

The Common Factor in Volatility Risk Premia*

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Abstract

We show that firm-level volatility risk premium obeys a strong factor structure. Such factor structure is also present in a decomposition of firms' volatility risk premium into good and bad counterparts, capturing compensation for realized volatility in positive and negative returns. Stocks with the weakest exposures to the common bad volatility risk premium factor earn average returns 7.32% higher than those with the strongest exposures. The common factor in total (bad) volatility risk premium predicts stock market returns at all horizons up to 24 months (from 6 months) both in-sample and out-of-sample. This predictive power is incremental to existing predictors.

Keywords: Firm volatility risk premia; cross-section of stock returns; market return predictability

JEL: E44, G11, G12, G14

*The support from the Czech Science Foundation under the 19-28231X (EXPRO) project is gratefully acknowledged.

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1 Introduction

This paper presents new empirical evidence on the behavior of individual stock volatility risk premia and examines the quantitative implications of this behavior in the two canonical tasks in finance: cross-sectional asset pricing and stock market forecasting. We focus on the “total” volatility risk premium of individual stocks, which we define as the difference between the risk-neutral and physical expectations of return volatility (Bollerslev, Tauchen, and Zhou, 2009), and the “good” and “bad” premia, which capture the compensation for realized volatility in positive and negative returns (Kilic and Shaliastovich, 2019). We document three novel findings. First, the volatility risk premia of US firms are synchronized. Second, only exposure to the common bad volatility risk premium is priced into the cross-section of stocks. Third, the common total (bad) volatility risk premium predicts excess market returns with statistically significant coefficients at all horizons up to 24 months (longer horizons from 6 months).

Although a large body of research focuses on higher-moment premiums in the equity index market, the stock-level volatility risk premium (VRP) is much less well understood.¹ The main contribution of this paper is to examine commonalities in firm-level VRPs across a cross-section of US firms. We begin our empirical investigation by constructing the daily volatility risk premium of 507 stocks over the period from January 2000 to December 2020. For each stock, we decompose its total volatility premium into good and bad components. Panel A in Figure 1 shows a cross-sectional distribution of annualized volatility risk premia. We observe a strong synchronicity in the dynamics of the individual premia. The first principal component explains about 60% (80%) of the time variation in daily firm-level total and good (bad) VRPs. Panels B and C further show that the commonality is even more pronounced within size and industry groups, clearly suggesting a strong factor structure in firms’ volatility risk premia.

We then compare the common factor of different components with the corresponding counterparts on the market index. Although the common and market volatility risk premiums exhibit similarities in dynamics, the comovement is far from perfect as we show in Figure 2. Prior literature (Bakshi and Kapadia, 2003) already documents this wedge

¹See, e.g., Kozhan, Neuberger, and Schneider (2013) for a skew premium in the US equity index market similar to a variance premium documented by Bollerslev et al. (2009).

between firm-level volatility risk premiums and those we extract from index options. This phenomenon is shown to be attributable to correlation risk (Driessen, Maenhout, and Vilkov, 2009), investor disagreement (Buraschi, Trojani, and Vedolin, 2014), and the default premium (González-Urteaga and Rubio, 2016). We complement these findings by linking the common factor in firm-level total, good, and bad volatility risk premia ($\mathcal{CVRP}^T, \mathcal{CVRP}^G, \mathcal{CVRP}^B$) to the cross-section of asset returns and the time-series predictability of aggregate market returns.

We begin by examining the link between the firms' sensitivities to common volatility risk premia and differences in expected returns. Specifically, we show this association for stocks in the Center for Research in Security Prices (CRSP) over a period spanning 2000-2020. We estimate the firms' betas on $\mathcal{CVRP}^T, \mathcal{CVRP}^G$, and \mathcal{CVRP}^B whilst controlling for the exposure to market volatility premium. The cross-sectional analysis demonstrates that the common bad volatility risk premium is strongly negatively priced in the cross-section of stock returns in both economic and statistical terms. The top \mathcal{CVRP}^B -beta quintile earns average raw returns of 6.48% and 7.32% per annum lower than the bottom quintile for equal- and value-weighted portfolios, respectively. The firms in the top \mathcal{CVRP}^G -beta quintile outperform those in the bottom quintile on average by 6.48% and 6.00% for the two weighting schemes. However, the statistical significance is weak. We find no evidence of statistical or economic significance of portfolio return spreads formed on \mathcal{CVRP}^T -betas.

