Branch and Bound (2)

By: Aminul Islam

Based on Chapter 6 of Foundations of Algorithms

Objectives

Objectives

- Describe the branch-and-bound technique for solving optimization problems
- Contrast the branch-and-bound technique with the backtracking
- Apply the branch-and-bound technique to solve the 0-1 Knapsack Problem, the Travelling Salesman Problem and Assignment Problem

Travelling salesman problem (TSP)

Travelling salesman problem (TSP)

Given a list of cities and the distances between each pair of cities, the task is to find the shortest possible route that visits each city exactly once and returns to the origin city.

Travelling salesman problem (TSP) (2)

Travelling salesman problem (TSP) (2)

- Goal: find an optimal tour
 - Starts at a given city
 - Visits every city exactly once
 - Returns to the starting city
- Such that the total distanced traveled is minimal

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\begin{bmatrix} 0 & 14 & 4 & 10 & 20 \\ 14 & 0 & 7 & 8 & 7 \\ 4 & 5 & 0 & 7 & 16 \\ 11 & 7 & 9 & 0 & 2 \\ 18 & 7 & 17 & 4 & 0 \end{bmatrix}
```

minlength =

Γ0	14	4	10	20 7
14	0	7	8	7
0 14 4 11 18	5	0	7	16 2
11	7	9	0	2
18	7	17	4	0

minlength =

Γ0	14	4	10	207
14 4	0	7	8	7
4	5	0	7	20 7 16 2 0
11 18	7	9	0	2
_18	7	17	4	0]



$\mathsf{minlength} = \infty$

- Determine promising, unexpanded node with the smallest bound
- Promising: Bound < minlength

	4	5	0 9 17	7
	11	7	9	0
	L18	7	17	4
_				

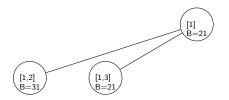


 $\mathsf{minlength} = \infty$



Γ0	14	4	10	207 7
14	0	7	8	
4	5	0	7	16 2 0
11	7	9	0	2
18	7	17	4	0]

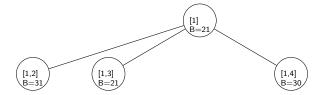
$\mathsf{minlength} = \infty$



Γ.0	14	4	10	207 7
14	0	1	8	
4	5	0	7	16 2 0
11	7	9	0	2
L18	7	17	4	0]

$\mathsf{minlength} = \infty$

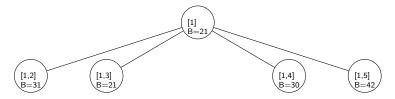
Γ0	14	4	10	207	
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11	7	9	0	2	
18	7	17	4	0 l	



$\mathsf{minlength} = \infty$

- Determine promising, unexpanded node with the smallest bound
- \bullet Promising: Bound < minlength

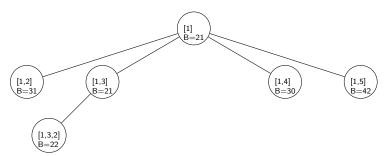
ΓO	14	4	10	207
0 14 4	0	7	8	20 7 16 2 0
4	5	0	7	16
11 18	7	9	0	2
18	7	17	4	0



• Promising: Bound < minlength

 $minlength = \infty$

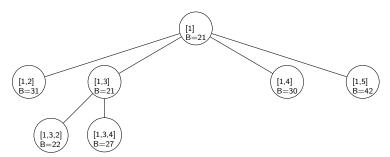
Γ0	14	4	10	207 7	
