CS-134/PSY-141: Cognitive Modeling
Homework 2: Modeling Response Times

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November 2025

1 Exercise A: Fitting the half-normal model

In this section section, we attempt to fit a model to reaction times such that the distribution can be approximated with a Half Normal (HN) distribution, which takes only one parameter, standard deviation σ.

1.1 Parameter Estimation

In order to understand a reasonable *mean* value of σ, I first had to select a prior distribution and see which range of σs would produce a distribution of RTs that matched my expectations. I accomplished the latter in Mathematica using the **Manipulate**[] function on the **HalfNormal**[] function, adjusting the mean, $1/\sigma$, until I got values between 100 and 1000 ms (in my limited knowledge of reaction time, this seemed safe). Then I chose the exponential distribution as a prior, given that it would produce strictly positive values over the range of values I wanted. From this, I decided on a reasonable mean σ, around 500, and randomly sampled from the exponential distribution with $\lambda = 1/\sigma = 1/500$. Finally, I applied these sampled parameters to the HN function to simulated a new sample of RTs. This allowed me to sanity check the shape of the model itself.

1.2 Prior and Posterior of the Model Parameter

Having made these decisions, I used PyMC to again sample from an exponential prior over the model parameter σ. With these values and our data, I used the MCMC simulator to generate samples of posterior distribution of σ (**Fig. 1**).

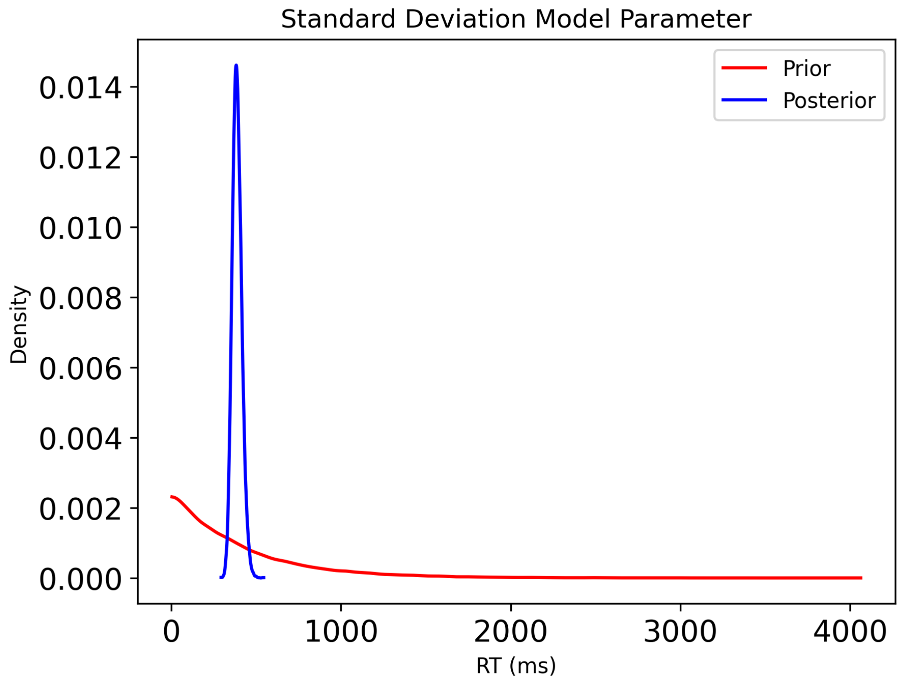

Figure 1: Prior and posterior of model parameter σ

1.3 Prior Predictive and Posterior Predictive

I could then use this sample of σ values to generate a prior predictive (i.e., the RT distribution based on my expectations of the Half Normal model alone) and a posterior predictive (i.e., the RT distribution when the prior is weighted by likelihood of the observed data). The results are shown in **Fig. 2**

