Lecture 10. Types and Type Checking

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Type

- Type is a property of program constructs such as expressions
- It defines a set of values (range of variables) and a set of operations on those values
- Classes are one instantiation of the modern notion of the type
  - fields and methods of a Java class are meant to correspond to values and operations

Type System

- A type system is a collection of rules that assign types to program constructs (more constraints added to checking the validity of the programs, violation of such constraints indicate errors)
- A languages type system specifies which operations are valid for which types
- Type systems provide a concise formalization of the semantic checking rules
- Type rules are defined on the structure of expressions
- Type rules are language specific

Why do we need type systems

Consider the assembly language fragment

addi $r1, $r2, $r3

What are the types of $r1, $r2, $r3?

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- Assembly language is untyped (MIPS assembly)

Why do we need type systems

Consider the assembly language fragment

addi \$r1, \$r2, \$r3

What are the types of \$r1, \$r2, \$r3?

- Assembly language is untyped (MIPS assembly)
- This instruction allows you to add the contents of a register to an immediate value (a constant) and store the result in a (possibly) another register.

Why do we need type systems

- It doesnt make sense to add a function pointer and an integer in C
- It does make sense to add two integers
- But both have the same assembly language implementation!

Use of Types

- Detect errors:
  - Memory errors, such as attempting to use an integer as a pointer.
  - Violations of abstraction boundaries, such as using a private field from outside a class.
- Help compilation:
  - When Python sees x+y, its type systems tells it almost nothing about types of x and y, so code must be general.
  - In C, C++, Java, code sequences for x+y are smaller and faster, because representations are known.

Type Checking and Type Inference

- Type Checking is the process of verifying fully typed programs
- Type Inference is the process of filling in missing type information
- The two are different, but are often used interchangeably

Inference Rule

- Types of expressions and parameters need not be explicit to have static typing. With the right rules, might *infer* their types.
- The appropriate formalism for type checking is logical rules of inference having the form

  If Hypothesis is true, then Conclusion is true
- For type checking, this might become:

  If E_1 and E_2 have certain types, then E_3 has a certain type.
- Given proper notation, easy to read (with practice), so easy to check that the rules are accurate.
- Can even be mechanically translated into programs.

From English to an Inference Rule

- Rules of inference are a compact notation of *if-then* statements
- Symbol $\wedge$ is "and"
- Symbol $\Rightarrow$ is "if-then"
- $x : T$ is "x has type T"

From English to an Inference Rule

If e_1 has type Int and e_2 has type Int,
then $e_1 + e_2$ has type Int

$$(e_1 \text{ has type Int} \wedge e_2 \text{ has type Int}) \Rightarrow e_1 + e_2 \text{ has type Int}$$

$$(e_1\text{: Int} \wedge e_2\text{: Int}) \Rightarrow e_1 + e_2\text{: Int}$$

From English to an Inference Rule

The statement

$$(e_1 : \text{Int} \wedge e_2 : \text{Int}) \Rightarrow e_1 + e_2 : \text{Int}$$

is a special case of

$$(\text{Hypothesis}_1 \wedge \ldots \wedge \text{Hypothesis}_n) \Rightarrow \text{Conclusion}$$

This is an *inference rule*

From English to an Inference Rule

- By tradition inference rules are written

$$\frac{\vdash \text{Hypothesis}_1 \quad \ldots \quad \vdash \text{Hypothesis}_n}{\vdash \text{Conclusion}}$$

- Cool type rules have hypotheses and conclusions of the form:

$$\vdash e : T$$

- $\vdash$ means "it is provable that . . ."

