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CS331 (Spring 2021): Introduction to Artificial Intelligence
Written Assignment #2

Date handed out: April 21, 2021
Date due: April 28, 2021 at 10:00am
Total: 25 points

The written portion of this assignment is to be done individually. Please hand in a pdf on Canvas. Assignments done on a word processor are preferred but not mandatory. For hand written assignments, if we cannot read your writing, we cannot mark your assignment.

1. Prove the following entailment in three different ways.
a) Prove that $(A \Leftrightarrow B) \vDash (A \vee \neg B)$ with truth tables. [2 points]
b) Prove that $(A \Leftrightarrow B) \vDash (A \vee \neg B)$ with logical equivalences. [2 points]
c) Prove that $(A \Leftrightarrow B) \vDash (A \vee \neg B)$ with the resolution algorithm. [3 points]

2. Decide whether each of the following sentences is valid, unsatisfiable or neither. Verify your decisions using truth tables or equivalence rules.
a) $(Study \Leftrightarrow Pass) \wedge \neg(\neg Study \Leftrightarrow \neg Pass)$ [2 points]
b) $((Study \wedge Rest) \Rightarrow Pass) \wedge (Study \Rightarrow \neg Rest) \wedge (Rest \Rightarrow \neg Study) \wedge Pass$ [2 points]

3. Convert the following sentences to CNF. [4 points]

S1. $P \Rightarrow (Q \vee R)$
S2. $\neg(P \wedge Q) \Leftrightarrow S$

4. Consider the following KB:

$$S1.\ A \vee B$$
$$S2.\ \neg B \vee \neg C$$
$$S3.\ \neg C \vee D$$
$$S4.\ B \vee \neg E$$
$$S5.\ \neg D \vee E$$

a) Use the resolution algorithm to determine whether the following KB entails $\neg E$. [5 points]
b) Use the resolution algorithm to determine whether the following KB entails $\neg C$. [5 points]

(a)

$$\neg A \vee B \qquad \neg B \vee A \qquad A \Longleftrightarrow B$$

¬A	A	¬B	B	A⇒B	B⇒A	(A⇒B)∧(B⇒A)	A∨¬B
F	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	F	F	T	F	T
T	F	F	T	T	F	F	F

$$\therefore (A \Longleftrightarrow B) \rightarrow (A \vee \neg B)$$

(b)

$$(A \Longleftrightarrow B) \models (A \vee \neg B)$$

$$\equiv (A \Rightarrow B) \wedge (B \Rightarrow A) \rightarrow (A \vee \neg B)$$

$$\equiv (\neg A \vee B) \wedge (\neg B \vee A) \rightarrow (A \vee \neg B)$$

$$\equiv \neg\big((\neg A \vee B) \wedge (\neg B \vee A)\big) \vee (A \vee \neg B) \qquad \text{impl elim}$$

$$\equiv \neg(\neg A \vee B) \vee \neg(\neg B \vee A) \vee (A \vee \neg B) \qquad \text{de Morgen}$$

$$\equiv (A \wedge \neg B) \vee (B \wedge \neg A) \vee A \vee \neg B \qquad \text{de Morgen}$$

$$\equiv (A \land \neg B) \lor \big((B \land \neg A) \lor A \big) \lor \neg B \quad \text{assoc}$$

$$\equiv (A \land \neg B) \lor \big((B \lor A) \lor \neg B \big) \quad \substack{\text{distr} \\ \text{and} \\ \text{assoc}}$$

$$\equiv (A \land \neg B) \lor \underbrace{(\underbrace{B \lor \neg B}_{True}) \lor A}_{} \quad \text{assoc}$$

$$\underbrace{}_{True}$$

(C) KB: $\neg \big((A \Longleftrightarrow B) \vDash (A \lor \neg B) \big)$

$$\equiv \neg \Big(\neg \big((\neg A \lor B) \land (\neg B \lor A) \big) \lor (A \lor \neg B) \Big)$$

$$\equiv \neg \Big(\big(\neg (\neg A \lor B) \lor \neg (\neg B \lor A) \big) \lor (A \lor \neg B) \Big)$$

$$\equiv \neg \big(\neg (\neg A \lor B) \lor \neg (\neg B \lor A) \big) \land \neg (A \lor \neg B)$$

$$\equiv (\neg A \lor B) \land (\neg B \lor A) \land (\neg A \land B)$$

CNF $= (\neg A \lor B) \land (\neg B \lor A) \land \neg A \land B$

$S_1 = \neg A \lor B$

$S_2 = \neg B \lor A$

$S_3 = \neg A$

$S_4 = B$

$S_5 = A \qquad (S_2 + S_4)$

$S_6 = \{\} \qquad (S_3 + S_5)$

$\therefore \ \neg((A \Leftrightarrow B) \models (A \lor \neg B))$ is false.

