Intelligent Systems
Exercises Block 2 Chapter 2
Learning discriminant functions: Perceptron algorithm

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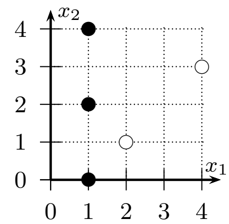
Questions

1 $\boxed{\text{A}}$ The perceptron algorithm is a ...

A) supervised and linear classifier
B) supervised and non-linear classifier
C) non-supervised and linear classifier
D) non-supervised and non-linear classifier

2 $\boxed{\text{C}}$ The figure on the right represents five two-dimension training samples that belong to two classes ○ and ●. We want to build a linear classifier for any $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, as follows:



$$c(\mathbf{x}) = \begin{cases} \circ & \text{if } \mathbf{w}^t\,\mathbf{x} > 0 \\ \bullet & \text{if } \mathbf{w}^t\,\mathbf{x} \leq 0 \end{cases} \quad \text{where} \quad \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \text{ is a weight vector to be selected}$$

If our learning criterion is the minimum classification error (over the learning samples), we will choose ...:

A) $\mathbf{w} = (1, 0)^t$
B) $\mathbf{w} = (1, 1)^t$
C) $\mathbf{w} = (1, -1)^t$
D) None of the above because we can find other weight vectors that produce a lower number of errors on the given training samples (there are better choices).

3 $\boxed{\text{D}}$ In the Perceptron algorithm:

A) there are mainly two parameters: the number of classes and the number of prototypes
B) the learning step α must be as large as possible in order to learn as much as possible
C) the margin must be zero when the classes are no linear-separable
D) there are mainly two parameters: the learning step α and the margin b.

4 $\boxed{\text{B}}$ In the Perceptron algorithm:

A) there are mainly two parameters: the number of classes and the number of prototypes
B) the margin b allows for finding adequate solutions when the problem is non-linearly separable
C) the margin b depends on the learning rate α
D) there are mainly two parametes: the learning rate α and the number of iterations

5 $\boxed{\text{B}}$ The parameter of the Perceptron algorithm that we name *margin*, b, is a real value that, assuming it is positive (as it usually is), reduces he number of possible solutions that the algorithm can find. Concretely, given N training samples, $(\mathbf{x}_1, c_1), \ldots, (\mathbf{x}_N, c_N)$ from C classes, the Perceptron algorithm will find linear discriminant functions $g_1(\cdot), \ldots, g_C(\cdot)$ such that for every $n = 1, \ldots, N$:

A) $g_{c_n}(\mathbf{x}_n) > g_c(\mathbf{x}_n)$ for every class $c \neq c_n$
B) $g_{c_n}(\mathbf{x}_n) > g_c(\mathbf{x}_n) + b$ for every class $c \neq c_n$
C) $g_{c_n}(\mathbf{x}_n) > g_c(\mathbf{x}_n) - b$ for every class $c \neq c_n$
D) None of the above

6 $\boxed{\text{B}}$ Given a classification problem of C classes $C \in \{1, 2, \ldots, C\}$, where objects are represented with a feature vector of D dimensions, $\mathbf{x} \in \mathbb{R}^D$, and assuming that a given $\mathbf{x}$ belongs to class 1, the Perceptron algorithm:

A) Modifies the linear discriminant $g_1(\mathbf{x})$ in any case.
B) Modifies the linear discriminant $g_1(\mathbf{x})$ if it exists $c \neq 1, g_c(\mathbf{x}) > g_1(\mathbf{x})$.
C) Modifies the linear discriminant $g_c(\mathbf{x})$ if $g_c(\mathbf{x}) < g_1(\mathbf{x})$ with $c \neq 1$.
D) Modifies the linear discriminant $g_1(\mathbf{x})$ only if $g_c(\mathbf{x}) > g_1(\mathbf{x})$ for every $c \neq 1$.

7 $\boxed{\text{C}}$ Given a classification problem of two classes, the following two-dimensional samples are provided: $\mathbf{x}_1 = (1,1)^t, \mathbf{x}_2 = (2,2)^t, \mathbf{x}_3 = (2,0)^t$; $\mathbf{x}_1$ and $\mathbf{x}_2$ belong to class A and $\mathbf{x}_3$ to class B. Taking into account that we are using a classifier based on linear discriminant functions with weight vectors $\mathbf{w}_A$ and $\mathbf{w}_B$ corresponding to classes A and B respectively, which of the following statements is *false*:

A) It is possible to find a linear discriminant function that classifies $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ with error=2/3.
B) Weight vectors $\mathbf{w}_A = (1, -1, 1)^t$ and $\mathbf{w}_B = (1, 2, -4)^t$ classify $\mathbf{x}_1$, $\mathbf{x}_2$ y $\mathbf{x}_3$ without errors.
C) Weight vectors $\mathbf{w}_A = (1, -1, 1)^t$ and $\mathbf{w}_B = (1, 2, -4)^t$ classify $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ with error=1/3.
D) It is possible to find a discriminant function that classifies $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ with error=1/3.

8 $\boxed{\text{D}}$ Let be a classification problem of three classes $\{A, B, C\}$ where objects are represented in a two-dimensional space $\mathbb{R}^2$. We want to use a classifier based on linear discriminant functions with the following weight vector for each class, $\mathbf{w}_A = (1, 1, 0)^t$, $\mathbf{w}_B = (-1, 1, -1)^t$ and $\mathbf{w}_C = (1, -2, 2)^t$. Which is the classification of $\mathbf{x}_1 = (1,1)^t$ and $\mathbf{x}_2 = (0, -1)^t$?

A) $c(\mathbf{x}_1) = B \quad c(\mathbf{x}_2) = C$
B) $c(\mathbf{x}_1) = A \quad c(\mathbf{x}_2) = B$
C) $c(\mathbf{x}_1) = B \quad c(\mathbf{x}_2) = A$
D) $c(\mathbf{x}_1) = A \quad c(\mathbf{x}_2) = A$

9 $\boxed{\text{D}}$ (Exam 18th January 2013) Let $g_1(\mathbf{y}) = y_1^2 + 2\,y_2^2$ and $g_2(\mathbf{y}) = 2\,y_1^2 + y_2^2$ be two discriminant functions for classes 1 and 2, respectively. The decision boundary between these two classes is:

A) A parabola
B) Hyperspherical
C) It is given by the equation $y_1^2 + y_2^2 = 0$.
D) A straight line: $y_2 = y_1$ – (by doing $g_1(\mathbf{y}) = g_2(\mathbf{y})$)

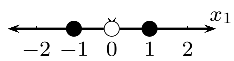
10 $\boxed{\text{A}}$ (Exam 30th January 2013) For a two-class classification problem in $\Re^2$ we have three different classifiers. One is formed by the two linear discriminant functions: $g_1(y) = 3 + 4\,y_1 - 2\,y_2$ and $g_2(y) = -3 + 1.5\,y_1 + 5\,y_2$. The second classifier is formed by $g_1'(y) = 6 + 8\,y_1 - 4\,y_2$ and $g_2'(y) = -6 + 3\,y_1 + 10\,y_2$. And the third by $g_1''(y) = -6 - 8\,y_1 + 4\,y_2$ and $g_2''(y) = 6 - 3\,y_1 - 10\,y_2$. Are the three classifiers equivalent?

A) (g_1, g_2) y (g_1', g_2') are equivalent.
B) The three of them are equivalent.
C) (g_1, g_2) y (g_1'', g_2'') are equivalent.
D) (g_1', g_2') y (g_1'', g_2'') are equivalent.

11 $\boxed{\text{A}}$ (Exam 30th January 2013) The perceptron algorithm is a ...

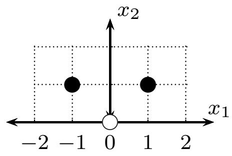
A) supervised and lineal classifier
B) supervised and quadratic classifier
C) non-supervised and linear classifier
D) non-supervised and quadratic classifier

12 $\boxed{\text{B}}$ (Exam January 15, 2014) The figure on the right shows three one-dimensional samples classified in two classes ○ and ●. Which is the number of errors of a minimum-error linear classifier?



A) 0
B) 1
C) 2
D) 3

13 $\boxed{A}$ (Exam January 15, 2014) Consider that we add a new feature x_2 to the samples of the above question. x_2 is defined as $x_2 = x_1^2$. Thus, we have now three two-dimensional samples as can be observed in the figure on the right. In this case, which is the number of errors of a minimum-error linear classifier?



A) 0
B) 1
C) 2
D) 3

14 $\boxed{A}$ (Exam January 15, 2014) Consider a classification problem in 2 classes, $c = 1, 2$, for objects represented by means of two-dimensional feature vectors. We have two training samples: $\mathbf{x}_1 = (0, 0)^t$ belongs to class $c_1 = 1$, and $\mathbf{x}_2 = (1, 1)^t$ belongs to class $c_2 = 2$. Likewise, we have the following linear classifier: $\mathbf{w}_1 = (w_{10}, w_{11}, w_{12}) = (1, -1, -1)^t$ and $\mathbf{w}_2 = (w_{20}, w_{21}, w_{22}) = (-1, 1, 1)^t$. If we apply one iteration of the Perceptron algorithm with learning speed $\alpha = 1$ and margin $b = 0.1$, then:

A) None of the weight vectors will be modified.
B) The weight vector of class 1 will be modified.
C) The weight vector of class 2 will be modified.
D) Both weight vectors will be modified.

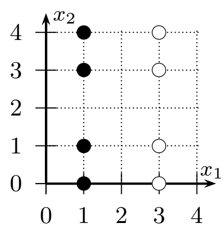
15 $\boxed{C}$ (Exam January 15, 2014) The Perceptron algorithm is controlled by two parameters, the *learning speed*, α, and the *margin*, b, both real values. Assuming we don't know whether the training samples are linearly separable or not, which values of the parameters α and b provide decision boundaries of better quality?