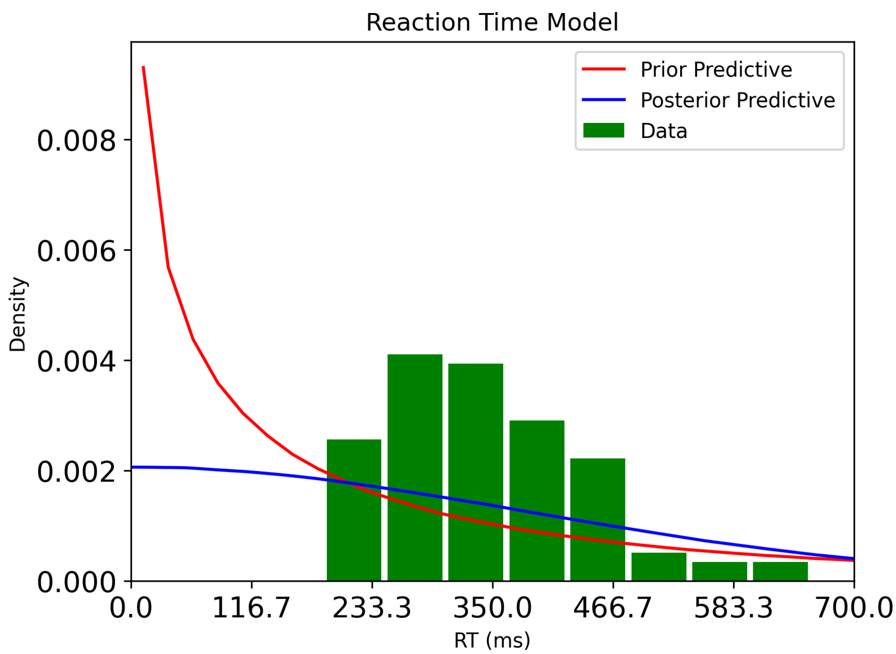

Figure 2: Prior predictive and posterior predictive of the HN model given sampled σ, compared against the observed data

1.4 Qualitative Model Fit of the Half Normal

In my opinion, this model provides a poor match for the shape (frequency and range) of the RTs. I believe I have either selected an inadequate prior (Exponential) over the parameters of my model and/or the wrong likelihood model (Half Normal) to fit the data. I think that the Half Normal itself is not well suited to the data because the HN has most of its density concentrated around smaller values, whereas the empirical data only exist within a range of about $200 - 600$ ms. My prior over the standard deviation of the half normal might be wrong because I actually have no reason to believe that the standard deviation is produced from a generative model with a majority of its probability density positively skewed toward smaller values.

2 Exercise B: Fitting a Gamma distribution

In this section, we attempt to fit a model to reaction times such that the distribution can be approximated with a Gamma distribution, which takes two parameters, which in PyMC are mean μ and standard deviation σ.

2.1 Prior and Posterior of the Model Parameters

Based on my observation of the empirical data, I guessed that the mean parameter would be Normal and centered around 350 with a lower standard deviation of 10 (**Fig. 3**). Because I was more confident that the mean might be around this point, I went for a Normal prior with more probability density toward my best guess (rather than a completely uninformed prior). On the other hand, as per the instructions, I defined a prior over the standard deviation of the Gamma model as a uniform distribution between the ranges of 50 to 100 (**Fig. 4**). Because I had a stronger guess for μ, an uninformed prior over σ meant that the model would be more flexible in this case.