Furthermore, we show that the significance of expected return differences is not driven by firms' exposure to market bad volatility risk premium (Kilic and Shaliastovich, 2019), innovations to the VIX index (Ang, Hodrick, Xing, and Zhang, 2006), common idiosyncratic volatility (Herskovic, Kelly, Lustig, and Van Nieuwerburgh, 2016), or common bad implied volatility (Barunik, Bevilacqua, and Ellington, 2023). The differences in average returns of extreme \mathcal{CVRP}^B -beta quintiles generate significant alphas relative to the five-factor model of Fama and French (2015) and six factors that add momentum of Jegadeesh and Titman (1993). In addition, we estimate significant risk premiums for the \mathcal{CVRP}^B factor using the Fama-MacBeth regression and three-pass regression procedure (Giglio and Xiu, 2021) using a set of one- and two-way portfolios sorted on \mathcal{CVRP}^B -betas and sensitivities to other key variables and firm-characteristics.

We next conduct in- and out-of-sample regression analyses to explore the relationships of commonalities among firm-level VRPs with stock market returns. Our analysis produces three key results. First, we show that common total and bad volatility risk premiums are strong predictors of stock market returns. The in-sample predictive power of \mathcal{CVRP}^T is statistically significant at all horizons considered up to 24 months, while \mathcal{CVRP}^B is a strong predictor at longer-term horizons starting from 6 months. The common good component weakly predicts the stock market returns only one month ahead. Quantitatively, the monthly in-sample R^2 statistics of univariate regressions with \mathcal{CVRP}^T and \mathcal{CVRP}^G increase monotonically with the horizon to 20% and 24% in the 24-month horizon case.

Second, a strong predictive power holds in out-of-sample estimation using only historical returns. The monthly out-of-sample R^2 statistics are positive at all horizons and monotonically increase to 17% and 19% when predicting two-year market returns. These values surpass those from corresponding statistics when using market volatility premia whose total and bad components also generate some out-of-sample benefits. Furthermore, our findings are in stark contrast to the extant predictors in [Welch and Goyal \(2008\)](#) that tend to produce a sound in-sample performance, which vanishes in an out-of-sample test.

Third, the bivariate regression analysis shows that common total and bad components of firm-level VRPs provide incremental predictive power relative to the market volatility risk premia whose total (good and bad) component is shown to predict the stock market returns at shorter (longer) horizons ([Bollerslev et al., 2009](#); [Kilic and Shaliastovich, 2019](#)). The predictive coefficients on \mathcal{CVRP}^T and \mathcal{CVRP}^B remain significant when we control for other volatility-related variables and the existing predictors.

Our paper links to several strands of the literature. We connect with the studies on the common factors in physical and risk-neutral return volatility. [Ang et al. \(2006\)](#) considers the pricing of aggregate volatility risk proxied by the VIX index in the cross-section of stock returns. [Chen and Petkova \(2012\)](#) decomposes aggregate market variance into an average correlation component and an average variance component, showing that only the latter commands a negative price of risk in the cross-section of portfolios sorted on idiosyncratic volatility. [Christoffersen, Fournier, and Jacobs \(2018\)](#) document a strong factor structure in the equity volatility levels, skews, and term structures. Our paper is the closest to

Herskovic et al. (2016) and Baruník et al. (2023) documenting that the common factor in idiosyncratic volatility and downside option-implied volatility is priced in the cross-section of stocks. We contribute to this literature by documenting a strong factor structure in firms' volatility risk premia. We find that the common factor in spreads between risk-neutral and physical volatilities has additional information content for the cross-section of asset returns relative to innovations in realized, option-implied, or residual volatility based on different factor models. Further, none of these other studies document a predictive power of common factors in volatilities for the stock market predictability.

We contribute to the literature on second-moment risk premia at the stock level. Driessen, Maenhout, and Vilkov (2009) attribute the differences between the index and individual stock variance premia to heterogeneous exposures to market-wide correlation shocks and connect this evidence to expected option returns. Buraschi et al. (2014) show that investor disagreement significantly drives the wedge between firms' and market volatility premiums. Focusing on stocks in the S&P 100 index, González-Urteaga and Rubio (2016) show that the difference in exposures of stock-level volatility risk premia to the market volatility premium can be explained by the default risk. Unlike these studies, our paper does not study the determinants of the cross-sectional variation of firms' VRPs. The key focus of our paper is the common factor of stock-level volatility risk premia and its implications for both individual stock returns and aggregate market predictability.

