Comprehensive Analysis on Meshing Method on Finite Element Method and S-N Curve Fitting

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Abstract: Finite element method is a computational technique used to obtain approximate solutions of engineering problems, especially when there are no analytical solutions. Generally, users try to solve a set of partial differential equations based on the profile of the displacements after the object is discretized into small elements. In this paper, a crankshaft from a diesel engine is regarded as the research object and ANSYS is also being used. The researchers discuss about the accuracy of different meshing method and compare the results to the standard one. In order to calculate out the life cycles of the crankshaft, the authors apply the rain flow counting method, S-N curve and the principle of linear fatigue damage accumulation. For a more accurate result, we estimated the confidence interval of S-N curve based on normal T distribution of residual.

Keywords: Crankshaft, life cycles, Meshing method, S-N curve fitting, linear fatigue damage accumulation

1 Introduction

1.1 Finite element method and shape function

Three factors are known to affect the accuracy of the finite element method results: the number of meshes, the mesh shape, and the number of nodes. In the original cylinder model, we used the default value 77 which is given in the reference book, and according to the available information, the increase in the number of nodes increases the accuracy of the model. In order to simplify matters, we assumed that the initial grid number value of 77 is convergent and investigated the effect of grid shape and the effect of the number of nodes while keeping the grid in the same shape.

One of the fundamental ideas of finite element method is to discretize the object, and then calculate the value of the nodes in each mesh using boundary conditions. The coordinates of the discretized object can then be distributed and linked to the whole object by the correspondence law of the corresponding shape functions.

The cell displacement function depends on the interpolation function. which is a shape function. A shape function is a continuous function defined by the coordinates within a cell. For, for two-dimensional problems in particular, the unit displacement determined by the shape function can be expressed as:

$$u = \sum_{i=1}^{n} N_i u_i \tag{1}$$

$$\mathrm{v} = \sum_{i=1}^{n} N_i v_i \tag{2}$$

The defined shape function should ensure that the displacement function defined by it should meet the convergence requirements, i.e., satisfy the completeness condition and the coordination condition. Therefore, in order to meet relevant requirements concerning interpolation completeness and coordination of unit displacement, the conditions that the shape function must satisfy are as follows:

$$\sum_{i=1}^{n} N_i(x,y) = 1 \tag{3}$$

$$\sum_{i=1}^{n} N_i(x,y)x_i = x \tag{4}$$

$$\sum_{i=1}^{n} N_i(x,y)y_i = y \tag{5}$$

The fundamental idea of the shape function is to approximate the result of the rest of the points from a known node by means of an interpolation algorithm. This algorithm is constructed in two main ways, either by natural coordinates or by generalized coordinates. The construction in generalized coordinates is completed by converting the cell displacement and representing it by polynomial means, and then using the geometric parameters of the cell as well as the node displacements to determine the coefficients of the polynomial. The natural coordinate method makes use of the nature of the nodes of the shape function and is constructed using the geometric method and then calibrating it. The most commonly used interpolation method is the Lagrangian interpolation method [1], whose formula is shown below:

$$N_i(x,y) = \frac{\prod_{k=1}^{m} F_k(x,y)}{\prod_{k=1}^{m} F_k(x_i,y_i)} \tag{6}$$

1.2 Rain flow counting

The rain flow was named from a comparison of this method to the flow of rain falling on a pagoda and running down the edges of the roof. The implement of rain flow counting is based on the stress-strain behavior of material. Rain flow counting method reflects the memory characteristics of materials and has a clear mechanical concept, consequently it has been widely recognized.

The rain flow counting algorithm is summarized as follows:

1. Rotate the loading history 90° such that the time axis is vertically downward and the load time history resembles a pagoda roof.
2. Imagine a flow of rain starting at each successive extremum point.
3. Define a loading reversal (half-cycle) by allowing each rain flow to continue to drip down these roofs until:
    a. It falls opposite a larger maximum (or smaller minimum) point.
    b. It meets a previous flow falling from above.

c. It falls below the roof.

4. Identify each hysteresis loop (cycle) by pairing up the same counted reversals.

1.3 Calculating the Confidence Interval of S-N Curve

The traditional formula for fitting S-N curve is to assume that there is a linear relationship between *S* and *ln(N)* as the Eq. (7):

$$\ln(N) = \alpha + \beta S \tag{7}$$

In Eq. (7), *N* indicates life cycles. S means the stress, and α and β are paraments.

According to Eq. (7), we can take *N* as the independent variable and *S* as the dependent variable to make regression analysis on the statistical data and get the formula of S-N data.

However, in reality, there must be errors in the data measurement. For example, when the stress amplitude is greater than 600 or less than 200, the actual measured value may not be perfectly fitted by the above formula. Therefore, how to perform curve fitting and quantify the uncertainty in the curve fitting process and express it is very important.

2.Model Formulation

In this paper, to begin with, the researchers utilize finite element method to analysis the stress distribution of a crankshaft from a diesel engine in working condition. In this process, different meshing methods are applied respectively [2]. By means of the finite element method we can get the difference of stress distribution in crankshaft. Nevertheless, it's difficult to estimate the influence on the life cycles. In this paper, we utilize rain flow counting method to extract closed loading cycles [3].

The next step is to fit the S-N curve based on the given material. In this section, we applied a new fitting method according to the distribution characteristics of data points and provided the confidence interval. In the end, we calculate out the life cycles on different occasions and compare them to the standard one.

2.1 Influence of Meshing Paraments in Finite Element Method

The shape and size of the meshing grid are set as tetrahedron and 77 respectively with the quadric elements, and the stress distribution on this occasion is regarded as the result with the highest accuracy. Afterwards, the researchers carried out three set of tests and the experimental scheme as well as the standard condition are illustrated in table 1.