Example

$$\frac{i \text{ is an integer}}{\vdash i : \text{Int}} \text{[Int]}$$

$$\frac{\begin{array}{c} \vdash e_1 : \text{Int} \\ \vdash e_2 : \text{Int} \end{array}}{\vdash e_1 + e_2 : \text{Int}} \text{[Add]}$$

Two Rules

- These rules give templates describing how to type integers and + expressions
- By filling in the templates, we can produce complete typings for expressions

Example

$$\dfrac{\dfrac{1 \text{ is an integer}}{\vdash 1 : \texttt{Int}} \qquad \dfrac{2 \text{ is an integer}}{\vdash 2 : \texttt{Int}}}{\vdash 1 + 2 : \texttt{Int}}$$

Static and Dynamic Typed Languages

- Statically typed languages: all or almost all type checking occurs at compilation time. (C, Java)
- Dynamically typed languages: almost all checking of types is done as part of program execution (Scheme)
- Untyped languages: no type checking (assembly, machine code)

Static and Dynamic Types

- The <u>dynamic type</u> of an object is the class *C* that is used in the "new *C*" expression that creates the object
  - A run-time notion
  - Even languages that are not statically typed have the notion of dynamic type
- The <u>static type</u> of an expression is a notation that captures all possible dynamic types the expression could take
  - A compile-time notion

Relations of Static and Dynamic Types in Simple Type Systems

Soundness theorem: for all expressions E

dynamic_type(E) = static_type(E)

(in **all** executions, E evaluates to values of the type inferred by the compiler)

So far:

A set of basic concepts:

- type
- type systems
- type checking
- type inference
- inference rules
- static and dynamic typed languages
- static and dynamic types

Semantic Analysis Related to Types

Goals:

- What is the type of the expression – type inference (what is the value the expression potentially produces? based on its range, what type it is?)
- Do we have a correct assignment (following type rules) of types on all the expressions in the program? – type checking

Perspectives of studying types:

- Language designers: designing the type systems
- Compilers: type checking programs

Next:

- We do an overview of the two perspectives

Designing a Type System

Two Conflict Goals:

- Give flexibility to the programmer
- Prevent valid programs to "go wrong"
  - Milner, 1981: "Well-typed programs do not go wrong"

- An active line of research is in the area of inventing more flexible type systems while preserving soundness

In another word: There is a tradeoff between

- Flexible rules that do not constrain programming
- Restrictive rules that ensure safety of execution

Soundness

- It is a property of the type system
- Intuitively, a sound type system can correctly predict the type of a variable at runtime
- There can be many sound type rules, we need to use the most precise ones so it can be useful

- A type system is <u>sound</u> if
  - Whenever $\vdash e : T$
  - Then e evaluates to a value of type T

- We only want sound rules
  - But some sound rules are better than others:

$$\frac{i \text{ is an integer}}{\vdash i : \text{Object}}$$

Tradeoffs of Static and Dynamic Type Checking Systems

- static type system does not have knowledge of input values or execution behaviors
- static type system disallows some correct programs, cannot predict precisely all the behaviors (some program runs correctly will be rejected)
- better static type system or dynamic type system

Static

- Static checking catches many programming errors at compile time
- Avoids overhead of runtime type checking
- Using various devices to recover the flexibility lost by "going static:" subtyping, coercions, type parameterization

Dynamic:

- Static type systems are restrictive; can require more work to do reasonable things.
- Rapid prototyping easier in a dynamic type system.

Using Subtypes

- In languages such as Java, can define types (classes) either to
  - Implement a type, or
  - Define the operations on a family of types without (completely) implementing them
  - Hence, relaxes static typing a bit: we may know that something is a **Y** without knowing precisely which subtype it has

Implicit Coercions

- In Java, can write

```
int x = 'c';
float y = x;
```

- But relationship between **char** and **int**, or **int** and **float** not usually called subtyping, but rather *conversion* (or *coercion*).
- Such implicit coercions avoid cumbersome casting operations.
- Might cause a change of value or representation,
- But usually, such coercions allowed implicitly only if type coerced to contains all the values of the that coerced from (a *widening coercion*).
- Inverses of widening coercions, which typically lose information (e.g., **int**$\longrightarrow$**char**), are known as *narrowing coercions.* and typically required to be explicit.
- **int**$\longrightarrow$**float** a traditional exception (implicit, but can lose information and is neither a strict widening nor a strict narrowing.)

Coercion Examples

```
Object x = ...;   String y = ...;
int a = ...;  short b = 42;
x = y; a = b;     // OK
y = x; b = a;     // ERRORS{ x = (Object) y; // {OK
a = (int) b;      // OK
y = (String) x;   // OK but may cause exception
b = (short) a;    // OK but may lose information
```

Possibility of implicit coercion complicates type-matching rules (see C++).

