



UNIVERSITAT
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Workbook: Breadth-first search

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DSIC

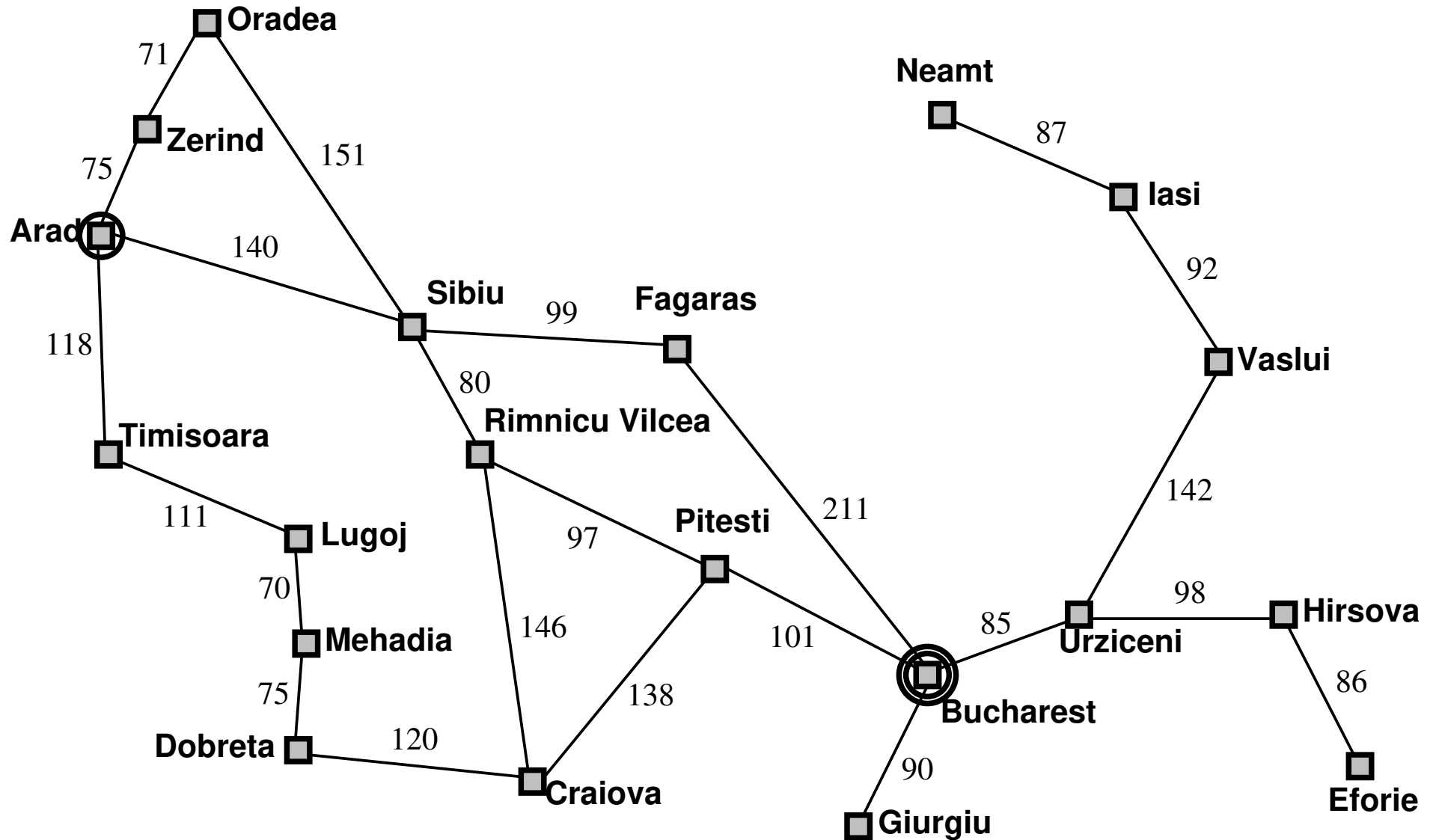
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Learning objectives

- ▶ To describe breadth-first search.
- ▶ To draw the tree of breadth-first search.
- ▶ To apply breadth-first search to a well-known problem.
- ▶ To analyze the quality of breadth-first search.

