Calculo diferencial para cursos por competencias – Jorge Luis Gil Sevilla y Rebeca Días Téllez

DERIVADAS

1. $\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

Derivadas de funciones básicas

2. $\frac{d}{dx} x = 1$
3. $-\frac{d(c)}{dx} = 0$
4. $\frac{d(cx)}{dx} = c$
5. $\frac{d(cx^n)}{dx} = ncx^{n-1}$
6. $\frac{d(cu^n)}{dx} = ncu^{n-1}\frac{du}{dx}$
7. $\frac{d}{dx} cu = c \cdot \frac{du}{dx}$
8. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
9. $\frac{d}{dx}\sqrt{u} = \frac{u'}{2\sqrt{u}}$
10. $\frac{d}{dx}(u)^{\frac{1}{n}} = \frac{u'}{n(u)^{1-\frac{1}{n}}}$
11. $\frac{d(uv)}{dx} = uv' + u'v$
12. $\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vu' - v'u}{v^2}$

Derivadas de funciones exponenciales

13. $\frac{d(a^u)}{dx} = a^u \frac{du}{dx}\ln a$
14. $\frac{d(e^u)}{dx} = e^u \frac{du}{dx}$
15. $\frac{d(u^v)}{dx} = vu^{n-1} + u^v \frac{dv}{dx}\ln u$

Derivadas de funciones logaritmicas

16. $\frac{d(\log u)}{dx} = \frac{u' \log e}{u}$ o $\frac{1}{\ln(10)x}$
17. $\frac{d(\ln u)}{dx} = \frac{u'}{u}$

Derivadas de funciones trigonométricas

18. $\frac{d(\operatorname{sen} u)}{dx} = \cos u \frac{du}{dx}$
19. $\frac{d(\cos u)}{dx} = -\operatorname{sen} u \frac{du}{dx}$
20. $\frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx}$
21. $\frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx}$
22. $\frac{d(\sec u)}{dx} = \sec u \tan u \frac{du}{dx}$
23. $\frac{d(\csc u)}{dx} = -\csc u \cot u \frac{du}{dx}$

Derivadas de funciones trigonométricas inversas

24. $\frac{d(\operatorname{sen}^{-1} u)}{dx} = \frac{u'}{\sqrt{1-u^2}}$
25. $\frac{d(\cos^{-1} u)}{dx} = -\frac{u'}{\sqrt{1-u^2}}$
26. $\frac{d(\tan^{-1} u)}{dx} = \frac{u'}{1+u^2}$
27. $\frac{d(\cot^{-1} u)}{dx} = -\frac{u'}{1+u^2}$
28. $\frac{d(\sec^{-1} u)}{dx} = \frac{u'}{|u|\sqrt{u^2-1}}$
29. $\frac{d(\csc^{-1} u)}{dx} = -\frac{u'}{|u|\sqrt{u^2-1}}$

Derivadas de funciones hiperbólicas

30. $\frac{d}{dx}\operatorname{senh}(u) = \cosh(u) \cdot u'$
31. $\frac{d}{dx}\cosh(u) = \operatorname{senh}(u) \cdot u'$
32. $\frac{d}{dx}\tanh(u) = \operatorname{sech}^2(u) \cdot u'$
33. $\frac{d}{dx}\coth(u) = -\operatorname{csch}^2(u) \cdot u'$
34. $\frac{d}{dx}\operatorname{sech}(u) = -\operatorname{sech}(u) \cdot \tanh(u) \cdot u'$
35. $\frac{d}{dx}\operatorname{csch}(u) = -\operatorname{csch}(u) \cdot \coth(u) \cdot u'$

$\operatorname{senh}(u) = \frac{e^u - e^{-u}}{2}$ $\quad \operatorname{csch}(u) = \frac{2}{e^u - e^{-u}}$

$\cosh(u) = \frac{e^u + e^{-u}}{2}$ $\quad \operatorname{sech}(u) = \frac{2}{e^u + e^{-u}}$

