Ejercicios mm1

2) a) $\cos(3\alpha) + \operatorname{sen}(3\alpha)i = [\cos(\alpha) + i\operatorname{sen}(\alpha)]^3$

$= \cos^3(\alpha) + 3\cos^2(\alpha)\operatorname{sen}(\alpha)i + \underbrace{i^2}_{-1} 3\cos(\alpha)\operatorname{sen}^2(\alpha)$

$+ \underbrace{i^3}_{-i} \operatorname{sen}^3(\alpha)$

$\Rightarrow \underbrace{\cos^3(\alpha) - 3\cos(\alpha)\operatorname{sen}^2(\alpha)}_{Re(z)} + i\underbrace{(3\cos^2(\alpha) - \operatorname{sen}^3(\alpha))}_{Im(z)}$

$\Rightarrow \cos(3\alpha) = Re(z) = \cos^3(\alpha) - 3\cos(\alpha)\operatorname{sen}^2(\alpha)$

b) $\Rightarrow \operatorname{sen}(3\alpha) = Im(z) = 3\cos^2(\alpha) - \operatorname{sen}^3(\alpha)$

5) a) $(2i)^{1/2} \Rightarrow \sqrt{2i} = \left\{ \sqrt{2}\, e^{i\frac{\frac{\pi}{2} + 2K\pi}{2}} : 0 \le K \le 1 \right\}$

$\Rightarrow \underbrace{\sqrt{2}\, e^{\frac{\pi}{4}i}}_{K=0}$ y $\underbrace{\sqrt{2}\, e^{\frac{5\pi}{4}i}}_{K=1}$ son las raíces

b)

$\theta = \tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$ $\quad \sqrt{1+3} = |2|$

$(1-\sqrt{3}i)^{1/2} \Rightarrow \sqrt[2]{1-\sqrt{3}i} = \left\{\sqrt{2}\, e^{\frac{i\left(2k\pi - \frac{\pi}{3}\right)}{2}} : 0 \le k \le 1\right\}$

$k=0 \Rightarrow \sqrt{2}e^{i\frac{-\pi}{6}} \qquad k=1 \Rightarrow \sqrt{2}e^{i\frac{5\pi}{6}}$

c) $(-1)^{1/3} \Rightarrow \sqrt[3]{-1} = \left\{e^{i\frac{2k\pi}{2}} : 0 \le k \le 2\right\}$

$\Rightarrow k_0 = -1 \; ; \; k_1 = e^{i\pi} \; ; \; k_2 = e^{2i\pi}$

d) $8^{1/6} \Rightarrow \sqrt[6]{8} = \left\{\sqrt{2}\, e^{ik\pi} : 0 \le k \le 5\right\}$

$k_0 = \sqrt{2} \; ; \; k_1 = \sqrt{2}e^{i\pi} \; ; \; k_2 = \sqrt{2}e^{i2\pi}$

$0 \le \theta \le 2\pi$

e) $(-8-8\sqrt{3}i)^{1/4} \Rightarrow \sqrt[4]{-8-8\sqrt{3}i} = \left\{2e^{i\frac{\frac{4\pi}{3}+2k\pi}{4}} : 0 \le k \le 3\right.$

$\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3} \qquad \sqrt{8^2+8^2\cdot 3} = 16 \; \text{y} \; \sqrt[4]{16} = 2$

$k_0 = 2e^{i\frac{\pi}{6}} \; ; \; k_1 = 2e^{i\frac{7\pi}{12}} \; ; \; k_2 = 2e^{i\frac{13\pi}{12}} \; ; \; k_3 = 2e^{i\frac{19\pi}{12}}$

6) a) $\log(-ie) = \log\left[e\, e^{i\left(-\frac{\pi}{2}+2n\pi\right)}\right] = \underbrace{1}_{\ln(e)} + i\left(\frac{-\pi}{2}+2n\pi\right)$

con valor principal (V.P): $1 - i\frac{\pi}{2}$

b) $\log(1-i) = \log\left[\sqrt{2}\, e^{i\left(-\frac{\pi}{4}+2n\pi\right)}\right] = \frac{1}{2}\ln(2) + i\left(\frac{\pi}{4}+2n\pi\right)$

$= \frac{1}{2}\ln(2) - \frac{\pi}{4}i \rightarrow V.P$

c) $\log(e) = \log\left[e\, e^{i(2n\pi i)}\right] = 1 + 2n\pi i$

$\downarrow$

$V.P = 1$

d) $\log(i) = \log\left[e^{i\left(\frac{\pi}{2}+2n\pi\right)}\right] = \frac{\pi}{2} + 2n\pi = \dots$

$\dots = \pi\left(\frac{1}{2}+2n\right)$