Heuristic functions: admissibility, consistency, dominance

Albert Sanchis
Alfons Juan
Jorge Civera

DSIC
Departamento de Sistemas Informáticos y Computación

Learning objectives

- Describe the concept of heuristic function.
- Propose admissible (lower bound) heuristic functions
  as a relaxation of the original problem
- Evaluate the admissibility and consistency of a heuristic function
- Compare heuristic functions in terms of dominance

Contents

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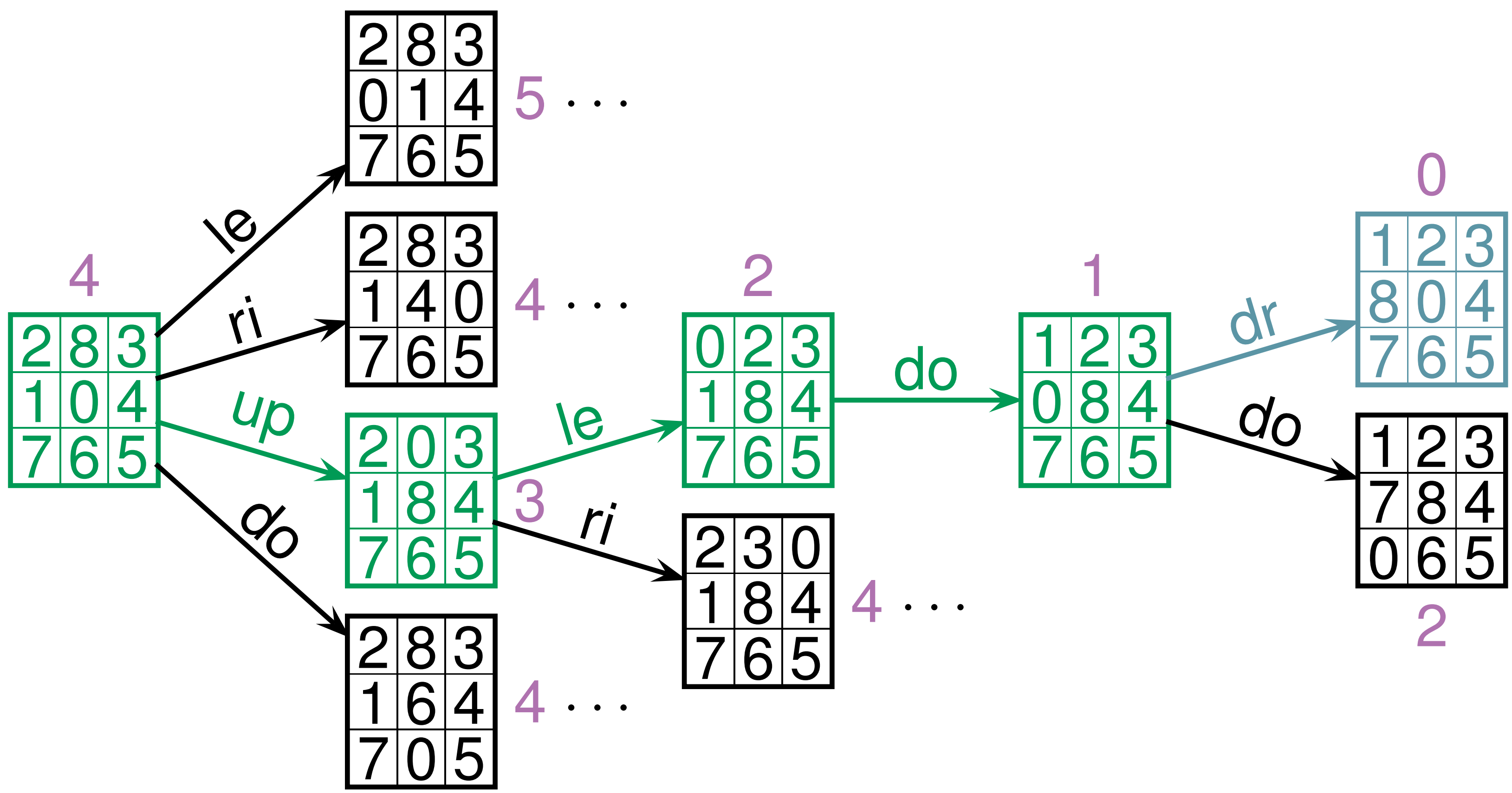
1 Concept of heuristic function

Provided a search problem on a state graph, a ***heuristic function*** h is a function that (efficiently) approximates the minimum cost h^* of reaching the goal state from a state n:

Example: sum of Manhattan distances in the 8-puzzle problem



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2 Admissibility

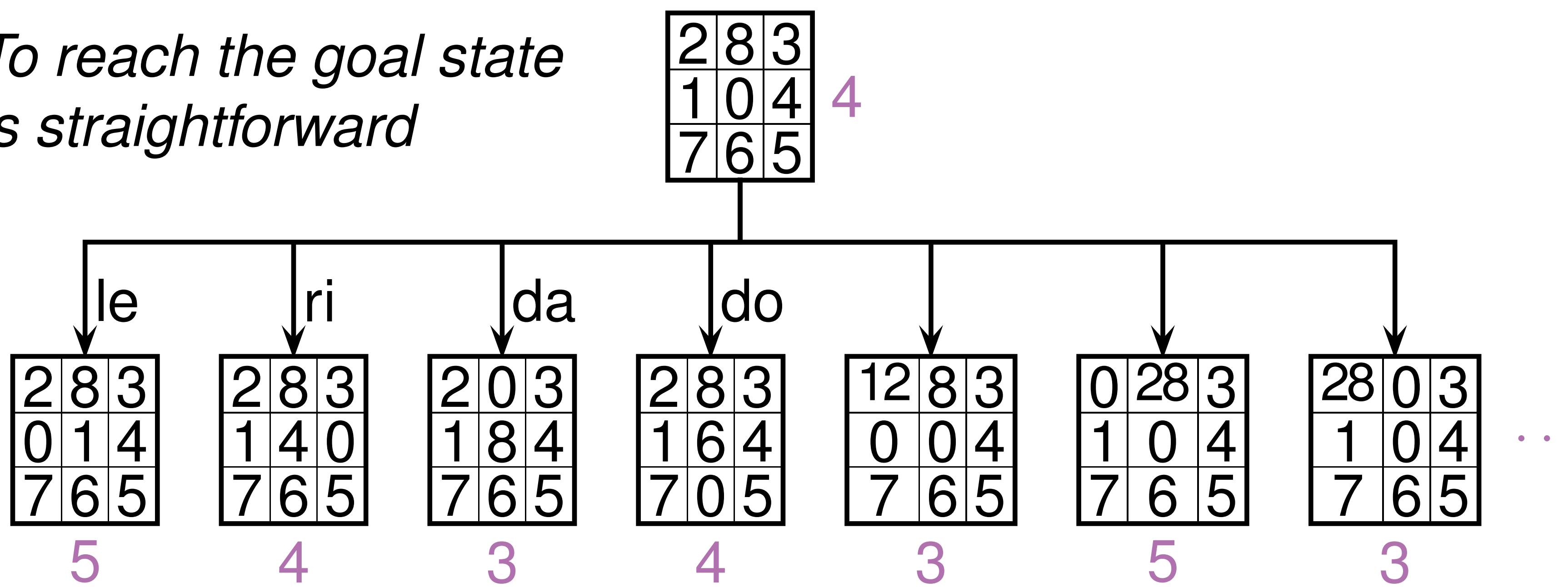
h is ***admissible*** (***lower bound***) if $h(n) \leq h^*(n)$ $\forall$ node n.

It is usually obtained as a ***relaxation of constraints*** in the original problem, that is, smoothing or eliminating constraints to ease reaching the goal state.

Example: sum of Manhattan distances in the 8-puzzle problem

A can be moved to B if: B is adjacent to A and ~~B is a blank~~

To reach the goal state is straightforward



3 Consistency or monotonicity

h is ***consistent*** if, $\forall$ node n [1, pp82–83]:

$$h(n) \leq k(n, n') + h(n') \quad \forall \text{ node } n'$$

where $k(n, n')$ is the minimum cost to go from n to n'. Therefore,

$$\begin{aligned} g(n) + h(n) \leq & \; g(n) + k(n, n') + h(n') \; \forall \text{ node } n' \\ f(n) \leq & \qquad\qquad f(n') \qquad\qquad\quad \forall \text{ node } n' \end{aligned}$$

Equivalently, h is ***monotone*** if, $\forall n$ [1, pp82-83]:

$$h(n) \leq c(n, n') + h(n') \quad \forall \; n' \text{ adjacent to } n.$$

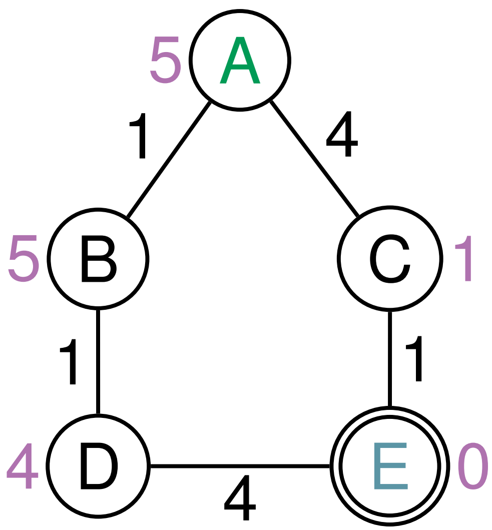
Consistency and ***monotonicity*** are equivalent properties.

4 Admissibility and consistency

Consistency $\Rightarrow$ ***Admissibility*** [1, pp83]

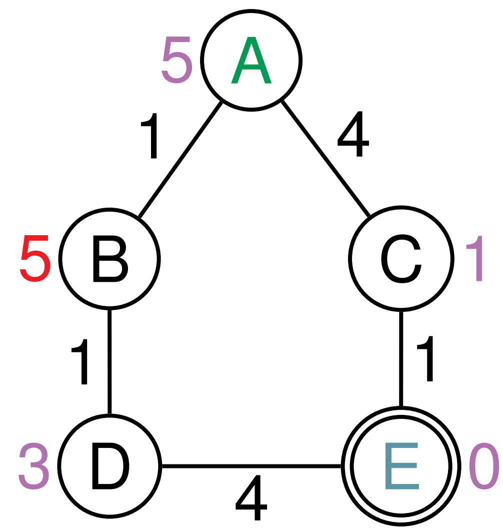
Admissibility $\not\Rightarrow$ ***Consistency*** is not always true:

Admissible and consistent



$$h(A) \leq w(A,B) + h(B)$$
$$h(A) \leq w(A,C) + h(C)$$
$$h(B) \leq w(B,A) + h(A)$$
$$h(B) \leq w(B,D) + h(D)$$
$$h(C) \leq w(C,A) + h(A)$$
$$h(C) \leq w(C,E) + h(E)$$
$$h(D) \leq w(D,B) + h(B)$$
$$h(D) \leq w(D,E) + h(E)$$
$$h(E) \leq w(E,C) + h(C)$$
$$h(E) \leq w(E,D) + h(D)$$

Admissible and not consistent



$$h(B) \not\leq w(B,D) + h(D)$$

5 Dominance

Dominance: $h_2(n)$ dominates $h_1(n)$ if:

$$h_1(n) \leq h_2(n) \leq h^*(n)\ \forall n$$

Example: Manhattan dominates Misplaced tiles in 8-puzzle

2	**8**	3
1	0	4
7	6	5

Misplaced tiles: $1+1+1=3$

Manhattan: $1+1+2=4$

- A* search with $h_2(n)$ expands fewer nodes than $h_1(n)$ [3].
- The closer $h(n)$ approximates $h^*(n)$, fewer nodes are expanded

Non-admissible heuristics cannot be compared with admissible heuristics in terms of number of expanded nodes

6 Conclusions

We have studied:

- The concept of heuristic function
- Admissible heuristic functions as a relaxation of constraints
- Admissibility and consistency of a heuristic function
- Compare heuristic functions in terms of dominance

References

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