Tabel 1. Experimental Scheme

Name	Mesh shape	Mesh size	Order of elements
Linear Tetrahedron	Tetrahedron	77	Linear
Quadric Tetrahedron	Tetrahedron	100	Quadric
Quadric Hexahedron	Hexahedron	77	Quadric
Standard	Tetrahedron	77	Quadric

2.2 Implement of Rain Flow Counting Algorithm

In this article, we utilize the library 'rainflow 3.0.1' in python. By inputting the stress-time distribution on the crankshaft, we can extract cycles from a complicated loading history, where each cycle is associated with a closed stress-strain hysteresis loop. In the next step, we select loads which go through a complete cycle and calculate out the corresponding total stress under various conditions by means of linear fatigue damage accumulation principle.

2.3 Details in Curve Fitting

In this experiment, by fitting the stress life cycle curve, we can get a standard S-N curve. At the same time, we estimate the overall interval of S-N curve and put forward the confidence interval. In view of the uncertainty in S-N curve estimation, we give up the assumption that the logarithm of S and N satisfies the linear relationship. Instead, we use experience formula for linear fitting and utilize SPSS for regression to estimate the curve.

3 Experiment

3.1 Dataset

3.1.1 Stress-Time dataset

In this part, stress-time distribution dataset is obtained by finite element method in ANSYS and they were calculated out under 4 conditions illustrated in Table. 1. The data is consisted of 5 columns: order number (No.), time(s), maximum stress, minimum stress and mean stress. All the dataset are exhibited in Appendix after the Conclusion part as well as the Figure 1. The magnitude of these stress dataset which is extracted by rain flow counting algorithm are illustrated in Figure 2:

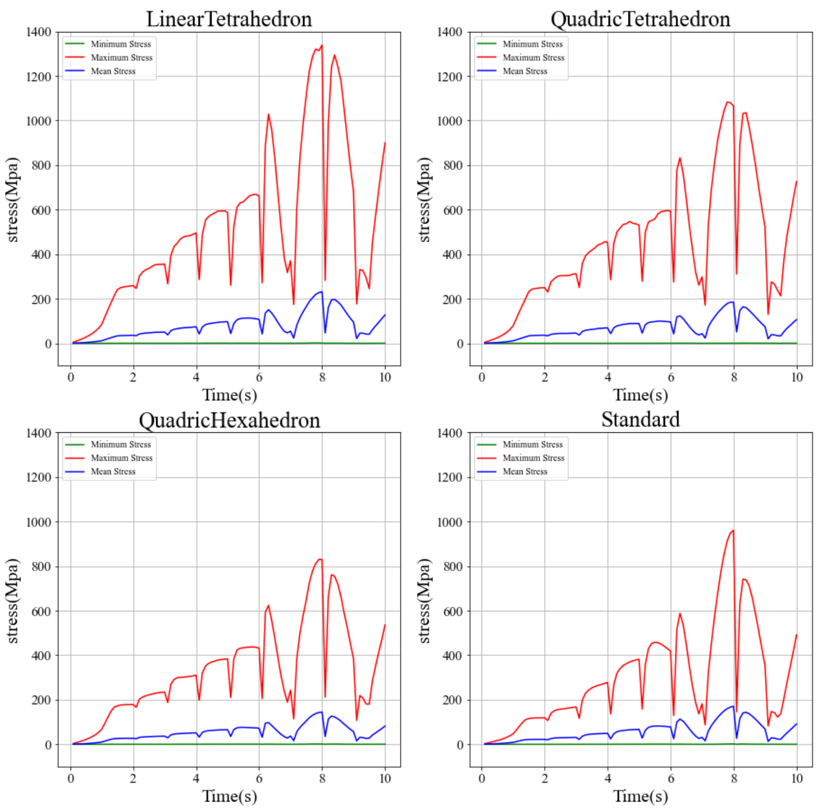

Figure 1. Stress-Time Distribution

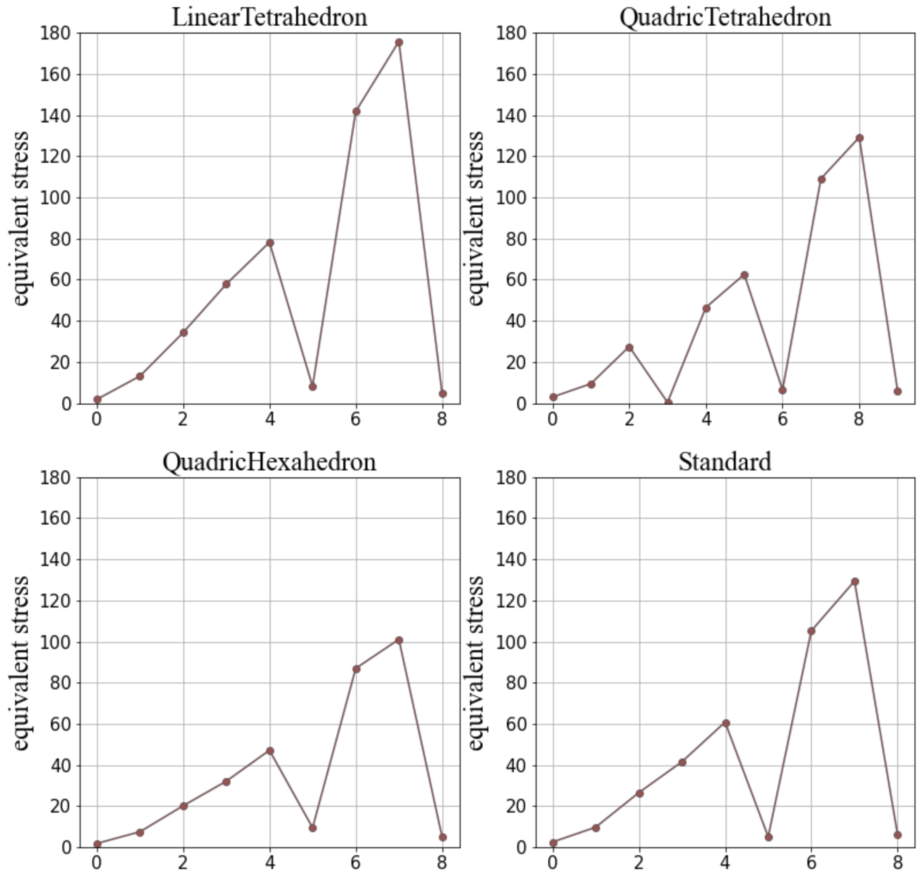

Figure 2. Amplitude of Stress

3.1.2 Stress-Life Cycles Dataset

Dataset applied to fit the S-N curve are illustrated in table 2:

Tabel 2. Stress-Life Cycles Dataset

N	ln(N)	S (Mpa)	ln(S)
149000	11.91	729.83	6.59
300000	12.61	729.83	6.59
626000	13.35	729.83	6.59
1130000	13.94	649.64	6.48
1170000	13.97	689.98	6.54
1410000	14.16	649.64	6.48
1470000	14.20	609.79	6.41
2060000	14.54	590.12	6.38
2210000	14.61	649.64	6.48
2760000	14.83	550.29	6.31
2960000	14.90	590.12	6.38
4410000	15.30	689.98	6.54
5070000	15.44	689.98	6.54
5670000	15.55	569.95	6.35
5960000	15.60	629.96	6.45
7450000	15.82	569.95	6.35

8160000	15.91	629.96	6.45
10000000	16.12	569.95	6.35
10000000	16.12	509.93	6.23
10000000	16.12	629.96	6.45
10000000	16.12	609.79	6.41
10000000	16.12	590.12	6.38
10000000	16.12	550.27	6.31
10000000	16.12	530.10	6.27
2960000	14.90	609.79	6.41

Under the same stress level, 2 or 3 times of tests were carried out to eliminate the randomness of the test results. Due to the limitation of the test funds, we truncated the life cycles. When $N > 10000000$, we regarded N as 10000000.

While estimating the interval of curve fitting, there is a certain error between the estimated value and the real value, that is, the residual of curve fitting. Commonly known that the error of data comes from 2 perspectives such as data measurement and estimation. Next, we give the confidence interval of curve fitting based on the residual of curve fitting, and estimate the curve again under degree of confidence of 95%.

3.2 Experimental Results and Analysis

3.2.1 S-N Curve Fitting

In this paper, linear function, logarithmic function, inverse function, quadratic function, cubic function, compound function, power function, S function, growth function, exponential function, and logistic function are applied for the original data. The fitting results are shown in Figure 3:

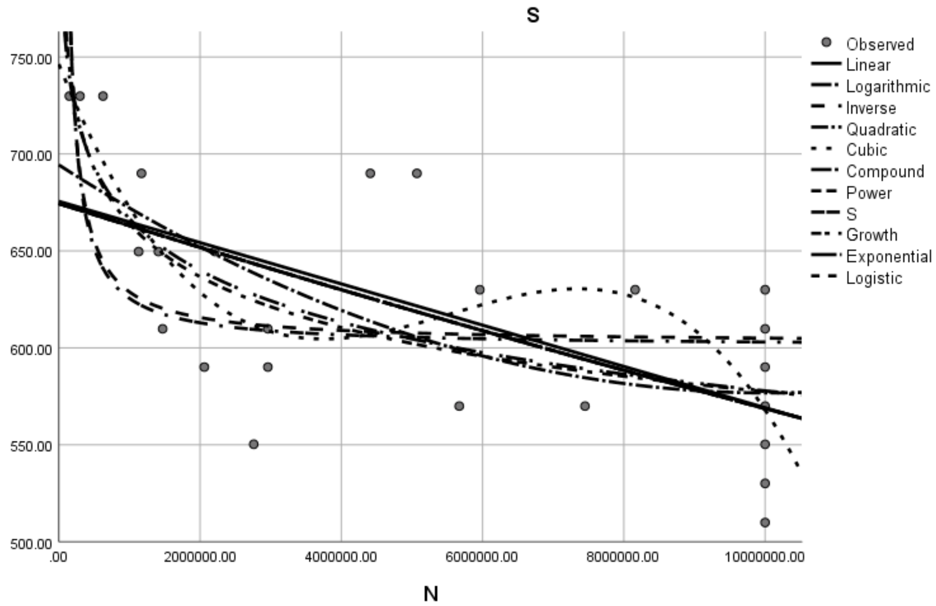

Figure 3. Curve Fitting in Various Method

According to the analysis of variance of multiple fitting results, it can be discovered that the best fitting function is the power function which is illustrated in Eq. (8)

$$N = \alpha S^{\beta} \tag{8}$$

According to the coefficient table of fitting results, the significance of fitting coefficient is very low, the confidence is very high which indicates that the fitting results are shown in Eq. (9):

$$N = 1543.47 S^{-0.061} \tag{9}$$

The power function fitting is illustrated in Figure 4:

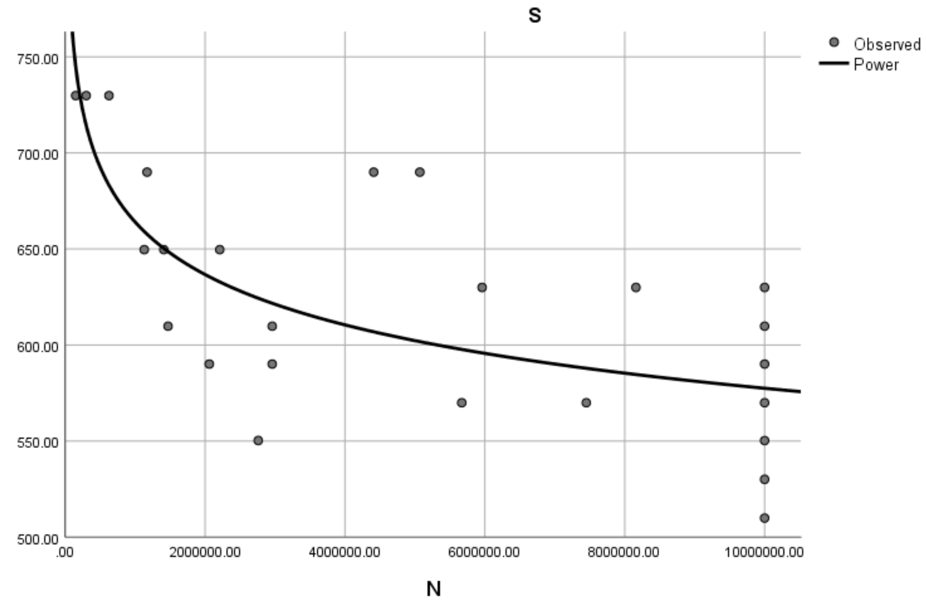

Figure 4. Power Function Fitting

In order to calculate out the residuals conveniently, we take logarithms on the left and right sides of the formula, and then perform linear curve fitting operation. The results are shown in Figure 5:

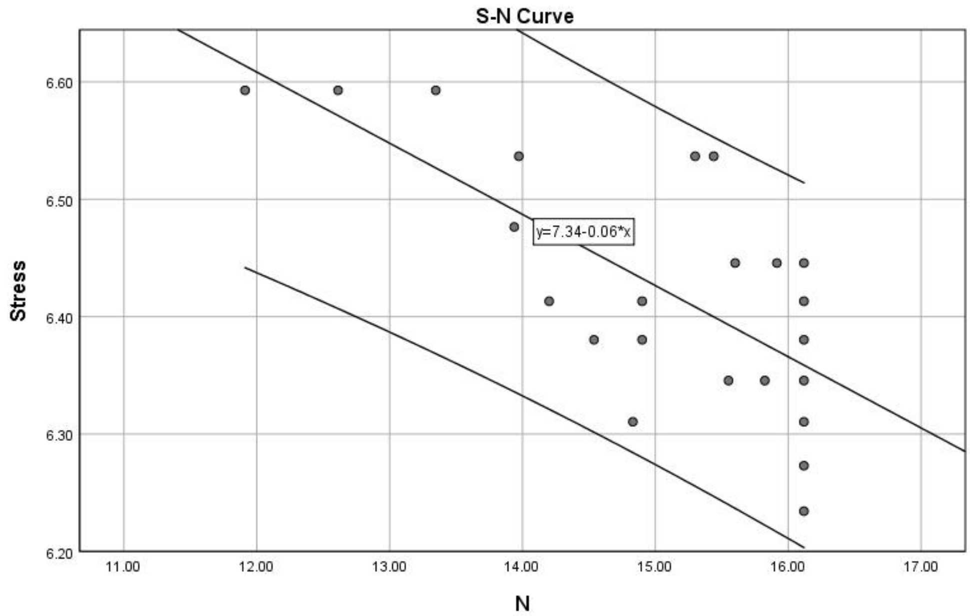

Figure 5. Linear Curve Fitting

The result of fitting is Eq. (10):

$$\ln(N) = -0.061 \times \ln(S) + 7.336 \tag{10}$$

The coefficients and model summary generated in SPSS are illustrated in Figure 6 and table 7.

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95.0% Confidence Interval for B Lower Bound	95.0% Confidence Interval for B Upper Bound
1	(Constant)	7.336	.189		38.879	.000	6.946	7.726
	LnN	-.061	.013	-.709	-4.826	.000	-.087	-.035

a. Dependent Variable: LnS

Figure 6. Coefficients

Model Summary[b]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics R Square Change	F Change	df1	df2	Sig. F Change	Durbin-Watson
1	.709[a]	.503	.482	.07230	.503	23.292	1	23	.000	1.936

a. Predictors: (Constant), LnN

b. Dependent Variable: LnS

Figure 7. Model Summary

According to the ANOVA table of linear fitting, the coefficient significance of linear estimation parameters is less than 0.01, and the R and R square of linear fitting model are greater than 0.4, which indicates that this model so it passes the goodness of fit test.

The distribution of residuals is illustrated in figure 8:

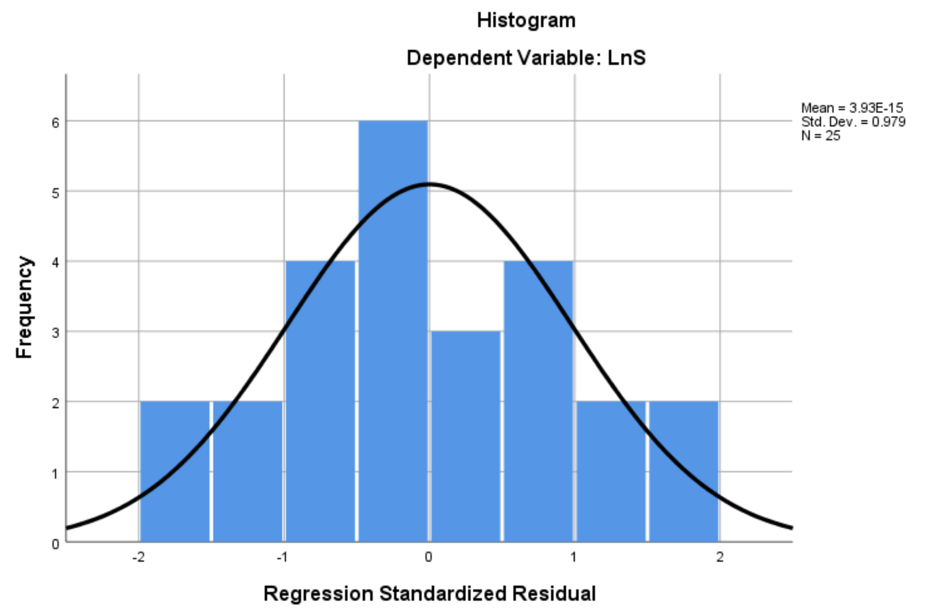

Figure 8. Distribution of Residuals

It can be found in the residual histogram that the residual distribution is approximate to the t distribution. Due to the sample size of dataset is relatively small (n = 25 < 30), t-distribution should be applied to analysis the problem [4].