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0 14 4 11 18	5	0	7	16 2 0	
11	7	9	0	2	
18	7	17	4	οl	

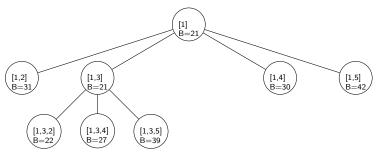


• Promising: Bound < minlength

 $minlength = \infty$

_				
Γ0	14	4	10	207
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4	5	0	7	16
11	7	9	0	16 2 0
18	7	17	4	0 l

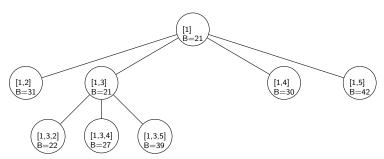


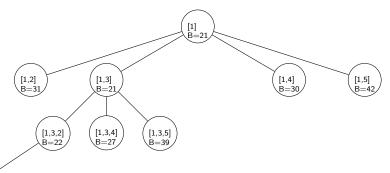


$\mathsf{minlength} = \infty$

- Determine promising, unexpanded node with the smallest bound
- Promising: Bound < minlength

Γ0	14	4	10	20 ₇
14	0	7	8	7
4	5	0	7	16
11 18	7	9	0	2 0
18	7	17	4	0]





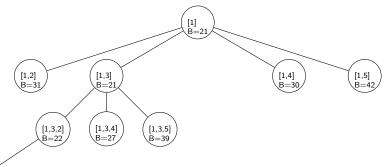
Length=37

minlength = 37

Length=37

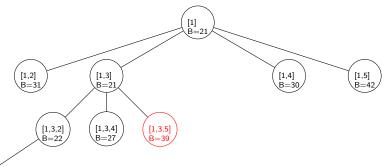
- Promising: Bound < minlength
- Start finding non-promising nodes

Γ0	14	4	10	207 7
14	0	7	8	7
4	5	0	7	16
11 18	7	9	0	2 0
L18	7	17	4	0]



- Promising: Bound < minlength
- Start finding non-promising nodes

1	Γ0	14	4	10	207 7
	14	0	7	8	
	4	5	0	7	16
	11 18	7	9	0	2 0
	L18	7	17	4	0]

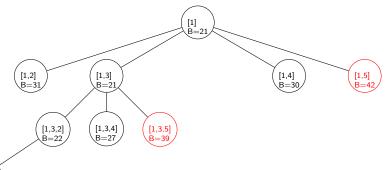


minlength = 37

Length=37

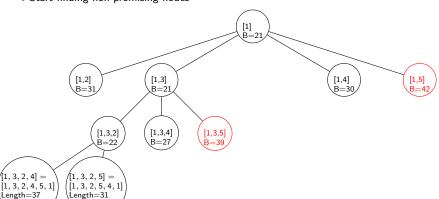
- Promising: Bound < minlength
- Start finding non-promising nodes

Γ0	14	4	10	207 7
14	0	7	8	
4	5	0	7	16
11 18	7	9	0	2 0
L18	7	17	4	0]



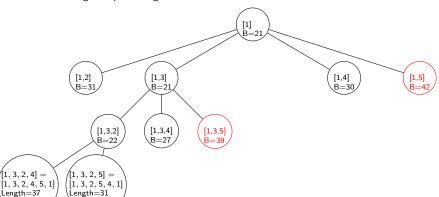
- Promising: Bound < minlength
- Start finding non-promising nodes

Γ0	14	4	10	20 7
14	0	7	8	
4	5	0	7	16 2 0
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L18	7	17	4	0]



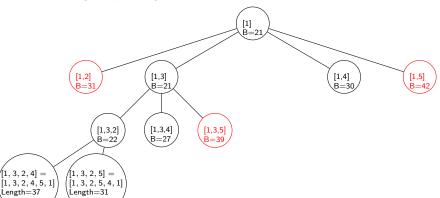
- Promising: Bound < minlength
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Γ0	14	4	10	207
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14 4	5	0	7	16 2 0
11 18	7	9	0	2
L18	7	17	4	0]



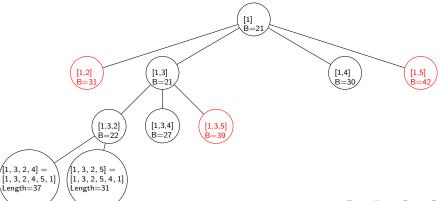
- Promising: Bound < minlength
- Start finding non-promising nodes

ΓO	14	4	10	20 7
14	0	7	8	
14 4 11 18	5	0	7	16 2 0
11	7	9	0	2
L18	7	17	4	0]



- Determine promising, unexpanded node with the smallest bound
- Promising: Bound < minlength
- Start finding non-promising nodes

Γ0	14	4	10	207
14	0	7	8	207 7
4	5	0	7	16
11	7	9	0	2 0
18	7	17	4	0]



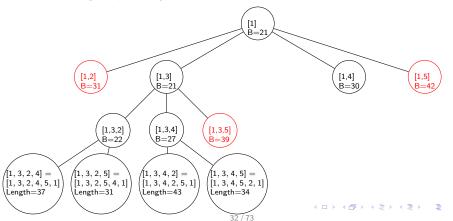