$\tanh(u) = \frac{e^u - e^{-u}}{e^u + e^{-u}}$ $\quad \coth(u) = \frac{e^u + e^{-u}}{e^u - e^{-u}}$

Derivadas de funciones hiperbólicas inversas

36. $\operatorname{senh}^{-1}(u) = \frac{u'}{\sqrt{u^2+1}}$
37. $\cosh^{-1}(u) = \frac{u'}{\sqrt{u^2-1}}$
38. $\tanh^{-1}(u) = \frac{u'}{1-u^2}$
39. $\coth^{-1}(u) = \frac{u'}{1-u^2}$
40. $\operatorname{sech}^{-1}(u) = \frac{-u'}{|u|\sqrt{1-u^2}}$
41. $\operatorname{csch}^{-1}(u) = \frac{-u'}{|u|\sqrt{1+u^2}}$

Derivadas de funciones implícitas

$$\frac{dy}{dx} = -\frac{\frac{\delta}{\delta x}}{\frac{\delta y}{\delta x}}$$

Derivadas

1. $\frac{d}{dx}(x) = 1$
2. $\frac{d}{dx}(ax) = a$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}(\cos x) = -\sin x$
5. $\frac{d}{dx}(\sin x) = \cos x$
6. $\frac{d}{dx}(\tan x) = \sec^2 x$
7. $\frac{d}{dx}(\cot x) = -\csc^2 x$
8. $\frac{d}{dx}(\sec x) = \sec x \tan x$
9. $\frac{d}{dx}(\csc x) = -\csc x(\cot x)$
10. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
11. $\frac{d}{dx}(e^x) = e^x$
12. $\frac{d}{dx}(a^x) = (\ln a)a^x$
13. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
14. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
15. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

Integrales

1. $\int 1\,dx = x + C$
2. $\int a\,dx = ax + C$
3. $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
4. $\int \sin x\,dx = -\cos x + C$
5. $\int \cos x\,dx = \sin x + C$
6. $\int \sec^2 x\,dx = \tan x + C$
7. $\int \csc^2 x\,dx = -\cot x + C$
8. $\int \sec x(\tan x)\,dx = \sec x + C$
9. $\int \csc x(\cot x)\,dx = -\csc x + C$
10. $\int \frac{1}{x}dx = \ln|x| + C$
11. $\int e^x dx = e^x + C$
12. $\int a^x dx = \frac{a^x}{\ln a} + C\ a > 0,\ a \neq 1$
13. $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C$
14. $\int \frac{1}{1+x^2}dx = \tan^{-1} x + C$
15. $\int \frac{1}{|x|\sqrt{x^2-1}}dx = \sec^{-1} x + C$

Homogeneas

Y=ux

dy= udx + xdu

u= y/x

X= uy

dx= udy + xdu

u= x/y

FUNCIONES TRIGONOMÉTRICAS

$$\text{sen} = \frac{o}{h} = \frac{1}{\text{csc}}$$
$$\cos = \frac{a}{h} = \frac{1}{\text{sec}}$$
$$\tan = \frac{o}{a} = \frac{\text{sen}}{\cos} = \frac{1}{\text{cot}}$$
$$\text{cot} = \frac{a}{o} = \frac{\cos}{\text{sen}} = \frac{1}{\tan}$$
$$\text{sec} = \frac{h}{a} = \frac{1}{\cos}$$
$$\text{csc} = \frac{h}{o} = \frac{1}{\text{sen}}$$

IDENTIDADES TRIGONOMÉTRICAS

De ángulo doble

$$\text{sen } 2A = 2 \text{ sen } A \cos A$$
$$\cos 2A = \cos^2 A - \text{sen}^2 A$$
$$\cos 2A = 1 - 2\,\text{sen}^2 A$$
$$\cos 2A = 2\cos^2 A - 1$$

