Search: the state graph

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Learning objectives

- To describe conventional search over a state graph.
- To know some popular problems in conventional search.

Contents

1 Conventional search over a state graph

Formal definition of conventional search problems [1]:

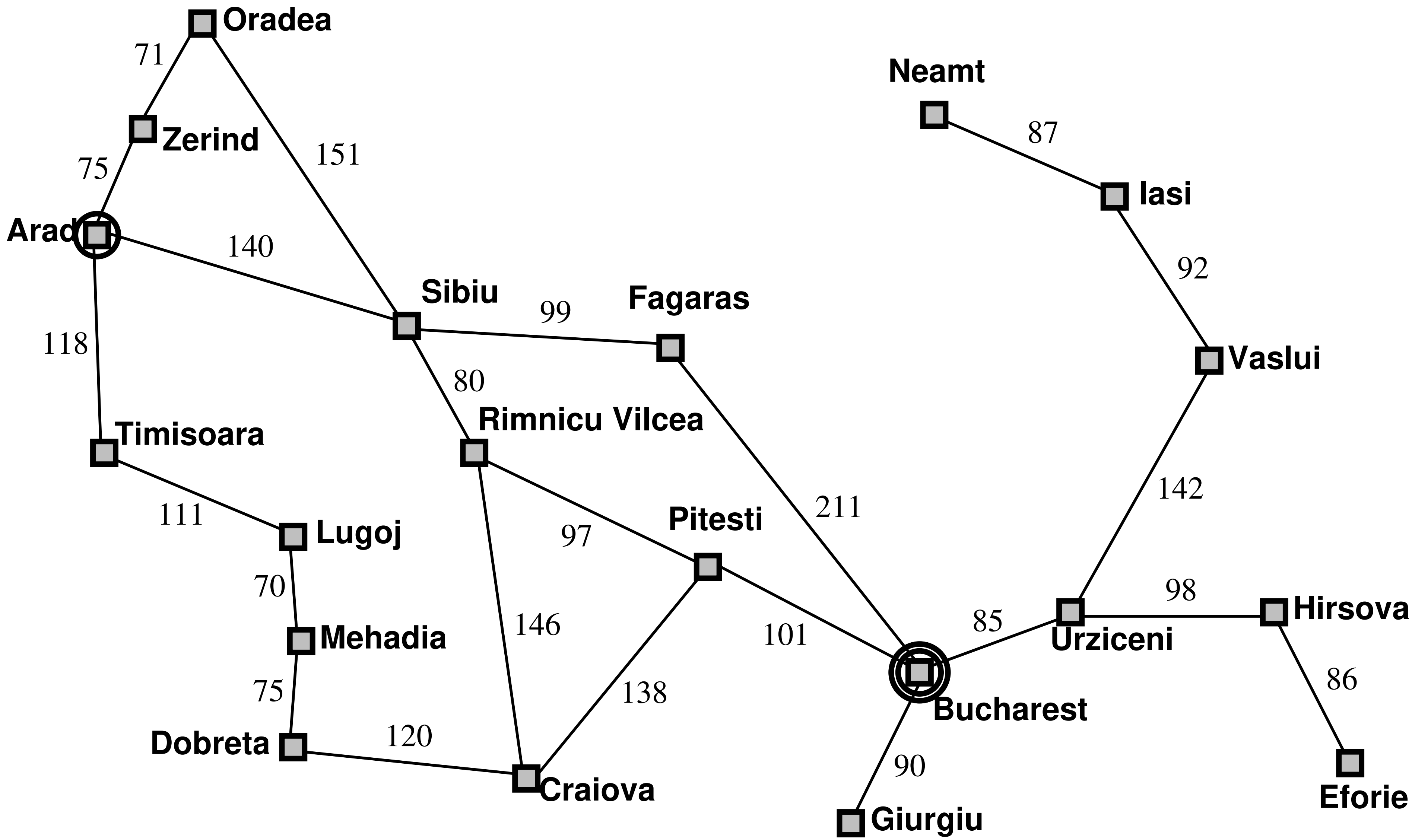
- *State space:* set of possible “world states”.
- *Initial state* s_0*:* state from which search starts.
- *Actions(*s*):* actions applicable to state s.
- *Result(*s, a*):* successor state resulting from applying of a to s.
- *Goal(*s*):* true if and only if state s is a solution.
- *Cost(*c*):* path cost c (sequence of actions).