minlength = 31 $\begin{bmatrix} 0 & 14 & 4 & 10 & 20 \\ 14 & 0 & 7 & 8 & 7 \\ 4 & 5 & 0 & 7 & 16 \\ 11 & 7 & 9 & 0 & 2 \\ 18 & 7 & 17 & 4 & 0 \end{bmatrix}$ • Promising: Bound < minlength • Start finding non-promising nodes [1] B=21 [1,3] [1,5] [1,4]B=21 [1,3,4] [1,3,2] B=22 [1,3,5] R-30 /[1, 3, 2, 4] = /[1, 3, 2, 5] =/[1, 3, 4, 2] = [1, 3, 2, 4, 5, 1][1, 3, 2, 5, 4, 1][1, 3, 4, 2, 5, 1]Length=31 Length=37 Length=43

minlength = 31 $\begin{bmatrix} 0 & 14 & 4 & 10 & 20 \\ 14 & 0 & 7 & 8 & 7 \\ 4 & 5 & 0 & 7 & 16 \\ 11 & 7 & 9 & 0 & 2 \\ 18 & 7 & 17 & 4 & 0 \end{bmatrix}$ • Promising: Bound < minlength • Start finding non-promising nodes [1] B=21 [1,2] [1,3] [1,5] [1,4] B=21 [1,3,4] [1,3,2] B=22 [1,3,5] /[1, 3, 2, 4] =/[1, 3, 2, 5] =/[1, 3, 4, 2] =/[1, 3, 4, 5] =[1, 3, 2, 4, 5, 1][1, 3, 2, 5, 4, 1][1, 3, 4, 2, 5, 1][1, 3, 4, 5, 2, 1]Length=37 Length=31 Length=43 Length=34

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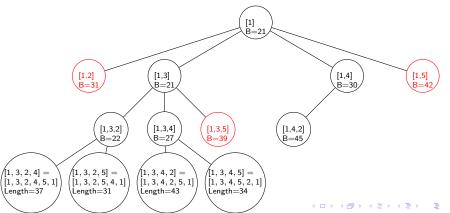
- Determine promising, unexpanded node with the smallest bound
- Promising: Bound < minlength
- Start finding non-promising nodes

Г 0	14	4	10	207
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4	5	0	7	16 2 0
11 18	7	9	0	2
_18	7	17	4	0]



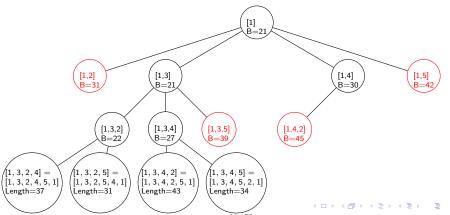
minlength = 31

- Promising: Bound < minlength
- Start finding non-promising nodes



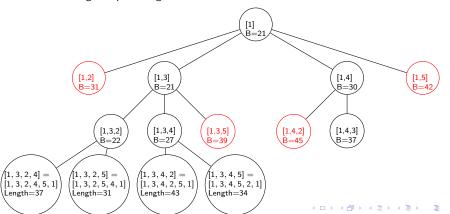
minlength = 31

- Promising: Bound < minlength
- Start finding non-promising nodes



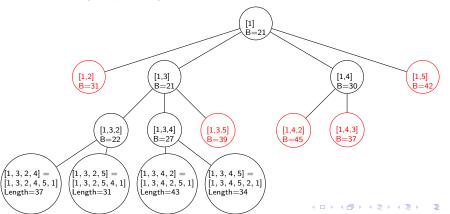
minlength = 31

- Promising: Bound < minlength
- Start finding non-promising nodes



minlength = 31

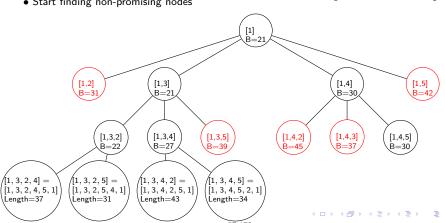
- Promising: Bound < minlength
- Start finding non-promising nodes



minlength = 31

- Promising: Bound < minlength
- Start finding non-promising nodes

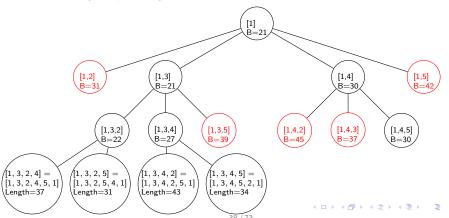
Γ0	14	4	10	20 7
14	0	7	8	
0 14 4	5	0	7	16 2 0
11 18	7	9	0	2
L18	7	17	4	0]



minlength = 31

- Determine promising, unexpanded node with the smallest bound
- Promising: Bound < minlength
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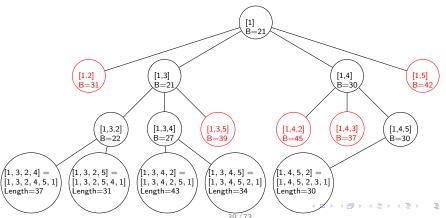
Г 0	14	4	10	207
14	0	7	8	20 ₇
4	5	0	7	16 2 0
11 18	7	9	0	2
_18	7	17	4	0]

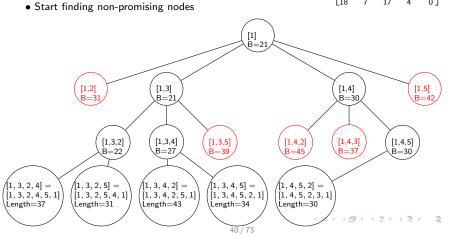


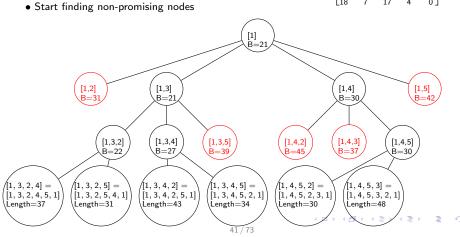
minlength = 31

- Promising: Bound < minlength
- Start finding non-promising nodes