A) $\alpha = 0.1$ and $b = 0.0$.
B) $\alpha = 0.0$ and $b = 0.0$.
C) $\alpha = 0.1$ and $b = 1.0$.
D) $\alpha = 0.0$ and $b = 1.0$.

16 $\boxed{B}$ (Exam January 28, 2014) Consider a classification problem in two classes, $c = A, B$, for objects represented by means of two-dimensional feature vectors. The weight vectors that result from applying the Perceptron algorithm with a training data set are $\mathbf{w}_A = (1, 1, 0)^t$ and $\mathbf{w}_B = (-1, 0, 1)^t$. Given these results, which class do samples $\mathbf{x}_1 = (-1, 0)^t$ and $\mathbf{x}_2 = (0, 3)^t$ belong to?

A) $\hat{c}(\mathbf{x}_1) = A$ and $\hat{c}(\mathbf{x}_2) = A$. $\quad \mathbf{x}_1 \;:\; \mathbf{w}_A^t \cdot (1, -1, 0)^t = 0 \quad \mathbf{w}_B^t \cdot (1, -1, 0)^t = -1 \;\Rightarrow\; \mathbf{x}_1 \in A$
B) $\hat{c}(\mathbf{x}_1) = A$ and $\hat{c}(\mathbf{x}_2) = B$.
C) $\hat{c}(\mathbf{x}_1) = B$ and $\hat{c}(\mathbf{x}_2) = A$. $\quad \mathbf{x}_2 \;:\; \mathbf{w}_A^t \cdot (1, 0, 3)^t = 1 \quad \mathbf{w}_B^t \cdot (1, 0, 3)^t = 2 \;\Rightarrow\; \mathbf{x}_2 \in B$
D) $\hat{c}(\mathbf{x}_1) = B$ and $\hat{c}(\mathbf{x}_2) = B$.

17 $\boxed{D}$ (January 13, 2015) The figure on the right shows four two-dimensional training samples of 2 classes: $\circ$ or $\bullet$. Assuming our learning criteria is to minimize the number of misclassified samples, we will select as the weight vector of each class ...



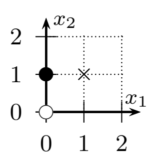
A) $\mathbf{a}_\circ = (3, 1, 1)^t$ y $\mathbf{a}_\bullet = (1, 2, 1)^t$ $\quad x_1 = 2 \quad R_\circ = \{x : x_1 < 2\}$ y $R_\bullet = \{x : x_1 > 2\}$

B) $\mathbf{a}_\circ = (1, 1, 2)^t$ y $\mathbf{a}_\bullet = (3, 1, 1)^t$ $\quad x_2 = 2 \quad R_\bullet = \{x : x_2 < 2\}$ y $R_\circ = \{x : x_2 > 2\}$

C) $\mathbf{a}_\circ = (3, 1, 1)^t$ y $\mathbf{a}_\bullet = (1, 1, 2)^t$ $\quad x_2 = 2 \quad R_\circ = \{x : x_2 < 2\}$ y $R_\bullet = \{x : x_2 > 2\}$

D) $\mathbf{a}_\circ = (1, 2, 1)^t$ y $\mathbf{a}_\bullet = (3, 1, 1)^t$ $\quad x_1 = 2 \quad R_\bullet = \{x : x_1 < 2\}$ y $R_\circ = \{x : x_1 > 2\}$

18 $\boxed{B}$ (January 13, 2015) The figure on the right shows three two-dimensional training samples of three classes: $\circ$, $\bullet$ and $\times$. Given the weight vectors $\mathbf{a}_\circ = (-2, -1, -3)^t$, $\mathbf{a}_\bullet = (-1, -3, 1)^t$ and $\mathbf{a}_\times = (-3, 3, -1)^t$, how many misclassification errors are generated?



A) 0
B) 1
C) 2
D) 3

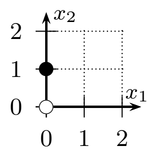
19 $\boxed{A}$ (January 13, 2015) If we apply one iteration of the Perceptron algorithm with learning rate $\alpha = 1.0$ and margin $b = 0.0$ over the samples and weight vectors of the above question, how many misclassification errors are generated with the new weight vectors?

A) 0 $\mathbf{a}_\circ = (-1,-1,-3)^t$, $\mathbf{a}_\bullet = (-2,-3,1)^t$ y $\mathbf{a}_\times = (-3,3,-1)^t$
B) 1
C) 2
D) 3

20 $\boxed{\text{C}}$ (January 26, 2015) Given a linear classifier of two classes $\circ$ and $\bullet$ with weight vectors $\mathbf{a}_\circ = (0,-1,1)^t$ and $\mathbf{a}_\bullet = (0,1,-1)^t$, which of the following weight vectors does ***NOT*** define an equivalent classifier to this one?

A) $\mathbf{a}_\circ = (1,-1,1)^t$ and $\mathbf{a}_\bullet = (1,1,-1)^t$ $\quad f(z) = az + b$ with $a = 1$ and $b = 1$
B) $\mathbf{a}_\circ = (-1,-2,2)^t$ and $\mathbf{a}_\bullet = (-1,2,-2)^t$ $\quad f(z) = az + b$ con $a = 2$ and $b = -1$
C) $\mathbf{a}_\circ = (0,2,-2)^t$ and $\mathbf{a}_\bullet = (0,-2,2)^t$ $\quad f(z) = az + b$ with $a = -2$ and $b = 0$
D) $\mathbf{a}_\circ = (0,-2,2)^t$ and $\mathbf{a}_\bullet = (0,2,-2)^t$ $\quad f(z) = az + b$ with $a = 2$ and $b = 0$

21 $\boxed{\text{A}}$ (January 26, 2015) The figure on the right shows two two-dimensional training samples of two classes: $\circ$, and $\bullet$. Given the weight vectors $\mathbf{a}_\circ = (0,1,-2)^t$, and $\mathbf{a}_\bullet = (0,0,1)^t$, if we apply one iteration of the Perceptron algorithm with learning rate $\alpha = 1.0$ and margin $b = 0.5$ over the training samples and weight vectors provided, how many misclassification errors will be generated with the new weight vectors resulting from the Perceptron algorithm?



A) 0 $\mathbf{a}_\circ = (1,1,-2)^t$ y $\mathbf{a}_\bullet = (-1,0,1)^t$
B) 1
C) 2
D) 3

22 $\boxed{\text{D}}$ (January, 2016) Let's consider a typical classification problem in C classes and objects represented through D-dimensional real feature vectors. In general, we can say that it it more difficult to find an accurate classifier when ...

A) the values of C and D are smaller
B) the value of C is smaller and the value of D is larger
C) the value of C is larger and the value of D is smaller
D) the values of C and D are larger

23 $\boxed{\text{B}}$ (January, 2016) We have learnt two different classifiers, c_A and c_B, for a classification problem. The probability of error of c_A has been empirically estimated from 100 test samples, obtaining an empirical estimate of error $\hat{p}_A = 0.10$ (10 %). Similarly, the probability of error of c_B has been empirically estimated but with a set of 200 test samples, obtaining an empirical estimate of error of 10 %, too ($\hat{p}_B = 0.10$). Based on these estimations, we can affirm with a 95 % of confidence that:

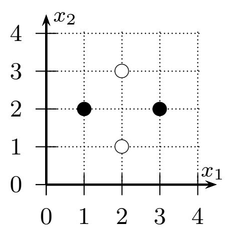
A) The confidence intervals of $\hat{p}_A$ y $\hat{p}_B$ are identical.

B) The confidence interval of $\hat{p}_A$ is larger than the one of $\hat{p}_B$. $\quad I_A = \hat{p}_A \pm 1.96\sqrt{\frac{\hat{p}_A\,(1-\hat{p}_A)}{100}} = 0.10 \pm 0.06$

C) The confidence interval of $\hat{p}_B$ is larger than the one of $\hat{p}_A$. $\quad I_B = \hat{p}_B \pm 1.96\sqrt{\frac{\hat{p}_B\,(1-\hat{p}_B)}{200}} = 0.10 \pm 0.04$

D) In this case, the confidence intervals of $\hat{p}_A$ and $\hat{p}_B$ are irrelevant because the estimate of error is the same.

24 $\boxed{\text{C}}$ (January, 2016) The figure on the right shows 4 two-dimensional samples classified in 2 classes: $\circ$ and $\bullet$. If we apply the Perceptron algorithm with initial weight vectors $\mathbf{a}_\circ = (0,1,0)^t$ and $\mathbf{a}_\bullet = (0,0,1)^t$, a learning rate $\alpha > 0$ and a margin b, indicate which assertion is **CORRECT**:



A) The algorithm will converge for some $b > 0$
B) The algorithm only converges if $b \leq 0$
C) If $b > 0$ there is no convergence but, by adjusting α, we can obtain good solutions after a finite number of iterations with respect to the probability of classification error (with 25 % of misclassification error)
D) The algorithm is not applicable in this case because the classes are non-linearly separable.

25 $\boxed{\text{B}}$ (January, 2016) Which is the number of errors of a minimum-error linear classifier for the training samples of the above question?

A) 0
B) 1
C) 2
D) 3

26 $\boxed{\text{B}}$ (January, 2016) Given a linear classifier of two classes $\circ$ and $\bullet$ with weight vectors $\mathbf{a}_\circ = (3,1,1)^t$ and $\mathbf{a}_\bullet = (1,2,1)^t$, respectively (the first component is the threshold or independent term of the linear function), which assertion is **CORRECT**?