Our paper is also related to Bali and Hovakimian (2009) and Han and Zhou (2012), who connect the second-moment risk premium at the stock level to expected returns.² They do not consider the common factor in volatility risk premia, a central driver of asset prices in this paper, and do not examine the implications for stock market forecasting. While in this paper cross-sectional asset pricing predictability is driven by the exposure to the common factor, it only arises from the commonalities in bad volatility risk premia, highlighting the importance of asymmetric components of volatility risk premium. Further, our conclusions apply to the whole cross-section of CRSP firms and are not limited to optionable stocks. Finally, our paper is, to the best of our knowledge, the first study to examine the good and bad components of volatility premia at the firm level.

²It is worth emphasizing that Bali and Hovakimian (2009) use the average implied volatility across all eligible options, which is different from the model-free implied volatility. Thus, their predictor is arguably different from the variance risk premium.

Our article relates to a plethora of research on the market variance risk premium. In relation to our focus on market return predictability, the market variance risk premium predicts aggregate market returns up to six months (Bollerslev et al., 2009; Bekaert and Hoerova, 2014), while the good and bad components possess a long-term predictive power (Feunou, Jahan-Parvar, and Okou, 2018; Kilic and Shaliastovich, 2019). Bollerslev, Todorov, and Xu (2015) decomposes the total variance into its continuous- and jump-variance components and finds that much of this predictability is attributable to jump (tail) risk. Hollstein and Simen (2020) decomposes the market variance risk premium into the VRP of individual equities and the correlation risk premium (CRP) factors and shows that the two factors improve the predictability of the S&P 500 excess returns. We contribute to this literature by showing that the common factors in firms' volatility risk premia yield higher predictability for market returns both in-sample and out-of-sample. Unlike the market second-moment premia, the predictive power of common volatility premiums is incremental to existing predictors. Further, we provide novel evidence that this is the commonality in the firms' bad volatility premium, which is priced in the cross-section of stocks.

Finally, our work directly builds on the literature that examines the information content of decomposed volatility risk measures, primarily uncovering the important role of downside risk for asset pricing and return predictability.³ We also relate to the literature showing superior predictive information of option prices for stock returns.⁴ Using options data to measure the volatility risk premium at the firm level, our key contribution is to provide the first examination of the common factor among firms' volatility premia and study the implications for the cross-sectional predictability of stock returns and time-series forecastability of aggregate market returns.

The rest of the paper proceeds as follows. Section 2 describes the data. Section 3 reports the cross-sectional asset pricing implications of common volatility risk premia using portfolio and regression analyses. Section 4 investigates the predictive power of common volatility risk premia for market returns. Section 5 concludes. Additional results are relegated to the Appendix.

³A non-exhaustive list includes Ang, Chen, and Xing (2006), Barndorff-Nielsen, Kinnebrock, and Sheppard (2010), Bollerslev, Todorov, and Xu (2015), Segal, Shaliastovich, and Yaron (2015); Patton and Sheppard (2015), Farago and Tédongap (2018), Bollerslev, Li, and Zhao (2020), Baruník, Bevilacqua, and Tunaru (2022).

⁴A voluminous literature includes Dennis and Mayhew (2002), Xing, Zhang, and Zhao (2010), Cremers and Weinbaum (2010), An, Ang, Bali, and Cakici (2014), Muravyev, Pearson, and Pollet (2022), among others.