Type Checking Algorithm

- Type checking proves facts $e : T$
  - Proof is on the structure of the AST
  - Proof has the shape of the AST
  - One type rule is used for each kind of AST node
- In the type rule used for a node e:
  - The hypotheses are the proofs of types of e's subexpressions
  - The conclusion is the proof of type of e
- Types are computed in a bottom-up pass over the AST

One Pass Type Checking for Cool

- COOL type checking can be implemented in a single traversal over the AST
- Type environment is passed down the tree
  - From parent to child
- Types are passed up the tree
  - From child to parent

So far:

- Concepts
- Overview of type system design and type checking algorithms

Next: COOL (Why Cool is a good language to learn when learning basic concepts?)

- Cool type system touches all the key concepts: e.g., subtyping, method dispatch

Cool Types

- Class names
- SELF_TYPE Note: there are no base types (as in Java int)
- The user declares types for all identifiers
- The compiler infers types for expressions (Infers a type for every expression)

Rules for Constant

$$\frac{}{\vdash \text{false} : \text{Bool}} \; [\text{Bool}]$$

$$\frac{s \text{ is a string constant}}{\vdash s : \text{String}} \; [\text{String}]$$

Rules for New

new T produces an object of type T

- Ignore SELF_TYPE for now ...

$$\frac{}{\vdash \text{new } T : T} \quad [\text{New}]$$

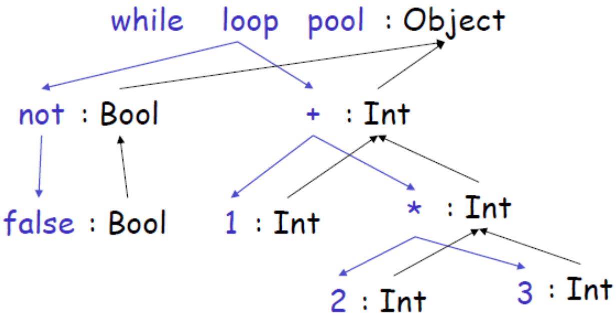
Two More Rules

$$\frac{\vdash e : \text{Bool}}{\vdash \text{not } e : \text{Bool}} \quad [\text{Not}]$$

$$\frac{\begin{array}{c}\vdash e_1 : \text{Bool} \\ \vdash e_2 : T\end{array}}{\vdash \text{while } e_1 \text{ loop } e_2 \text{ pool} : \text{Object}} \quad [\text{Loop}]$$

Type Inference: Determining Types for Every AST Node

- Typing for while not false loop 1 + 2 * 3 pool



Type Derivations

- The typing reasoning can be expressed as a tree:

$$\dfrac{\dfrac{\vdash \text{false} : \text{Bool}}{\vdash \text{not false} : \text{Bool}} \qquad \dfrac{\vdash 1 : \text{Int} \qquad \dfrac{\vdash 2 : \text{Int} \qquad \vdash 3 : \text{Int}}{\vdash 2 * 3 : \text{Int}}}{\vdash 1 + 2 * 3 : \text{Int}}}{\vdash \text{while not false loop } 1 + 2 * 3 : \text{Object}}$$

- The root of the tree is the whole expression
- Each node is an instance of a typing rule
- Leaves are the rules with no hypotheses

Get Into More Complicated Rules

Important ones:

- Let
- If-then-else, case
- Method
- Self_Type

Pay attention to:

- notation and concept development
- typing rule design

Type Rules and Type Environment

- The type rules define the type of every Cool expression in a given context.
- The context is the type environment, which describes the type of every unbound identifier appearing in an expression.

Type Rules and Type environment

Type rules have general format:

$$\frac{\vdots}{O, M, C \vdash e : T}$$

- O environment for object
- M environment for methods
- C containing class
- The dots above the horizontal bar stand for other statements about the types of sub-expressions of e. These other statements are hypotheses of the rule; if the hypotheses are satisfied, then the statement below the bar is true.