De Pitágoras

$$\text{sen}^2 A + \cos^2 A = 1$$
$$\sec^2 A - \tan^2 A = 1$$
$$\csc^2 A - \cot^2 A = 1$$

Reducción de exponente

$$\text{sen}^2 A = \tfrac{1}{2} - \tfrac{1}{2}\cos 2A$$
$$\cos^2 A = \tfrac{1}{2} + \tfrac{1}{2}\cos 2A$$
$$\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$$

De multiplicación

$$\text{sen } A \csc A = 1$$
$$\tan A \cot A = 1$$
$$\text{sen } A \csc A = 1$$
$$\cos A \sec A = 1$$

Mitad de un ángulo

$$\text{sen}^2 \frac{x}{2} - \tfrac{1}{2} = -\tfrac{1}{2}\cos x$$
$$\cos^2 \frac{x}{2} - \tfrac{1}{2} = \tfrac{1}{2}\cos x$$
$$2\,\text{sen}^2 \frac{x}{2} - 1 = -\cos x$$

$$\text{sen}(A \pm B) = \text{sen } A \cos B \pm \cos A \text{ sen } B$$
$$\cos(A \pm B) = \cos A \cos B \pm \text{sen } A \pm \text{sen } B$$
$$\text{sen } A + \text{sen } B = 2 \text{ sen } \tfrac{1}{2}(A+B)\cos \tfrac{1}{2}(A-B)$$
$$\cos A + \cos B = 2\cos \tfrac{1}{2}(A+B)\cos \tfrac{1}{2}(A-B)$$
$$\text{sen } \alpha \cos \beta = \tfrac{1}{2}\text{sen}(\alpha+\beta) + \tfrac{1}{2}\text{sen}(\alpha-\beta)$$
$$\cos \alpha \cos \beta = \tfrac{1}{2}\cos(\alpha+\beta) + \tfrac{1}{2}\cos(\alpha-\beta)$$
$$\text{sen } \alpha \text{ sen } \beta = \tfrac{1}{2}\cos(\alpha-\beta) - \tfrac{1}{2}\cos(\alpha-\beta)$$

Fórmulas de ángulos compuestos

1. $\text{sen}(x+y) = \text{sen } x \cos y + \cos x \text{ sen } y$
2. $\text{sen}(x-y) = \text{sen } x \cos y - \cos x \text{ sen } y$
3. $\text{sen } 2x = 2 \text{ sen } x \cos x$
4. $\text{sen } x = \pm\sqrt{\dfrac{1-\cos x}{2}}$
5. $\cos(x+y) = \cos x \cos y - \text{sen } x \text{ sen } y$
6. $\cos(x-y) = \cos x \cos y + \text{sen } x \text{ sen } y$
7. $\cos 2x = \cos^2 x - \text{sen}^2 x$
8. $\cos \dfrac{x}{2} = \pm\sqrt{\dfrac{1+\cos x}{2}}$
9. $\text{sen } x + \text{sen } y = 2 \text{ sen } \dfrac{x+y}{2}\cos\dfrac{x-y}{2}$
10. $\text{sen } x - \text{sen } y = 2\cos\dfrac{x+y}{2}\text{sen}\dfrac{x-y}{2}$
11. $\cos x - \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$
12. $\cos x - \cos y = -2 \text{ sen } \dfrac{x+y}{2}\cos\dfrac{x-y}{2}$
13. $2 \text{ sen } x \text{ sen } y = -\cos(x+y) + \cos(x-y)$
14. $2 \text{ sen } x \cos y = \text{sen}(x+y) + \text{sen}(x-y)$
15. $2\cos x \text{ sen } y = \text{sen}(x+y) - \text{sen}(x-y)$
16. $2\cos x \cos y = \cos(x+y) + \cos(x-y)$
17. $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$
18. $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$
19. $\tan 2x = \dfrac{2\tan x}{1 - \tan x^2}$
20. $\tan \dfrac{x}{2} = \dfrac{1-\cos x}{\text{sen } x}$