The pivot that being used is shown in Eq. (11):

$$t = \frac{\overline{x} - \mu_0}{s\sqrt{n}} \tag{11}$$

The confidence interval of linear fitting parameters with 95% confidence can be obtained and were exhibited in figure 9:

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	7.336	.189		38.879	.000	6.946	7.726
	LnN	-.061	.013	-.709	-4.826	.000	-.087	-.035

a. Dependent Variable: LnS

Figure 9. Confidence IntervalS with 95% Confidence

The confidence upper and lower bound of α are 7.726 and 6.946. The confidence upper and lower bound of β are -0.035 and -0.087.

3.2.2 Fatigue life cycles calculation

By means of predicting the corresponding fatigue life cycles at certain stress level, we can get the exact fatigue life cycles and their confidence upper and lower bound which are illustrated in table 2:

Table 2. Results of Fatigue Life

	Linear Tetrahedron	Quadric Tetrahedron	Quadric Hexahedron	Standard
Stress in one period	665.99	461.25	549.86	536
Fatuige Life	1177307	536674384	28691700	43047857
Upper Limit	1299243	11920750642	78670162	1577709933
Lower Limit	329856	88584966	11753033	15547620
Relative Error	-97.26%	11466.93%	33.35%	0

The definition of relative error is Eq. (12):

$$Relative\ Error = \frac{Fatigue\ Life - Standard\ \ Fatigue\ Life}{Standard\ \ Fatigue\ Life} \times 100\% \tag{12}$$

It can be seen from table 2 that there is a large fluctuation range in the fatigue life and its upper and lower limits under various conditions. Among them, the relative error in the Quadric Hexahedron is the lowest, only 33.35%, while the calculated relative error in the other two cases is so large that these two conditions should be avoided. The reasons for these results will be discussed in the following section.

4 Conclusion

The test results are dramatic, and we speculate that the reason for such a large error comes from the finite element method and S-N curve fitting, so we will discuss the test results from these two perspectives.

4.1 Problems in S-N Curve Fitting

It is obvious that curve fitting results are not accurate. We attribute the problems in dataset to the following aspects.

4.1.1 The Distribution of Data Points is Relatively Discrete

It is obvious that even though the optimum power function is obtained, the distance from many points to the line (i.e., the residuals) is still large. The residuals present an approximate T distribution, which implies that there is no obvious human intervention in the process and all the errors come from the system. When estimating the confidence interval, researchers found out that the confidence interval of the parameter estimates is relatively large, which also causes the difference between the upper and lower limits of fatigue life to range dramatically from one percent to 100 times. The result is not accurate enough to solve the engineering problems, and the underlying reason comes from the scatter of the dataset.

4.1.2 The Dataset is Relatively Small

Due to the cost of the test, there are only 25 data points in total, and only 3-4 tests under the same stress level. The number of samples is less than threshold 30 which is commonly used in small sample analysis methods. As a result of scant data points and the limited information they contain, we cannot solve this problem with various data processing methods that usually applied in large sample problems.

4.1.3 The Fixed Number Truncated Data

Through the distribution of data, we can figure out that there are 7 data points with 10,000,000 fatigue life cycles, and 10,000,000 is also the largest fatigue life cycle among all points. Therefore, we can judge that this test is a fix number truncated test. The loss of information in truncated data is also inevitable, which has a certain impact on the curve fitting results.

4.2 Problems in Finite Element Method

While using ANSYS to analyze the stress of crankshaft in diesel engine, if we only analyze the relative error from the perspective of stress magnitude, the relative error of Quadric Tetrahedron is 2.2%, the relative error of Linear Tetrahedron is 22.0%, and Quadric Hexahedron’s is 12.9%. The magnitude of these relative errors is much smaller than that of the relative errors of fatigue life. It can be demonstrated that the S-N curve is very sensitive to the magnitude of the stress values, which means slight fluctuations can cause large differences in the final results.

As for the reason of poor result, we analyzed and attributed it to the structure of the crankshaft. Since diesel engine and crankshaft have more irregular surfaces and more contact relations, the influence of different parameter settings on the final results in the finite element method process is inevitable, especially in the stage of meshing and approaching the results by shape function. Due to the lack of ANSYS basic layer algorithm, we are unable to analyze this quantitatively and hope to supplement it in future studies.

5 References

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Appendix

No.	Time (S)	Minimum (MPa)	Maximum (MPa)	Mean (MPa)
1	0.1	3.8794e-003	4.9033	0.67896
2	0.2	7.4742e-003	9.933	1.3684
3	0.3	1.1125e-002	15.241	2.0912
4	0.4	1.5162e-002	21.01	2.8818
5	0.5	1.9676e-002	27.406	3.7654
6	0.6	2.5156e-002	35.132	4.8185
7	0.7	3.1366e-002	43.82	6.0123
8	0.8	3.8821e-002	54.318	7.4382
9	0.9	4.8053e-002	67.259	9.1909
10	1.	6.0402e-002	84.079	11.469
11	1.1	8.6163e-002	117.09	16.011
12	1.2	0.11155	150.85	20.783
13	1.3	0.13373	182.26	25.205
14	1.4	0.15584	213.22	29.586
15	1.5	0.17118	241.39	33.22
16	1.6	0.17615	249.48	34.323
17	1.7	0.17904	252.87	34.832
18	1.8	0.18155	255.05	35.192
19	1.9	0.18413	257.03	35.526
20	2.	0.18658	258.84	35.844
21	2.1	0.18126	246.39	34.145
22	2.2	0.21661	301.56	41.862
23	2.3	0.20356	319.54	44.333
24	2.4	0.18811	330.28	45.97
25	2.5	0.1753	335.85	46.82
26	2.6	0.16771	345.11	48.144
27	2.7	0.1603	352.76	49.298
28	2.8	0.15213	353.99	49.673
29	2.9	0.14568	354.14	49.982
30	3.	0.14272	355.76	50.396
31	3.1	0.13189	266.76	37.932
32	3.2	0.14963	394.41	56.67
33	3.3	0.18413	434.38	62.83
34	3.4	0.21042	449.11	65.411
35	3.5	0.23799	467.78	68.049
36	3.6	0.26222	477.64	69.635
37	3.7	0.28887	481.11	70.575
38	3.8	0.30706	482.78	71.413