State graph: nodes are states and edges are actions.

Conventional search: find an ***optimal path*** in the state graph.

2 Shortest path between two locations

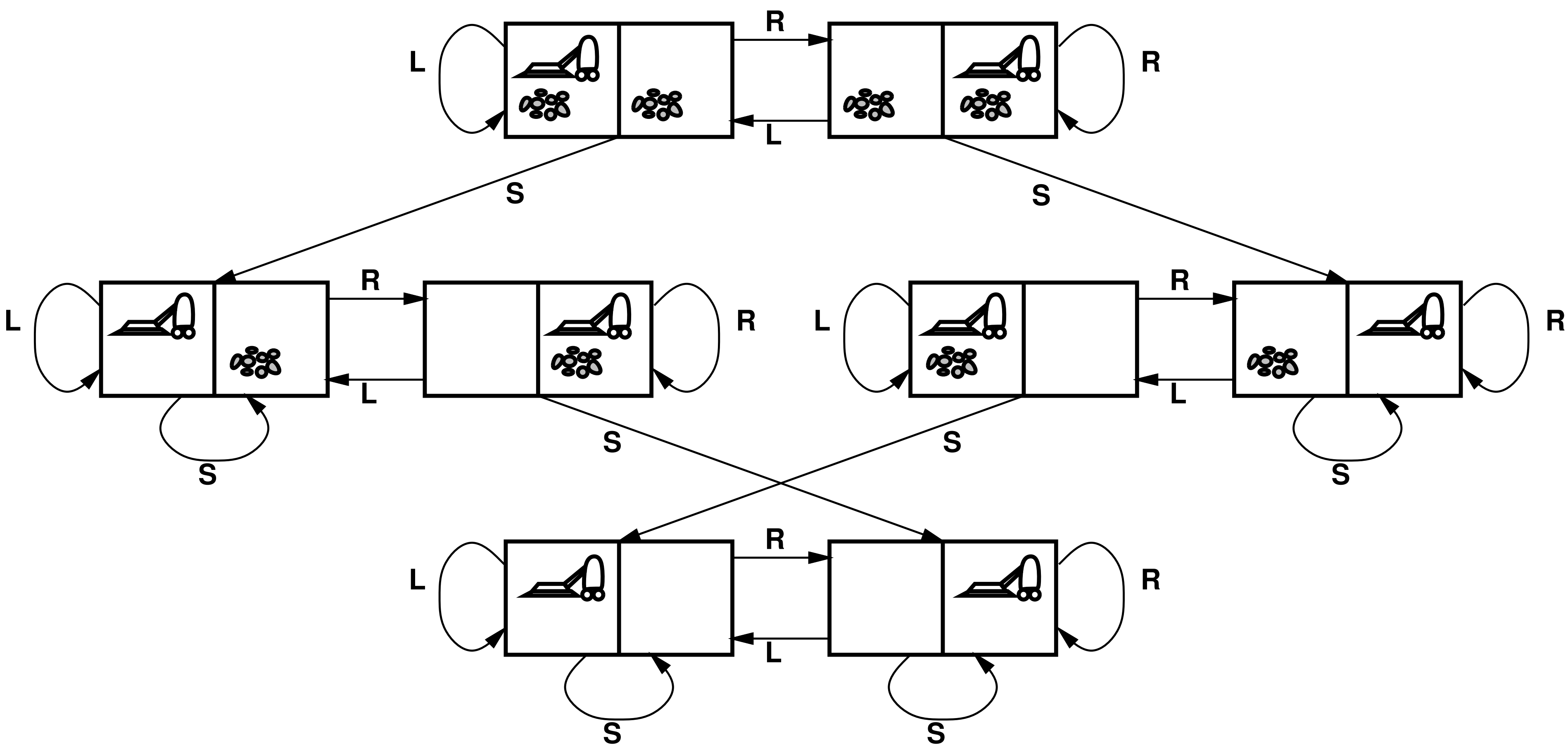
Search for a shortest path from Arad to Bucarest [1]:



Actions(Arad) = {Move(Sibiu), Move(Timisoara), Move(Zerind)}.