 $\begin{bmatrix} 0 & 14 & 4 & 10 & 20 \\ 14 & 0 & 7 & 8 & 7 \\ 4 & 5 & 0 & 7 & 16 \\ 11 & 7 & 9 & 0 & 2 \\ 18 & 7 & 17 & 4 & 0 \end{bmatrix}$



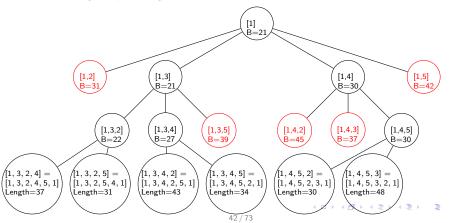


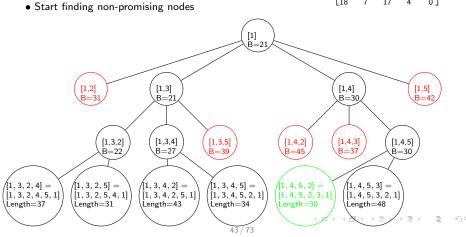


minlength = 30

- Determine promising, unexpanded node with the smallest bound
- Promising: Bound < minlength
- Start finding non-promising nodes

Γ0	14	4	10	207
14	0	7	8	20 ₇
4	5	0	7	16
11 18	7	9	0	2 0
L18	7	17	4	0]





Exercise

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Use the Best-First Search with Branch-and-Bound Pruning Algorithm for the Travelling Salesperson problem to find an optimal tour and the length of the optimal tour for the graph whose adjacency matrix is given by the following matrix. Show the actions step by step. (Show the pruned state space tree produced by using the above mentioned algorithm. Specify 'city number' and 'bound'/'length' from top to bottom at each node in the pruned state space tree. Mark each nonpromising node with a cross. Mark the node(s) with optimal solution.)

Assignment Problem Using B&B

Assignment Problem Using B&B

■ What is the best assignment of jobs to different people?

Assignment Problem Using B&B

- What is the best assignment of jobs to different people?
- Consider the following example:

	job1	job2	job3	job4
Α	9	2	7	8
В	6	4	3	7
C	5	8	1	8
D	7	6	9	4

	job1	job2	job3	job4
Α	9	2	7	8
В	6	4	3	7
B C	5	8	1	8
D	7	6	9	4

mintime =

• Promising: Bound < mintime

	job1	job2	job3	job4
Α	9	2	7	8
В	6	4	3	7
C	5	8	1	8
D	7	6	9	4

mintime =

• Promising: Bound < mintime



	job1	job2	job3	job4
Α	9	2	7	8
В	6	4	3	7
B C D	5	8	1	8
D	7	6	9	4

$mintime = \infty$

- Determine promising, unexpanded node with the smallest bound
- Promising: Bound < mintime



	job1	job2	job3	job4
Α	9	2	7	8
В	6	4	3	7
B C D	5	8	1	8
D	7	6	9	4

$mintime = \infty$

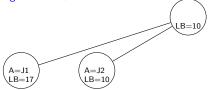
• Promising: Bound < mintime



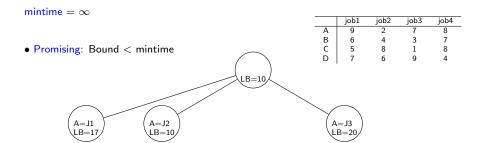
	job1	job2	job3	job4
A	9	2	7	8
В	6	4	3	7
C	5	8	1	8
D	7	6	9	4

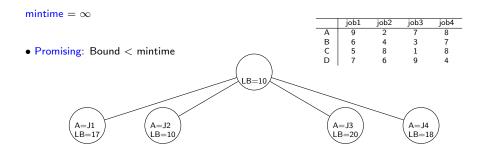
$mintime = \infty$

• Promising: Bound < mintime



	job1	job2	job3	job4
Α	9	2	7	8
В	6	4	3	7
C	5	8	1	8
D	7	6	9	4





 $mintime = \infty$ job1 iob2 job3 job4 • Determine promising, unexpanded node with the smallest bound В • Promising: Bound < mintime C LB=10 A=J2 A=J1 A=J3A=J4LB=10