2 The factor structure in volatility risk premia

2.1 Data

We compute firm-level implied volatilities using daily data from OptionMetrics over the sample from January 03, 2000, to December 31, 2020.⁵ We include all stocks from the time of their IPO and listing with good options data coverage (we require stocks to have data spanning more than 5 years of continuous data). We exclude stocks due to i) bankruptcy; ii) delisting; and iii) mergers and acquisitions.⁶

We apply common options filtering rules to further exclude stock options with i) missing deltas; ii) missing implied volatility; iii) bid prices equal to 0; iv) nil volume; v) nil open interest; vi) negative bid-ask spread; and that vii) violate arbitrage conditions ([Bakshi, Kapadia, and Madan, 2003](#); [Carr and Wu, 2011](#); [Christoffersen, Jacobs, and Ornthanalai, 2012](#)). Following these filtering criteria, we then remove options with less than 4 contracts on a specific day and are left with 507 firms.⁷ Approximately 90% of these firms are large-cap with the remaining 10% being mid-cap stocks. Most stocks in our sample appear as a constituent of the S&P500 throughout our sample. Other stocks come from the Russell 1000 for which there is sufficient data coverage. To proxy the market volatility risk premium, we use the same filtering criteria for S&P500 index options.

Each day t , our data sample contains daily stock options observations for which we are able to calculate values of the implied variance and semi-variance measures. We consider call and put option prices with a maturity of around 30 days, considering all available strikes for each option. We keep implied variance measures within 23 and 37 days of maturity to represent a proxy of investor expectations of the one-month ahead fluctuations in the underlying asset.

In addition to option prices, we also use 5-minute returns from Kibot to construct the

⁵This period allows us to have good data coverage, which was insufficient to compute implied variances prior to January 2000.

⁶Examples of bankruptcies are General Motors, Lehman Brothers, and Merrill Lynch; examples of M&As are Raytheon and United Technologies, Dow Chemical and DuPont, and Walt Disney Company and 21st Century Fox.

⁷Most of these data filters are common in the option pricing literature. The volume and open interest constraints ensure that there is genuine interest in the option contract. Options that are close to maturity are removed ([Carr and Wu, 2011](#); [Christoffersen, Jacobs, and Ornthanalai, 2012](#)). We remove options with a negative bid-ask spread and those that violate no-arbitrage constraints, as these option prices are invalid and inconsistent with theory. Finally, we remove ITM contracts, as they tend to be more illiquid than OTM and ATM options ([Christoffersen et al., 2012](#)).

realized variance measures for all stocks that estimate physical expected variance. We use a 5-minute sampling frequency that has an optimal trade-off between the precision of the estimators and the impact of microstructure noise (Liu, Patton, and Sheppard, 2015).

2.2 Implied variances

We use the methods in Bakshi and Madan (2000) and Bakshi et al. (2003) to extract variance measures from the cross-section of option prices in a model-free manner. We consider the price of a variance contract that pays the squared logarithm of the return at time $t + 1$, which in our case corresponds to a fixed horizon of the next 30 days. Let $s_{i,t}$ denote the natural logarithm of the price $S_{i,t}$ of the i -th asset at time t . The payoff of the variance contract is $r_{i,t+1}^2 = (s_{i,t+1} - s_{i,t})^2$ and we define the total implied variance, $\mathcal{IV}_{i,t}^T$, as the price of the contract:

$$\mathcal{IV}_{i,t}^T \equiv e^{-r_t^f} \mathbb{E}_t^Q [r_{i,t+1}^2] \quad (1)$$

where $\mathbb{E}_t^Q[\cdot]$ is the expectation operator under the risk-neutral measure conditional on time t information and r_t^f is the risk-free rate. Kilic and Shaliastovich (2019) and Baruník et al. (2022) decompose the total implied variance measure given by Equation (1) into two components associated with positive and negative returns of the variance contract, respectively. In the absence of arbitrage, the sum of these components is the total implied variance. One obtains the prices of these components from OTM call and put options.

Implied variance measures the expectations of fluctuations in the underlying asset over a given horizon. Furthermore, Bakshi and Madan (2000) and Bakshi et al. (2003) show that one can compute $\mathcal{IV}_{i,t}^T$ from the prices OTM call and put options:

$$\mathcal{IV}_{i,t}^T = \underbrace{\int_{S_{i,t}}^{\infty} \frac{2(1 - \log(K/S_{i,t}))}{K^2} C(t, t+1, K) dK}_{\mathcal{IV}_{i,t}^G} + \underbrace{\int_0^{S_{i,t}} \frac{2(1 + \log(S_{i,t}/K))}{K^2} P(t, t+1, K) dK}_{\mathcal{IV}_{i,t}^B}, \quad (2)$$

where $C(\cdot)$ and $P(\cdot)$ denote the prices at time t of a call and put contract with a time to the expiration of one period and a strike price of K . Call option prices reflect a good state for the stock, while the prices of a put option reflect a bad state for the stock. The two states, most of the time, relate to contrasting investors' future expectations (Buraschi and Jiltsov, 2006). OTM puts are usually linked with hedging and insurance against equity

market drops ([Han, 2008](#)), whereas OTM calls are commonly associated with optimistic beliefs ([Buraschi and Jiltsov, 2006](#)).