3 The vacuum-cleaner

Search for a shortest cleaning path (*Left, Right, Suck*) [1]:



States for n locations: $n \times 2^n$ (vacuum-cleaner and dirt location).

4 The 8-puzzle

Search for a shortest sequence of blank space (0) movements [1, 2]:

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

***n*-puzzle:** $(n+1)!$ states (permutations of "$01 \cdots n$")

$n = 3$

0	1
2	3

24

$n = 8$

0	1	2
3	4	5
6	7	8

$362\,880$

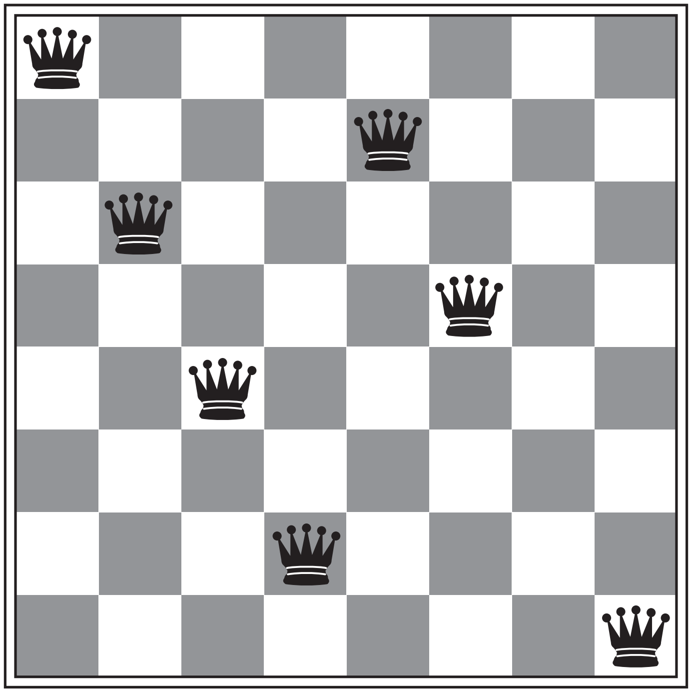
$n = 15$

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

$2 \cdot 10^{14}$

5 The eight queens

Search for an arrangement such that no queen is attacked [1, 3]:



Complete formulation:

States: any arrangement from 0 to 8 queens; $\approx 2 \cdot 10^{14}$ states.

Actions: add a queen to an empty square.

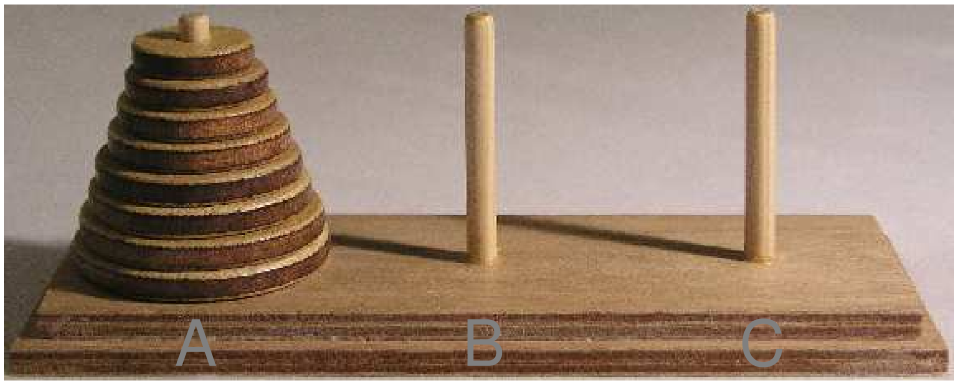
Incremental formulation:

States: arragements of n $(0 \leq n \leq 8)$ non-attacked queens, one per column in the leftmost n columns; 2057 states.