A) There are four decision regions because there are two weight vectors and it is a two-dimensional representation space

B) The weight vectors $\mathbf{a}_\circ = (2,-2,-2)^t$ and $\mathbf{a}_\bullet = (-2,0,-2)^t$ define the same decision boundary than the weight vectors given in the question statement The decision boundary equation is: $\mathbf{a}_\circ^t\mathbf{y} = \mathbf{a}_\bullet^t\mathbf{y}$. In both cases, we have: $y_1 = 2$.

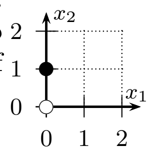
C) The weight vectors $\mathbf{a}_\circ = (1,2,1)^t$ y $\mathbf{a}_\bullet = (3,1,1)^t$ define an equivalent classifier to the one given in the statement Opposed decision regions.

D) The decision boundary is defined as a plane in $\mathbb{R}^3$ because the weight vectors are three-dimensional

27 $\boxed{\text{C}}$ (January, 2016) We have three different classifiers for a two-class problem in $\Re^2$. One classifier is formed by the linear functions: $g_1(y) = 2\,y_1 + y_2 + 3$ and $g_2(y) = y_1 + 2$. The second classifier is formed by: $g_1'(y) = -2\,y_1 + y_2 - 1$ and $g_2'(y) = -y_1 + 2\,y_2$. The third classifier is formed by: $g_1''(y) = -2\,y_1 - y_2 - 3$ and $g_2''(y) = -y_1 - 2$. Which assertion is TRUE?

A) (g_1, g_2) y (g_1', g_2') are equivalent, but (g_1, g_2) y (g_1'', g_2'') are not.
B) (g_1, g_2) y (g_1', g_2') are not equivalent, but (g_1, g_2) y (g_1'', g_2'') are equivalent.
C) (g_1, g_2) y (g_1', g_2') are not equivalent, but (g_1', g_2') y (g_1'', g_2'') are equivalent. Common boundary $y_2 = -y_1 - 1$ but $R \neq R' = R''$
D) The three classifiers are not equivalent to each other.

28 $\boxed{\text{C}}$ (January, 2016) The figure on the right shows two bi-dimensional samples in 2 classes: $(x_1,\circ)$ and $(x_2,\bullet)$. Given the weight vectors $\mathbf{a}_\circ = (0,1,-2)^t$ and $\mathbf{a}_\bullet = (0,0,1)^t$, if we apply the Perceptron algorithm only to the sample x_1, we obtain the new weight vectors $\mathbf{a}_\circ = (1,1,-2)^t$ and $\mathbf{a}_\bullet = (-1,0,1)^t$. Which is the value of the learning factor α and margin b?

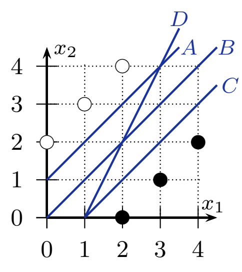


A) $\alpha = 1.0$ y $b = 0.0$
B) $\alpha = -1.0$ y $b = 0.5$
C) $\alpha = 1.0$ y $b = 0.5$
D) It is not possible to determine the value of α and b

29 $\boxed{\text{A}}$ (January 2017) Let be a classification problem of two classes in $\mathbb{R}^2$. We have a classifier made up of two discriminant linear functions with weight vectors $\mathbf{a}_\circ = (-1,1,2)^t$ and $\mathbf{a}_\bullet = (1,1,1)^t$. Indicate the decision regions defined by this classifier.

A) $R_\circ = \{\mathbf{x} \in \mathbb{R}^2 : x_2 > 2\}$ y $R_\bullet = \{\mathbf{x} \in \mathbb{R}^2 : x_2 < 2\}$ $g_\circ(\mathbf{x}) = g_\bullet(\mathbf{x}) \rightarrow x_2 = 2 \ \wedge \ g_\circ((0,0)^t) < g_\bullet((0,0)^t)$
B) $R_\circ = \{\mathbf{x} \in \mathbb{R}^2 : x_1 > 2\}$ y $R_\bullet = \{\mathbf{x} \in \mathbb{R}^2 : x_1 < 2\}$
C) $R_\circ = \{\mathbf{x} \in \mathbb{R}^2 : x_1 < 2\}$ y $R_\bullet = \{\mathbf{x} \in \mathbb{R}^2 : x_1 > 2\}$
D) $R_\circ = \{\mathbf{x} \in \mathbb{R}^2 : x_2 < 2\}$ y $R_\bullet = \{\mathbf{x} \in \mathbb{R}^2 : x_2 > 2\}$

30 $\boxed{\text{B}}$ (January 2017) The figure on the right shows 6 two-dimensional samples of two classes ($\circ$ and $\bullet$). After applying the Perceptron algorithm with different values of the parameter b, we obtain the 4 classifiers that appear in the response choices. Which classifier returns the most centered decision boundary and consequently the lowest expected error?



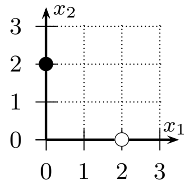
A) $\mathbf{a}_\circ = (-1,1,2)^t$ y $\mathbf{a}_\bullet = (\ \ 0,2,1)^t$ $g_\circ(\mathbf{x}) = g_\bullet(\mathbf{x}) \rightarrow x_2 = x_1 + 1$

B) $\mathbf{a}_\circ = (\ \ 1,1,2)^t$ y $\mathbf{a}_\bullet = (\ \ 1,2,1)^t$ $g_\circ(\mathbf{x}) = g_\bullet(\mathbf{x}) \rightarrow x_2 = x_1$

C) $\mathbf{a}_\circ = (\ \ 1,1,2)^t$ y $\mathbf{a}_\bullet = (\ \ 0,2,1)^t$ $g_\circ(\mathbf{x}) = g_\bullet(\mathbf{x}) \rightarrow x_2 = x_1 - 1$

D) $\mathbf{a}_\circ = (\ \ 1,1,1)^t$ y $\mathbf{a}_\bullet = (-1,3,0)^t$ $g_\circ(\mathbf{x}) = g_\bullet(\mathbf{x}) \rightarrow x_2 = 2x_1 - 2$

31 $\boxed{\text{C}}$ (January 2017) The figure on the right shows 2 bidimensional samples of two classes ($\mathbf{x}_1$,$\circ$) and ($\mathbf{x}_2$,$\bullet$). Given the weights $\mathbf{a}_\circ = (0,1,0)^t$ and $\mathbf{a}_\bullet = (1,0,0)^t$, if we apply one iteration of the Perceptron algorithm using only the sample $\mathbf{x}_1$, which is the minimum value of the margin b for which the weight vectors are updated?



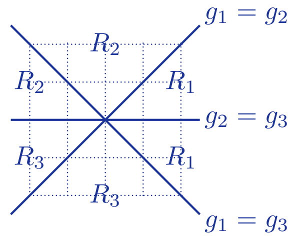
A) $b = 0.5$

B) $b = 1.0$

C) $b = 1.5$ $\quad g_\circ(\mathbf{x}_1) = 2 \quad g_\bullet(\mathbf{x}_1) = 1 \quad$ if $(g_\bullet(\mathbf{x}_1) + b > g_\circ(\mathbf{x}_1))$

D) None of the above

32 $\boxed{\text{C}}$ (January 2017) For a three-class problem in $\mathbb{R}^2$, we have a classifier defined by three discriminant linear functions: $g_1(\mathbf{x}) = x_1$, $g_2(\mathbf{x}) = x_2$ and $g_3(\mathbf{x}) = -x_2$. Show the expression that **IS NOT CORRECT** for the defined classifier.



A) It defines three decision boundaries that intersect in the origin coordinate $(0,0)$.

B) The decision region of class 1 is defined as $R_1 = \{\mathbf{x} \in \mathbb{R}^2 : x_1 > 0 \wedge x_1 > |x_2|\}$

C) In the decision region R_2, x_2 is lower than 0 and in R_3, x_2 is greater than 0.

D) In the decision region R_2, x_2 is greater than 0 and in R_3, x_2 is lower than 0.

33 $\boxed{\text{B}}$ (January 2017) Let $\hat{p}$ be the probability of error of a classifier estimated on a test set of size N and let $I = [\hat{p} \pm \epsilon]$ be the confidence interval of this estimation. Indicate the **CORRECT** answer.

A) If $N = 160$ and the classifier makes at least one error, ϵ will be less than 1 %.

B) If $N > 150$ and the probability of error is $\hat{p} = 0.1$, ϵ will be less than 5 %.

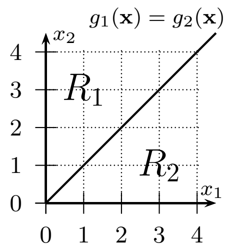
C) If N_e is the number of errors made by the classifier, then $\hat{p} = N/N_e$ and ϵ is inversely proportional to N.

D) It is not possible to determine ϵ if $\hat{p} = 0$.

34 $\boxed{\text{B}}$ (January 2018) In a 4-class classification problem for objects in $\mathbb{R}^3$, we have a linear classifier with weight vectors: $\mathbf{w}_1 = (-2,1,2,0)^t$, $\mathbf{w}_2 = (0,2,2,0)^t$, $\mathbf{w}_3 = (1,1,1,0)^t$ and $\mathbf{w}_4 = (3,0,0,1)^t$ (the first component is the threshold or independent term of the linear function). Mark the class that the object $\mathbf{x} = (1,2,2)^t$ will be assigned to.

A) 1. $\quad -2 + 1 \cdot 1 + 2 \cdot 2 + 0 \cdot 2 = 3$
B) 2. $\quad 0 + 2 \cdot 1 + 2 \cdot 2 + 0 \cdot 2 = 6$
C) 3. $\quad 1 + 1 \cdot 1 + 1 \cdot 2 + 0 \cdot 2 = 4$
D) 4. $\quad 3 + 0 \cdot 1 + 0 \cdot 2 + 1 \cdot 2 = 5$

35 $\boxed{\text{C}}$ (January 2018) The figure on the right represents the decision boundary and the two regions of a binary classifier. Which of the following weight vectors represents the classifier of the figure?



