Ejercicios mm1

2. a) $\nabla(\phi\psi) = \hat{e}_x \frac{\partial(\phi\psi)}{\partial x} + \hat{e}_y \frac{\partial(\phi\psi)}{\partial y} + \hat{e}_z \frac{\partial(\phi\psi)}{\partial z}$

$= \partial^i(\phi^K \psi^K)\,\hat{e}_i \Rightarrow$ Aplicando derivación usual para campos escalares (regla de leibniz)

$= (\partial^i \phi^K)\psi^K + (\partial^i \psi^K)\phi^K = \psi \nabla\phi + \phi\nabla\psi$

d) $\nabla\cdot(\nabla\times a) = (\nabla\cdot(\nabla\times a))^i = \partial^i(\nabla\times a)_i$

$= \partial^i \varepsilon_{ijk}\partial^j a^k = \varepsilon_{ijk}\partial^i\partial^j a^k$

$$= \begin{vmatrix} \partial^1 & \partial^2 & \partial^3 \\ \partial^1 & \partial^2 & \partial^3 \\ a^1 & a^2 & a^3 \end{vmatrix} = 0$$

$\nabla\times(\nabla\cdot a) = (\nabla\times(\nabla\cdot a))^i = \varepsilon^{ijk}\partial_j(\nabla\cdot a)$

$= \varepsilon^{ijk}\partial_j \partial^l a_l$

Se puede decir que son operaciones distintas. No son intercambiables.

f) $\nabla \times (\nabla \times a) = (\nabla \times (\nabla \times a))^i = \cdots$

$$\cdots = \varepsilon^{ijk} \partial_j (\nabla \times a) = \varepsilon^{ijk} \partial_j \varepsilon_{kmn} \partial^m a^n$$

$$= \varepsilon^{ijk} \varepsilon_{mnk} \partial^m \partial_j a^n = (\delta^i_m \delta^j_n - \delta^i_n \delta^j_m) \partial^m \partial_j a^n$$

$$= \delta^i_m \partial^m \delta^j_n \partial_j a^n - \delta^j_m \partial^m \partial_j \delta^i_n a^n$$

$$= \partial^i \partial_n a^j - \partial^j \partial_m a^i = \nabla (\nabla \cdot a) - \nabla^2 a$$