We follow [Kilic and Shaliastovich \(2019\)](#) and decompose the payoff from the variance contract into two intuitive measures of expectations of good and bad events for the stock:

$$\mathcal{IV}_{i,t}^{\mathcal{T}} \equiv \underbrace{e^{-r_t^f} \mathbb{E}_t^Q \left[r_{i,t+1}^2 \mathbb{I}_{\{r_{i,t+1} > 0\}} \right]}_{\mathcal{IV}_{i,t}^{\mathcal{G}}} + \underbrace{e^{-r_t^f} \mathbb{E}_t^Q \left[r_{i,t+1}^2 \mathbb{I}_{\{r_{i,t+1} \leq 0\}} \right]}_{\mathcal{IV}_{i,t}^{\mathcal{B}}} \quad (3)$$

Intuitively, good and bad components of the payoff add to the total, and we can obtain the prices of its components in a model-free manner from a bundle of option prices upon a discretization of Equation (2). Appendix provides details of the procedure. The total implied variance is the weighted sum of option prices, and its components are identifiable by claims that have payoffs relating to the sign of the realized return. Good (bad) implied variance is identifiable from call (put) options that pay off when we realize a positive (negative) return. Consequently, the first (second) term in Equation (2) refers to a positive (negative) component of the payoff of the variance contract. Taking the square roots of respective implied variances gives us total implied volatility $\sqrt{\mathcal{IV}_{i,t}^{\mathcal{T}}}$, good implied volatility $\sqrt{\mathcal{IV}_{i,t}^{\mathcal{G}}}$, and bad implied volatility $\sqrt{\mathcal{IV}_{i,t}^{\mathcal{B}}}$.

2.3 Realized variances

In addition to the risk-neutral variance measures, we construct measures of the physical expected variance based on high-frequency data. Let $q_{i,k,t}$ denote the high-frequency logarithmic return of an asset i over the k -th time interval within some fixed time period t . In our case, k is fixed to a 5-minute time interval with N such intervals available over a day t . Following [Andersen, Bollerslev, Diebold, and Labys \(2003\)](#) and [Barndorff-Nielsen et al. \(2010\)](#), we construct the realized variance of returns on a given trading day t for a given stock i as $\mathcal{RV}_{i,t}^{\mathcal{T}} = \sum_{k=1}^N q_{i,k,t}^2$. We add the squared intraday log returns (difference in log prices when the market opens and closes). Similarly, we decompose realized variance into good and bad realized variances as:

$$\mathcal{RV}_{i,t}^{\mathcal{G}} = \sum_{k=1}^N q_{i,k,t}^2 \mathbb{1}_{(q_{i,k,t} > 0)} \quad \wedge \quad \mathcal{RV}_{i,t}^{\mathcal{B}} = \sum_{k=1}^N q_{i,k,t}^2 \mathbb{1}_{(q_{i,k,t} \leq 0)} \quad (4)$$

Intuitively, the good and bad realized variance measures capture information about

time variation in the positive and negative components of the physical distribution of stock returns. By construction, the cumulative realized variance adds up the cumulative good and bad realized variances. Barndorff-Nielsen et al. (2010) provide the theoretical underpinning of this decomposition based on a jump-diffusion process for a stock price, and under general assumption demonstrate that with $N \rightarrow \infty$ both good and bad variances converge to half of the Gaussian diffusion in the returns and positive and negative quadratic jump variation, respectively.

2.4 Firm-level volatility risk premia

Our analysis resonates with Bakshi and Kapadia (2003), one of the first papers examining the differences between the firm-level and market volatility risk premiums, and a recent study on the determinants of the cross-sectional variation of volatility risk premia by González-Urteaga and Rubio (2016). This article complements the extant literature by examining the commonalities in volatility risk premia of individual firms and their incremental information relative to the market volatility premium through the asset pricing context. In addition, this paper examines the good and bad components of volatility premia at the firm level.