Actions: add a queen to any square in the leftmost empty column such that it is not attacked by any other queen.

6 Tower of Hanoi

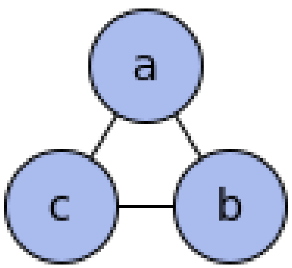
Search for a shortest sequence of single-disk movements to move an n-disk stack from rod A to C [4]:

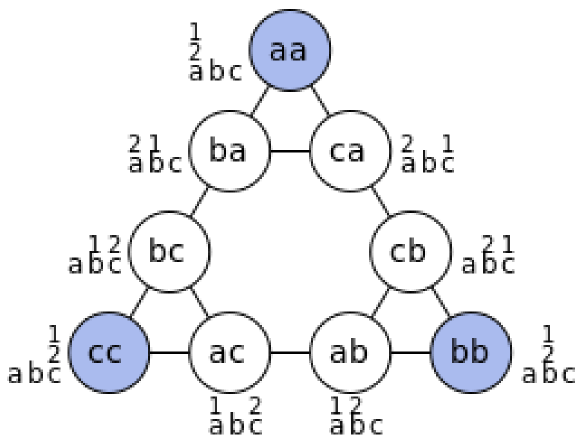


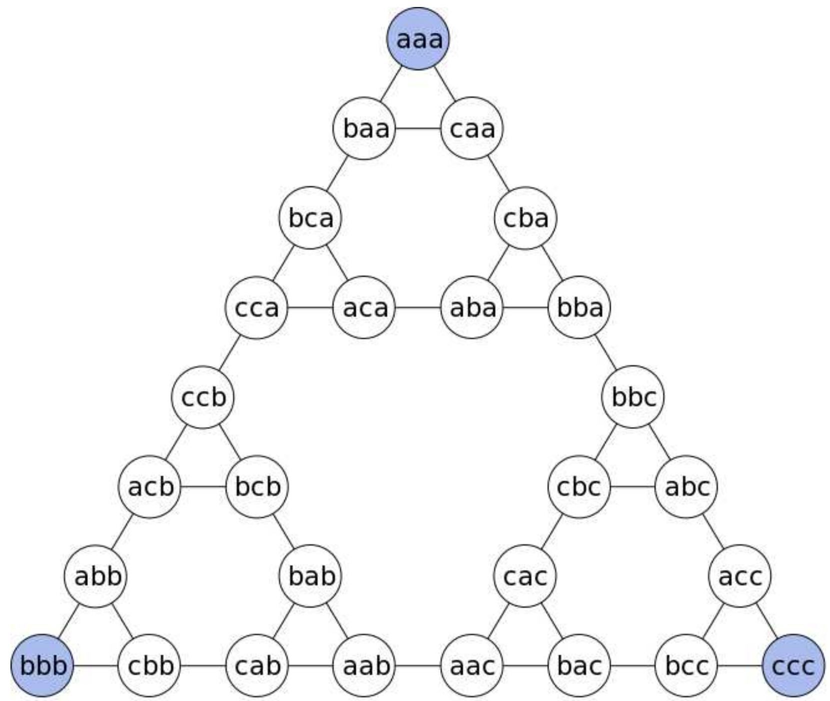
Rule 1: only disks at the top of a stack can be moved.

Rule 2: no disk can be placed on top of a smaller disk.

Graphs of 3^n nodes:







Optimal path: $2^n - 1$ movements!

References

[1] S. Russell and P. Norvig. *Artificial Intelligence: A Modern Approach*. Pearson, third edition, 2010.

[2] J. Slocum and D. Sonneveld. *The 15 Puzzle*. Slocum Puzzle Foundation, 2006.

[3] A000170: Number of ways of placing n nonattacking queens on an n X n board. https://oeis.org/A000170.

[4] Tower of Hanoi. https://en.wikipedia.org.