A) $\mathbf{w}_1 = (-1,-1,-2)^t$ and $\mathbf{w}_2 = (-1,-2,-1)^t$ $\quad x_2 = x_1 \quad R_1 : x_2 < x_1 \quad R_2 : x_2 > x_1$
B) $\mathbf{w}_1 = (\ \ 1,-1,-2)^t$ and $\mathbf{w}_2 = (\ \ 0,-2,-1)^t$ $\quad x_2 = x_1 + 1 \quad R_1 : x_2 < x_1 + 1 \quad R_2 : x_2 > x_1 + 1$
C) $\mathbf{w}_1 = (\ \ 1,\ \ 1,\ \ 2)^t$ and $\mathbf{w}_2 = (\ \ 1,\ \ 2,\ \ 1)^t$ $\quad x_2 = x_1 \quad R_1 : x_2 > x_1 \quad R_2 : x_2 < x_1$
D) $\mathbf{w}_1 = (-1,\ \ 1,\ \ 2)^t$ and $\mathbf{w}_2 = (\ \ 0,\ \ 2,\ \ 1)^t$ $\quad x_2 = x_1 + 1 \quad R_1 : x_2 > x_1 + 1 \quad R_2 : x_2 < x_1 + 1$

36 $\boxed{\text{D}}$ (January 2018) Let be a 3-class classification problem ($c = 1,2,3$) for two-dimensional objects ($\mathbb{R}^2$). We have 3 training samples: $\mathbf{x}_1 = (1,2)^t$ belongs to class $c_1 = 1$, $\mathbf{x}_2 = (2,3)^t$ belongs to class $c_2 = 2$ and $\mathbf{x}_3 = (3,1)^t$ belongs to class $c_3 = 3$. We also have a linear classifier defined with weight vectors: $\mathbf{w}_1 = (w_{10}, w_{11}, w_{12}) = (2,-8,0)^t$, $\mathbf{w}_2 = (w_{20}, w_{21}, w_{22}) = (-5,-2,-1)^t$ and $\mathbf{w}_3 = (w_{30}, w_{31}, w_{32}) = (-2,1,-10)^t$ (the first component is the threshold of the linear function). If we apply an iteration of the Perceptron algorithm from the given weight vectors with learning rate $\alpha = 1$ and margin $b = 1.5$, then:

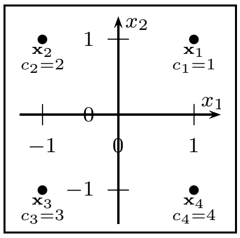
A) $\mathbf{w}_1$ and $\mathbf{w}_2$ will be modified.
B) $\mathbf{w}_1$ and $\mathbf{w}_3$ will be modified.
C) $\mathbf{w}_2$ and $\mathbf{w}_3$ will be modified.
D) No weight vector will be modified.

37 $\boxed{\text{B}}$ (January 2018) Let be a 3-class classification problem, $c = 1,2,3$, for two-dimensional objects, $\mathbf{x} = (x_1, x_2)^t \in \mathbb{R}^2$. We have a linear classifier with the following weight vectors: $\mathbf{w}_1 = (w_{10}, w_{11}, w_{12})^t = (2,0,0)^t$, $\mathbf{w}_2 = (0,1,1)^t$ and $\mathbf{w}_3 = (0,1,-1)^t$. The decision region of the class 1 defined by this classifier is:

A) $\{\mathbf{x} : x_1 \geq 0 \wedge x_2 < -x_1 + 2\} \cup \{\mathbf{x} : x_1 < 0 \wedge x_2 < x_1 + 2\}$.
B) $\{\mathbf{x} : x_2 \geq 0 \wedge x_2 < -x_1 + 2\} \cup \{\mathbf{x} : x_2 < 0 \wedge x_2 > x_1 - 2\}$.

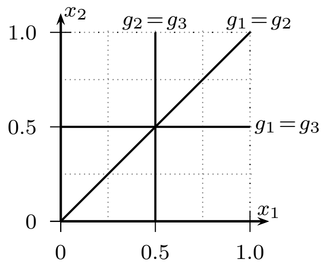
C) $\{\mathbf{x} : x_1 \geq 0 \wedge x_2 < -x_1 + 1\} \cup \{\mathbf{x} : x_1 < 0 \wedge x_2 < x_1 + 1\}$.
D) $\{\mathbf{x} : x_2 \geq 0 \wedge x_2 < -x_1 + 1\} \cup \{\mathbf{x} : x_2 < 0 \wedge x_2 > x_1 - 1\}$.

38 $\boxed{D}$ (January 2018) The figure shows 4 samples, each belonging to one different class among 4 classes: $\mathbf{x}_1 = (1,1)^t$ belongs to class $c_1 = 1$, $\mathbf{x}_2 = (-1,1)^t$ belongs to class $c_2 = 2$, $\mathbf{x}_3 = (-1,-1)^t$ belongs to class $c_3 = 3$, and $\mathbf{x}_4 = (1,-1)^t$ belongs to class $c_4 = 4$. Let's assume we apply the Perceptron algorithm to these samples with learning rate $\alpha = 1$, margin $b = 0.1$ and initial null weight vectors. Once the *first 3 samples* have been processed at the first iteration of the algorithm, we get the weight vectors $\mathbf{w}_1 = (w_{10}, w_{11}, w_{12})^t = (0,2,0)^t$, $\mathbf{w}_2 = (-1,-1,1)^t$, $\mathbf{w}_3 = (-1,-1,-3)^t$ and $\mathbf{w}_4 = (-3,1,-1)^t$. Finish the first iteration of the Perceptron and mark, from the resulting weight vectors, the number of samples that are **CORRECTLY** classified:



A) 1.
B) 2. $\mathbf{w}_1 = (-1,1,1)^t$, $\mathbf{w}_2 = (-1,-1,1)^t$, $\mathbf{w}_3 = (-2,-2,-2)^t$, $\mathbf{w}_4 = (-2,2,-2)^t$
C) 3. $\mathbf{x}_1$: $g_1 = 1$, $g_2 = -1$, $g_3 = -6$, $g_4 = -2$ $\quad$ $\mathbf{x}_2$: $g_1 = -1$, $g_2 = 1$, $g_3 = -2$, $g_4 = -6$
D) 4. $\mathbf{x}_3$: $g_1 = -3$, $g_2 = -1$, $g_3 = 2$, $g_4 = -2$ $\quad$ $\mathbf{x}_4$: $g_1 = -1$, $g_2 = -3$, $g_3 = -2$, $g_4 = 2$

39 $\boxed{B}$ The figure on the right represents the decision boundaries of a 3-class classifier. Which of the following weight vectors define these boundaries?



A) $\mathbf{w}_1 = (0,0,1)^t$ $\quad$ $\mathbf{w}_2 = (0,1,0)^t$ y $\mathbf{w}_3 = (1,0,0)^t$

B) $\mathbf{w}_1 = (0,0,1)^t$ $\quad$ $\mathbf{w}_2 = (0,1,0)^t$ y $\mathbf{w}_3 = (0.5,0,0)^t$

C) $\mathbf{w}_1 = (0.5,0,0)^t$ $\mathbf{w}_2 = (0,1,0)^t$ y $\mathbf{w}_3 = (0,0,1)^t$

D) $\mathbf{w}_1 = (0,0,1)^t$ $\quad$ $\mathbf{w}_2 = (1,0,0)^t$ y $\mathbf{w}_3 = (0,1,0)^t$

40 $\boxed{A}$ We have a linear classifier for 2 classes, $\circ$ and $\bullet$, with weight vectors $\mathbf{a}_\circ = (2,-5,4)^t$ and $\mathbf{a}_\bullet = (5,1,1)^t$, respectively. Which of the following assertions is TRUE?

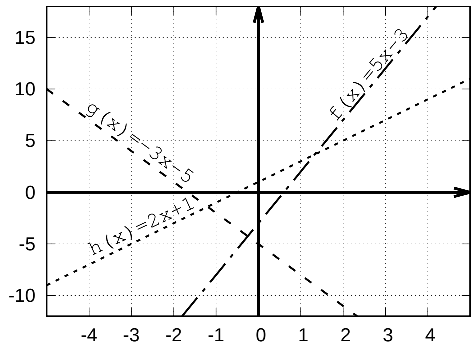
A) The weight vectors $\mathbf{a}_\circ = (3,4,1)^t$ and $\mathbf{a}_\bullet = (2,2,2)^t$ define the same decision boundary than the weight vectors shown in the question wording.

B) The weight vectors $\mathbf{a}_\circ = (-2,5,-4)^t$ and $\mathbf{a}_\bullet = (-5,-1,-1)^t$ define a classifier which is equivalent to the classifier of the question wording.

C) The point $\mathbf{x}' = (1,2)^t$ belongs to class $\circ$.

D) The point $\mathbf{x}' = (-2,0)^t$ is on the decision boundary.

41 $\boxed{C}$ The figure on the right shows the discriminant linear functions that result after training a classifier using the Perceptron algorithm with a set of $\mathbb{R}$ objects. The obtained functions are: $g(x) = -3x - 5$, $h(x) = 2x + 1$ and $f(x) = 5x - 3$. Mark the CORRECT decision boundaries between $g(x)$ and $h(x)$, and between $h(x)$ and $f(x)$.



A) $x = -5/3$ and $x = 3/5$.

B) $x = -1/2$ and $x = 3/5$.

C) $x = -6/5$ and $x = 4/3$.