Type Environment for Object

Let O be a function from ObjectIdentifiers to Types

The sentence $O \vdash e : T$

is read: Under the assumption that variables have the types given by O, it is provable that the expression e has the type T

Type Environment for Object

A *type environment* gives types for *free* variables

- A type environment is a function from ObjectIdentifiers to Types
- A variable is free in an expression if:
  - It occurs in the expression
  - It is declared outside the expression
- E.g. in the expression "x", the variable "x" is free
- E.g. in "let x : Int in x + y" only "y" is free

Notation Understanding

- O is a function (implemented in the symbol table: mapping between variables and types)
- $O[T/x]$ is a also function, extend O with a pair of: x of type T
- The type environment provides the type of free variables in the current scope
- During type checking, you can look for the type of a particular variable on the AST in the type environment O
- $O[T/x] \vdash e : T$ execute the expression e in this environment, and get the type T

Example:

$$O[T_0/x]\,(x) = T_0$$
$$O[T_0/x]\,(y) = O(y)$$

An Example

$$\frac{i \text{ is an integer}}{O \vdash i : \text{Int}} \text{[Int]}$$

$$\frac{\begin{array}{c} O \vdash e_1 : \text{Int} \\ O \vdash e_2 : \text{Int} \end{array}}{O \vdash e_1 + e_2 : \text{Int}} \text{[Add]}$$

New Rule

$$\frac{O(x) = T}{O \vdash x : T} \quad [\mathrm{Var}]$$

Let: No Initialization

$$\frac{O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \text{ in } e_1 : T_1} \quad \text{[Let-No-Init]}$$

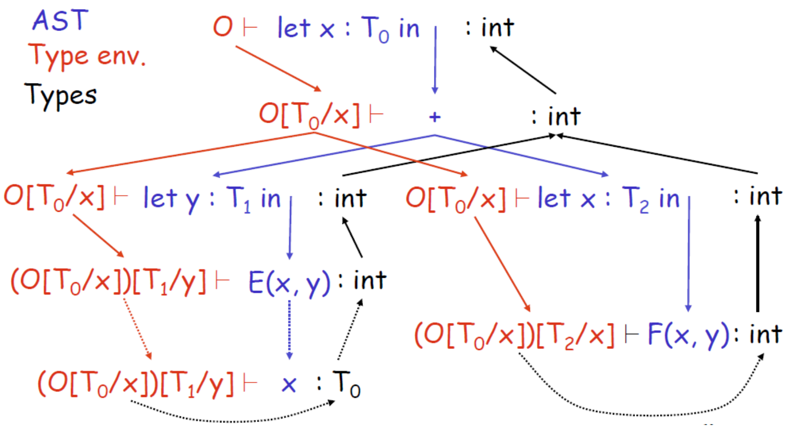
Let Example

- Consider the Cool expression

  $\text{let } x : T_0 \text{ in } (\text{let } y : T_1 \text{ in } E_{x,y}) + (\text{let } x : T_2 \text{ in } F_{x,y})$

  (where $E_{x,y}$ and $F_{x,y}$ are some Cool expression that contain occurrences of "x" and "y")
- Scope
  - of "y" is $E_{x,y}$
  - of outer "x" is $E_{x,y}$
  - of inner "x" is $F_{x,y}$
- This is captured precisely in the typing rule

Let Example



AST
Type env.
Types
O ⊢ let x : T_0 in : int
O[T_0/x] ⊢ + : int
O[T_0/x] ⊢ let y : T_1 in : int
O[T_0/x] ⊢ let x : T_2 in : int
(O[T_0/x])[T_1/y] ⊢ E(x, y) : int
(O[T_0/x])[T_2/x] ⊢ F(x, y) : int
(O[T_0/x])[T_1/y] ⊢ x : T_0

Type Inference Approach

- The type environment gives types to the free identifiers in the current scope
- The type environment is passed down the AST from the root towards the leaves
- Types are computed up the AST from the leaves towards the root

Let With Initialization

Now consider let with initialization:

$$\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O[T_0/x] \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad \text{[Let-Init]}$$

This rule is weak. Why?