39	3.9	0.32204	488.65	72.806
40	4.	0.339	495.63	74.156
41	4.1	0.14077	285.98	42.22
42	4.2	0.33137	488.97	73.227
43	4.3	0.42165	553.21	84.068
44	4.4	0.48337	569.05	87.756
45	4.5	0.51294	578.88	90.047
46	4.6	0.53149	585.77	92.15
47	4.7	0.54874	593.87	94.143
48	4.8	0.56419	593.35	95.178
49	4.9	0.56208	594.88	96.566
50	5.	0.55343	587.57	97.01
51	5.1	0.20402	259.65	43.907
52	5.2	0.45056	523.79	89.117
53	5.3	0.55374	611.28	105.65
54	5.4	0.56704	630.47	110.94
55	5.5	0.55194	634.72	112.82
56	5.6	0.5337	649.06	113.21
57	5.7	0.51133	661.86	113.34
58	5.8	0.49383	667.62	112.45
59	5.9	0.43249	668.25	110.04
60	6.	0.37506	661.85	107.34
61	6.1	0.138	271.48	42.237
62	6.2	0.32159	888.51	135.98
63	6.3	0.35635	1028.6	150.91
64	6.4	0.2119	961.77	136.53
65	6.5	0.15065	832.91	116.74
66	6.6	0.23455	672.79	93.424
67	6.7	0.18668	519.65	72.129
68	6.8	0.12855	384.08	53.818
69	6.9	0.12976	317.25	46.872
70	7.	0.25373	370.72	54.501
71	7.1	0.14011	174.27	23.471
72	7.2	0.3742	607.98	82.197
73	7.3	0.32982	833.35	115.78
74	7.4	0.49259	987.59	141.57
75	7.5	0.74692	1114.	164.86
76	7.6	0.98372	1225.1	188.17
77	7.7	1.2084	1287.8	204.65
78	7.8	1.3231	1320.8	219.49
79	7.9	1.2813	1313.5	227.6
80	8.	1.192	1338.6	231.54
81	8.1	0.18889	282.22	46.166

82	8.2	0.57723	987.71	161.05
83	8.3	0.52821	1240.5	194.75
84	8.4	0.45988	1294.5	197.14
85	8.5	0.44266	1243.2	186.54
86	8.6	0.40885	1178.8	172.57
87	8.7	0.30306	1055.	151.96
88	8.8	0.18841	924.26	131.36
89	8.9	0.16682	792.75	112.07
90	9.	0.206	683.15	95.22
91	9.1	0.13525	176.22	21.315
92	9.2	0.12322	331.13	45.479
93	9.3	0.11633	326.54	45.178
94	9.4	6.2692e-002	294.46	41.354
95	9.5	0.19156	244.87	40.693
96	9.6	0.21542	449.66	61.146
97	9.7	0.30696	572.19	78.096
98	9.8	0.3381	686.25	94.332
99	9.9	0.34078	796.33	110.8
100	10.	0.45731	899.37	127.17

Stress-Time Distribution in Linear Tetrahedron Model

No.	Time (S)	Minimum (MPa)	Maximum (MPa)	Mean (MPa)
1	0.1	2.6354e-003	4.3514	0.61171
2	0.2	5.3065e-003	8.9308	1.2581
3	0.3	8.2413e-003	13.869	1.9556
4	0.4	1.138e-002	19.244	2.7214
5	0.5	1.4951e-002	25.207	3.5793
6	0.6	1.892e-002	32.049	4.5595
7	0.7	2.3681e-002	40.063	5.7052
8	0.8	2.9245e-002	49.652	7.0555
9	0.9	3.6422e-002	61.923	8.7709
10	1.	4.6261e-002	78.235	11.035
11	1.1	6.6768e-002	110.01	15.545
12	1.2	8.1219e-002	142.11	20.241
13	1.3	9.6025e-002	172.09	24.601
14	1.4	0.11251	201.94	28.955
15	1.5	0.13005	231.78	33.308
16	1.6	0.13624	242.88	34.975
17	1.7	0.13773	246.06	35.474
18	1.8	0.13774	247.76	35.803
19	1.9	0.13812	249.21	36.09
20	2.	0.13812	249.89	36.23
21	2.1	0.12981	230.21	33.436

22	2.2	0.1466	275.25	40.016
23	2.3	0.15102	289.05	42.174
24	2.4	0.15371	299.71	43.814
25	2.5	0.15355	303.39	44.419
26	2.6	0.15126	303.68	44.427
27	2.7	0.1463	304.15	44.565
28	2.8	0.14152	304.83	44.704
29	2.9	0.14029	310.72	45.648
30	3.	0.14314	312.19	45.901
31	3.1	0.11248	250.69	36.964
32	3.2	0.13995	359.02	53.144
33	3.3	0.14302	392.26	58.398
34	3.4	0.1346	407.46	60.729
35	3.5	0.13168	417.7	62.396
36	3.6	0.13005	428.56	64.022
37	3.7	0.1282	441.99	66.514
38	3.8	0.12685	446.19	67.227
39	3.9	0.12805	455.43	68.666
40	4.	0.12958	454.62	69.079
41	4.1	8.0721e-002	284.85	43.406
42	4.2	0.11638	450.75	69.871
43	4.3	0.13077	500.28	78.39
44	4.4	0.13555	518.1	81.799
45	4.5	0.13564	533.5	85.086
46	4.6	0.13691	537.06	86.314
47	4.7	0.14033	546.8	89.017
48	4.8	0.1476	539.02	88.702
49	4.9	0.15743	536.22	89.007
50	5.	0.1658	530.7	88.639
51	5.1	9.3674e-002	278.5	46.174
52	5.2	0.17674	498.84	83.831
53	5.3	0.22214	544.67	92.977
54	5.4	0.24923	551.57	96.084
55	5.5	0.26624	557.73	97.257
56	5.6	0.28677	580.65	99.548
57	5.7	0.30942	590.68	99.641
58	5.8	0.33004	593.22	98.432
59	5.9	0.35033	596.32	97.474
60	6.	0.37012	593.54	95.946
61	6.1	0.20656	275.13	42.239
62	6.2	0.61454	777.	118.69
63	6.3	0.43418	832.81	123.13
64	6.4	0.57159	763.57	111.12