D) $x = -5/3$ and $x = 4/3$.

42 $\boxed{A}$ Regarding the Perceptron algorithm (hereafter, we will refer to the algorithm as P), show the **TRUE** statement when P is applied to a sample set S of labeled vectors:

A) If the samples of S are linearly separable, P ends after a finite number of iterations and the resulting weights classify S without errors.

B) The number of vectors of S that are correctly classified with the weights obtained in each iteration of P is higher than the number of vectors correctly classified in the previous iteration.

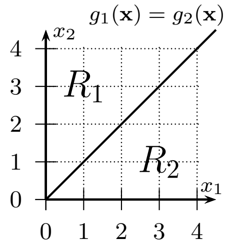
C) P always converges after a finite number of iterations although it is possible that the final weights do not classify all the vectors of S correctly.

D) The larger the set S is, the higher the number of iterations that P needs to reach convergence.

43 $\boxed{\text{B}}$ In a 4-class classification problem for objects in $\mathbb{R}^3$, we have a linear classifier with weight vectors: $\mathbf{w}_1 = (-2, 1, 2, 0)^t$, $\mathbf{w}_2 = (0, 2, 2, 0)^t$, $\mathbf{w}_3 = (1, 1, 1, 0)^t$ and $\mathbf{w}_4 = (3, 0, 0, 1)^t$ (the first component is the threshold or independent term of the linear function). Mark the class that the object $\mathbf{x} = (1, 2, 2)^t$ will be assigned to.

A) 1. $-2 + 1 \cdot 1 + 2 \cdot 2 + 0 \cdot 2 = 3$
B) 2. $0 + 2 \cdot 1 + 2 \cdot 2 + 0 \cdot 2 = 6$
C) 3. $1 + 1 \cdot 1 + 1 \cdot 2 + 0 \cdot 2 = 4$
D) 4. $3 + 0 \cdot 1 + 0 \cdot 2 + 1 \cdot 2 = 5$

44 $\boxed{\text{C}}$ The figure on the right represents the decision boundary and the two regions of a binary classifier. Which of the following weight vectors represents the classifier of the figure?



A) $\mathbf{w}_1 = (-1, -1, -2)^t$ and $\mathbf{w}_2 = (-1, -2, -1)^t$ $x_2 = x_1$ $R_1 : x_2 < x_1$ $R_2 : x_2 > x_1$
B) $\mathbf{w}_1 = (\ 1, -1, -2)^t$ and $\mathbf{w}_2 = (\ 0, -2, -1)^t$ $x_2 = x_1 + 1$ $R_1 : x_2 < x_1 + 1$ $R_2 : x_2 > x_1 + 1$
C) $\mathbf{w}_1 = (\ 1, \ 1, \ 2)^t$ and $\mathbf{w}_2 = (\ 1, \ 2, \ 1)^t$ $x_2 = x_1$ $R_1 : x_2 > x_1$ $R_2 : x_2 < x_1$
D) $\mathbf{w}_1 = (-1, \ 1, \ 2)^t$ and $\mathbf{w}_2 = (\ 0, \ 2, \ 1)^t$ $x_2 = x_1 + 1$ $R_1 : x_2 > x_1 + 1$ $R_2 : x_2 < x_1 + 1$

45 $\boxed{\text{D}}$ Let be a 3-class classification problem ($c = 1, 2, 3$) for two-dimensional objects ($\mathbb{R}^2$). We have 3 training samples: $\mathbf{x}_1 = (1, 2)^t$ belongs to class $c_1 = 1$, $\mathbf{x}_2 = (2, 3)^t$ belongs to class $c_2 = 2$ and $\mathbf{x}_3 = (3, 1)^t$ belongs to class $c_3 = 3$. We also have a linear classifier defined with weight vectors: $\mathbf{w}_1 = (w_{10}, w_{11}, w_{12}) = (2, -8, 0)^t$, $\mathbf{w}_2 = (w_{20}, w_{21}, w_{22}) = (-5, -2, -1)^t$ and $\mathbf{w}_3 = (w_{30}, w_{31}, w_{32}) = (-2, 1, -10)^t$ (the first component is the threshold of the linear function). If we apply an iteration of the Perceptron algorithm from the given weight vectors with learning rate $\alpha = 1$ and margin $b = 1.5$, then:

A) $\mathbf{w}_1$ and $\mathbf{w}_2$ will be modified.
B) $\mathbf{w}_1$ and $\mathbf{w}_3$ will be modified.
C) $\mathbf{w}_2$ and $\mathbf{w}_3$ will be modified.
D) No weight vector will be modified.

46 $\boxed{\text{D}}$ (January 2019) Let be a classification problem in 4 classes for objects represented in $\mathbb{R}^3$. We have a linear classifier with weight vectors:

$$\mathbf{a}_1 = (-2, 1, 2, 0)^t \qquad \mathbf{a}_2 = (0, 2, 2, 0)^t \qquad \mathbf{a}_3 = (1, 1, 1, 0)^t \qquad \mathbf{a}_4 = (3, 0, 0, 2)^t$$

Which class will the object $\mathbf{x} = (1, 2, 2)^t$ be assigned to?.

A) 1. $-2 + 1 \cdot 1 + 2 \cdot 2 + 0 \cdot 2 = 3$
B) 2. $0 + 2 \cdot 1 + 2 \cdot 2 + 0 \cdot 2 = 6$
C) 3. $1 + 1 \cdot 1 + 1 \cdot 2 + 0 \cdot 2 = 4$
D) 4. $3 + 0 \cdot 1 + 0 \cdot 2 + 2 \cdot 2 = 7$

47 $\boxed{\text{D}}$ (January 2019) Let's assume that we are applying the Perceptron algorithm with $b = 1.5$ and that the current weight vectors are the ones given in the above question. Let's also assume that the object $\mathbf{x} = (1, 2, 2)^t$ belongs to class 3 and that $\mathbf{x}$ is the next sample to be analyzed by the algorithm. After analyzing $\mathbf{x}$, we have that:

A) The weight vectors $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{a}_4$ are modified.
B) Only the weight vector $\mathbf{a}_3$ is modified.
C) None of the weight vectors is modified.
D) All of the weight vectors are modified.

$g_1(\mathbf{x}) + b > g_3(\mathbf{x})? \rightarrow 4.5 > 4?$ Yes $\rightarrow$ mod $\mathbf{a}_1$
$g_2(\mathbf{x}) + b > g_3(\mathbf{x})? \rightarrow 7.5 > 4?$ Yes $\rightarrow$ mod $\mathbf{a}_2$
$g_4(\mathbf{x}) + b > g_3(\mathbf{x})? \rightarrow 8.5 > 4?$ Yes $\rightarrow$ mod $\mathbf{a}_4$
An error has been produced? Yes $\rightarrow$ mod $\mathbf{a}_3$

48 $\boxed{\text{A}}$ (January 2019) The homogeneous notation is used to describe discriminant linear functions $g(\cdot)$ in a compact way. Let E be a three-dimension representation space; $\mathbf{y} \in E$ a point in E; a_0, a_1, a_2 y a_3 four real coefficients; $\mathbf{w} \stackrel{\text{def}}{=} (a_1, a_2, a_3)^t$ a three-dimension real vector; and $\mathbf{a} \stackrel{\text{def}}{=} (a_0, a_1, a_2, a_3)^t$ a four-dimension real vector. Show which of the following expressions makes an **INCORRECT** use of the homogeneous notation:

A) $g(\mathbf{y}) = \mathbf{a}^t\,\mathbf{y}$
B) $g(\mathbf{y}) = \mathbf{w}^t\,\mathbf{y} + a_0$
C) $g(\mathbf{y}) = \mathbf{a}^t\,\mathbf{x}$, donde $\mathbf{x} \stackrel{\text{def}}{=} (1, y_1, y_2, y_3)^t$
D) $g(\mathbf{y}) = a_0 + (a_1, a_2, a_3)^t\,\mathbf{y}$

49 $\boxed{\text{C}}$ (January 2019) Let $S = \{(\mathbf{y}_1, c_1), \ldots, (\mathbf{y}_N, c_N)\}$, $1 \leq c_j \leq C, 1 \leq j \leq N$, be a set of linearly separable samples in homogeneous notation. We use S as input for the Perceptron algorithm initialized with $\mathbf{a}'_j = \mathbf{0}$, $1 \leq j \leq C$. After a sufficiently large number of iterations with a learning factor $\alpha = 1$ and margin $b = 10$, the algorithm finishes and returns C weight vector in homogeneous notation, $\mathbf{a}_j$, $1 \leq j \leq C$. Show the expression that is **CORRECT**:

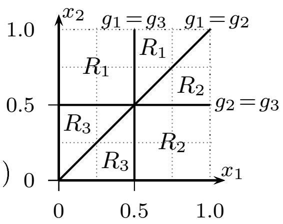
A) It holds the $N \cdot C$ inequations:
$\mathbf{a}_i^t \mathbf{y}_i > \mathbf{a}_j^t \mathbf{y}_j \;\; 1 \leq i \leq N, \; 1 \leq j \leq C, \; i \neq j$

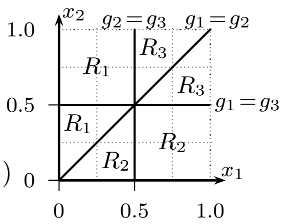
B) The N samples get correctly classified; that is, it holds the N inequations:
$\mathbf{a}_{c_i}^t \mathbf{y}_i > \mathbf{a}_j^t \mathbf{y}_i, \;\; 1 \leq i \leq N, \; j \neq c_i$ hold

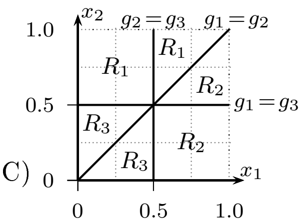
C) The N samples get correctly classified; that is, it holds the $N \cdot (C-1)$ inequations:
$\mathbf{a}_{c_i}^t \mathbf{y}_i > \mathbf{a}_j^t \mathbf{y}_i, \;\; 1 \leq i \leq N, \; 1 \leq j \leq C, \; j \neq c_i$

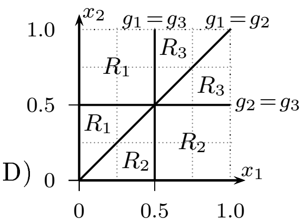
D) Although it is separable, we cannot affirm all the samples get correctly classified with the weight vectors $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_C$ because $b \gg \alpha > 0$.

