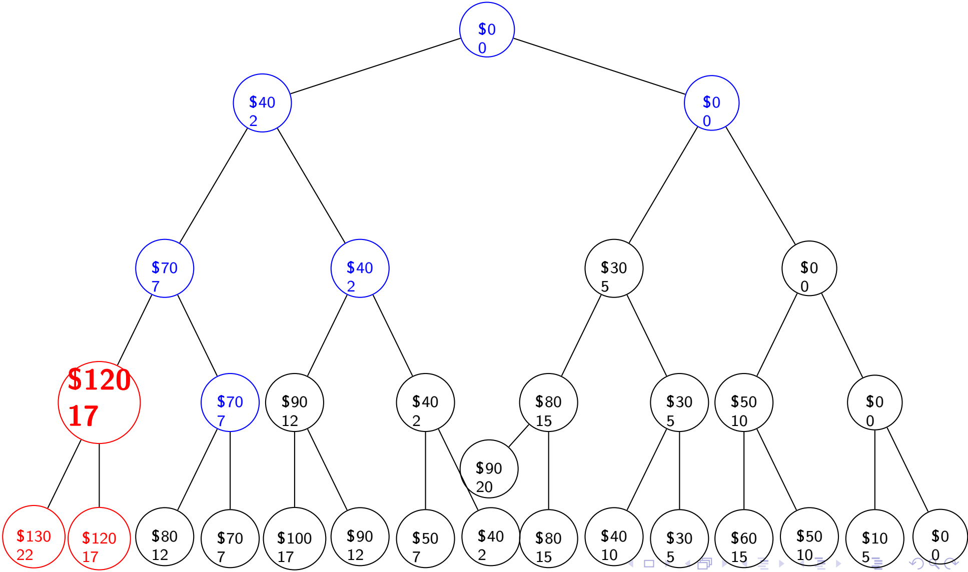

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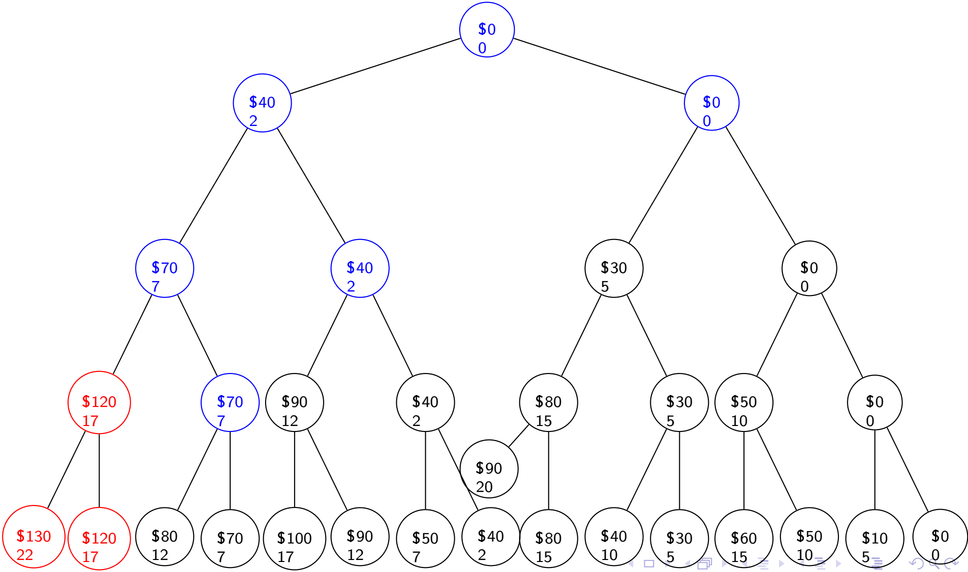

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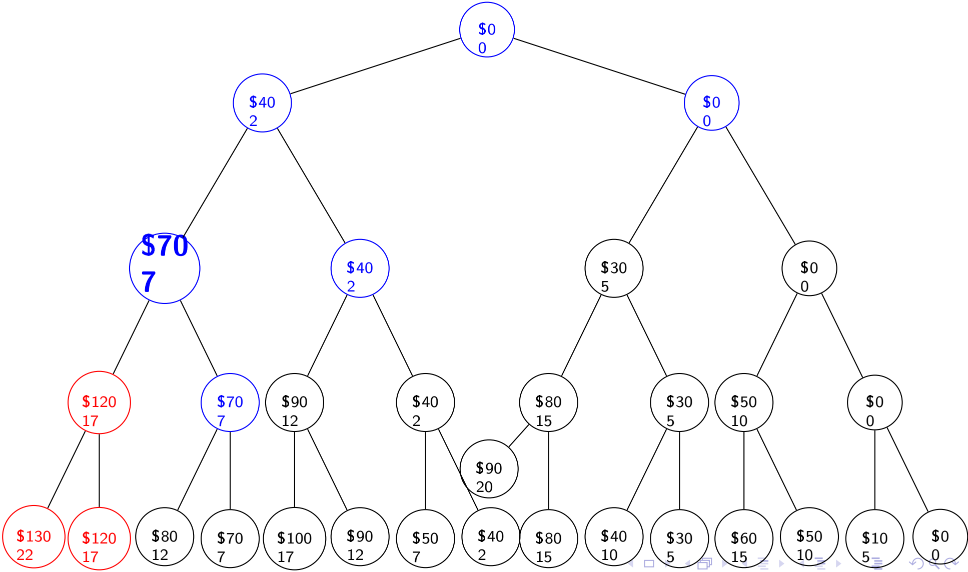

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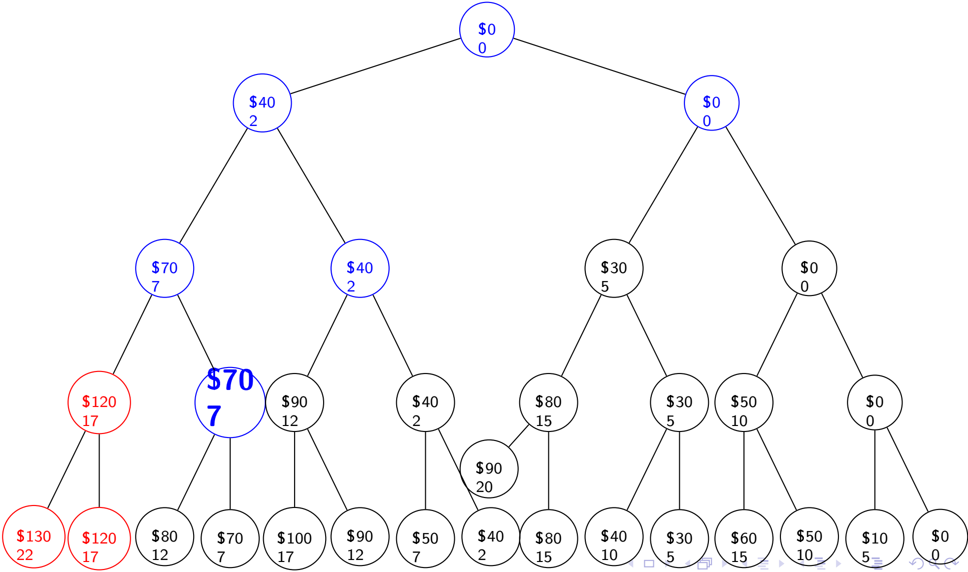

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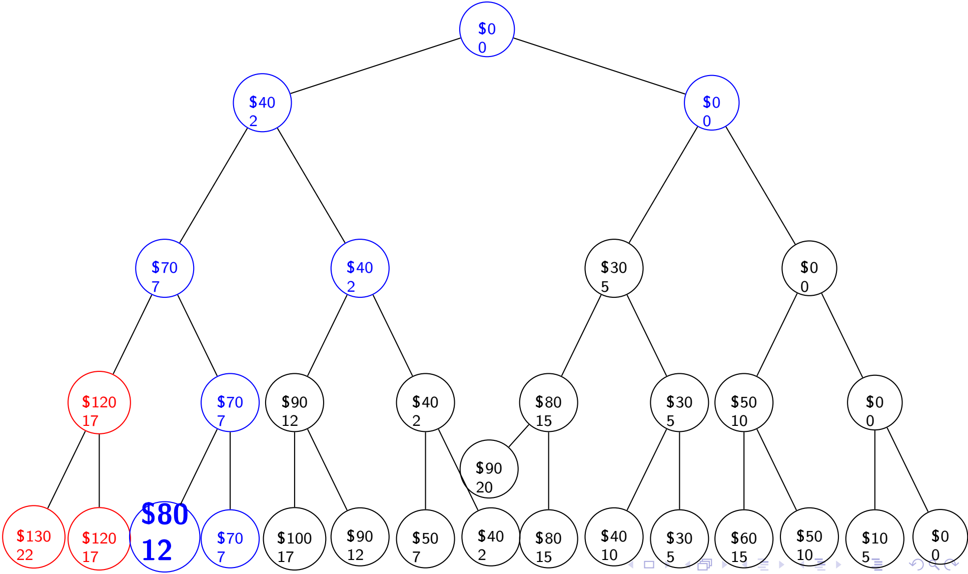

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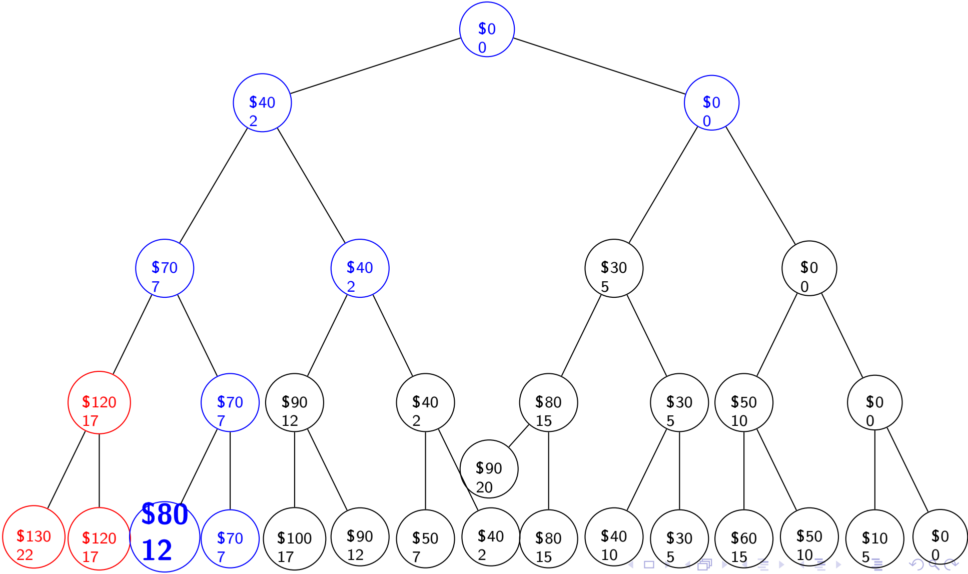

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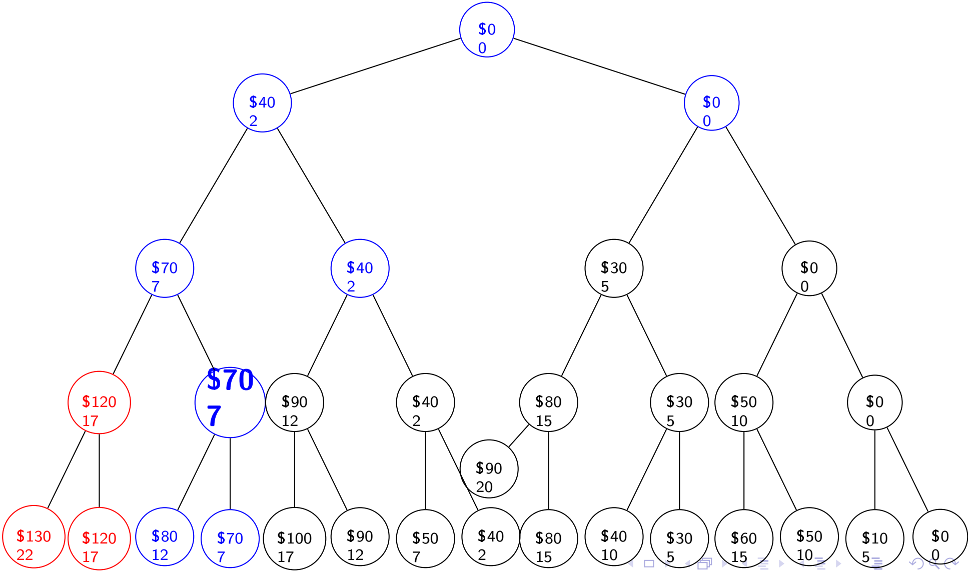

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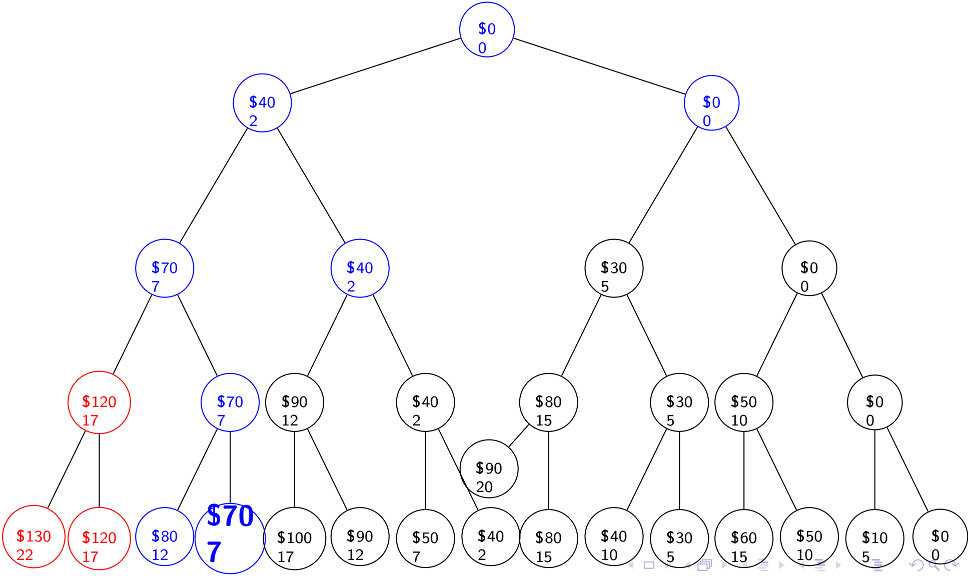