65	6.5	0.49578	649.94	92.968
66	6.6	0.41664	525.72	74.142
67	6.7	0.34942	418.36	58.339
68	6.8	0.26314	317.24	44.398
69	6.9	0.17144	261.06	37.332
70	7.	8.0206e-002	298.38	43.467
71	7.1	0.16042	171.3	23.321
72	7.2	0.28774	530.16	73.825
73	7.3	0.32892	695.01	98.806
74	7.4	0.29947	813.87	118.49
75	7.5	0.25189	914.1	137.04
76	7.6	0.25638	985.75	152.27
77	7.7	0.27223	1045.2	166.51
78	7.8	0.33279	1083.1	178.63
79	7.9	0.42065	1079.6	184.31
80	8.	0.51224	1065.6	184.67
81	8.1	0.23437	311.14	50.937
82	8.2	0.60999	892.29	146.85
83	8.3	0.72525	1031.5	164.04
84	8.4	0.79099	1034.7	160.14
85	8.5	0.66866	968.02	147.43
86	8.6	0.50213	887.2	132.98
87	8.7	0.59376	799.48	118.1
88	8.8	0.51983	699.5	102.59
89	8.9	0.45643	609.02	88.123
90	9.	0.40383	523.11	74.961
91	9.1	0.14286	130.06	20.063
92	9.2	0.26133	275.87	38.905
93	9.3	0.22636	265.93	37.557
94	9.4	0.18987	237.71	33.778
95	9.5	6.636e-002	212.71	33.253
96	9.6	0.20136	373.33	51.853
97	9.7	0.29899	484.75	67.758
98	9.8	0.32993	565.83	80.007
99	9.9	0.33433	649.29	93.533
100	10.	0.31192	726.25	106.03

Stress-Time Distribution in Quadric Tetrahedron Model

No.	Time (S)	Minimum (MPa)	Maximum (MPa)	Mean (MPa)
1	0.1	3.5134e-003	3.8155	0.57449
2	0.2	7.0036e-003	7.7098	1.1627
3	0.3	1.0664e-002	11.866	1.791
4	0.4	1.4691e-002	16.469	2.4898

5	0.5	1.9241e-002	21.622	3.2714
6	0.6	2.4461e-002	27.538	4.1694
7	0.7	3.0679e-002	34.464	5.2212
8	0.8	3.8243e-002	42.784	6.4845
9	0.9	4.8183e-002	53.125	8.0528
10	1.	6.2365e-002	66.708	10.119
11	1.1	8.7994e-002	93.596	14.234
12	1.2	0.10753	121.81	18.593
13	1.3	0.13088	148.65	22.724
14	1.4	0.14272	166.78	25.363
15	1.5	0.14457	172.6	26.22
16	1.6	0.14513	176.37	26.782
17	1.7	0.14391	177.52	26.973
18	1.8	0.14247	178.04	27.074
19	1.9	0.14116	178.39	27.143
20	2.	0.14018	178.54	27.198
21	2.1	0.13506	166.55	25.466
22	2.2	0.14614	202.68	30.986
23	2.3	0.15202	212.39	32.62
24	2.4	0.15735	217.55	33.53
25	2.5	0.16462	222.18	34.295
26	2.6	0.17238	225.54	34.869
27	2.7	0.18034	229.2	35.553
28	2.8	0.18851	231.68	36.023
29	2.9	0.19642	233.31	36.336
30	3.	0.2047	234.62	36.585
31	3.1	0.15672	187.78	29.379
32	3.2	0.28537	269.93	42.443
33	3.3	0.34942	290.88	46.027
34	3.4	0.38763	298.63	47.589
35	3.5	0.38702	301.22	48.52
36	3.6	0.38931	301.36	49.19
37	3.7	0.39686	303.16	49.776
38	3.8	0.40728	304.69	50.38
39	3.9	0.42479	307.27	51.12
40	4.	0.44411	310.24	51.696
41	4.1	0.27078	198.34	32.564
42	4.2	0.4972	320.24	53.57
43	4.3	0.59081	351.94	59.246
44	4.4	0.6564	363.11	61.324
45	4.5	0.70173	369.98	62.554
46	4.6	0.70422	373.54	63.463
47	4.7	0.71268	378.5	64.462

48	4.8	0.70999	380.29	64.91
49	4.9	0.7071	381.58	65.253
50	5.	0.71058	383.57	65.653
51	5.1	0.38231	210.27	35.452
52	5.2	0.71027	383.51	66.543
53	5.3	0.7723	423.01	74.07
54	5.4	0.78141	431.26	75.868
55	5.5	0.77615	433.42	76.127
56	5.6	0.76673	435.78	75.711
57	5.7	0.75562	436.36	75.047
58	5.8	0.75252	437.3	74.49
59	5.9	0.7364	435.51	73.557
60	6.	0.63638	432.02	71.999
61	6.1	0.32558	205.39	31.966
62	6.2	0.96498	592.44	95.06
63	6.3	0.87607	624.18	97.663
64	6.4	0.59784	558.22	84.929
65	6.5	0.49277	476.81	70.921
66	6.6	0.33067	389.28	57.138
67	6.7	0.19884	307.11	44.659
68	6.8	0.15864	239.17	34.762
69	6.9	0.21062	189.4	28.125
70	7.	0.2283	243.25	37.227
71	7.1	9.7929e-002	114.53	16.479
72	7.2	0.26536	385.8	56.661
73	7.3	0.43701	500.14	75.927
74	7.4	0.71993	578.57	90.839
75	7.5	0.86795	648.38	105.44
76	7.6	1.2157	727.2	120.18
77	7.7	1.4227	778.7	130.75
78	7.8	1.458	811.7	138.09
79	7.9	1.4833	830.22	143.03
80	8.	1.4777	829.26	144.98
81	8.1	0.32012	211.79	34.99
82	8.2	1.0164	664.1	113.03
83	8.3	1.1188	761.37	126.55
84	8.4	1.2296	754.93	123.04
85	8.5	1.1625	719.66	114.73
86	8.6	0.88074	663.67	104.26
87	8.7	0.6557	591.69	90.991
88	8.8	0.533	525.38	79.017
89	8.9	0.4806	452.57	67.423
90	9.	0.34538	386.01	57.012