50 $\boxed{C}$ Let be a three-class classifier based on the following bi-dimensional linear functions of weight vectors: $\mathbf{w}_1 = (0,0,1)^t$, $\mathbf{w}_2 = (0,1,0)^t$ and $\mathbf{w}_3 = (0.5,0,0)^t$. Which of the following figures is consistent with the decision boundaries and decision regions defined by the given classifier?

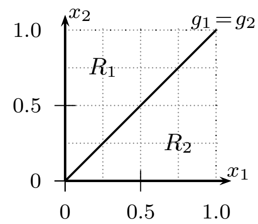
A) 

B) 

C) 

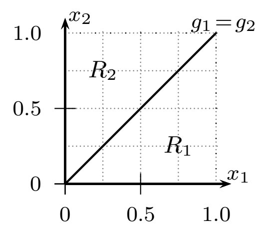
D) 

51 $\boxed{B}$ The figure on the right shows the decision boundary and decision regions of a two-class classifier. Which of the following weight vectors **IS NOT** a classifier equivalent to the one in the figure?



A) $\mathbf{w}_1 = (0,0,1)^t$ and $\mathbf{w}_2 = (0,1,0)^t$.
B) $\mathbf{w}_1 = (0,1,0)^t$ and $\mathbf{w}_2 = (0,0,1)^t$.
C) $\mathbf{w}_1 = (0,-1,0)^t$ and $\mathbf{w}_2 = (0,0,-1)^t$.
D) All three above weight vectors define a classifier equivalent to the one shown in the figure.

52 $\boxed{D}$ During the application of the Perceptron algorithm ($\alpha = 1.0$ and $b = 0$) to a two-class classification problem, we have obtained the weight vectors $\mathbf{w}_1 = (-1,1,0)^t$ and $\mathbf{w}_2 = (1,0,1)^t$. Let's assume the next step in the application of the Perceptron algorithm is to process a particular training sample $\mathbf{x}$ of class c. After processing $\mathbf{x}$, which of the following options produces the weight vectors that define the decision boundary and decision regions of the figure on the right?.



A) $\mathbf{x} = (-1,1)^t$ and $c = 2$.
B) $\mathbf{x} = (0,0)^t$ and $c = 2$.
C) $\mathbf{x} = (-1,1)^t$ and $c = 1$
D) $\mathbf{x} = (0,0)^t$ and $c = 1$.

53 $\boxed{D}$ The expression $\hat{c} = \arg\max_{1 \leq c \leq C} P(c \mid \mathbf{y})$, where $\mathbf{y} \in \mathbb{R}^d$ is an object to classify, corresponds to a minimum-error or Bayes classifier in C classes. Under certain assumptions, this classifier is equivalent to a classifier based on C *Discriminant Functions*, defined by $\hat{c} = \arg\max_{1 \leq c \leq C} g_c(\mathbf{y})$. Show which of the following assumptions **IS NOT** a generally correct statement:

A) $g_c(\mathbf{y}) = P(c \mid \mathbf{y})$.
B) $g_c(\mathbf{y}) = \log P(c \mid \mathbf{y})$.
C) $g_c(\mathbf{y}) = 0.5 \cdot P(c \mid \mathbf{y}) + 0.5$.
D) $g_c(\mathbf{y}) = \sum_{j=1}^{d} a_j P(c \mid y_j) + a_0$, where $a_j, 0 \leq j \leq d$, are non-null real values.

54 $\boxed{B}$ Let S be a set of N pairs of training samples (object, class) and C the number of classes. Considering any iteration of the Perceptron algorithm, except the last one, applied to S, and that k weight vectors are modified in such iteration, which of the following statements is **INCORRECT**?

A) It holds $2 \leq k \leq C \cdot N$.
B) It holds $2 \leq k \leq (C-1) \cdot N$.
C) It holds $2 \leq k \leq C' \cdot N$, where C' is bounded by $1 \leq C' \leq C$.

D) It holds $2 \leq k \leq \sum_{n=1}^{N} k_n$, where $k_n, 1 \leq k_n \leq C$, is the number of modified vectors for the n^{th} data

55 $\boxed{D}$ The estimated probability of error of a classifier is 18 %. Which is the minimum number of testing samples, M, so that the 95 % confidence interval of this probability error is not higher than ±1 %; that is, $I = [17\%, 19\%]$: $M = 5671$

A) $M < 2000$.

B) $2000 \leq M < 3500$.

C) $3500 \leq M < 5000$.

D) $M \geq 5000$.

56 $\boxed{D}$ Let's suppose that we are applying the Perceptron algorithm, with learning rate $\alpha = 1$ and margin $b = 0.1$, to a set of 3 bidimensional learning samples for a problem of 2 classes. After processing the first 2 samples, the weight vectors $\mathbf{w}_1 = (0, 0, -3)^t$, $\mathbf{w}_2 = (0, 0, 3)^t$ were obtained. Next, the last sample $(\mathbf{x}_3, c_3)$ is processed and the following weight vectors $\mathbf{w}_1 = (1, 4, -1)^t$, $\mathbf{w}_2 = (-1, -4, 1)^t$ are obtained, which of the following samples is that last sample?

A) $((4, 5)^t, 1)$

B) $((2, 1)^t, 1)$

C) $((3, 1)^t, 2)$

D) $((4, 2)^t, 1)$

57 $\boxed{B}$ Given a classifier for 3 classes defined by their weight vectors $\mathbf{w}_1 = (2, 0, 0)^t$, $\mathbf{w}_2 = (-3, 3, 0)^t$, $\mathbf{w}_3 = (-2, 1, 1)^t$ in homogeneous notation, which of the following weight vectors do **not** define a classifier equivalent to the one given?

A) $\mathbf{w}_1 = (4, 0, 0)^t$, $\mathbf{w}_2 = (-1, 3, 0)^t$, $\mathbf{w}_3 = (0, 1, 1)^t$

B) $\mathbf{w}_1 = (-2, 0, 0)^t$, $\mathbf{w}_2 = (3, -3, 0)^t$, $\mathbf{w}_3 = (2, -1, -1)^t$

C) $\mathbf{w}_1 = (4, 0, 0)^t$, $\mathbf{w}_2 = (-6, 6, 0)^t$, $\mathbf{w}_3 = (-4, 2, 2)^t$

D) $\mathbf{w}_1 = (6, 0, 0)^t$, $\mathbf{w}_2 = (-4, 6, 0)^t$, $\mathbf{w}_3 = (-2, 2, 2)^t$

58 $\boxed{A}$ Given a classifier for 3 classes defined by their weight vectors $\mathbf{w}_1 = (-1, 1, -3)^t$, $\mathbf{w}_2 = (-3, 1, -3)^t$, $\mathbf{w}_3 = (0, -3, -2)^t$ in homogeneous notation, which of the following weight vectors do **not** define a classifier equivalent to the one given?

A) $\mathbf{w}_1 = (1, -1, 3)^t$, $\mathbf{w}_2 = (3, -1, 3)^t$, $\mathbf{w}_3 = (0, 3, 2)^t$

B) $\mathbf{w}_1 = (0, 2, -6)^t$, $\mathbf{w}_2 = (-4, 2, -6)^t$, $\mathbf{w}_3 = (2, -6, -4)^t$

C) $\mathbf{w}_1 = (-2, 2, -6)^t$, $\mathbf{w}_2 = (-6, 2, -6)^t$, $\mathbf{w}_3 = (0, -6, -4)^t$

D) $\mathbf{w}_1 = (1, 1, -3)^t$, $\mathbf{w}_2 = (-1, 1, -3)^t$, $\mathbf{w}_3 = (2, -3, -2)^t$

59 $\boxed{B}$ Let's suppose that we are applying the Perceptron algorithm, with learning rate $\alpha = 1$ and margin $b = 0.1$, to a set of 3 bidimensional learning samples for a problem of 2 classes. After processing the first 2 samples, the weight vectors $\mathbf{w}_1 = (0, 0, 3)^t$, $\mathbf{w}_2 = (0, 0, -3)^t$ were obtained. Next, the last sample $(\mathbf{x}_3, c_3)$ is processed and the following weight vectors $\mathbf{w}_1 = (-1, -5, -1)^t$, $\mathbf{w}_2 = (1, 5, 1)^t$ are obtained, which of the following samples is that last sample?