Let With Initialization

- Consider the example:

```
class C inherits P { … }
…
let x : P ← new C in …
 …
```

- The previous let rule does not allow this code
  - We say that the rule is too weak

Subtyping

- Define a relation $\leq$ on classes

  $X \leq X$

  $X \leq Y$ if X inherits from Y

  $X \leq Z$ if $X \leq Y$ and $Y \leq Z$

- Reflexive
- Transitive

Let with Initialization Modified

$$\frac{\begin{array}{c} O \vdash e_0 : T \\ T \leq T_0 \\ O[T_0/x] \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \text{[Let-Init]}$$

- Both rules for let are correct
- But more programs type check with the latter

Let with Initialization – More Examples

How it is different from the previous Let rule?

$$\frac{O \vdash e_0 : T \qquad T \leq T_0 \qquad O \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

Examples of Wrong Typing Rules

- The following good program does not typecheck

$$\text{let } x : \text{Int} \leftarrow 0 \text{ in } x + 1$$

- Why?

Examples of Wrong Typing Rules

- Consider the following Cool class definitions

```
Class A { a() : int { 0 }; }
Class B inherits A { b() : int { 1 }; }
```

- An instance of B has methods "a" and "b"
- An instance of A has method "a"
  - A type error occurs if we try to invoke method "b" on an instance of A

Examples of Wrong Typing Rules

- Now consider a hypothetical let rule:

$$\frac{O \vdash e_0 : T \qquad T_0 \leq T \qquad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?
- The following bad program is well typed

$$\text{let } x : B \leftarrow \text{new } A \text{ in } x.b()$$

- Why is this program bad?

Examples of Wrong Typing Rules

- Now consider a hypothetical let rule:

$$\frac{O \vdash e_0 : T \qquad T \leq T_0 \qquad O[T/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?
- The following good program is not well typed

  let x : A ← new B in {... x ← new A; x.a(); }

- Why is this program not well typed?

Typing Rules

- The typing rules use very concise notation
- They are very carefully constructed
- Virtually any change in a rule either:
  - Makes the type system unsound (bad programs are accepted as well typed)
  - makes the type system less usable (perfectly good programs are rejected)

Assignment

Very similar to let:

$$\frac{\begin{array}{c} O(id) = T_0 \\ O \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O \vdash id \leftarrow e_1 : T_1} \quad [Assign]$$

Initialized Attributes

- Let $O_C(x) = T$ for all attributes $x{:}T$ in class C

- Attribute initialization is similar to let, except for the scope of names

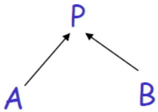
$$\frac{\begin{array}{c} O_C(id) = T_0 \\ O_C \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O_C \vdash id : T_0 \leftarrow e_1 \,;} \quad \text{[Attr-Init]}$$

If-then-else

- Consider:
  if e_0 then e_1 else e_2 fi
- The result can be either e_1 or e_2
- The type is either e_1's type or e_2's type
- The best we can do is the smallest supertype larger than the type of e_1 and e_2

If-then-else

- Consider the class hierarchy



- ... and the expression

  if ... then new A else new B fi

- Its type should allow for the dynamic type to be both A or B
  - Smallest supertype is P

Least Upper Bound: Operations

- $lub(X,Y)$, the least upper bound of X and Y, is Z if
  - $X \leq Z \wedge Y \leq Z$

    Z is an upper bound
  - $X \leq Z' \wedge Y \leq Z' \Rightarrow Z \leq Z'$

    Z is least among upper bounds
- In COOL, the least upper bound of two types is their least common ancestor in the inheritance tree

- inheritance tree (rooted at object)
- class hierarchy descent from the object
- walk back the tree to find the parent of the two types

If-then-else

$$\frac{\begin{array}{c} O \vdash e_0 : \text{Bool} \\ O \vdash e_1 : T_1 \\ O \vdash e_2 : T_2 \end{array}}{O \vdash \text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi} : \text{lub}(T_1, T_2)} \text{ [If-Then-Else]}$$

Case

- The rule for case expressions takes a lub over all branches

$$\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O[T_1/x_1] \vdash e_1 : T_1' \\ \dots \\ O[T_n/x_n] \vdash e_n : T_n' \end{array}}{O \vdash \text{case } e_0 \text{ of } x_1{:}T_1 \Rightarrow e_1; \dots; x_n : T_n \Rightarrow e_n; \text{ esac} : \text{lub}(T_1', \dots, T_n')} \quad [\text{Case}]$$