91	9.1	6.7471e-002	107.19	15.172
92	9.2	7.6421e-002	219.43	31.729
93	9.3	0.12236	207.19	30.064
94	9.4	0.18602	180.93	26.726
95	9.5	0.277	180.78	28.171
96	9.6	0.20781	283.81	41.515
97	9.7	0.26062	351.29	51.288
98	9.8	0.28152	412.95	60.996
99	9.9	0.3423	471.21	70.882
100	10.	0.48661	535.97	82.078

Stress-Time Distribution in Quadric Hexahedron Model

No.	Time (S)	Minimum (MPa)	Maximum (MPa)	Mean (MPa)
1	0.1	5.4492e-004	3.1279	0.59096
2	0.2	1.0219e-003	6.3089	1.192
3	0.3	1.3685e-003	9.6311	1.8175
4	0.4	1.899e-003	13.265	2.5045
5	0.5	2.6153e-003	17.103	3.2193
6	0.6	3.1534e-003	20.625	3.8791
7	0.7	4.0334e-003	25.674	4.8295
8	0.8	5.082e-003	31.581	5.9488
9	0.9	6.6849e-003	38.496	7.2603
10	1.	9.363e-003	46.833	8.8389
11	1.1	1.3098e-002	63.511	12.016
12	1.2	2.0537e-002	81.535	15.454
13	1.3	2.5133e-002	98.542	18.694
14	1.4	2.7499e-002	110.91	20.867
15	1.5	2.8511e-002	115.75	21.73
16	1.6	2.9633e-002	117.44	22.031
17	1.7	3.0782e-002	118.31	22.186
18	1.8	3.2029e-002	118.91	22.302
19	1.9	3.318e-002	119.02	22.348
20	2.	3.4327e-002	119.47	22.428
21	2.1	3.0127e-002	106.94	20.025
22	2.2	4.9312e-002	138.47	25.946
23	2.3	5.7391e-002	150.18	28.138
24	2.4	6.4752e-002	155.27	29.134
25	2.5	7.1331e-002	157.9	29.686
26	2.6	7.6967e-002	159.33	29.918
27	2.7	8.2739e-002	161.4	30.236
28	2.8	8.8634e-002	163.84	30.648
29	2.9	9.3794e-002	165.58	30.968
30	3.	9.991e-002	167.89	31.291

31	3.1	6.8953e-002	116.66	21.898
32	3.2	0.1374	199.08	36.634
33	3.3	0.168	229.27	42.004
34	3.4	0.1924	243.69	44.436
35	3.5	0.21428	253.35	46.022
36	3.6	0.23353	259.43	46.987
37	3.7	0.2523	264.95	47.797
38	3.8	0.27003	268.79	48.231
39	3.9	0.2879	273.26	48.944
40	4.	0.30608	277.83	49.658
41	4.1	0.15512	136.03	24.305
42	4.2	0.32392	268.95	47.744
43	4.3	0.39866	319.85	56.66
44	4.4	0.44486	340.71	60.191
45	4.5	0.48367	353.93	62.378
46	4.6	0.52021	362.29	63.864
47	4.7	0.55356	369.66	65.128
48	4.8	0.57933	374.51	65.84
49	4.9	0.60642	378.27	66.47
50	5.	0.63684	382.41	67.301
51	5.1	0.28722	158.27	28.093
52	5.2	0.65112	358.55	63.235
53	5.3	0.80091	429.95	75.986
54	5.4	0.87501	452.81	80.341
55	5.5	0.93147	457.67	81.988
56	5.6	0.90806	456.24	82.215
57	5.7	0.87461	449.86	81.757
58	5.8	0.8356	440.69	80.668
59	5.9	0.79329	429.64	79.456
60	6.	0.75186	418.26	78.316
61	6.1	0.1926	129.68	25.223
62	6.2	0.76534	515.29	99.953
63	6.3	0.66792	588.62	113.35
64	6.4	0.52711	538.9	103.96
65	6.5	0.41834	449.03	87.319
66	6.6	0.31002	348.98	68.869
67	6.7	0.23517	258.53	51.554
68	6.8	0.18377	185.64	36.784
69	6.9	0.1179	136.47	26.391
70	7.	0.13389	182.05	31.301
71	7.1	5.594e-002	87.593	16.029
72	7.2	6.4881e-002	319.25	59.557
73	7.3	0.16135	456.4	84.501

74	7.4	0.37166	576.48	103.98
75	7.5	0.62141	681.3	121.14
76	7.6	0.89578	774.81	136.89
77	7.7	1.177	853.66	150.07
78	7.8	1.4382	912.85	160.02
79	7.9	1.681	948.65	167.12
80	8.	1.8755	960.47	170.81
81	8.1	0.25985	145.68	27.036
82	8.2	1.1067	626.68	115.24
83	8.3	1.2682	741.64	140.93
84	8.4	1.1755	738.23	143.57
85	8.5	0.92698	710.49	137.
86	8.6	0.71225	655.48	125.89
87	8.7	0.5701	585.37	112.51
88	8.8	0.49827	508.72	98.067
89	8.9	0.41808	430.37	83.397
90	9.	0.32924	355.5	69.308
91	9.1	7.8719e-002	82.443	12.962
92	9.2	0.14317	147.2	28.881
93	9.3	0.15738	142.44	27.697
94	9.4	9.8757e-002	122.18	23.455
95	9.5	0.1496	136.92	23.052
96	9.6	2.6111e-002	205.09	37.963
97	9.7	4.5505e-002	279.12	51.741
98	9.8	4.381e-002	347.34	64.486
99	9.9	8.6966e-002	416.42	77.554
100	10.	0.19025	491.1	91.352

Stress-Time Distribution in Standard Model