A) $((1, 1)^t, 2)$

B) $((5, 4)^t, 2)$

C) $((3, 3)^t, 2)$

D) $((3, 2)^t, 2)$

Problems

1. Let be a classification problem among 3 classes, $c = \{A, B, C\}$, where the objects are represented using a vectorial space of three dimensions. The classifier is based on Linear Discriminant Functions (LDF):

$$g_c(\mathbf{x}) = \mathbf{w}_c \cdot \mathbf{x} \qquad \text{for every class } c$$

where $\mathbf{w}_c$ y $\mathbf{x}$ are represented in compact notation; that is: $\mathbf{w} = (w_0, w_1, w_2, w_3)^t \in \mathbb{R}^4$ and $\mathbf{x} = (x_0, x_1, x_2, x_3)^t \in \mathbb{R}^4$, con $x_0 = 1$. Taking into account:

$$\mathbf{w}_A = (1, 1, 1, 1)^t \qquad \mathbf{w}_B = (-1, 0, -1, -2)^t \qquad \text{y} \qquad \mathbf{w}_C = (-2, 2, -1, 0)^t$$

Solve the following:

a) Classify the point $\mathbf{x}' = (2, 1, 2)^t$.

b) We know that the point $\mathbf{x}' = (-1, 0, -1)^t$ belongs to the class A. Which values will $\mathbf{w}_A$,$\mathbf{w}_B$ and $\mathbf{w}_C$ have after the application of the Perceptron algorithm for this particular point using a learning rate $\alpha = 0.1$?

c) Given the point $\mathbf{x}' = (1, -1, 2)^t$ that belongs to class C, obtain one of the possible values of the LDFs that will classify it correctly

Solution:

a) Discriminant functions for $(2, 1, 2)^t$:

$$g_A(\mathbf{x}) = \mathbf{w}_A \cdot \mathbf{x} = 6$$
$$g_B(\mathbf{x}) = \mathbf{w}_B \cdot \mathbf{x} = -6$$
$$g_C(\mathbf{x}) = \mathbf{w}_C \cdot \mathbf{x} = 1$$

Classification:

$$c(\mathbf{x}) = \arg\max_c \; g_c(\mathbf{x}) = A$$

b) Discriminant functions for $(-1, 0, -1)^t$:

$$g_A(\mathbf{x}) = \mathbf{w}_A \cdot \mathbf{x} = -1$$
$$g_B(\mathbf{x}) = \mathbf{w}_B \cdot \mathbf{x} = 1$$
$$g_C(\mathbf{x}) = \mathbf{w}_C \cdot \mathbf{x} = -4$$

Since $\mathbf{x}$ belongs to class A, the Perceptron algorithm modifies the weight vectors of the discriminant functions that return a value higher than g_A as well as the weight vector of g_A itself.

$$\mathbf{w}_A^* = \mathbf{w}_A + \alpha\mathbf{x} = (1.1, 0.9, 1, 0.9)^t$$
$$\mathbf{w}_B^* = \mathbf{w}_B - \alpha\mathbf{x} = (-1.1, 0.1, -1, -1.9)^t$$
$$\mathbf{w}_C \text{ IS NOT MODIFIED}$$

c) For example:

$$\mathbf{w}_A = (0, 0, 0, 0)^t$$
$$\mathbf{w}_B = (0, 0, 0, 0)^t$$
$$\mathbf{w}_C = (-2, 2, -1, 0)^t$$

2. Let be a classification problem among 3 classes, $c = \{1, 2, 3\}$, for objects represented by means of two-dimensional feature vectors. Given 3 training samples: $\mathbf{x}_1 = (0, 0)^t$ belongs to class $c_1 = 1$, $\mathbf{x}_2 = (0, 1)^t$ belongs to $c_2 = 2$, and $\mathbf{x}_3 = (2, 2)^t$ belongs to $c_3 = 3$. Find a linear classifier with minimum error using the Perceptron algorithm. Set the initial weights to zero for all the classes, learning factor $\alpha = 1$ and margin $b = 0.1$. Show all the steps of the iterative algorithm, the value of the weights for all the classes, until convergence to the minimum error. Remember to use compact notation for the weights.

Solution:

Weight are shown in compact notation: $\mathbf{w}_c = (w_{c0}, w_{c1}, w_{c2})^t$

```
Iteration 1

Sample 1 belongs to class 1
g_1(x_1)=0
g_2(x_1)=0
Error: w_2=  -1   0   0
g_3(x_1)=0
Error: w_3=  -1   0   0
Error: w_1=   1   0   0
Sample 2 belongs to class 2
g_2(x_2)=-1
g_1(x_2)=1
Error: w_1=   0   0  -1
g_3(x_2)=-1.000000
Error: w_3=  -2   0  -1
Error: w_2=   0   0   1
Sample 3 belongs to class 3
```

```
g_3(x_3)=-4
g_1(x_3)=-2
Error: w_1=  -1  -2  -3
g_2(x_3)=2
Error: w_2=  -1  -2  -1
Error: w_3=  -1   2   1

Iteration 2

Sample 1 belongs to class 1
g_1(x_1)=-1
g_2(x_1)=-1
Error: w_2=  -2  -2  -1
g_3(x_1)=-1
Error: w_3=  -2   2   1
Error: w_1=   0  -2  -3
Sample 2 belongs to class 2
g_2(x_2)=-3
g_1(x_2)=-3
Error: w_1=  -1  -2  -4
g_3(x_2)=-1
Error: w_3=  -3   2   0
Error: w_2=  -1  -2   0
Sample 3 belongs to class 3
g_3(x_3)=1
g_1(x_3)=-13
g_2(x_3)=-5

Iteration 3

Sample 1 belongs to class 1
g_1(x_1)=-1
g_2(x_1)=-1
Error: w_2=  -2  -2   0
g_3(x_1)=-3
Error: w_1=   0  -2  -4
Sample 2 belongs to class 2
g_2(x_2)=-2
g_1(x_2)=-4
g_3(x_2)=-3
Sample 3 belongs to class 3
g_3(x_3)=1
g_1(x_3)=-12
g_2(x_3)=-6

Iteration 4

Sample 1 belongs to class 1
g_1(x_1)=0
g_2(x_1)=-2
g_3(x_1)=-3
```

3. Let be a classification problem between 2 classes, $c = \{1, 2\}$, where objects are represented by means of a two-dimensional feature vector. We have 2 training samples: $\mathbf{x}_1 = (0, 0)^t$ belongs to class $c_1 = 1$, and $\mathbf{x}_2 = (1, 1)^t$ belongs to class $c_2 = 2$. Find a minimum-error linear classifier by applying the Perceptron algorithm with initial weight vectors set to zero for the two classes, learning rate $\alpha = 1$ and margin $b = 0.1$. Show all the steps of the iterative algorithm and the successive updates of the weight vectors of the two classes, until convergence to the minimum error.

   Solution:

   Weight vectors are shown in compact notion: $\mathbf{w}_c = (w_{c0}, w_{c1}, w_{c2})^t$

```
Iteration 1

Sample 1 belongs to class 1
g_1(x_1)=0
```

```
g_2(x_1)=0
Error: w_2=  -1   0   0
Error: w_1=   1   0   0

Sample 2 belongs to class 2
g_2(x_2)=-1
g_1(x_2)=1
Error: w_1=   0  -1  -1
Error: w_2=   0   1   1

Iteration 2

Sample 1 belongs to class 1
g_1(x_1)=0
g_2(x_1)=0
Error: w_2=  -1   1   1
Error: w_1=   1  -1  -1

Sample 2 belongs to class 2
g_2(x_2)=1
g_1(x_2)=-1

Iteration 3

Sample 1 belongs to class 1
g_1(x_1)=1
g_2(x_1)=-1
```

4. We have a classification problem of two classes, A, B, where objects are represented by a two-dimensional feature vector. We have two training samples:

$$\mathbf{y}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \in A, \quad \mathbf{y}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \in B,$$

   a) Set the initial weight vectors to 0 and show a trace of the Perceptron algorithm with learning rate $\alpha = 1.0$ and margin $b = 0.1$. Show the successive updates of the weight vectors and their final values.

   b) Obtain the equation of the decision boundary between the two classes according to the solution returned by the Perceptron algorithm. Represent graphically the boundary and the two training samples. Does this solution return a satisfactory classification?

Solution:

1. The algorithm executes two iterations of the main loop, yielding the next sequence of weight vectors:

$$\begin{array}{ccccccccccc}
\mathbf{y}: & & \mathbf{y_1} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \in A & & & \mathbf{y_2} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \in B & & \mathbf{y_1} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \in A & & \mathbf{y_2} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \in B & \\
g_A: & & 0 & & & 1+2+2=5 & & 0-1+2=1 & & 0-2+1=-1 & \\
g_B: & & 0 & & & -1-2-2=-5 & & 0+1-2=-1 & & 0+2-1=1 & \\
\text{error}: & & true & & & true & & false & & false & \text{SOLUTION} \\
\mathbf{a}_A: & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & +\,\mathbf{y_1} = & \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} & -\,\mathbf{y_2} = & \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} & \longrightarrow & \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} & \longrightarrow & & \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \\
\mathbf{a}_B: & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & -\,\mathbf{y_1} = & \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix} & +\,\mathbf{y_2} = & \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} & \longrightarrow & \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} & \longrightarrow & & \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}
\end{array}$$

2. Boundary decision equation:

$$0 - y_1 + y_2 = 0 + y_1 - y_2 \quad \rightarrow \quad y_2 = y_1$$

The graphical representation is a straight line passing through the origin with slope 45^0. The learning points are equidistant to both sides of the boundary, which can be considered an entirely satisfactory solution.

5. Let us consider a classification problem of two classes, 0 and 1, for objects represented in $\{0,1\}^2$, that is, bit vectors defined as $\mathbf{x} = (x_1, x_2)^t$ with $x_1, x_2 \in \{0,1\}$. In addition, there are four training samples available:

$\mathbf{x}_n$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$
x_{n1}	0	0	1	1
x_{n2}	0	1	0	1
c_n	0	1	1	0

1 (0.75 points) Show a trace of the Perceptron algorithm running for one iteration with initial weight vectors equal to zero, learning rate $\alpha = 1$ and margin $b = 0.1$. What are the weight vectors obtained at the end of the iteration?

2 (0.50 points) From the initialization given above, will the Perceptron algorithm converge to a solution without misclassified training samples?
Please say yes or no and then briefly discuss the answer.