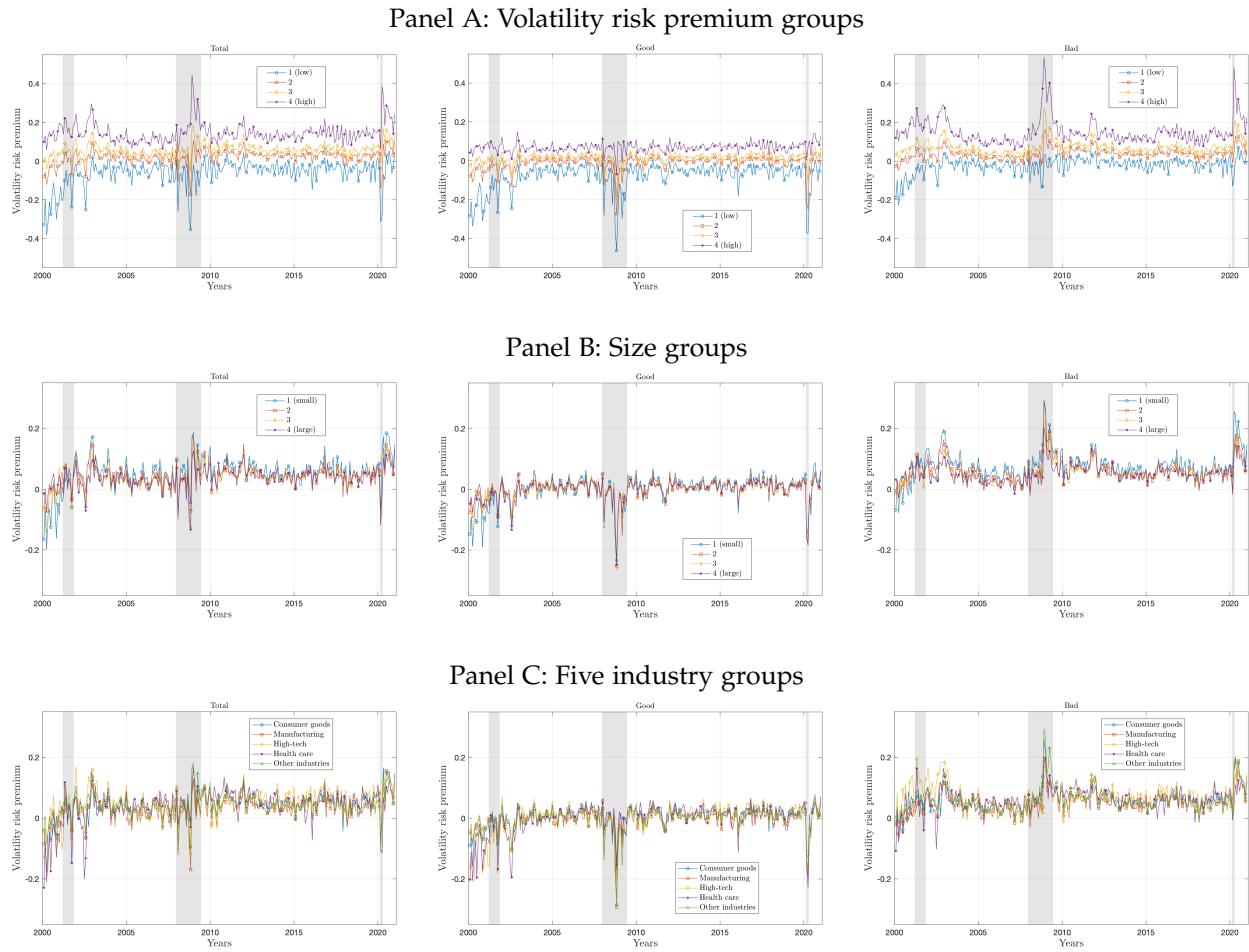
To formalize the discussion, we define the total, good, and bad volatility risk premiums of an asset i at time t as:

$$\mathcal{VRP}_{i,t}^{\mathcal{X}} = \sqrt{\mathcal{IV}_{i,t}^{\mathcal{X}}} - \sqrt{30 \times \mathcal{RV}_{i,t}^{\mathcal{X}}}, \quad \mathcal{X} \in \{\mathcal{T}, \mathcal{G}, \mathcal{B}\} \quad (5)$$