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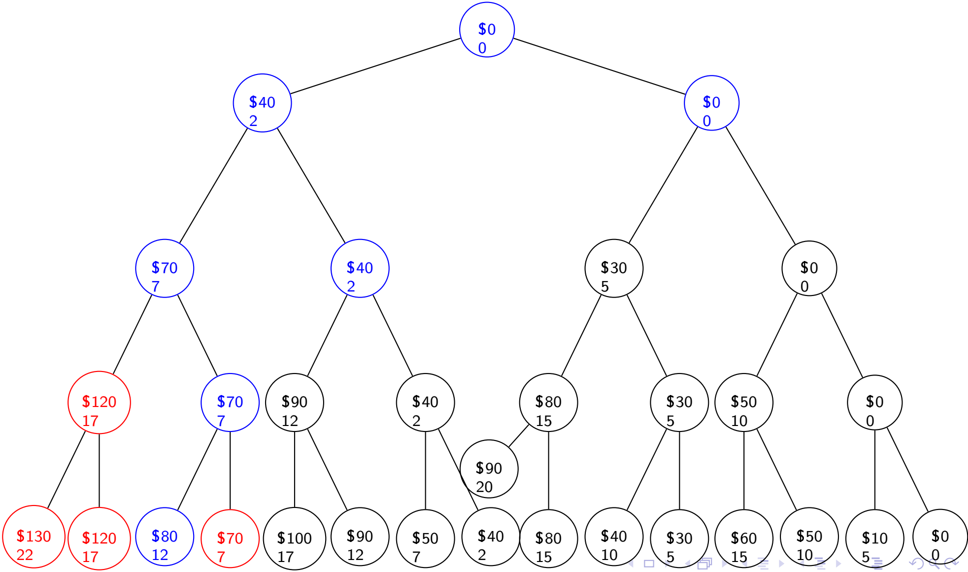

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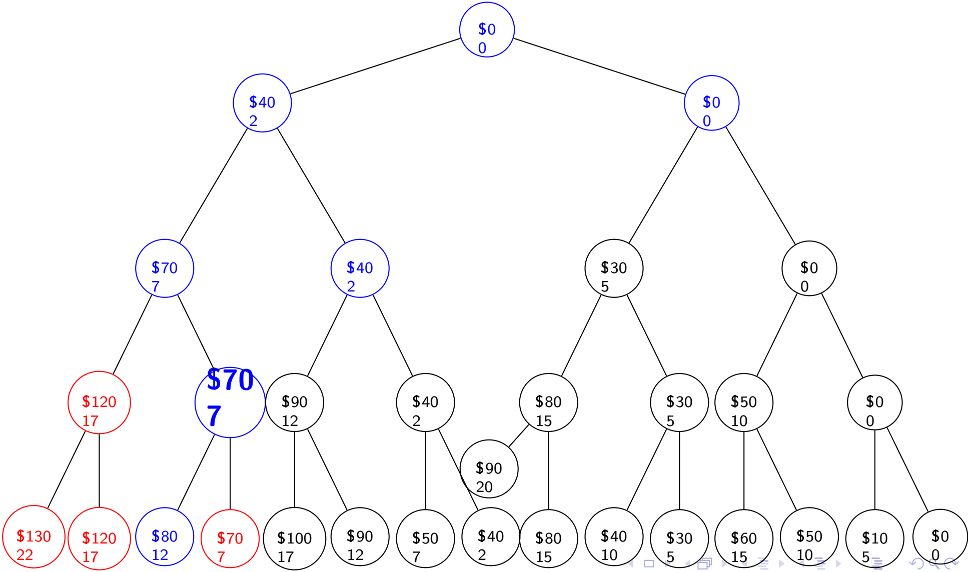

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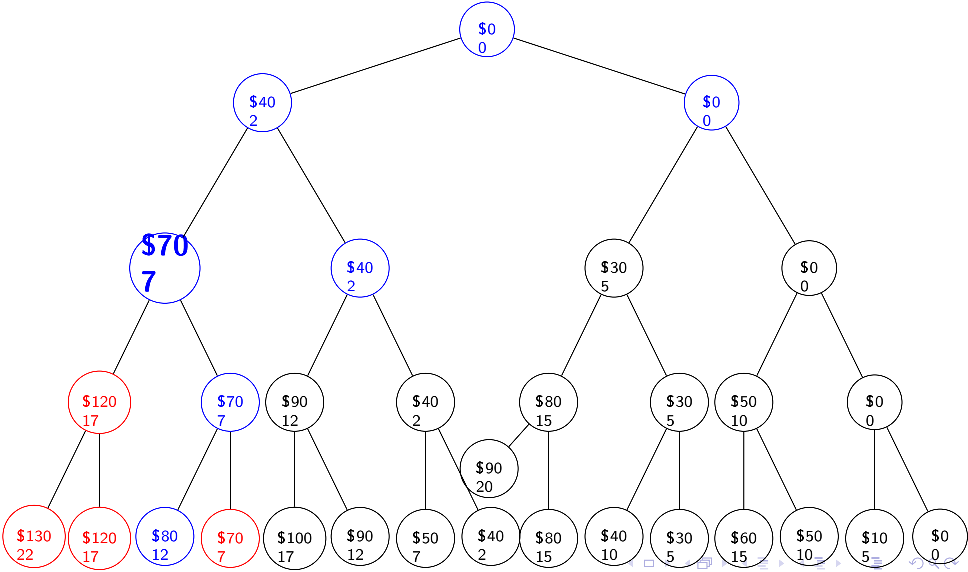

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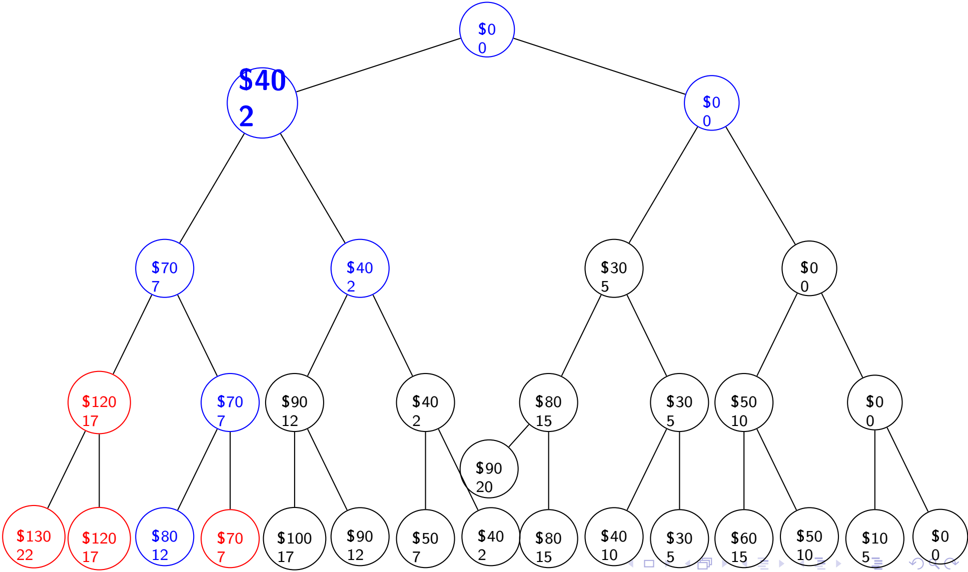

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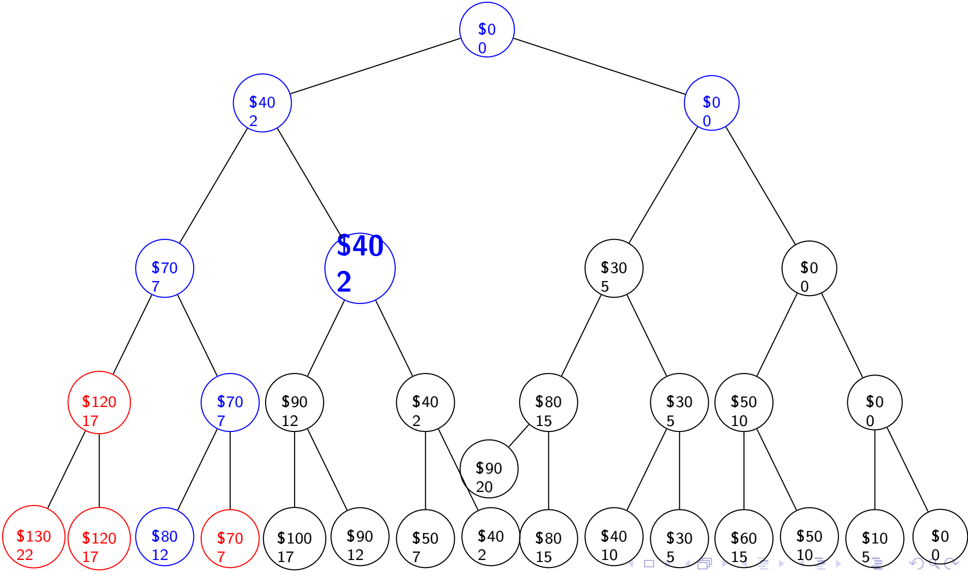

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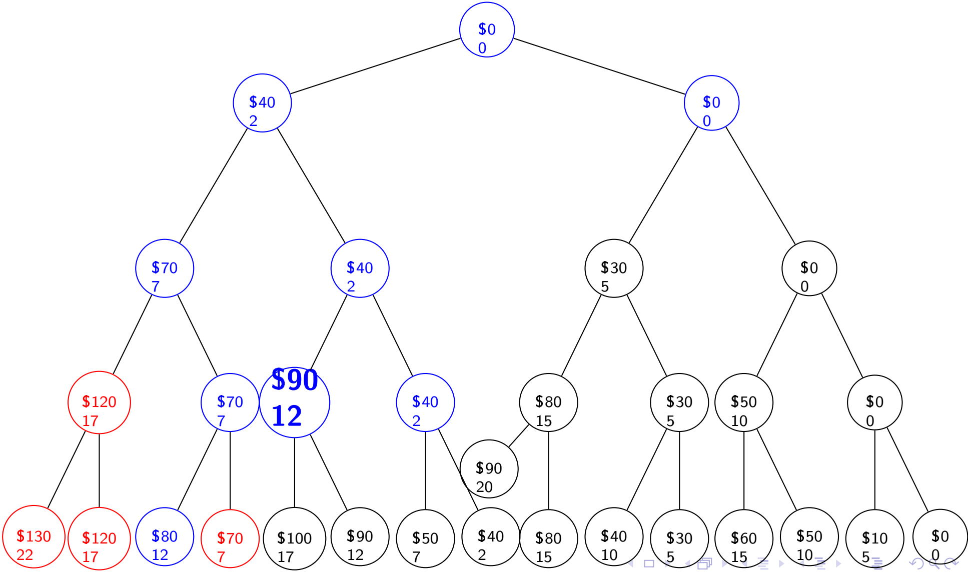

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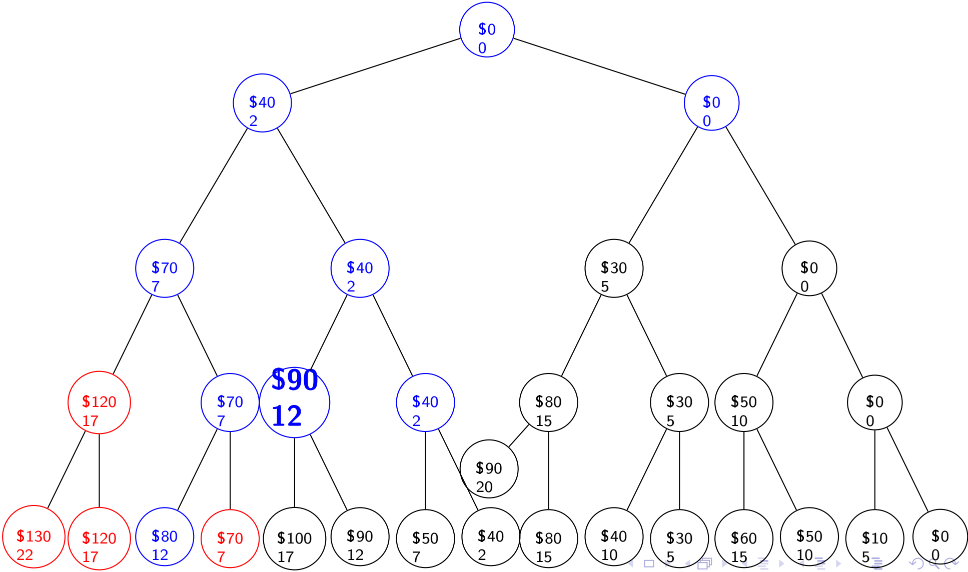