Method Dispatch

- There is a problem with type checking method calls:

$$\frac{O \vdash e_0 : T_0 \quad O \vdash e_1 : T_1 \quad \ldots \quad O \vdash e_n : T_n}{O \vdash e_0.f(e_1,\ldots,e_n) : ?} \quad \text{[Dispatch]}$$

- We need information about the formal parameters and return type of f

Notes on Method Dispatch

- In Cool, method and object identifiers live in different name spaces
  - A method foo and an object foo can coexist in the same scope
- In the type rules, this is reflected by a separate mapping M for method signatures

$$M(C,f) = (T_1, \ldots T_n, T_{n+1})$$

means in class C there is a method f

$$f(x_1{:}T_1, \ldots, x_n{:}T_n){:}\ T_{n+1}$$

Type Environment: Method

- The method environment must be added to all rules
- In most cases, M is passed down but not actually used
  - Example of a rule that does not use M:

$$\frac{\begin{array}{c} O, M \vdash e_1 : T_1 \\ O, M \vdash e_2 : T_2 \end{array}}{O, M \vdash e_1 + e_2 : \text{Int}} \quad \text{[Add]}$$

  - Only the dispatch rules uses M

Discussion

- study type system from programming languages and compiler points of view
- "type rule is weak?" vs "weakly typed programming languages?"

Three types of dispatch in Cool

We first discuss two:

```
<expr>.<id>(<expr>,...,<expr>)
```

Consider the dispatch $e_0.f(e_1, \ldots, e_n)$

$e@B.f()$ invokes the method f in class B on the object that is the value of e.

```
<expr>@<type>.id(<expr>,...,<expr>)
```

The Dispatch Rule

$$\frac{\begin{array}{c} O, M \vdash e_0 : T_0 \\ O, M \vdash e_1 : T_1 \\ \ldots \\ O, M \vdash e_n : T_n \\ M(T_0, f) = (T_1', \ldots, T_n', T_{n+1}') \\ T_i \leq T_i' \qquad (\text{for } 1 \leq i \leq n) \end{array}}{O, M \vdash e_0.f(e_1, \ldots, e_n) : T_{n+1}'} \quad \text{[Dispatch]}$$

Static Dispatch

The class T of the method f is given in the dispatch, and the type T_0 must conform to T.

$$
\frac{
\begin{array}{c}
O, M \vdash e_0 : T_0 \\
O, M \vdash e_1 : T_1 \\
\ldots \\
O, M \vdash e_n : T_n \\
T_0 \leq T \\
M(T, f) = (T_1', \ldots, T_n', T_{n+1}') \\
T_i \leq T_i' \quad (\text{for } 1 \leq i \leq n)
\end{array}
}{
O, M \vdash e_0@T.f(e_1, \ldots, e_n) : T_{n+1}'
} \quad [\text{StaticDispatch}]
$$

Self_Type

Tradeoffs between complexity and flexibility

Static and Dynamic Types in Cool

```
class A { ... }
class B inherits A {...}
class Main {
  A x ← new A;
  ...
  x ← new B;
  ...
}
```

x has static type A

Here, x's value has dynamic type A

Here, x's value has dynamic type B

- A variable of static type A can hold values of static type B, if B ≤ A

Static and Dynamic Types in Cool

Soundness theorem for the Cool type system:

$$\forall E. \quad \text{dynamic\_type}(E) \le \text{static\_type}(E)$$

Why is this Ok?

- All operations that can be used on an object of type C can also be used on an object of type $C' \le C$
  - Such as fetching the value of an attribute
  - Or invoking a method on the object
- Subclasses can only add attributes or methods
- Methods can be redefined but with same type !

