Tarea 5 mm1

Ejercicio 6 de la sección 2.2.4

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Mm1 2.2.4

6) a) $|a\rangle = a^0 + a^1|q_1\rangle + a^2|q_2\rangle + a^3|q_3\rangle$

Comprobando si es E.V.s al que se llamará: $\mathbb{E}$:

1) $|a\rangle + |b\rangle = (a^\alpha + b^\alpha)|q_\alpha\rangle = c^\alpha|q_\alpha\rangle = |c\rangle \in \mathbb{E}$

2) $|a\rangle + |b\rangle = |c\rangle = c^\alpha|q_\alpha\rangle = (a^\alpha + b^\alpha)|q_\alpha\rangle$

$= (b^\alpha + a^\alpha)|q_\alpha\rangle = c^\alpha|q_\alpha\rangle = |b\rangle + |a\rangle$

3) $(|a\rangle + |b\rangle) + |c\rangle = (a^\alpha + b^\alpha)|q_\alpha\rangle + c^\alpha|q_\alpha\rangle$

$= (a^\alpha + b^\alpha + c^\alpha)|q_\alpha\rangle = a^\alpha|q_\alpha\rangle + (b^\alpha + c^\alpha)|q_\alpha\rangle$

$= |a\rangle + (|b\rangle + |c\rangle)$

4) El neutro lo defino como:

$|a\rangle = a^\alpha|q_\alpha\rangle = a^0|q_0\rangle + a^j|q_j\rangle = 0 \iff a_0 = 0 \wedge a^j = 0 \ \forall j$

5) El simétrico es:

$-|a\rangle \Rightarrow |a\rangle - |a\rangle = (a^\alpha - a^\alpha)|q_\alpha\rangle = 0$

6) $\alpha|c\rangle = \alpha c^\alpha|q_\alpha\rangle \Rightarrow \alpha(c^\alpha|q_\alpha\rangle) = \cdots$

$\dots = \alpha c^{\alpha} |q_{\alpha}\rangle .$

7) $\alpha(\beta|c\rangle) = \alpha(\beta c^{\alpha}|q_{\alpha}\rangle) = \alpha\beta c^{\alpha}|q_{\alpha}\rangle$

8) $(\alpha+\beta)|c\rangle = \underbrace{(\alpha+\beta)}_{\gamma} c^{\alpha}|q_{\alpha}\rangle = \gamma c^{\alpha}|q_{\alpha}\rangle$

$= (\alpha c^{\alpha} + \beta c^{\alpha})|q_{\alpha}\rangle = \alpha|c\rangle + \beta|c\rangle$

9) $\alpha(|a\rangle + |b\rangle) = \alpha(a^{\alpha} + b^{\alpha}|q_{\alpha}\rangle) = \dots$

$\dots = (\alpha(a^{\alpha}+b^{\alpha})|q\rangle) = (\alpha a^{\alpha} + \alpha b^{\alpha})|q_{\alpha}\rangle$

$= \alpha|a\rangle + \alpha|b\rangle$

10) $\alpha = 1 \Rightarrow 1|a\rangle = 1a^{\alpha}|q_{\alpha}\rangle = |a\rangle$

Si forma E.V.

b) $|d\rangle = |b\rangle \odot |r\rangle \Leftrightarrow (d^0, d) = (b^0 r^0 - b \cdot r, r^0 b + b^0 r + b \times r)$

$\Rightarrow |d\rangle = (b^0 + b^i |q_i\rangle) \odot (r^0 + r^j |q_j\rangle)$

$= b^0 \odot r^0 + b^0 \odot r^j |q_j\rangle + b^i |q_i\rangle \odot r^0 + b^i |q_i\rangle \odot r^j |q_j\rangle$

$= b^0 r^0 + \underbrace{b^i |q_i\rangle \odot r^i |q_i\rangle}_{i=j} = b^0 r^0 - b \cdot r = d^0$

$= b^0 \odot r^j |q_j\rangle + b^i |q_i\rangle \odot r^0 + b^i |q_i\rangle \odot r^j |q_j\rangle$

$= b^0 r + b r^0 + b \times r$

c) $|d\rangle = |b\rangle \odot |r\rangle = b^\alpha |q_\alpha\rangle \odot r^\alpha |q_\alpha\rangle = \ldots$

$\ldots = (b^0 |q_0\rangle + b^i |q_i\rangle) \odot (r^0 |q_0\rangle + r^j |q_j\rangle)$

$= b^0 |q_0\rangle \odot r^0 |q_0\rangle + b^0 |q_0\rangle \odot r^j |q_j\rangle + b^i |q_i\rangle \odot \cdots$

$\cdots \odot r^0 |q_0\rangle + b^i |q_i\rangle \odot r^j |q_j\rangle$

$= \underset{\downarrow \; b^0 r^0}{a^0} |q_0\rangle$