Problem: Shortest path between two points

Shortest path from Arad to Bucarest [1]:



$\text{Actions}(\text{Arad}) = \{\text{Move}(\text{Sibiu}), \text{Move}(\text{Timisoara}), \text{Move}(\text{Zerind})\}.$

Breadth-first search [1, 2, 3, 4]

```
BFS( $G, s'$ )           // Breadth-first search;  $G$  graph and  $s'$  initial node
 $O = \text{InitQueue}(s')$            // Open: search frontier-queue
 $C = \emptyset$                  // Closed: set of explored nodes
while not  $\text{EmptyQueue}(O)$ :
     $s = \text{Unqueue}(O)$            // FIFO (First in, first out) selection
     $C = C \cup \{s\}$              //  $s$  is now explored
    forall  $(s, n) \in \text{Adjacents}(G, s)$ :           // generation:  $n$  child of  $s$ 
        if  $n \notin C \cup O$ :           //  $n$  not discovered until now
            if  $\text{Goal}(n)$  return  $n$            // solution found!
             $\text{Append}(O, n)$            //  $n$  added to the queue
return NULL                 // no solution found
```

- **Question 1:** Write a trace of the **BFS** algorithm applied to the problem of finding the shortest path from Arad to Bucarest.

O	C	s
{Arad}	{}	—
{Sibiu, Timisoara, Zerind}	{Arad}	Arad
{Timisoara, Zerind, Fagaras, Oradea, Rimnicu}	{Arad, Sibiu}	Sibiu
{Zerind, Fagaras, Oradea, Rimnicu, Lugoj}	{Arad, Sibiu, Timisoara}	Timisoara
{Fagaras, Oradea, Rimnicu, Lugoj}	{Arad, Sibiu, Timisoara, Zerind}	Zerind
{Oradea, Rimnicu, Lugoj}	{Arad, Sibiu, Timisoara, Zerind, Fagaras}	Fagaras

- **Question 2:** Draw the search tree as a result of applying the **BFS** algorithm to the problem of finding the shortest path from Arad to Bucarest.

- ▶ **Question 3:** Does the BFS algorithm find a solution? **Yes**
- ▶ **Question 4:** If the answer is “Yes”:
 - ▷ What is the solution found? ***The solution path is: Arad, Sibiu, Fagaras, Bucharest***
 - ▷ What is the cost of this solution? **450**
 - ▷ Is this the solution of minimum cost? ***No, because there is an alternative solution with lower cost of 418: Arad, Sibiu, Rimnicu, Pitesti, Bucharest***
 - ▷ What type of solution is found by the BFS algorithm? ***The shortest path in terms of number of movements***

BFS (check goal when expanding) [1, 2, 3, 4]

```
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- **Question 1:** Write a trace of the **BFS** algorithm applied to the problem of finding the shortest path from Arad to Bucarest.

O	C	s
{Arad}	{}	—
{Sibiu, Timisoara, Zerind}	{Arad}	Arad
{Timisoara, Zerind, Fagaras, Oradea, Rimnicu}	{Arad, Sibiu}	Sibiu
{Zerind, Fagaras, Oradea, Rimnicu, Lugoj}	{Arad, Sibiu, Timisoara}	Timisoara
{Fagaras, Oradea, Rimnicu, Lugoj}	{Arad, Sibiu, Timisoara, Zerind}	Zerind
{Oradea, Rimnicu, Lugoj, Bucharest}	{Arad, Sibiu, Timisoara, Zerind, Fagaras}	Fagaras
{Rimnicu, Lugoj, Bucharest}	{Arad, Sibiu, Timisoara, Zerind, Fagaras, Oradea}	Oradea
{Lugoj, Bucharest, Craiova, Pitesti}	{Arad, Sibiu, Timisoara, Zerind, Fagaras, Oradea, Rimnicu}	Rimnicu
{Bucharest, Craiova, Pitesti, Mehadia}	{Arad, Sibiu, Timisoara, Zerind, Fagaras, Oradea, Rimnicu, Lugoj}	Lugoj
{Craiova, Pitesti, Mehadia}	{Arad, Sibiu, Timisoara, Zerind, Fagaras, Oradea, Rimnicu, Lugoj, Bucharest}	Bucharest

- **Question 2:** Draw the search tree as a result of applying the **BFS** algorithm to the problem of finding the shortest path from Arad to Bucarest.

References

- [1] S. Russell and P. Norvig. *Artificial Intelligence: A Modern Approach*. Pearson, third edition, 2010.
- [2] E. Moore. The shortest path through a maze. In *Proc. of the Int. Symposium on the Theory of Switching, Part II*, pages 285–292. Harvard University Press, 1959.
- [3] C. Y. Lee. An algorithm for path connections and its applications. *IRE Trans. on Electronic Computers*, EC-10, 1961.
- [4] Bernhard Korte and Jens Vygen. *Combinatorial Optimization: Theory and Algorithms*. Springer, 2018.