Self_Type: a Motivating Example

```
class Count {
   i : int ← 0;
   inc () : Count {
       {
           i ← i + 1;
           self;
       }
   };
};
```

- Class Count incorporates a counter
- The inc method works for any subclass
- But there is disaster lurking in the type system

Self_Type: a Motivating Example

- Consider a subclass Stock of Count

```
class Stock inherits Count {
  name : String; -- name of item
};
```

- And the following use of Stock:

```
class Main {
  Stock a ← (new Stock).inc ();    Type checking error !
  ...  a.name ...
};
```

Self_Type: a Motivating Example

- (new Stock).inc() has dynamic type Stock
- So it is legitimate to write

  Stock a ← (new Stock).inc ()
- But this is not well-typed

  (new Stock).inc() has static type Count
- The type checker "loses" type information
- This makes inheriting inc useless
  - So, we must redefine inc for each of the subclasses, with a specialized return type

Self_Type: a Motivating Example

- We will extend the type system
- Insight:
  - inc returns "self"
  - Therefore the return value has same type as "self"
  - Which could be Count or any subtype of Count !
  - In the case of (new Stock).inc () the type is Stock
- We introduce the keyword SELF_TYPE to use for the return value of such functions
  - We will also need to modify the typing rules to handle SELF_TYPE

Self_Type: a Motivating Example

- SELF_TYPE allows the return type of inc to change when inc is inherited
- Modify the declaration of inc to read

  inc() : SELF_TYPE { ... }
- The type checker can now prove:

  $O, M \vdash$ (new Count).inc() : Count

  $O, M \vdash$ (new Stock).inc() : Stock
- The program from before is now well typed

Self_Type

- A special type
- A concept of static type not dynamic type
- Helps with the expressiveness and flexibility (accept more correct programs)
- It is like a "type variable"
- An example to show tradeoffs between complexity vs expressiveness of the type systems

Note: The meaning of SELF_TYPE depends on where it appears

- We write SELF_TYPE_C to refer to an occurrence of SELF_TYPE in the body of C

Important Typing Rule Regarding Self_Type

- This suggests a typing rule:

$$\text{SELF\_TYPE}_C \leq C$$

- This rule has an important consequence:
  - In type checking it is always safe to replace SELF_TYPE_C by C
- This suggests one way to handle SELF_TYPE :
  - Replace all occurrences of SELF_TYPE_C by C

- This would be correct but it is like not having SELF_TYPE at all

Operations on Self_Type

- Recall the operations on types
  - $T_1 \leq T_2$ T_1 is a subtype of T_2
  - $\text{lub}(T_1, T_2)$ the least-upper bound of T_1 and T_2
- We must extend these operations to handle SELF_TYPE

Operations on Self_Type

Let T and T' be any types but SELF_TYPE

There are four cases in the definition of ≤

1. $\text{SELF\_TYPE}_C \leq T$ if $C \leq T$
   - SELF_TYPE_C can be any subtype of C
   - This includes C itself
   - Thus this is the most flexible rule we can allow
2. $\text{SELF\_TYPE}_C \leq \text{SELF\_TYPE}_C$
   - SELF_TYPE_C is the type of the "self" expression
   - In Cool we never need to compare SELF_TYPEs coming from different classes

Operations on Self_Type

3. $T \leq SELF\_TYPE_C$ always false

   Note: $SELF\_TYPE_C$ can denote any subtype of C.

4. $T \leq T'$ (according to the rules from before)

Based on these rules we can extend lub ...

Operations on Self_Type

Let T and T' be any types but SELF_TYPE

Again there are four cases:

1. lub($SELF\_TYPE_C$, $SELF\_TYPE_C$) = $SELF\_TYPE_C$

2. lub($SELF\_TYPE_C$, T) = lub(C, T)

   This is the best we can do because $SELF\_TYPE_C \leq C$

3. lub(T, $SELF\_TYPE_C$) = lub(C, T)

4. lub(T, T') defined as before

Self_Type in Cool

- The parser checks that SELF_TYPE appears only where a type is expected
- But SELF_TYPE is not allowed everywhere a type can appear:

1. class T inherits T' {...}
   - T, T' cannot be SELF_TYPE
   - Because SELF_TYPE is never a dynamic type
2. x : T
   - T can be SELF_TYPE
   - An attribute whose type is SELF_TYPE_C

Self_Type in Cool

3. let x : T in E
   - T can be SELF_TYPE
   - x has type $SELF\_TYPE_C$
4. new T
   - T can be SELF_TYPE
   - Creates an object of the same type as self
5. $m@T(E_1,\ldots,E_n)$
   - T cannot be SELF_TYPE

Type Checking Rules with Self_Type

- Since occurrences of SELF_TYPE depend on the enclosing class we need to carry more context during type checking
- New form of the typing judgment:

$$O,M,C \vdash e : T$$

(An expression e occurring in the body of C has static type T given a variable type environment O and method signatures M)

Type Checking Rules with Self_Type

- The next step is to design type rules using SELF_TYPE for each language construct
- Most of the rules remain the same except that $\leq$ and lub are the new ones
- Example:

$$\frac{O(id) = T_0 \quad O \vdash e_1 : T_1 \quad T_1 \leq T_0}{O \vdash id \leftarrow e_1 : T_1}$$

Type Checking Rules with Self_Type

Old rule for method dispatch:

$$\frac{\begin{array}{c} O,M,C \vdash e_0 : T_0 \\ \ldots \\ O,M,C \vdash e_n : T_n \\ M(T_0, f) = (T_1',\ldots,T_n',T_{n+1}') \\ T_{n+1}' \neq \text{SELF\_TYPE} \\ T_i \leq T_i' \qquad 1 \leq i \leq n \end{array}}{O,M,C \vdash e_0.f(e_1,\ldots,e_n) : T_{n+1}'}$$

Type Checking Rules with Self_Type

- If the return type of the method is SELF_TYPE then the type of the dispatch is the type of the dispatch expression:

$$\frac{\begin{array}{c} O,M,C \vdash e_0 : T_0 \\ \ldots \\ O,M,C \vdash e_n : T_n \\ M(T_0, f) = (T_1', \ldots, T_n', \text{SELF\_TYPE}) \\ T_i \leq T_i' \qquad 1 \leq i \leq n \end{array}}{O,M,C \vdash e_0.f(e_1,\ldots,e_n) : T_0}$$

Type Checking Rules with Self_Type

- Note this rule handles the Stock example
- Formal parameters cannot be SELF_TYPE
- Actual arguments can be SELF_TYPE
  - The extended $\leq$ relation handles this case
- The type T_0 of the dispatch expression could be SELF_TYPE
  - Which class is used to find the declaration of f?
  - Answer: it is safe to use the class where the dispatch appears

Type Checking Rules with Self_Type

- Recall the original rule for static dispatch

$$\frac{\begin{array}{c} O,M,C \vdash e_0 : T_0 \\ \ldots \\ O,M,C \vdash e_n : T_n \\ T_0 \leq T \\ M(T, f) = (T_1', \ldots, T_n', T_{n+1}') \\ T_{n+1}' \neq \text{SELF\_TYPE} \\ T_i \leq T_i' \qquad 1 \leq i \leq n \end{array}}{O,M,C \vdash e_0@T.f(e_1, \ldots, e_n) : T_{n+1}'}$$

Type Checking Rules with Self_Type

- If the return type of the method is SELF_TYPE we have:

$$\frac{\begin{array}{c} O,M,C \vdash e_0 : T_0 \\ \ldots \\ O,M,C \vdash e_n : T_n \\ T_0 \leq T \\ M(T, f) = (T_1', \ldots, T_n', SELF\_TYPE) \\ T_i \leq T_i' \qquad 1 \leq i \leq n \end{array}}{O,M,C \vdash e_0@T.f(e_1, \ldots, e_n) : T_0}$$

Type Checking Rules with Self_Type

- Why is this rule correct?
- If we dispatch a method returning SELF_TYPE in class T, don't we get back a T?

- No. SELF_TYPE is the type of the self parameter, which may be a subtype of the class in which the method appears

- The static dispatch class cannot be SELF_TYPE

Type Checking Rules with Self_Type

- There are two new rules using SELF_TYPE

$$\frac{}{O,M,C \vdash \text{self} : \text{SELF\_TYPE}_C}$$

$$\frac{}{O,M,C \vdash \text{new SELF\_TYPE} : \text{SELF\_TYPE}_C}$$

- There are a number of other places where SELF_TYPE is used

Type Systems

- The rules in these lecture were COOL-specific
  - Other languages have very different rules
- General themes
  - Type rules are defined on the structure of expressions
  - Types of variables are modeled by an environment
- Types are a play between flexibility and safety