Intelligent Systems - Re-take Exam (Block 2): Test (1.75 points)

ETSINF, Universitat Politècnica de València, February 1st, 2024

Group, surname(s) and name: 1,

Tick only one choice among the given options. Score: $\max(0, (\text{correct_answers-wrong_answers}/3) \cdot 1.75 / 6)$.

1 Given the following probability distributions:

В	0	0	1	1
C	0	1	0	1
$P(A=0 \mid B,C)$	0.921	0.900	0.378	0.273
P(B,C)	0.322	0.412	0.108	0.157

Which is the value of $P(A = 1, B = 1 \mid C = 1)$?

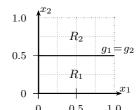
A)
$$P(A=1, B=1 \mid C=1) \le 0.25$$

B)
$$0.25 < P(A=1, B=1 \mid C=1) \le 0.50$$

C)
$$0.50 < P(A=1, B=1 \mid C=1) \le 0.75$$

D)
$$0.75 < P(A=1, B=1 \mid C=1) \le 1.00$$

2 The figure on the right represents the decision boundary and the two regions of a binary classifier. Which of the following weight vectors (in homogeneous notation) defines a classifier equivalent to the one of the figure?



A)
$$\mathbf{w}_1 = (-0.5, 0, 0)^t$$
 and $\mathbf{w}_2 = (0, 0, -1)^t$.

B)
$$\mathbf{w}_1 = (0.5, 0, 0)^t$$
 and $\mathbf{w}_2 = (0, 0, 1)^t$.

C)
$$\mathbf{w}_1 = (0, 0, 1)^t$$
 and $\mathbf{w}_2 = (0.5, 0, 0)^t$.

D) All the above weight vectors define an equivalent classifier.

Let's suppose that we are applying the Perceptron algorithm, with learning rate $\alpha = 1$ and margin b = 0.1, to a set of 4 bidimensional learning samples for a problem of 4 classes, c = 1, 2, 3, 4. At a given moment in the execution of the algorithm, we have obtained the weight vectors $\mathbf{w}_1 = (-2, -3, -9)^t$, $\mathbf{w}_2 = (-2, -5, -5)^t$, $\mathbf{w}_3 = (-2, -7, -11)^t$, $\mathbf{w}_4 = (-2, -3, -5)^t$. Assuming that the sample $(\mathbf{x}, c) = ((3, 4)^t, 3)$ is then going to be processed, how many weight vectors will be modified?

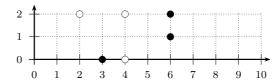
- A) 0
- B) 2
- C) 3
- D) 4

- 4 The estimated probability of error of a classifier is 7%. Which is the minimum number of testing samples, M, so that the 95% confidence interval of this estimated probability of error is not higher than $\pm 1\%$; that is, I = [6%, 8%]:
 - A) M < 1000.
 - B) $1000 \le M < 2000$.
 - C) $2000 \le M < 3000$.
 - D) $M \ge 3000$.
- 5 Given the following dataset to train a classification tree with 5 bidimensional samples that belong to 2 classes:

n	1	2	3	4	5
x_{n1}	2	3	5	5	3
x_{n2}	1	1	1	5	4
c_n	1	2	2	2	2

How many different partitions can be generated at the root node? Do not consider those partitions in which all data samples are assigned to the same child node.

- A) 4
- B) 5
- C) 2
- D) 3
- 6 The figure below shows a partition of 6 two-dimensional points in 2 clusters, \bullet and \circ :



What point when transferred minimises the variation of the Sum of Square Errors (SSE), $\Delta J = J - J'$ (SSE after the transfer minus SSE before the transfer)?

- A) $(3,0)^t$
- B) $(6,2)^t$
- C) $(4,0)^t$
- D) $(2,2)^t$

Intelligent Systems - Re-take Exam (Block 2): Problem (2 points) ETSINF, Universitat Politècnica de València, February 1st, 2024

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Problem: Logistic regression

The following table shows per rows a training set of 2 samples with 2 dimensions that belong to 2 classes:

$$\begin{array}{c|ccccc} n & x_{n1} & x_{n2} & c_n \\ \hline 1 & 0 & 1 & 1 \\ 2 & 0 & 0 & 2 \\ \end{array}$$

In addition, the following table represents an initial weight matrix with the weights of each class per columns:

\mathbf{w}_1	\mathbf{w}_2
0.	0.
0.	0.
0.25	-0.25

Answer the following questions:

- 1. (0.5 points) Compute the vector of logits for each training sample.
- 2. (0.25 points) Apply the softmax function to the vector of logits for each training sample.
- 3. (0.25 points) Classify every training sample. In case of a tie, choose any class.
- 4. (0.5 points) Compute the gradient of the function NLL at the point of the initial weight matrix.
- 5. (0.5 points) Update the initial weight matrix applying gradient descent with learning rate $\eta = 1.0$.