$\therefore \ (A \Leftrightarrow B) \models (A \lor \neg B)$ is True.

2. (a) $(study \Leftrightarrow Pass) \land \neg(study \Leftrightarrow \neg Pass)$

study (¬)	Pass (¬)	study ⇒ Pass	Pass ⇒ study	study ⇔ Pass	
T (F)	T (F)	T	T	T	
F (T)	F (T)	T	T	T	
T (F)	F (T)	f	T	f	
F (T)	T (F)	T	f	F	

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¬study ⇒¬Pass	¬Pass ⇒ ¬ study	¬ Study ⟺ ¬Pass	¬(¬study ⟺¬Pass)
T	T	T	F
T	T	T	F
T	F	F	T
F	T	F	T

This is an Unsatisfiable Sentence

(b)

((study ∧ Rest) ⇒ Pass) ∧ (Study ⇒¬Rest) ∧ (Rest ⇒ ¬study) ∧ Pass

A — ((study ∧ Rest) ⇒ Pass)
B — (Study ⇒¬Rest)
C — (Rest ⇒ ¬study)

A ≡¬(study ∧ Rest) ∨ Pass ≡ ¬study ∨ ¬Rest ∨ Pass

B ≡ ¬ study ∨ ¬Rest

C ≡ ¬ Rest ∨ ¬study

(¬study ∨ ¬Rest ∨ Pass) ∧ (¬study ∨ ¬Rest) ∧ (¬Rest ∨¬study) ∧ Pass

Same

≡ (¬study ∨¬Rest ∨Pass) ∧ (¬study ∨ ¬Rest) ∧ Pass

≡ (¬(study ∧ Rest) ∨ Pass) ∧ ¬(study ∧ Rest) ∧ Pass

$\equiv \neg (\text{study} \wedge \text{Rest}) \wedge \underline{\neg \text{Pass}} \quad \wedge \underline{\neg (\text{study} \wedge \text{Rest})} \wedge \underline{\text{Pass}}$

Same

contradiction

$\equiv \neg (\text{study} \wedge \text{Rest}) \wedge \neg \text{Pass} \wedge \text{Pass}$

is unsatisfiable

3.

$S_1 : P \Rightarrow (Q \vee R)$

$S_2 : \neg (P \wedge Q) \Longleftrightarrow S$

$S_1 \equiv \neg P \vee (Q \vee R) \equiv \neg P \vee Q \vee R$

$S_2 \equiv (\neg (P \wedge Q) \Rightarrow S) \wedge (S \Rightarrow \neg (P \wedge Q))$

$\equiv ((P \wedge Q) \vee S) \wedge (\neg S \vee (\neg P \vee \neg Q))$

$\equiv ((S \vee P) \wedge (S \vee Q)) \wedge (\neg S \vee \neg P \vee \neg Q)$

$\equiv (S \vee P) \wedge (S \vee Q) \wedge (\neg S \vee \neg P \vee \neg Q)$

4.

KB: $S_1 : A \lor B$

$S_2 : \neg B \lor \neg C$

$S_3 : \neg C \lor D$

$S_4 : B \lor \neg E$

$S_5 : \neg D \lor E$

(a)

$S_6 = E$ $KB \models \neg E$

$S_7 = \neg C \lor A$ $(S_1 + S_2)$

$S_8 = B$ $(S_6 + S_4)$

$S_9 = \neg C$ $(S_6 + S_2)$

$S_{10} = B \lor \neg D$ $(S_4 + S_5)$

$S_{11} = \neg C \lor \neg E$ $(S_2 + S_4)$

$S_{12} = \neg C \lor \neg D$ $(S_5 + S_{11})$

$S_{13} = \neg C \lor E$ $(S_3 + S_5)$

$S_{14} = \neg C \lor B$ $(S_8 + S_9)$ $(S_3 + S_{10})$

∴ KB doesn't enail $\neg E$

Because ran out of clauses.

(b) $KB \models \neg C$

$S6 = C$

$S7 = D$ $(S_3 + S6)$

$S8 = \neg B$ $(S_2 + S6)$

$S9 = \neg \bar{E}$ $(S_4 + S8)$

$S10 = \neg D$ $(S5 + S9)$

$S11 = \{\}$ $(S_{10} + S_7)$

$\therefore KB \models \neg C$ ✓