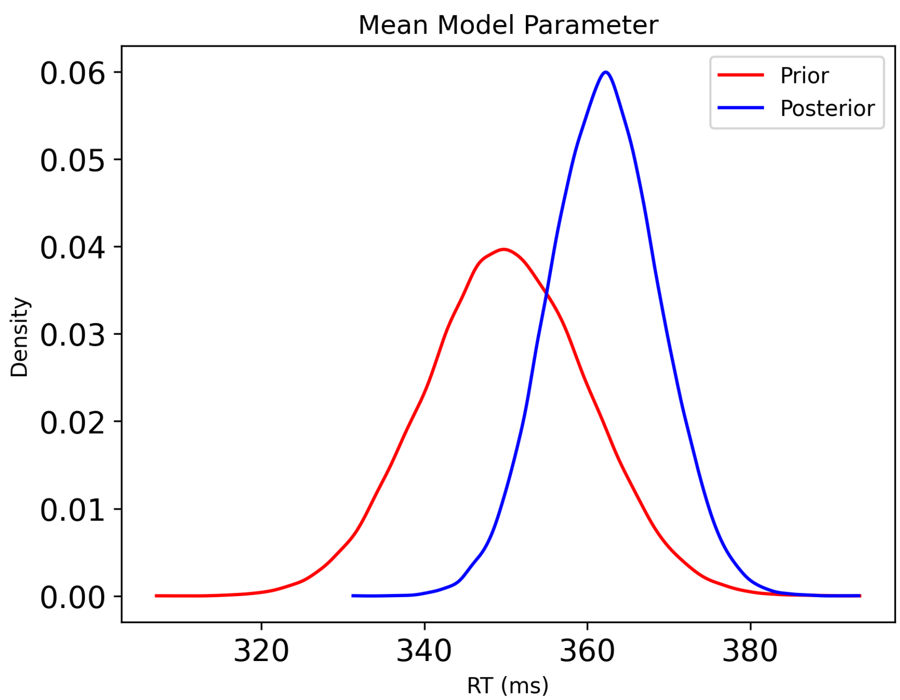

Figure 3: Prior and posterior of μ parameter

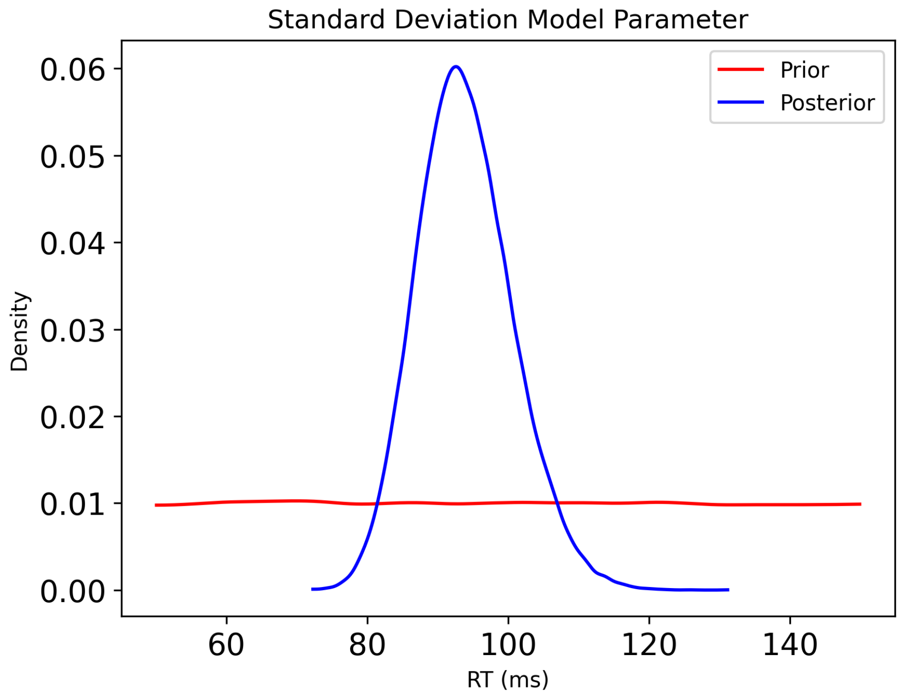

Figure 4: Prior and posterior of σ parameter

2.2 Prior Predictive and Posterior Predictive

This time, I sampled from both parameter priors μ and σ to pass through the Gamma function thus generating a prior predictive and a posterior predictive for the model (**Fig. 5**).