3 (0.25 points) Is there any initialization with non-null weight vectors, $\alpha > 0$ and $b = 0.1$, from which the Perceptron algorithm will converge to a solution without misclassified training samples?
Please say yes or no and then briefly discuss the answer.

1 $\mathbf{w}_0 = \mathbf{w}_1 = (0\ 0\ 0)^t$, $\alpha = 1$ y $b = 0.1$.

$$
\begin{aligned}
&g_0(\mathbf{x}_1) = (0\ 0\ 0)(1\ 0\ 0)^t = 0\\
&g_1(\mathbf{x}_2) = (0\ 0\ 0)(1\ 0\ 0)^t = 0\\
&g_1(\mathbf{x}_1) + b > g_0(\mathbf{x}_2)?\quad \text{Sí}\\
&\ \rightarrow \mathbf{w}_1 = \mathbf{w}_1 - \mathbf{x}_1 = (0\ 0\ 0)^t - (1\ 0\ 0)^t = (-1\ 0\ 0)^t\\
&\ \rightarrow \mathbf{w}_0 = \mathbf{w}_0 + \mathbf{x}_1 = (0\ 0\ 0)^t + (1\ 0\ 0)^t = (1\ 0\ 0)^t\\
&g_1(\mathbf{x}_2) = (-1\ 0\ 0)(1\ 0\ 1)^t = -1\\
&g_0(\mathbf{x}_2) = (1\ 0\ 0)(1\ 0\ 1)^t = 1\\
&g_0(\mathbf{x}_2) + b > g_1(\mathbf{x}_2)?\quad \text{Sí}\\
&\ \rightarrow \mathbf{w}_0 = \mathbf{w}_0 - \mathbf{x}_2 = (1\ 0\ 0)^t - (1\ 0\ 1)^t = (0\ 0\ -1)^t\\
&\ \rightarrow \mathbf{w}_1 = \mathbf{w}_1 + \mathbf{x}_2 = (-1\ 0\ 0)^t + (1\ 0\ 1)^t = (0\ 0\ 1)^t\\
&g_1(\mathbf{x}_3) = (0\ 0\ 1)(1\ 1\ 0)^t = 0\\
&g_0(\mathbf{x}_3) = (0\ 0\ -1)(1\ 1\ 0)^t = 0\\
&g_0(\mathbf{x}_3) + b > g_1(\mathbf{x}_3)?\quad \text{Sí}\\
&\ \rightarrow \mathbf{w}_0 = \mathbf{w}_0 - \mathbf{x}_3 = (0\ 0\ -1)^t - (1\ 1\ 0)^t = (-1\ -1\ -1)^t\\
&\ \rightarrow \mathbf{w}_1 = \mathbf{w}_1 + \mathbf{x}_3 = (0\ 0\ 1)^t + (1\ 1\ 0)^t = (1\ 1\ 1)^t\\
&g_0(\mathbf{x}_4) = (-1\ -1\ -1)(1\ 1\ 1)^t = -3\\
&g_1(\mathbf{x}_4) = (1\ 1\ 1)(1\ 1\ 1)^t = 3\\
&g_1(\mathbf{x}_4) + b > g_0(\mathbf{x}_4)?\quad \text{Sí}\\
&\ \rightarrow \mathbf{w}_1 = \mathbf{w}_1 - \mathbf{x}_4 = (1\ 1\ 1)^t - (1\ 1\ 1)^t = (0\ 0\ 0)^t\\
&\ \rightarrow \mathbf{w}_0 = \mathbf{w}_0 + \mathbf{x}_4 = (-1\ -1\ -1)^t + (1\ 1\ 1)^t = (0\ 0\ 0)^t
\end{aligned}
$$

The final weight vectors are null vectors, same as the initial vectors.

2 No. The set of training samples **IS NOT** linearly separable. Best case is to have three samples correctly classified.

3 No, same reason as in the above item.

6. The table below shows a training set of 4 two-dimension samples that belong to 3 classes, $c = 1, 2, 3$.

n	x_{n1}	x_{n2}	c_n
1	4	2	3
2	5	2	1
3	4	4	1
4	2	5	2

Answer the following questions:

a) (1.5 points) Show a trace of the Perceptron algorithm running for 3 iterations with $\alpha = 1$, $\beta = 0.1$ and null initial weight vectors.

b) (0.5 points) Classify the testing sample $\mathbf{x} = (2, 1)^t$ by using a linear classifier with the weight vectors learnt after the third iteration.

Solution:

a) Three iterations of the Perceptron algorithm.

- Iteration 1: 3 samples wrongly classified and resulting weights

d	w_{d1}	w_{d2}	w_{d3}
0	-1	-1	-1
1	-1	-7	-3
2	-5	1	-5

- Iteration 2: 3 samples wrongly classified and resulting weights

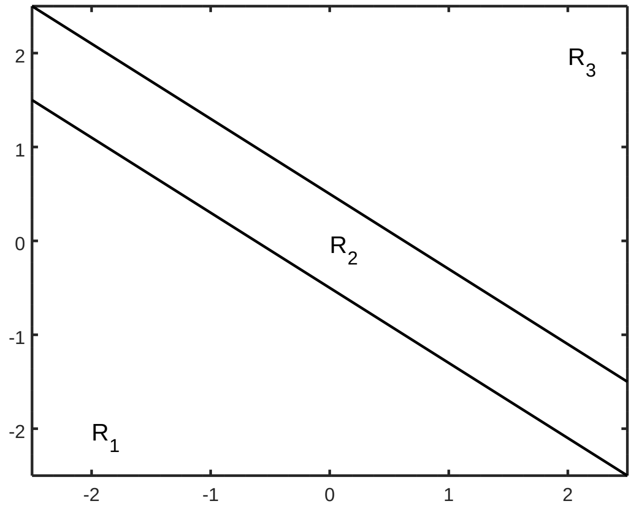
d	w_{d1}	w_{d2}	w_{d3}
0	-2	-1	-1
1	-2	-10	-4
2	-10	4	-5

- Iteration 3: 1 sample wrongly classified and resulting weights

d	w_{d1}	w_{d2}	w_{d3}
0	-1	-1	-2
1	3	-10	-9
2	-8	4	-7

b) Classification of the testing sample.

$$\begin{aligned} g_1(\mathbf{x}) &= -3 \\ g_2(\mathbf{x}) &= -17 \\ g_3(\mathbf{x}) &= -27 \\ c(\mathbf{x}) &= 1 \end{aligned}$$



7. The table on the left side shows a training set of 3 two-dimension samples that belong to 3 classes, while the table on the right side shows the initial weight vector for each class.

n	x_{n1}	x_{n2}	c_n
1	-2	-2	1
2	0	0	2
3	2	2	3

	$\mathbf{w}_1$	$\mathbf{w}_2$	$\mathbf{w}_3$
w_{c0}	0	-1	-1
w_{c1}	-2	0	4
w_{c2}	-2	0	4

Answer the following questions:

a) (1.5 points) Show a trace of the Perceptron algorithm running for one iteration with learning rate $\alpha = 1$, margin $\beta = 0.1$ using the initial weight vectors provided.

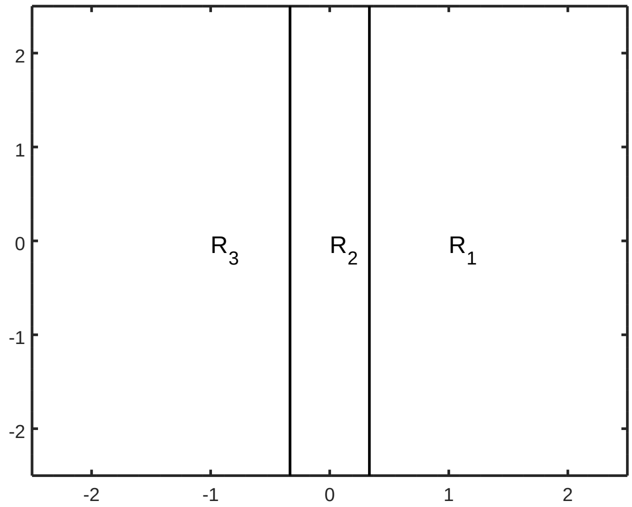
b) (0.5 points) Plot the decision regions of the resulting classifier, as well as the decision boundaries involved.

Solution:

a) One iteration of the Perceptron algorithm with 1 misclassified sample provides the following final weight vectors:

	$\mathbf{w}_1$	$\mathbf{w}_2$	$\mathbf{w}_3$
w_{c0}	-1	0	-2
w_{c1}	-2	0	4
w_{c2}	-2	0	4

b) The following plot shows the three decision regions along with the two decision boundaries involved:



8. The table on the left side shows a training set of 3 two-dimension samples that belong to 3 classes, while the table on the right side shows the initial weight vector for each class.

n	x_{n1}	x_{n2}	c_n
1	1	0	1
2	0	0	2
3	-1	0	3

	$\mathbf{w}_1$	$\mathbf{w}_2$	$\mathbf{w}_3$
w_{c0}	-2	-1	-1
w_{c1}	2	1	-3
w_{c2}	0	0	0

Answer the following questions:

a) (1.5 points) Show a trace of the Perceptron algorithm running for one iteration with learning rate $\alpha = 1$, margin $\beta = 0.1$ using the initial weight vectors provided.

b) (0.5 points) Plot the decision regions of the resulting classifier, as well as the decision boundaries involved.

Solution:

a) One iteration of the Perceptron algorithm with 2 misclassified samples provides the following final weight vectors:

	$\mathbf{w}_1$	$\mathbf{w}_2$	$\mathbf{w}_3$
w_{c0}	-2	-1	-2
w_{c1}	3	0	-3
w_{c2}	0	0	0

b) The following plot shows the three decision regions along with the two decision boundaries involved: