



UNIVERSITAT
POLITÈCNICA
DE VALÈNCIA

Workbook:

Uniform-cost search:

Dijkstra's algorithm

Albert Sanchis
Jorge Civera

DSIC

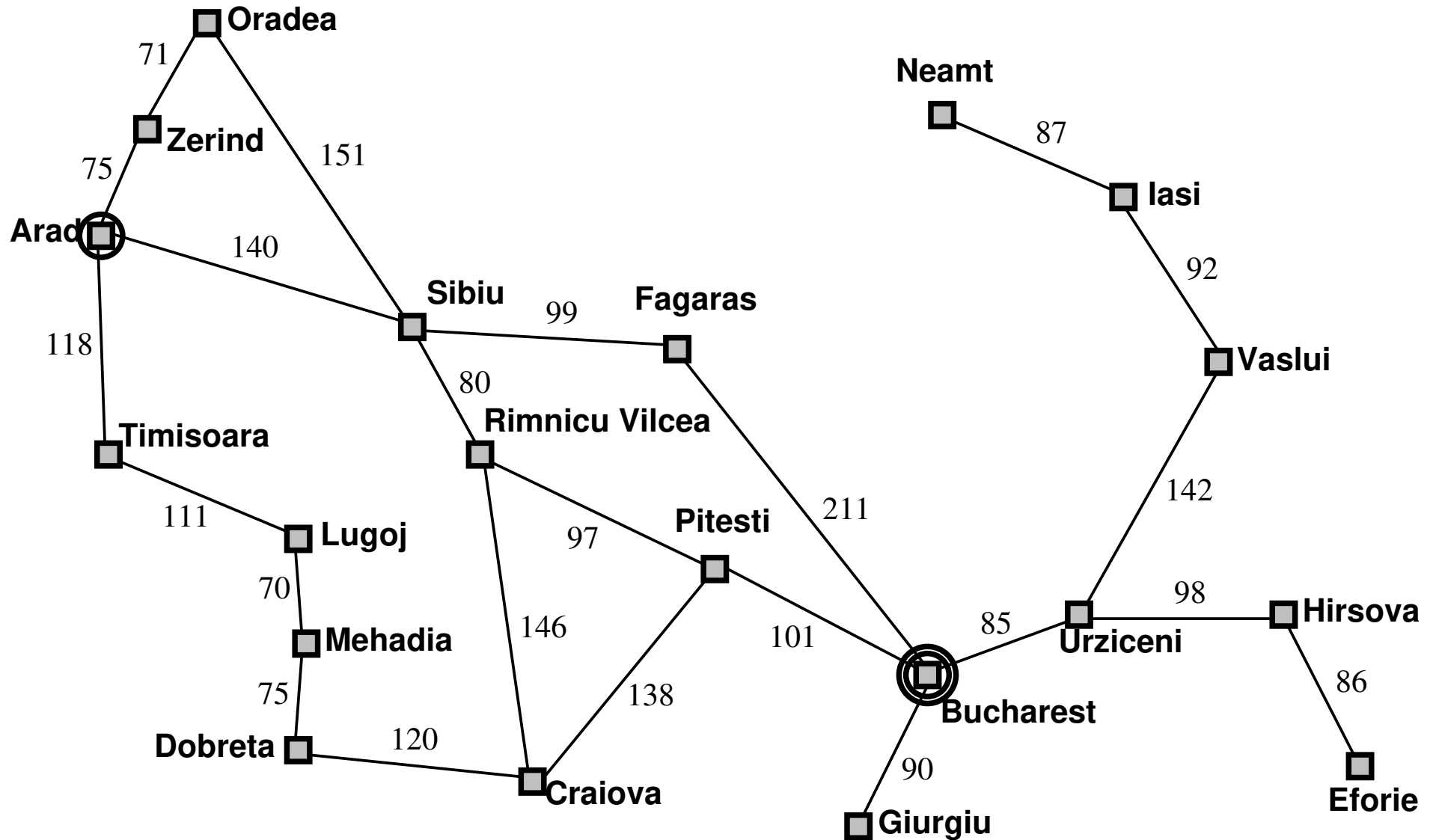
Departament de Sistemes
Informàtics i Computació

Learning objectives

- ▶ To describe uniform-cost search or Dijkstra's algorithm.
- ▶ To draw a uniform-cost search tree.
- ▶ To apply uniform-cost search to a well-known problem
- ▶ To analyze the quality of uniform-cost search.

Problem: Shortest path between two points

Shortest path from Arad to Bucarest [1]:



$\text{Actions}(\text{Arad}) = \{\text{Move}(\text{Sibiu}), \text{Move}(\text{Timisoara}), \text{Move}(\text{Zerind})\}.$

Uniform-cost or Dijkstra's algorithm [1, 2, 3]

```
UCS( $G, s'$ )      // Uniform-cost search;  $G$  weighted graph,  $s'$  start
 $O = \text{InitQueue}(s', g_{s'} \triangleq 0)$            // Open: priority queue  $g$ 
 $C = \emptyset$                                    // Closed: explored nodes
while not  $\text{EmptyQueue}(O)$ :                     // best-first:  $s = \arg \min_{n \in O} g_n$ 
     $s = \text{Pop}(O)$                                // ties solved in favor of goals
    if  $\text{Goal}(s)$  return  $s$                        // solution found!
     $C = C \cup \{s\}$                                //  $s$  explored
    forall  $(s, n) \in \text{Adjacents}(G, s)$ :         // generation:  $n$  child of  $s$ 
         $x = g_s + w(s, n)$                        // path cost from  $s'$  to  $n$  through  $s$ 
        if  $n \notin C \cup O$ :  $\text{Push}(O, n, g_n \triangleq x)$ 
        else if  $n \in O$  and  $x < g_n$ :  $\text{Update}(O, n, g_n \triangleq x)$ 
return NULL                                     // no solution found
```

- ▶ **Question 1:** Write a trace of the **UCS** algorithm applied to the problem of finding the shortest path from Arad to Bucarest.
- ▶ **Question 2:** Draw the search tree as a result of applying the **UCS** algorithm to the problem of finding the shortest path from Arad to Bucarest.
- ▶ **Question 3:** Does the IDS algorithm find a solution?
- ▶ **Question 4:** If the answer is “Yes”:
 - ▷ What is the solution found?
 - ▷ What is the cost of this solution?
 - ▷ Is this the solution of minimum cost?
 - ▷ What type of solution is found by the UCS algorithm?

References

- [1] S. Russell and P. Norvig. *Artificial Intelligence: A Modern Approach*. Pearson, third edition, 2010.
- [2] E. W. Dijkstra. A Note on Two Problems in Connexion with Graphs. *Numerische Mathematik*, 1959.
- [3] Bernhard Korte and Jens Vygen. *Combinatorial Optimization: Theory and Algorithms*. Springer, 2018.