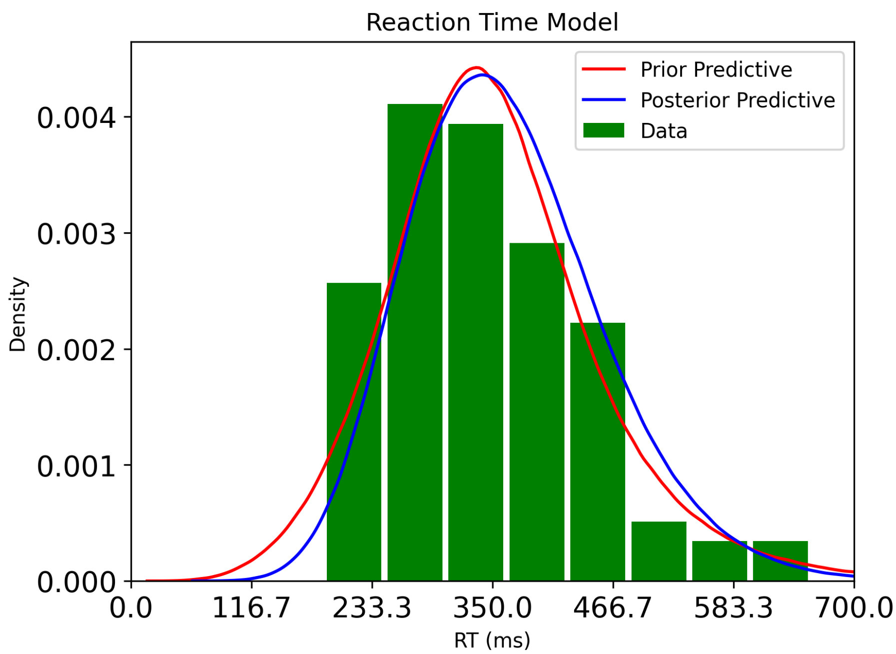

Figure 5: Prior and posterior predictive of the Gamma model

2.3 Qualitative Model Fit of the Gamma Normal

I think this model fits well! In terms of the relationship between the posterior predictive and the observed data, there is much more observed data predicted/explained by our model. I worry that the likelihood may dominate the model as the posterior and prior predictives are nearly overlapping, but perhaps this is a sign that my prior was well-suited to the data overall. Some aspects of the data are not adequately described by the model however. For instance, the posterior fails to capture the skew of the data, with a longer, positive tail.

3 Exercise C – Comparison and Discussion

3.1 Visual Comparison

Comparing the match between the posterior predictive and the histogram of observed reaction times for each model, it appears that the Gamma distribution is a better fit to the participant's data. The posterior predictive of the Half Normal model assigns roughly equal density across the RT range (and beyond) with a slight preference for smaller RTs. On the other hand, the Gamma Model fits not only the plausible range but also the general shape of the distribution.

3.2 Bayes Factor Comparison

The Bayes Factor (BF) was calculated as a ratio of marginal log likelihoods, approximated using the sequential Monte Carlo sampling method from PyMC. In the ratio, $H_0 = \Gamma(\mu, \sigma)$ i.e., the Gamma Model and $H_1 = HN(\sigma)$ i.e., the Half Normal Model. From this process, we see:

- $BF_{01} = 1.55e + 31$
- $BF_{10} = 6.46e - 32$

The BF indicates that the data are 1.55e+31 times more likely when modeled with the Gamma distribution than the Half Normal. This is consistent with my visual evaluation of the model fits. Despite having more parameters, the Gamma model explains the data better than the more simple Half Normal distribution. I think this is because although having an additional parameter further spreads the prior ("hedges our bets") with respect to the likelihood thus penalizing more complex models, the likelihood of the Gamma distribution better fit the data overall. I also think this may be because the Half Normal model and exponential prior distribution I used for the standard deviation was poorly suited to the problem.

3.3 Implications

What does this comparison tell us about human response times to the stopping of a sound? My conclusion has to do with the likelihood functions used in our models: the Half Normal seems poorly suited to the data. Because the HN likelihood function necessarily begins at zero (as it is a truncated Gaussian), the posterior predictive predicts the bulk of RTs to be concentrated at quite low values. However, based on the empirical control-condition data, it appears that RTs tend to be in the 250-400 ms range with no evidence to suggest that RTs close to 0 are even plausible. So, we can assume that human RTs, at least in the auditory domain and for this participant, have a clear psychological/physiological lower bound. This is where the Gamma distribution succeeds. Gamma distributions can gravitate across the x-axis, dependent on the μ parameter, so it can center itself at a higher value, such as 350 ms, with little plausibility granted to values below 200.

Additionally, there is a tailed drop-off at the upper end of our observed data, showing that human RTs may lag on occasion (consistent with my own experience of dazing off or growing tired when participating in psych experiments). This general behavior is captured by both models in theory. Of course, neither is a perfect model (for example, neither the HN nor the Gamma perfectly captures the stark drop-off that occurs for RTs below 200 ms), the Gamma tells us more about the behavior of human RTs to the stopping of a sound.