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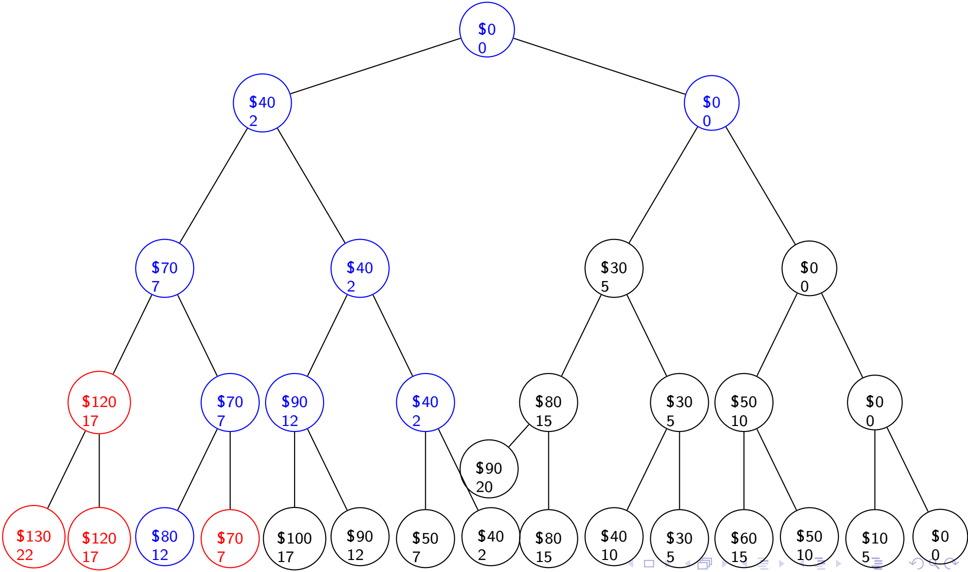

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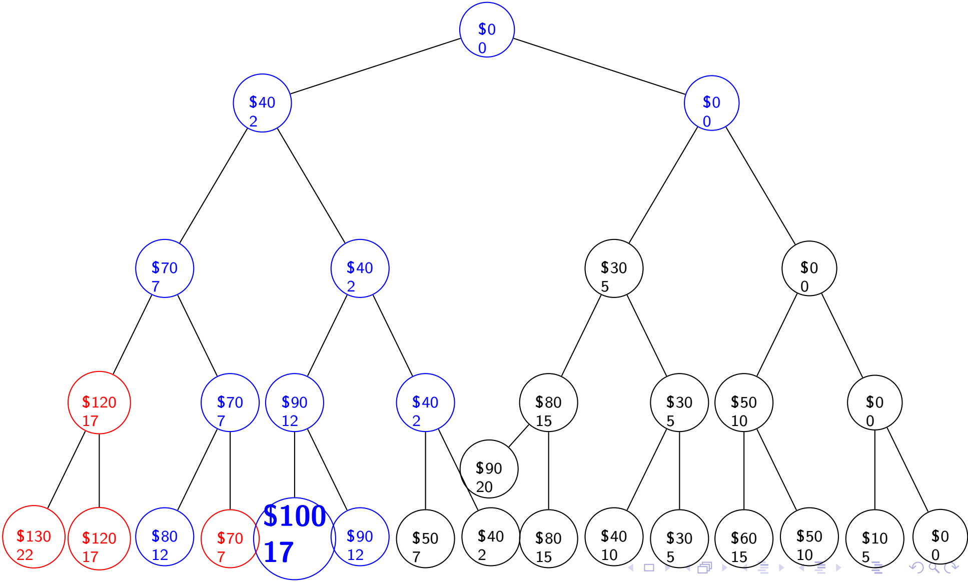

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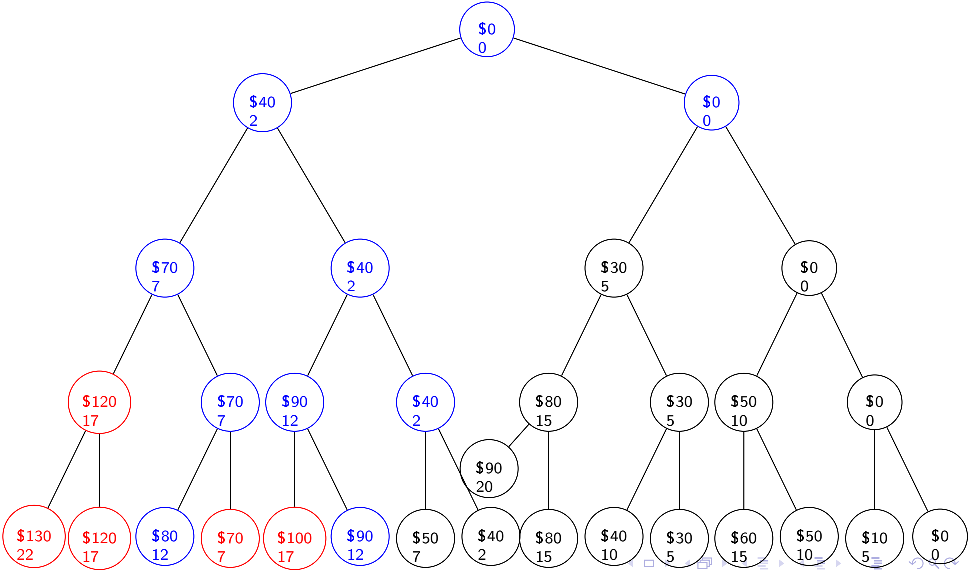

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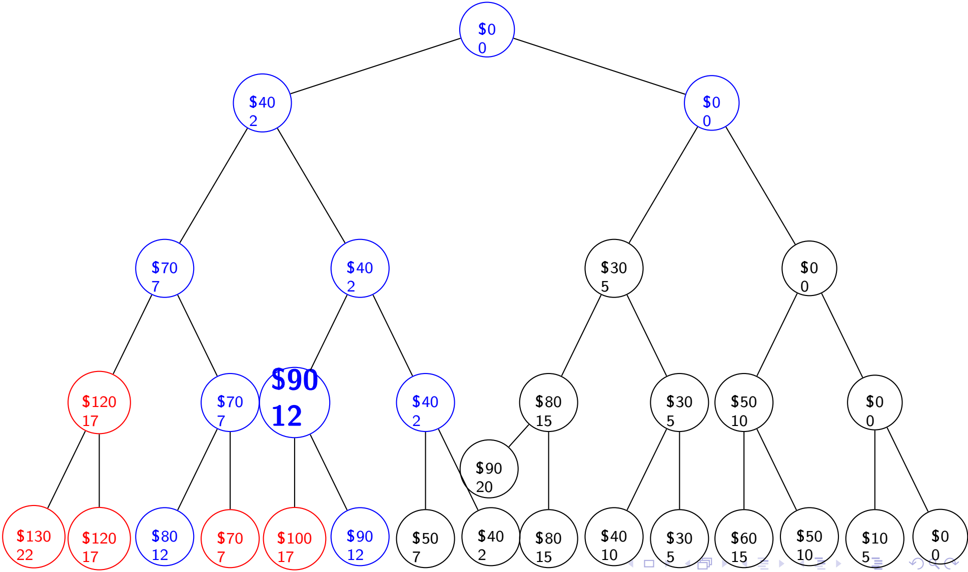

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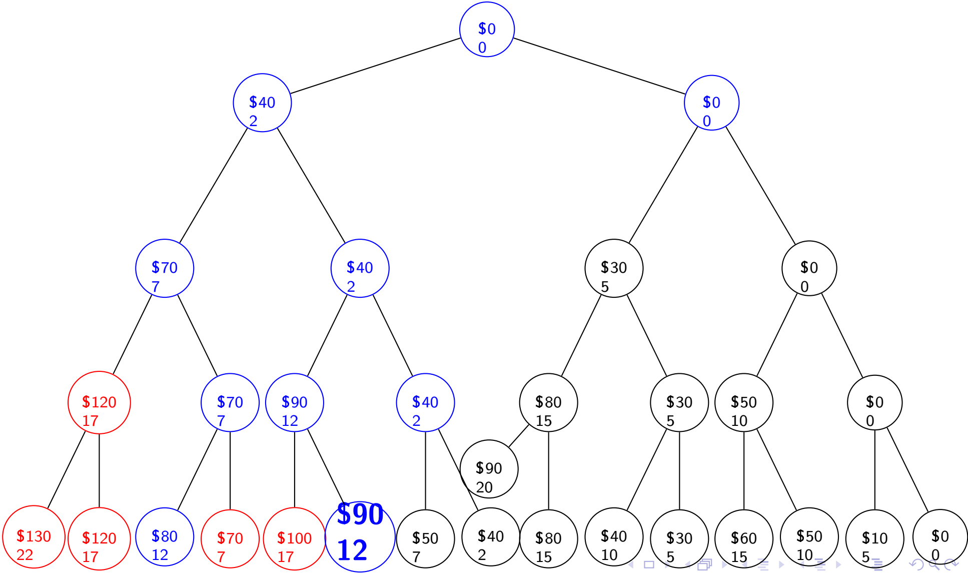

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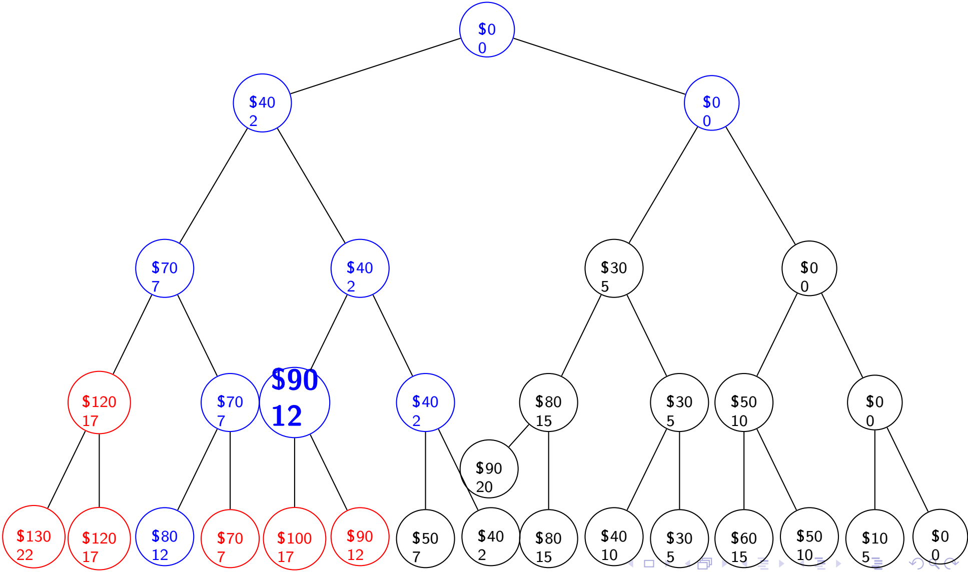

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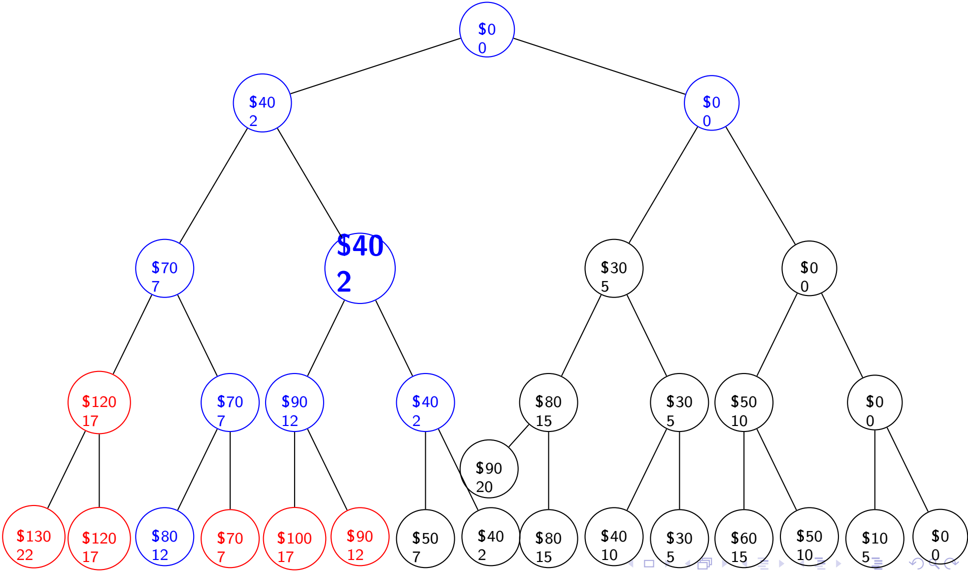

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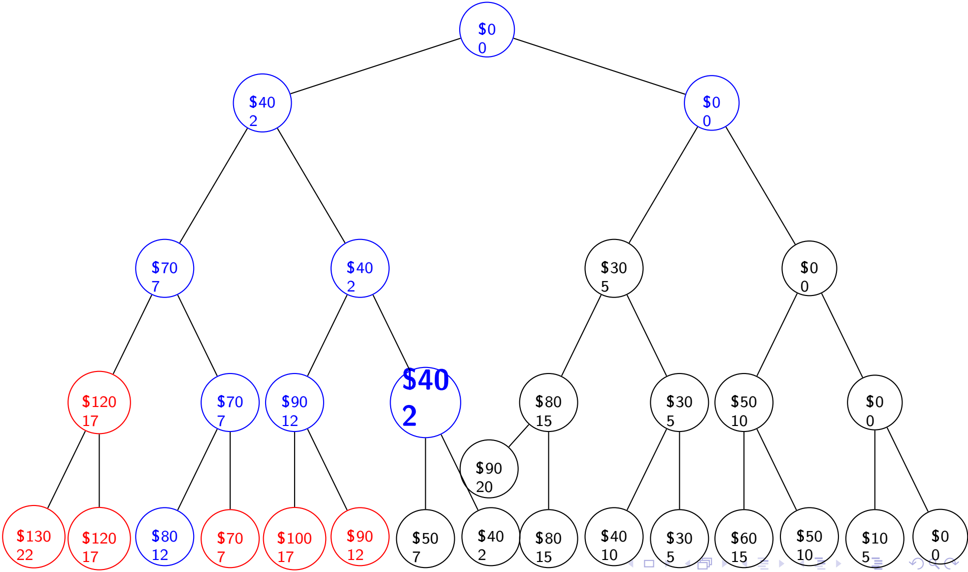

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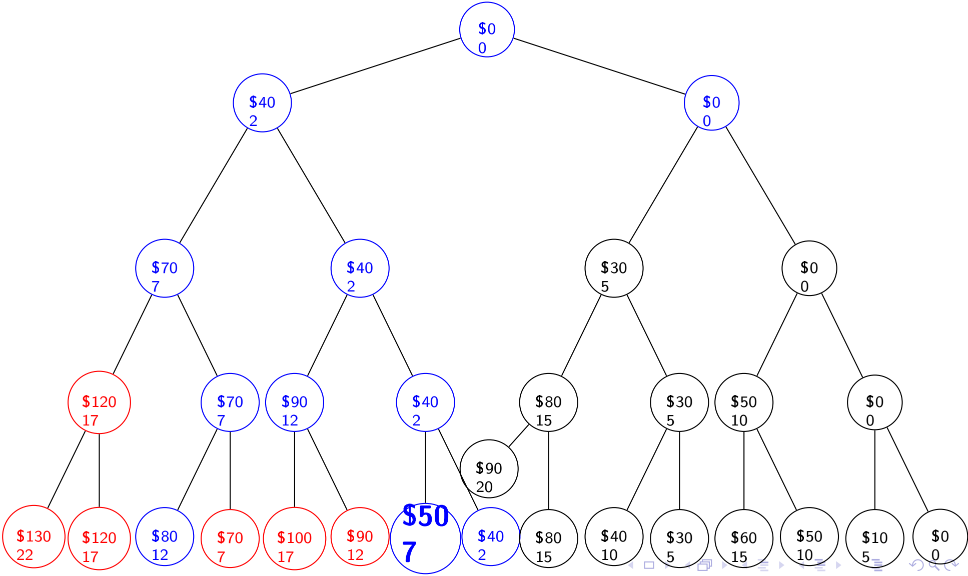

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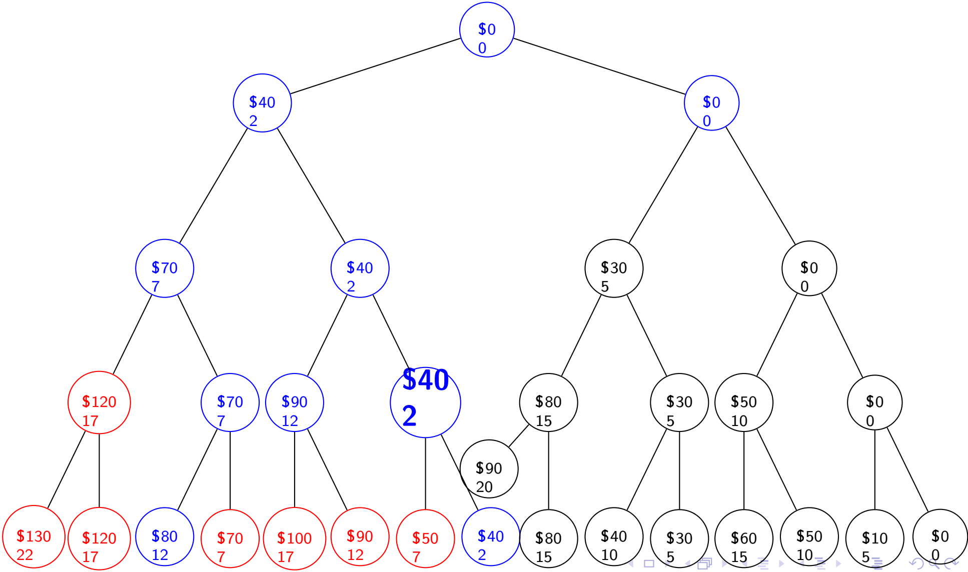

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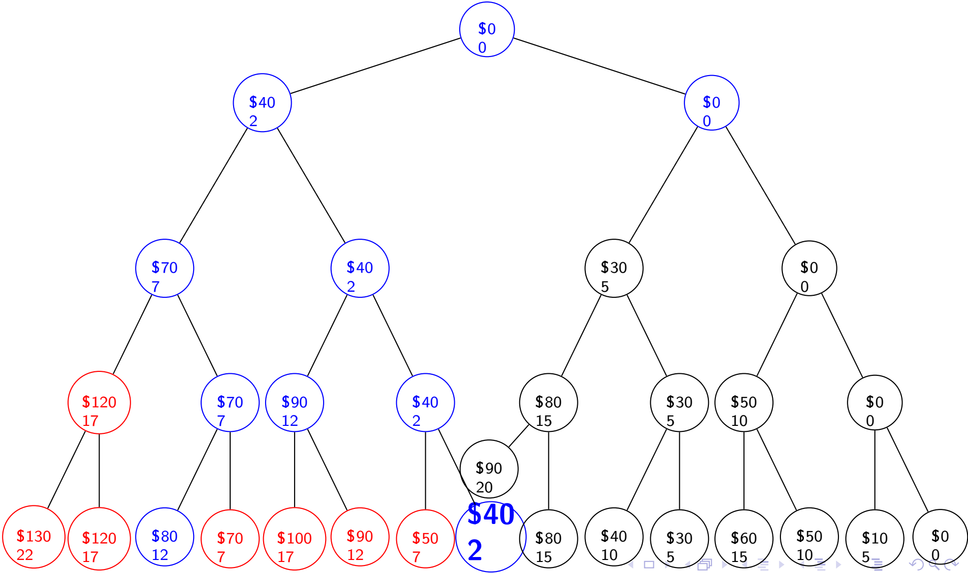

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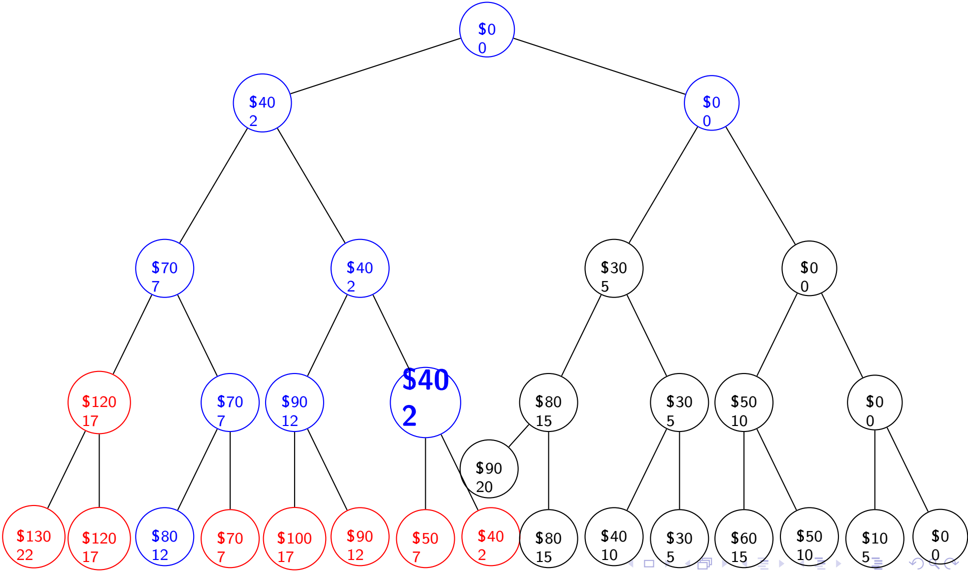

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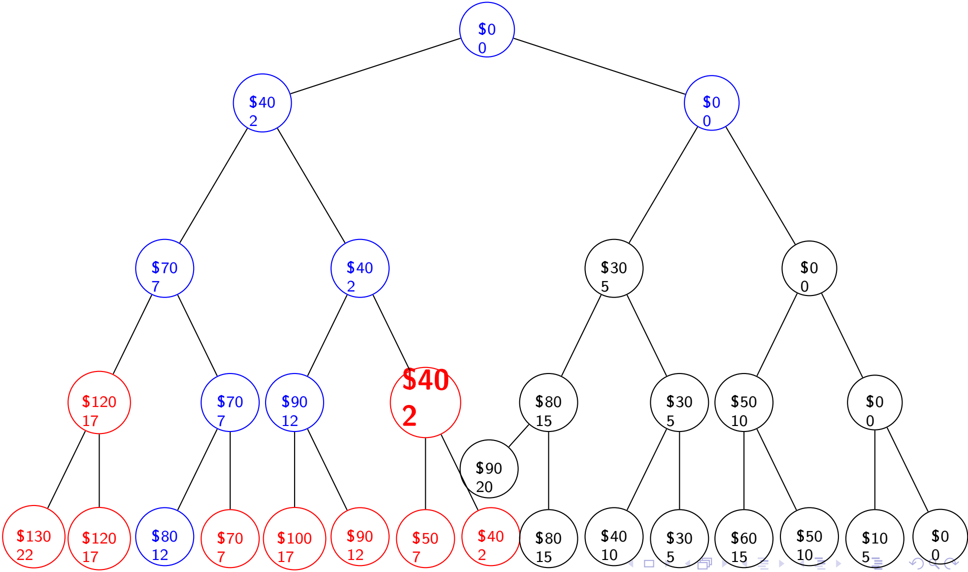

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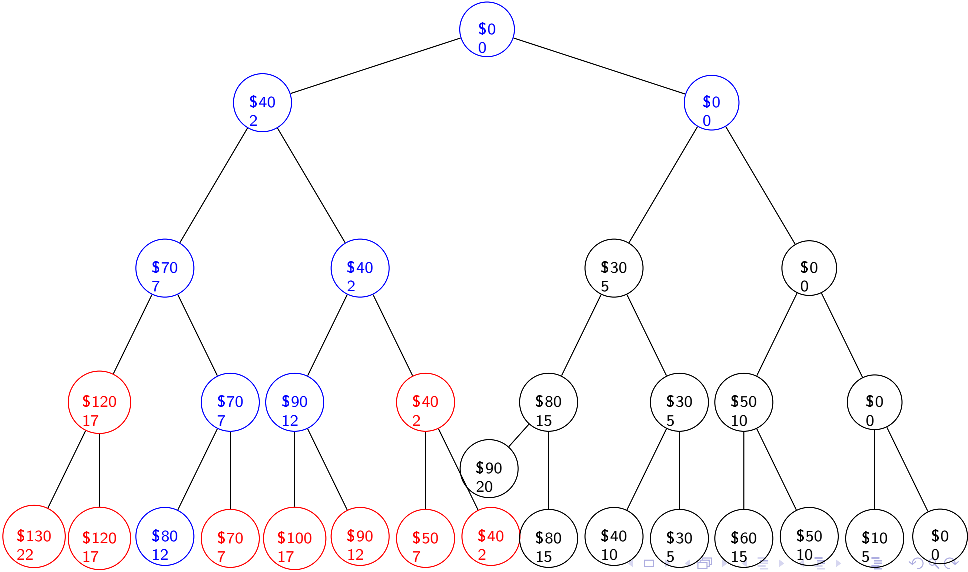

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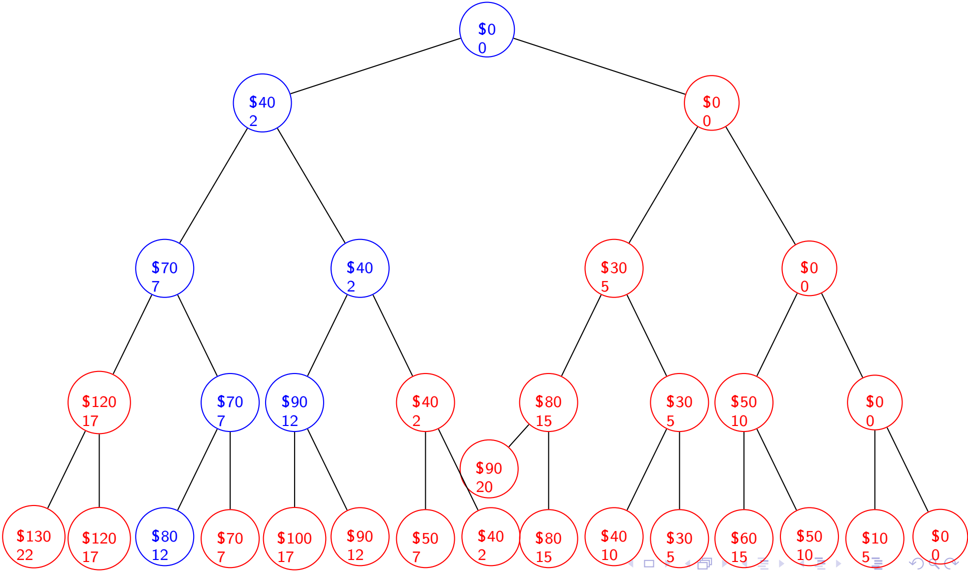

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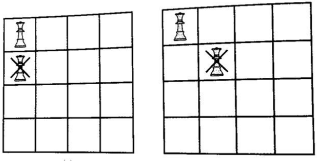

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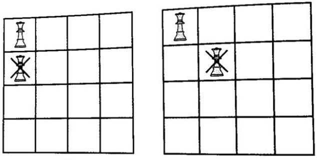
- Goal: position n queens on a $n \times n$ board such that no two queens threaten each other
  - No two queens may be in the same row, column, or diagonal



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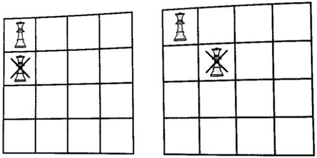
- Sequence: n positions where queens are placed



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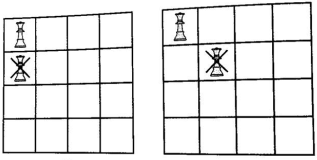

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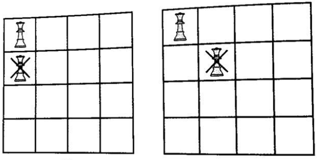
  - No two queens may be in the same row, column, or diagonal
- Sequence: n positions where queens are placed

- Set: n^2 positions on the board
- Criterion: no two queens threaten each other



4-Queen Problem (DFS)

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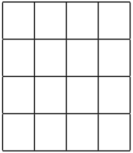
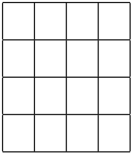
- Consider a 4-queen problem for simplicity



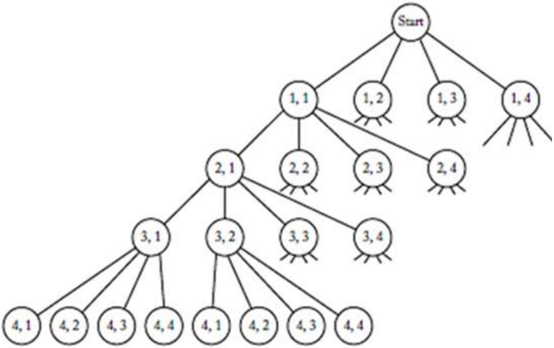
4-Queen Problem (DFS)

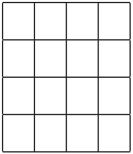
- Consider a 4-queen problem for simplicity
- Assign each queen a different row



4-Queen Problem (DFS)

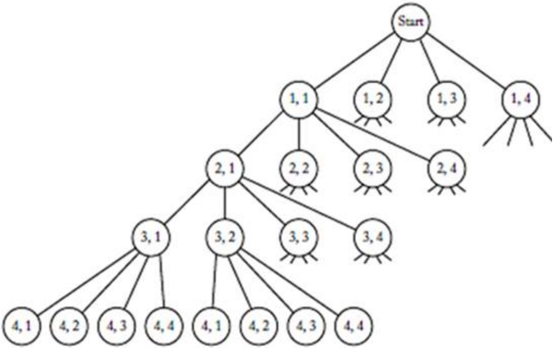
- Consider a 4-queen problem for simplicity
- Assign each queen a different row
- Check which column combinations yield solutions

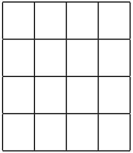




4-Queen Problem (DFS)

- Consider a 4-queen problem for simplicity
- Assign each queen a different row
- Check which column combinations yield solutions
- We can represent this idea in a 'State space tree'





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- Consider a 4-queen problem for simplicity
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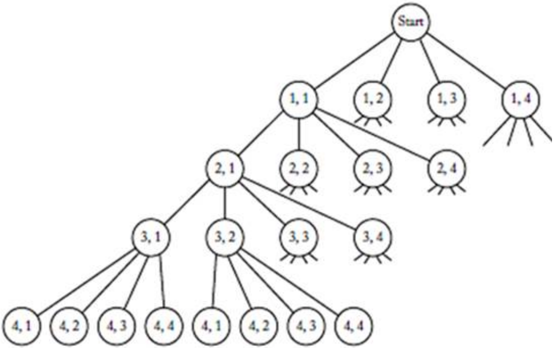
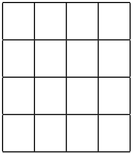
- What would be the total number of nodes in this 'State space tree'





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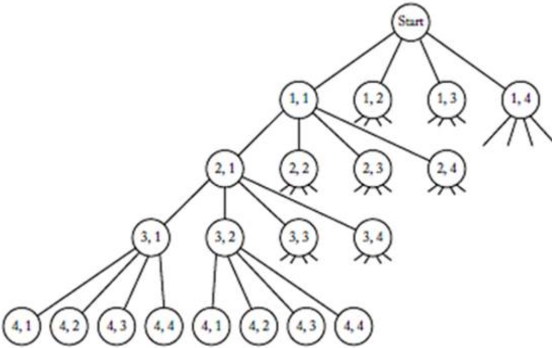
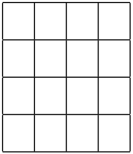
  $1 + 4 + 4^2 + 4^3 + 4^4$





4-Queen Problem (DFS)

- Consider a 4-queen problem for simplicity
- Assign each queen a different row
- Check which column combinations yield solutions
- We can represent this idea in a 'State space tree'
- What would be the total number of nodes in this 'State space tree'
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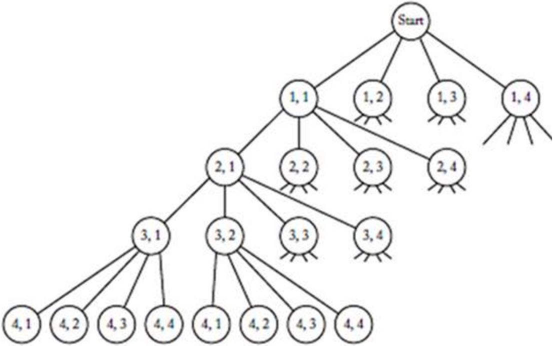

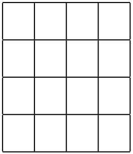

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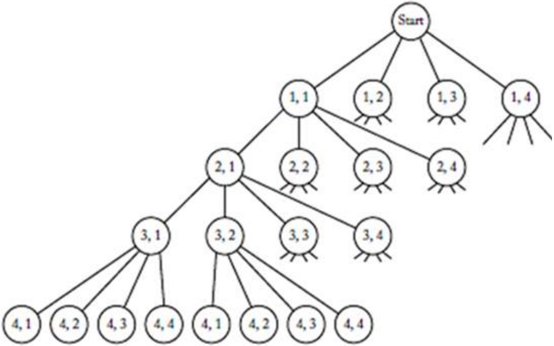

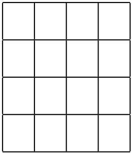

4-Queens problem (improving from DFS)

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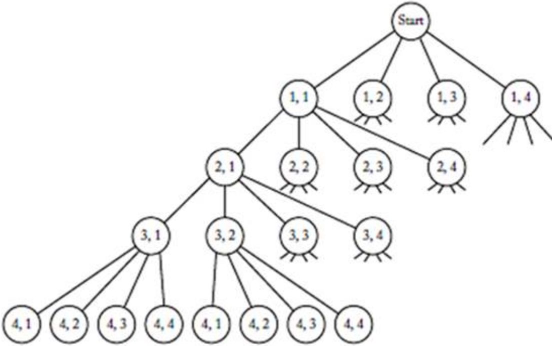

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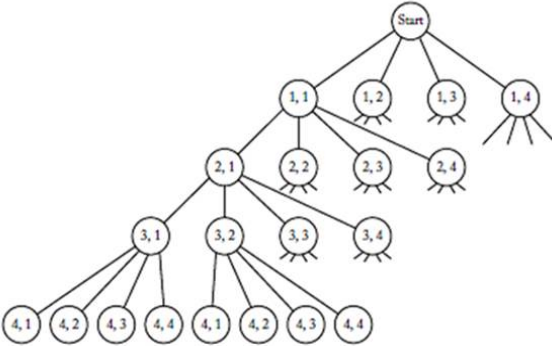

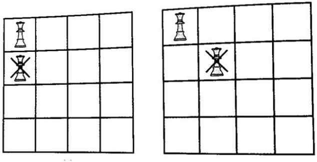

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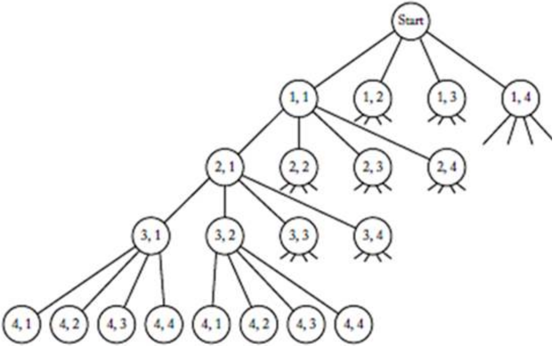

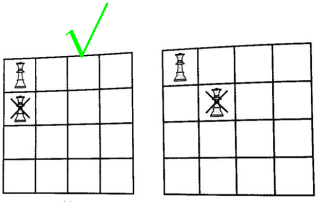

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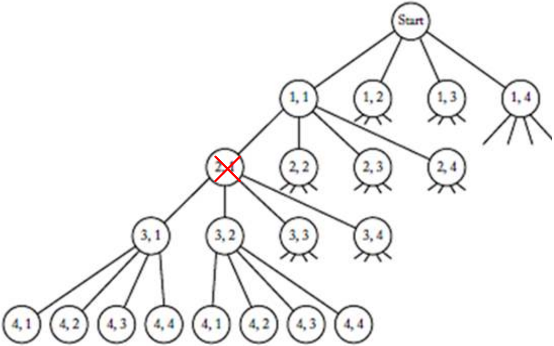

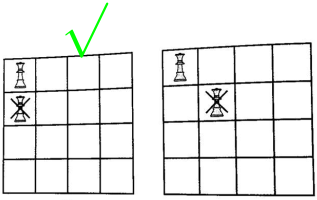

4-Queens problem (improving from DFS) (2)

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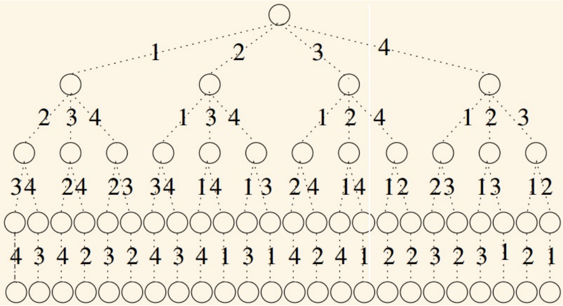

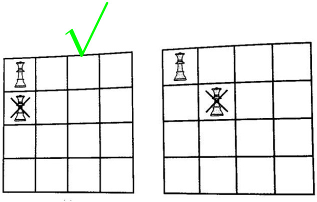

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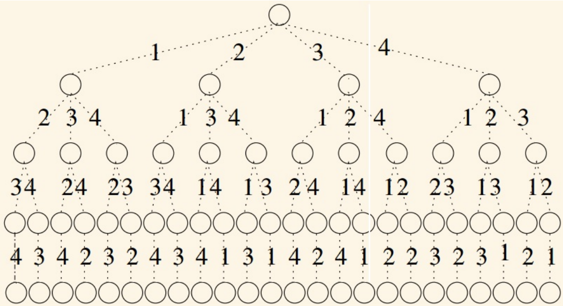
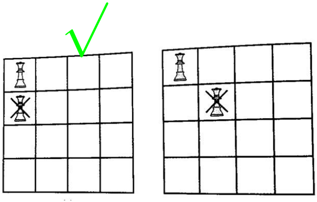
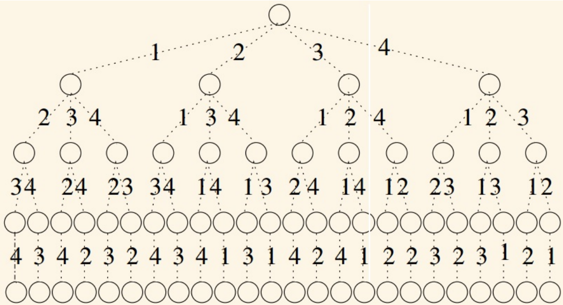
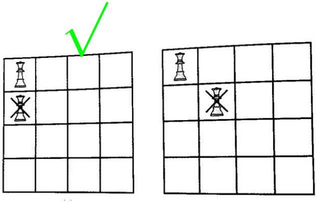




- What would be the total number of sequence in this 'State space tree'?

4-Queens problem (improving from DFS) (2)





- What would be the total number of sequence in this 'State space tree'? $4 \times 3 \times 2 \times 1$

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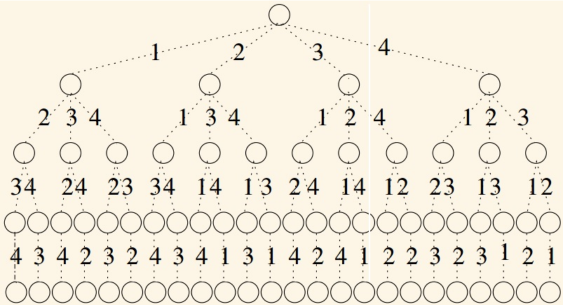

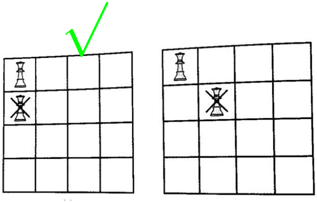

- What would be the total number of sequence in this 'State space tree'?
  $4 \times 3 \times 2 \times 1 \;= 4!$

4-Queens problem (improving from DFS) (3)

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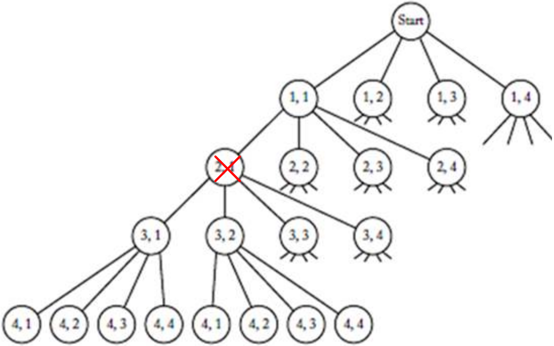

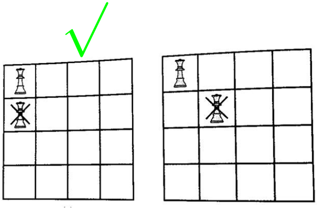

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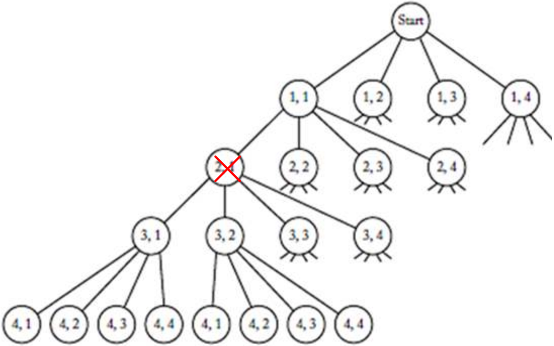

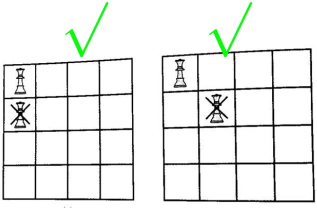

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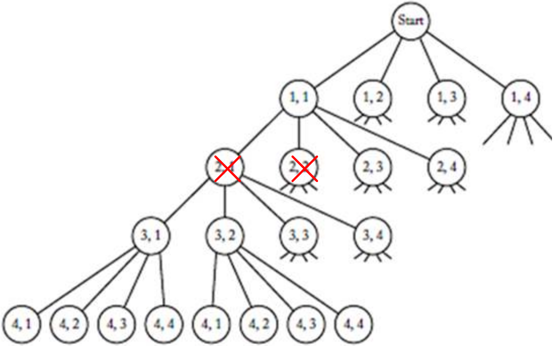

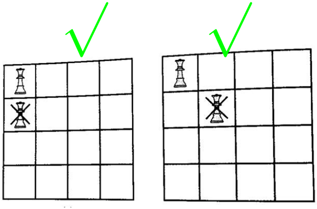

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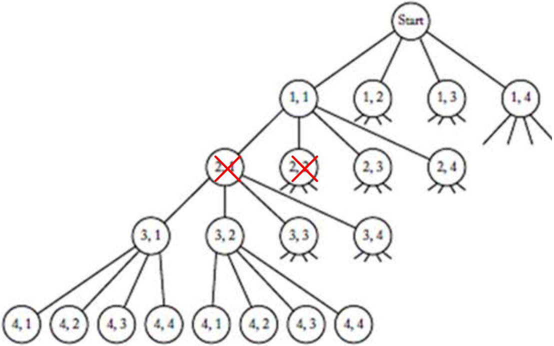
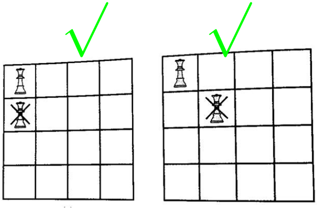
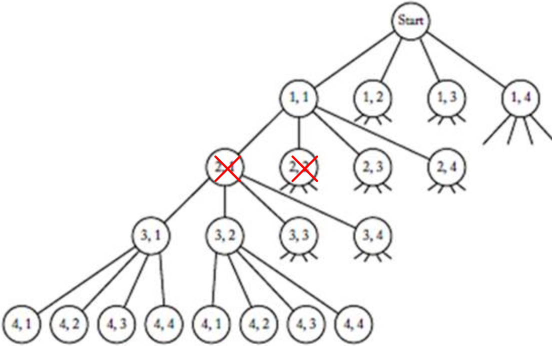
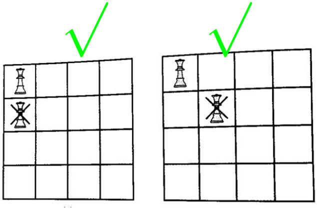




4-Queens problem (improving from DFS) (3)

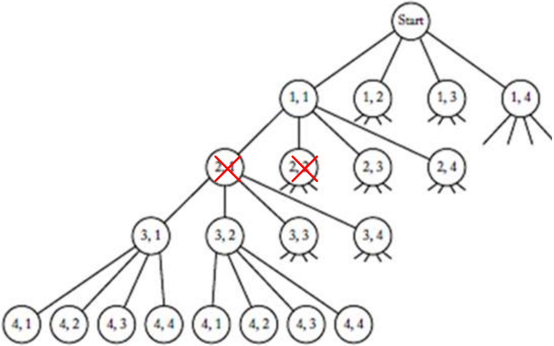
- After realizing a path is not promising
  - Backtrack to the parent

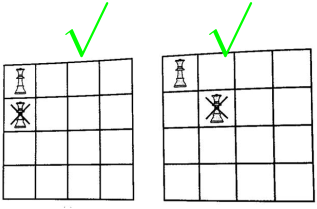




4-Queens problem (improving from DFS) (3)

- After realizing a path is not promising
  - Backtrack to the parent
  - Proceed with the next child





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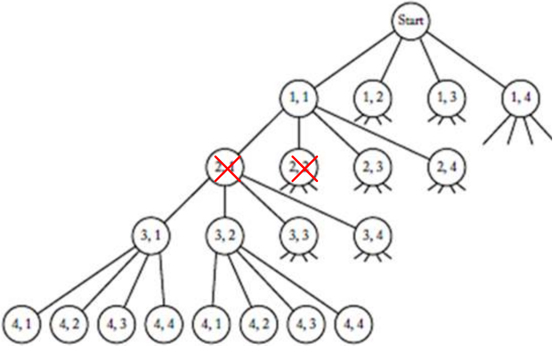

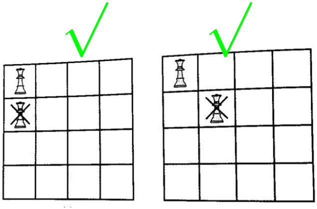

Promising Function

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- Promising function *n*-queen problem:

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- Promising function n-queen problem:
  - Returns false if a node and any of the nodes ancestors place queens in the same column or diagonal

Backtracking Alg. for the n-Queens Problem

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- All solutions to the n-Queens problem

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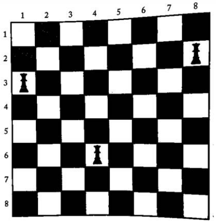

Backtracking Alg. for the n-Queens Problem

- All solutions to the n-Queens problem
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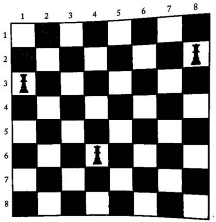
    - Queens of the same row or the same columns challenge each other



Backtracking Alg. for the n-Queens Problem

- All solutions to the n-Queens problem
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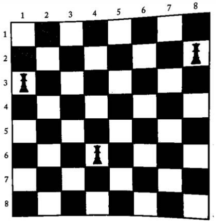
  - No two queens have the same diagonal



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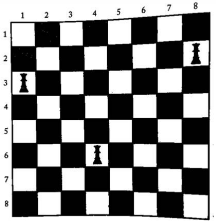

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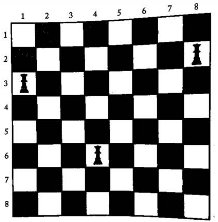

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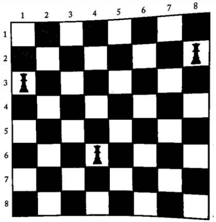
  - $col(i) \neq col(k)$ and $row(i) \neq row(k)$
  - How about 2 queens in the same diagonal?



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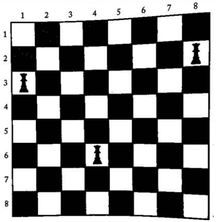
- Promising function:
  - $col(i) \neq col(k)$ and $row(i) \neq row(k)$
  - How about 2 queens in the same diagonal? $col(i) - col(k) \neq i - k$ (for left diagonal)



Backtracking Alg. for the n-Queens Problem

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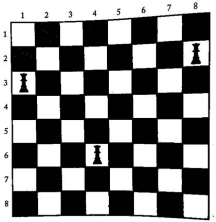
  - No two queens have the same diagonal

- Promising function:
  - $col(i) \neq col(k)$ and $row(i) \neq row(k)$
  - How about 2 queens in the same diagonal?
    $col(i) - col(k) \neq i - k$ (for left diagonal) or
    $col(i) - col(k) \neq k - i$ (for right diagonal)



Backtracking Alg. for the n-Queens Problem (2)

Some observations about solutions:

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Backtracking Alg. for the n-Queens Problem (2)

Some observations about solutions:

- Each Queen must be on a different row.
- Assume Queen i is placed on row i.
- So all we have to do is choose the columns.
- A candidate configuration can be represented by an n-tuple, $< x_1, x_2, \ldots, x_n >$, where x_i is the column on which Queen i is placed.

Example of How Backtracking works for 4-queen Problem

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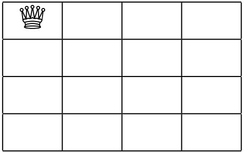

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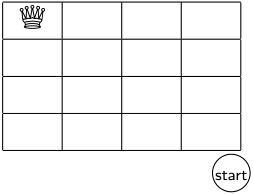

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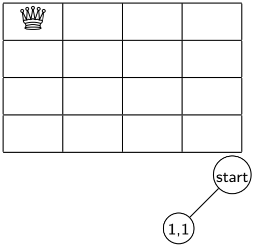

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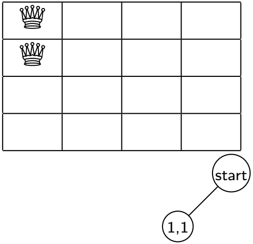

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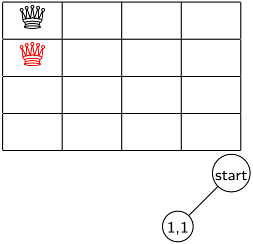

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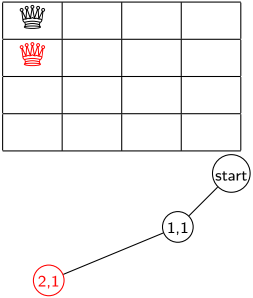

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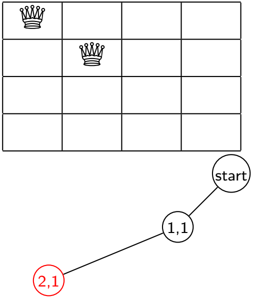

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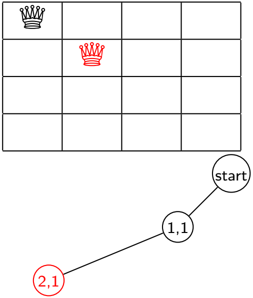

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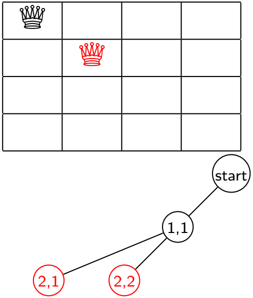

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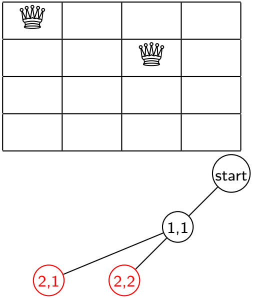

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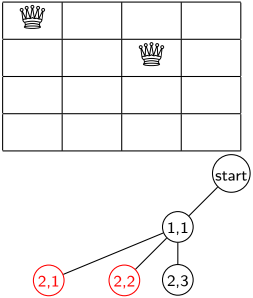

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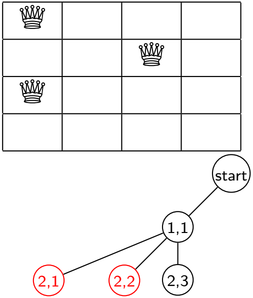

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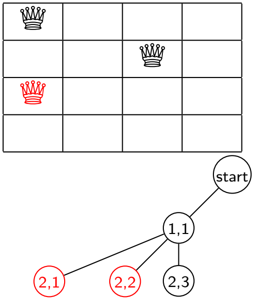

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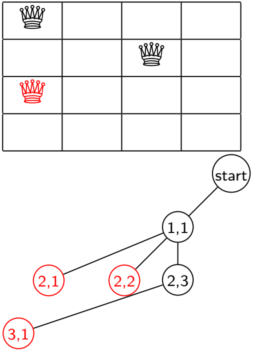

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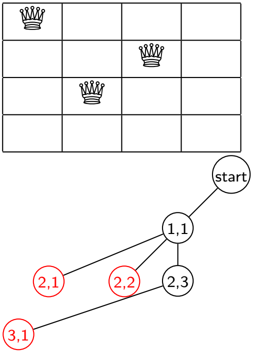

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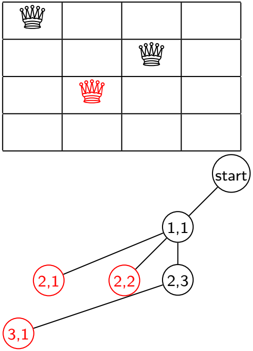

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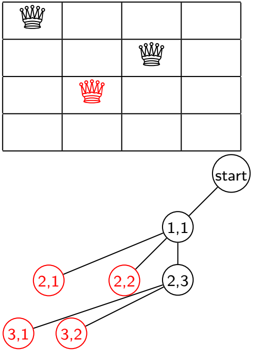

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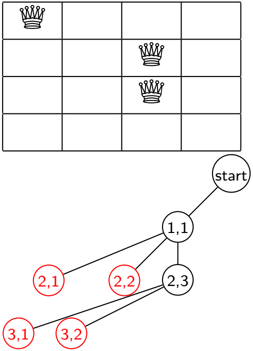

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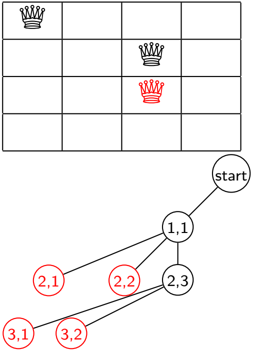

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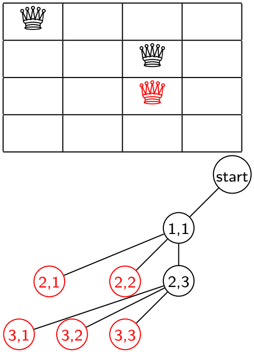

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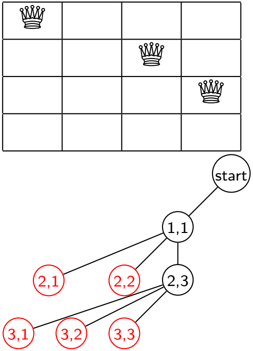

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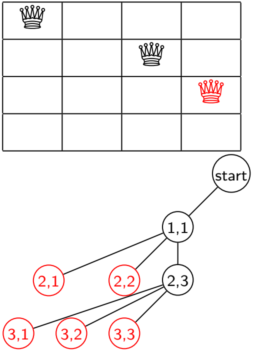

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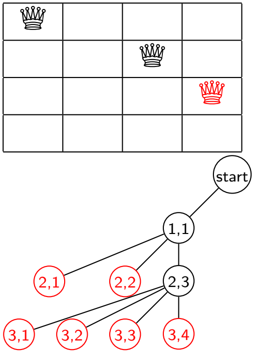

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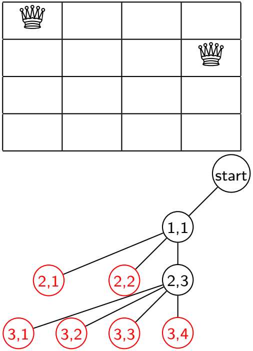

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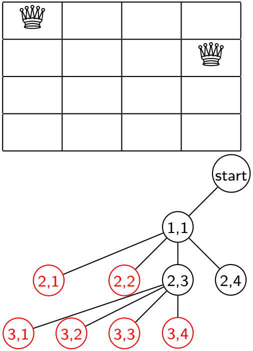

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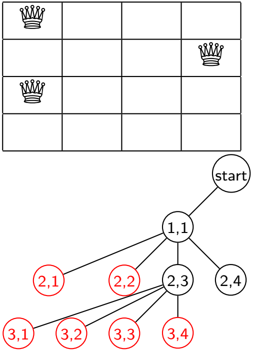

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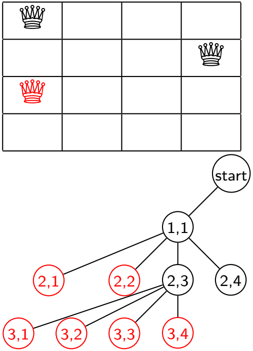

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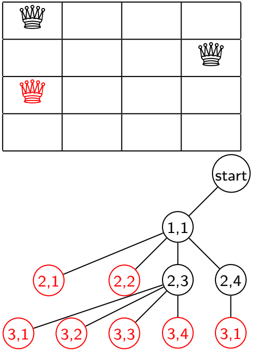

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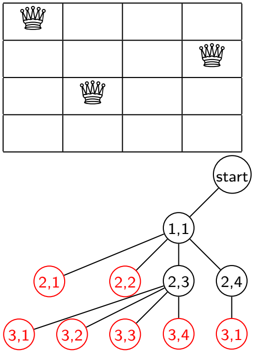

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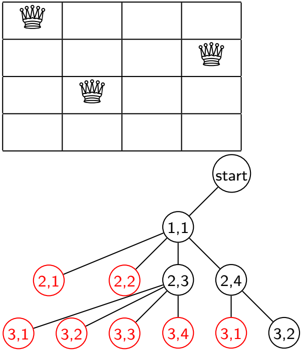

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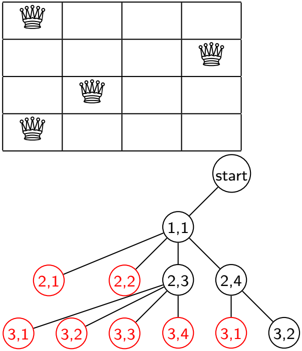

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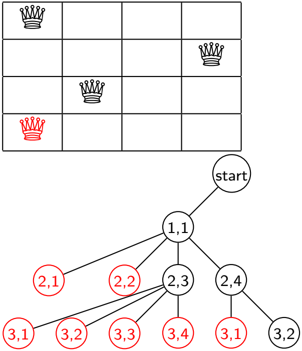

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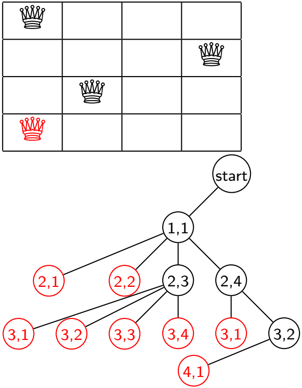

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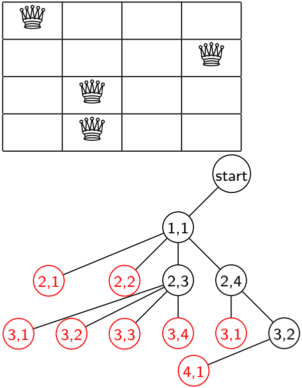

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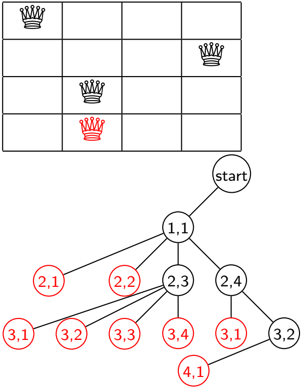

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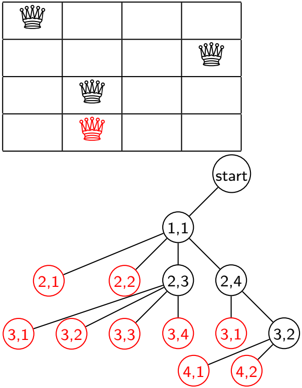

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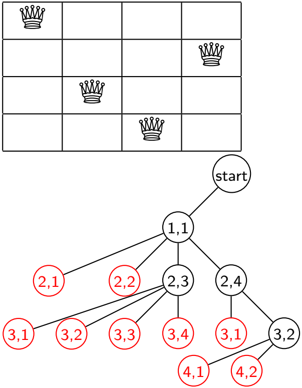

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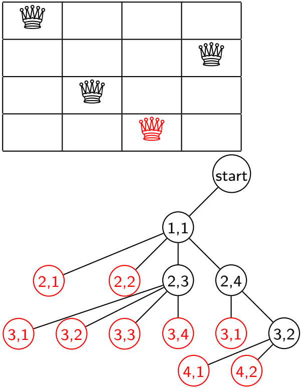

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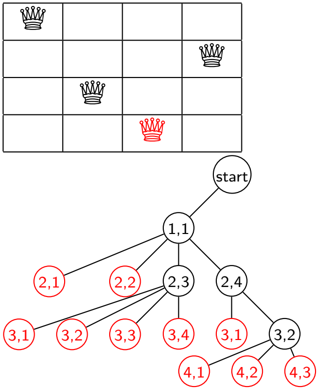

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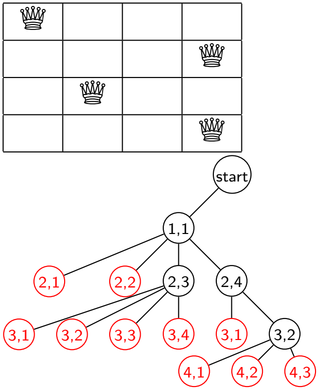

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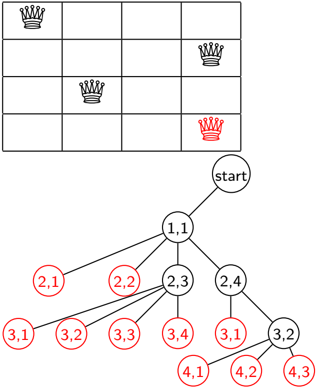

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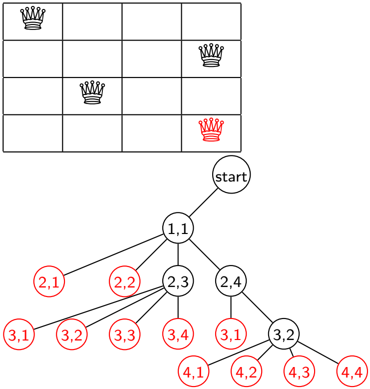

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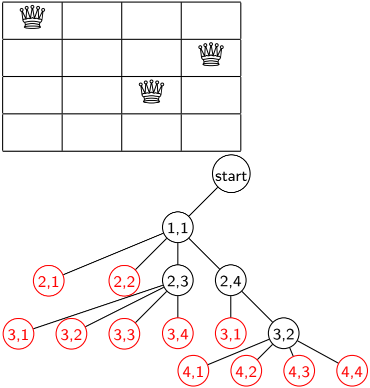

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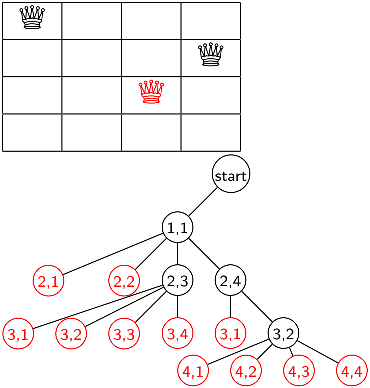

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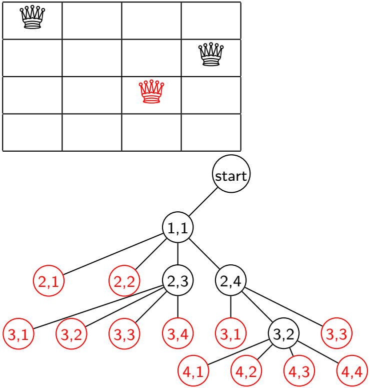

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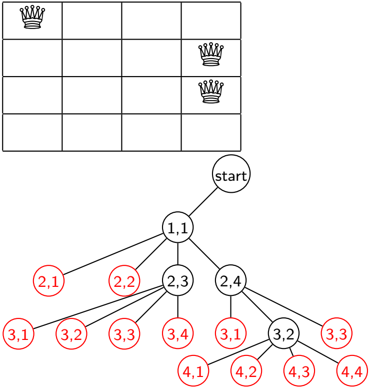

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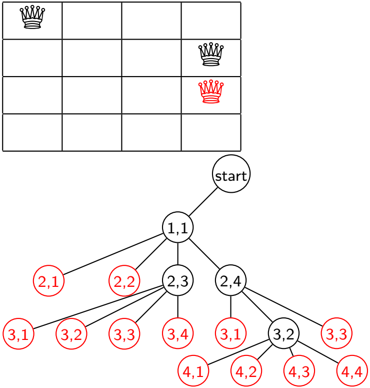

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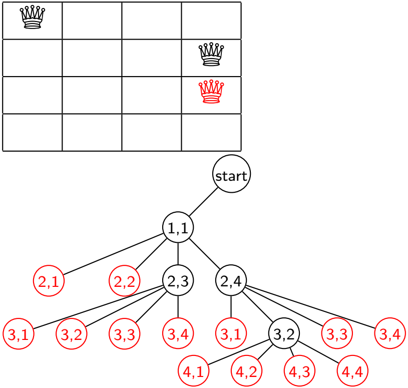

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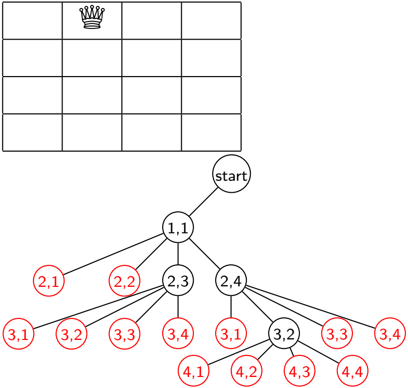

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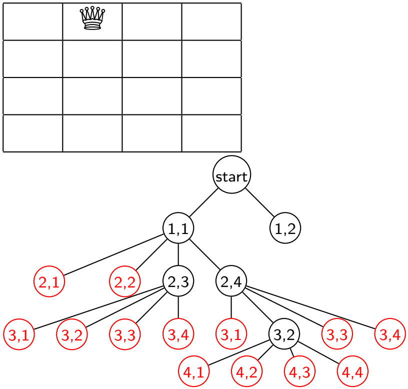

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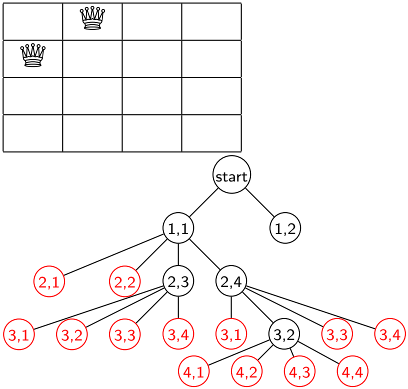

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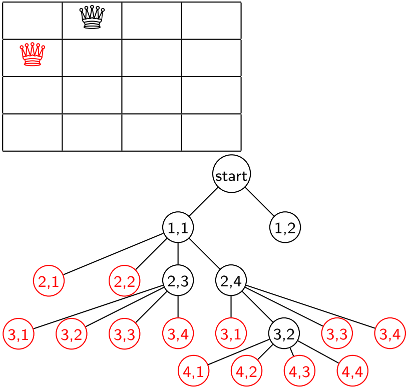

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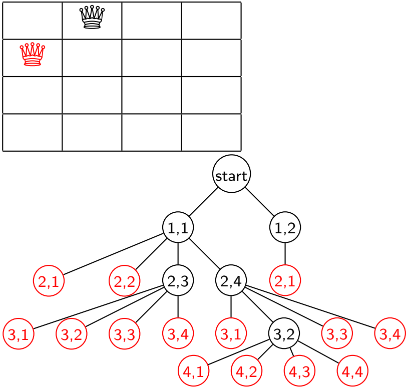

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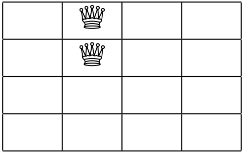

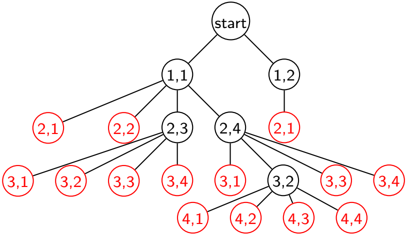

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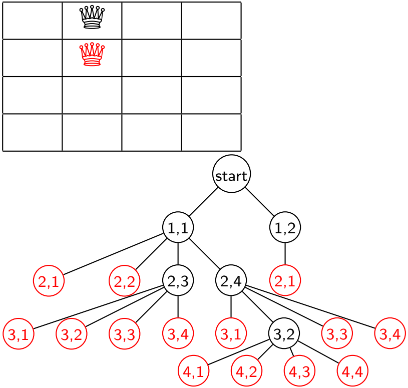

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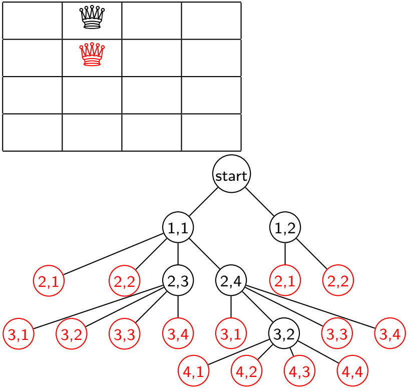

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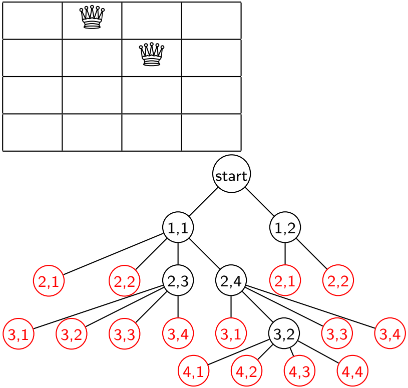

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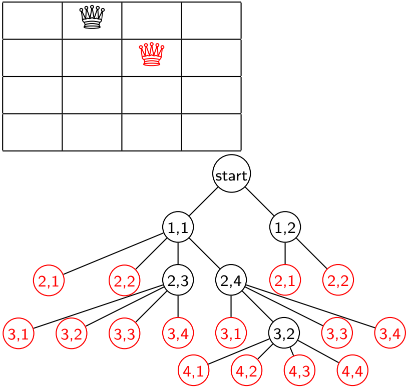

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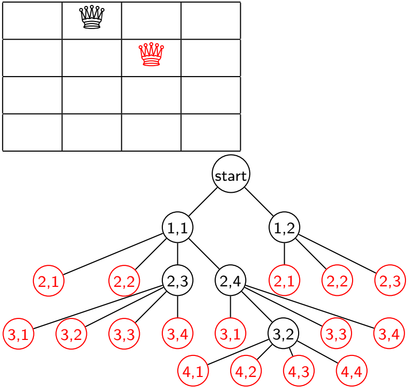

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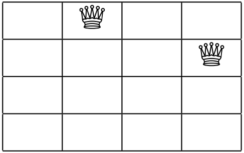

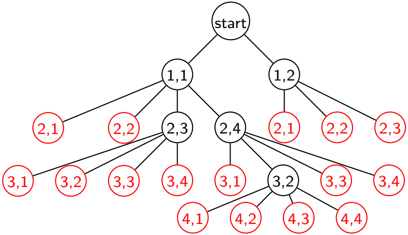

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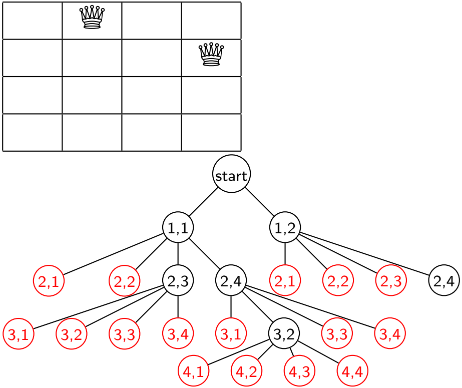

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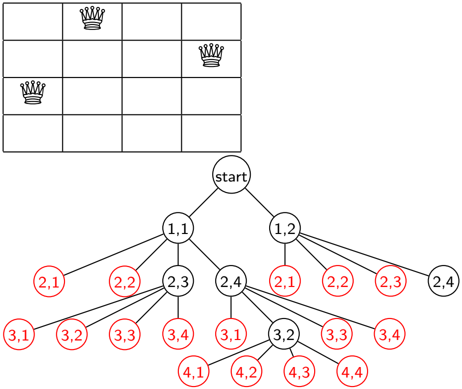

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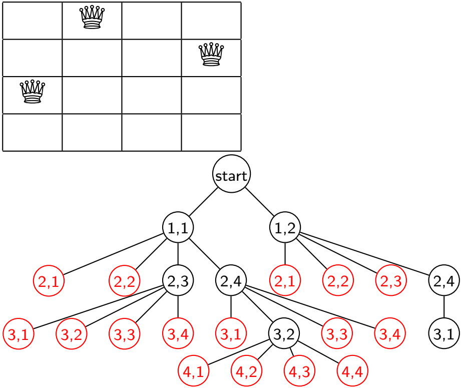

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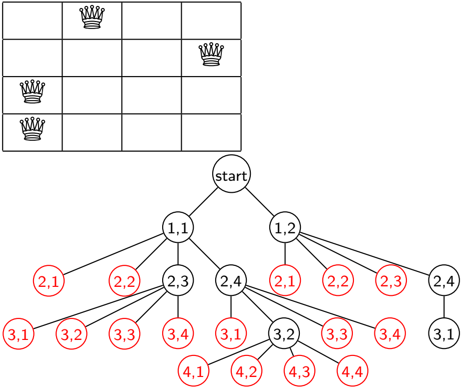

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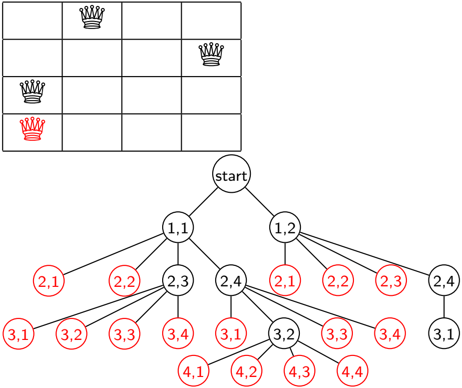

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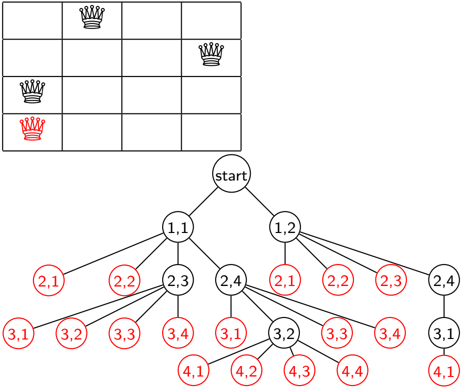

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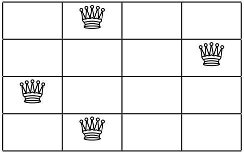

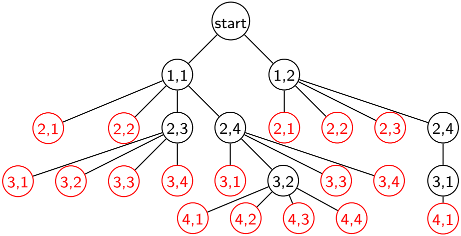

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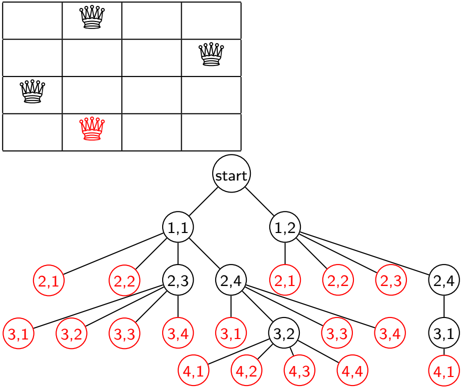

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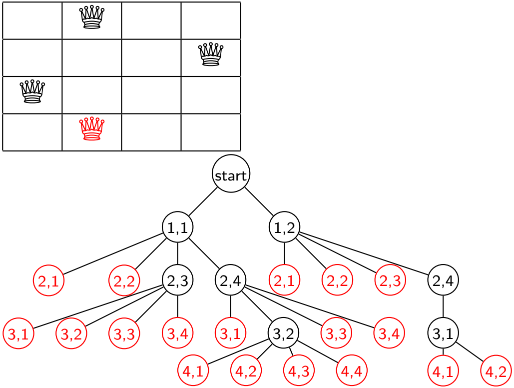

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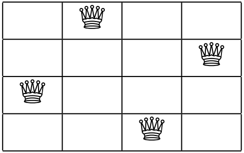

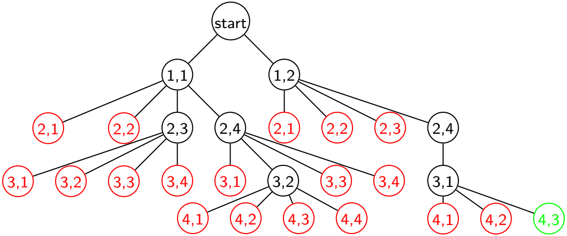

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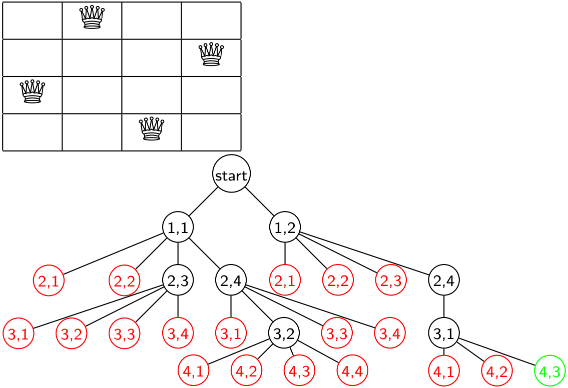

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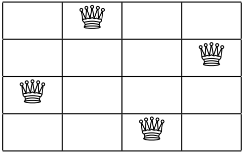

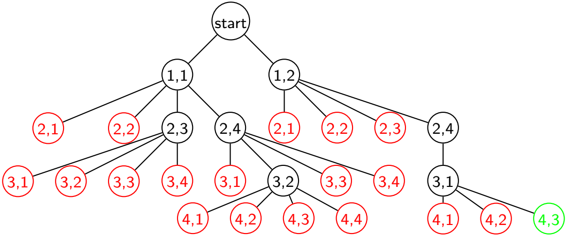

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- The backtracking algorithms checks 27 nodes before finding a solution.

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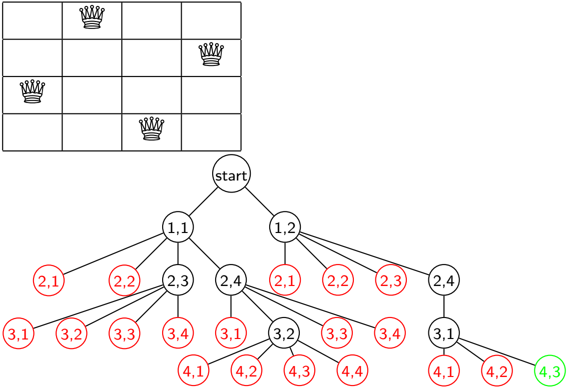

- The first solution is $<2, 4, 1, 3>$
- The backtracking algorithms checks 27 nodes before finding a solution.
- Without backtracking, a depth-first search of the state space tree checks 155 nodes before finding the same solution.

Backtracking Alg. for the n-Queens problem (3)

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```
void Queens (index i){
    index j;
    if (promising(i))
       if (i==n)
          cout<< col[1] through col[n]
       else
          for(j=1; j<=n; j++){
             col[i+1]=j;
             Queens(i+1);
          }
}
                             // Call the function with Queens(0);
bool promising (index i){
    index k=1;
    bool switch=true;
    while (k<i && switch){
       if (col[i]==col[k] || abs(col[i]-col[k])==i-k)
          switch = false;
       k++;
       }
    return switch;
}
```

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  - Pruned State Space Tree:
    sub-tree consisting of visited nodes