Workbook:

# A* Search (graph search)

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Learning objectives

► To describe A^* search (graph search).

► To draw the tree of A^* search (graph search).

► To apply A^* search (graph search) to a well-known problem.

► To analyze the quality of A^* search (graph search).

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Problem: Shortest path between two points

Shortest path from Arad to Bucarest [1]:
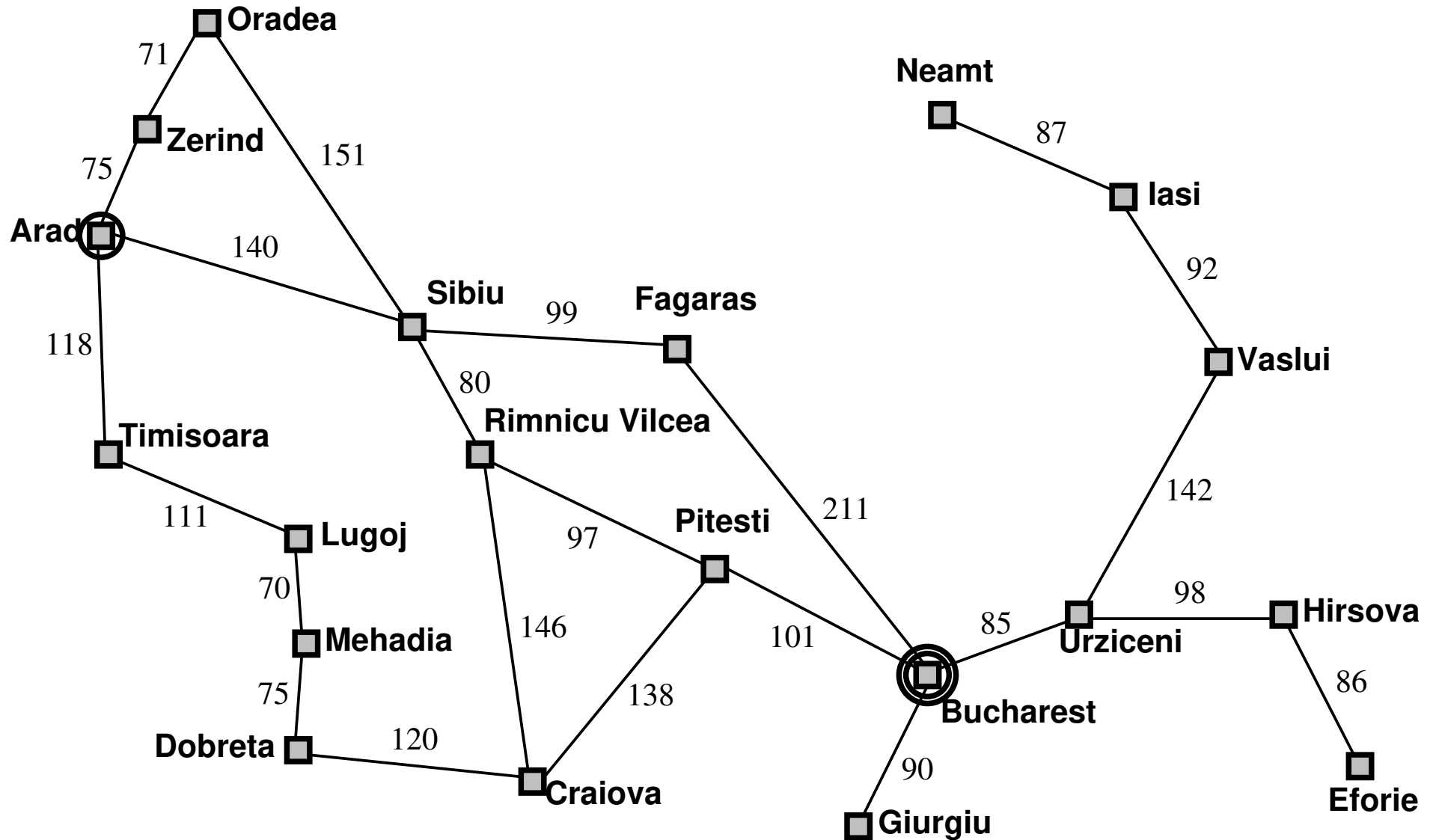
$$Actions(Arad) = \{Move(Sibiu),\ Move(Timisoara),\ Move(Zerind)\}.$$

Problem: Shortest path between two points

Straight-line distances to Bucharest:

	Bucharest		Bucharest
Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

1 The A* algorithm (graph search) [2]

A* (G, s', h) // G weighed graph, s' *start*, h **heuristic**

  $O = InitQueue(s', f_{s'} \triangleq 0 + h(s'))$ // **Open: priority queue** $f \triangleq g + h$

  $C = \emptyset$ // **Closed:** explored nodes

  while not $EmptyQueue(O)$:   // **best-first:** $s = \arg\min_{n \in O} f_n$

    $s = Pop(O)$ // draws in favour of goal state

    if $Goal(s)$ **return** s // solution found!

    $C = C \cup \{s\}$ // s explored

    for all $(s, n) \in Adjacents(G, s)$:   // generation: n is child of s

      $x = (g_s + w(s, n)) + h(n)$ // possibly new f_n

      if $n \notin C \cup O$: $Push(O, n, f_n \triangleq x)$

      else if $n \in O$ **and** $x < f_n$: $Update(O, n, f_n \triangleq x)$

      else if $n \in C$ **and** $x < f_n$: $C = C \setminus \{n\}$; $Push(O, n, f_n \triangleq x)$

 return NULL   // solution not found

Question 1: Write a trace of the A^* algorithm (graph search) applied to finding the shortest path from Arad to Bucarest.

O	C	s
{Arad (c=0+366=366)}	{}	–
{Sibiu (c=140+253=393), Timisoara (c=118+329=447), Zerind (c=75+374=449)}	{Arad (c=0+366=366)}	Arad
{Rimnicu (c=140+80+193=413), Fagaras (c=140+99+176=415), Timisoara (c=118+329=447), Zerind (c=75+374=449), Oradea (c=140+151+380=671)}	{Arad (c=0+366=366), Sibiu (c=140+253=393)}	Sibiu
{Fagaras (c=140+99+176=415), Pitesti (c=220+97+100=417), Timisoara (c=118+329=447), Zerind (c=75+374=449), Craiova (c=220+146+160=526), Oradea (c=140+151+380=671)}	{Arad (c=0+366=366), Sibiu (c=140+253=393), Rimnicu (c=220+193=413)}	Rimnicu
{Pitesti (c=220+97+100=417), Timisoara (c=118+329=447), Bucharest (c=239+211+0=450), Zerind (c=75+374=449), Craiova (c=220+146+160=526), Oradea (c=140+151+380=671)}	{Arad (c=0+366=366), Sibiu (c=140+253=393), Rimnicu (c=220+193=413), Fagaras (c=239+176=415)}	Fagaras
{Bucharest (c=317+101+0=418), Timisoara (c=118+329=447), Zerind (c=75+374=449), Craiova (c=220+146+160=526), Oradea (c=140+151+380=671)}	{Arad (c=0+366=366), Sibiu (c=140+253=393), Rimnicu (c=220+193=413), Fagaras (c=239+176=415), Pitesti (c=317+100=417)}	Pitesti
{Timisoara (c=447), Zerind (c=449), Craiova (c=526), Oradea (c=671)}	{Arad (c=366), Sibiu (c=393), Rimnicu (c=413), Fagaras (c=415), Pitesti (c=417)}	Bucharest (c=418)

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► *Question 2*: Draw the search tree as a result of applying the *A** algorithm (graph search) to the problem of finding the shortest path from Arad to Bucarest.

- ▶ *Question 3*: Does the A* algorithm (graph search) find a solution? *Yes*

- ▶ *Question 4*: If the answer is "Yes":

  - ▷ What is the solution found? *The solution path is: Arad, Sibiu, Rimnicu, Pitesti, Bucharest*

  - ▷ What is the cost of this solution? *418*

  - ▷ Is this the solution of minimum cost? *Yes*

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References

[1] S. Russell and P. Norvig. *Artificial Intelligence: A Modern Approach*. Pearson, third edition, 2010.

[2] P. E. Hart, N. J. Nilsson, and B. Raphael. A Formal Basis for the Heuristic Determination of Minimum Cost Paths. *IEEE Transactions on Systems Science and Cybernetics*, 1968.