where $\mathcal{IV}_{i,t}^{\mathcal{X}}$ and $\mathcal{RV}_{i,t}^{\mathcal{X}}$ denote the daily estimates of the corresponding implied and realized variances. Figure 1 further illustrates the cross-sectional distribution of annualized volatility risk premia. Panel A presents the daily averages of the total, good, and bad time series within quartiles formed on each day. Although we observe a high degree of variation and differences in the magnitude of firms' volatility premia, the averages for the four groups exhibit an extraordinary comovement at the daily frequency. To quantify this commonality, the first principal component explains around 80% (60%) of the variation in daily bad (total or good) VRPs. Panel B shows the mean volatility risk premia within the quartiles formed on the market capitalization, whereas Panel C reports the averages within five industries based on the standard industry classification (SIC). There are even smaller

Figure 1. The cross-sectional distribution of volatility risk premia

The figure plots annualized total (left plots), good (middle plots), and bad (right plots) volatility risk premia (VRP) averaged within VRP (Panel A), size (Panel B), and industry groups (Panel C). At the end of each month, we divide all stocks into four equal groups based on the VRP (market equity) quartiles. Panels A and B demonstrate the within-group averages of volatility risk premia for the two cases. We also divide stocks into five industry groups based on the standard industry classification (SIC). Panel C illustrates the average volatility risk premia within each group.



differences in the time-series dynamics across different size and industry groups.

Panel A in Table 1 reports summary statistics of individual time series. The firm-level volatility risk premiums are, on average, positive with a bad (good) component having a larger (smaller) average value. This evidence is consistent with positive average volatility premiums extracted from index options. Note that the bad and good components do not sum up to the total quantity because we work with the volatility risk premium similar to Bakshi and Kapadia (2003), Driessen et al. (2009), and González-Urteaga and Rubio (2016), among others. There is a considerable time-series variation in firm-level VRPs, resulting in a wide range of volatility estimates. On average, the volatility of the total

Table 1. Firm-level volatility risk premia

This table presents summary statistics (Panel A) and one-factor regression results (Panel B) of daily firm-level volatility risk premia. For each firm, we compute the sample mean, standard deviation, skewness, kurtosis, and autocorrelation of total, good, and bad volatility risk premiums. Panel A reports the cross-sectional averages and percentiles of these statistics. The mean and standard deviation are expressed in annual terms, whereas other statistics are based on daily series. For each firm, we also estimate a univariate time-series regression of the firm's volatility risk premia on the corresponding common factor – an equal-weighted cross-sectional average of individual time series. Panel B reports the cross-sectional averages and percentiles of intercepts, slopes, and R^2 s. The numbers in shaded rows are Newey-West adjusted t-statistics of regression coefficients. The last row in Panel B shows the R^2 statistics from a pooled regression. The sample is from January 2000 to December 2020.

	Total				Good				Bad			
	Mean	5%	50%	95%	Mean	5%	50%	95%	Mean	5%	50%	95%
Panel A: Summary statistics												
Mean	4.69	0.45	4.35	10.39	0.21	-2.52	0.19	3.57	6.36	2.17	5.99	11.85
Std	9.11	4.41	8.29	16.89	7.11	3.64	6.42	12.52	7.77	3.45	7.24	14.06
Skew	-0.72	-2.55	-0.78	1.28	-2.33	-4.66	-2.05	-0.64	0.57	-1.16	0.42	2.78
Kurt	8.96	3.45	7.54	19.14	14.80	3.94	11.08	34.92	7.88	3.23	6.43	18.38
AR(1)	0.22	-0.10	0.19	0.55	0.24	-0.07	0.22	0.59	0.33	-0.03	0.32	0.72
Panel B: One-factor regression results												
Intercept	0.00	-0.05	0.00	0.06	0.00	-0.03	0.00	0.03	0.00	-0.06	0.00	0.06
	-0.10	-4.38	-0.25	4.00	-0.15	-5.28	0.08	4.68	0.14	-3.81	0.17	4.28
Slope	1.00	0.35	0.96	1.89	0.99	0.48	0.94	1.71	0.98	0.19	0.88	2.16
	5.98	1.87	5.45	11.76	7.34	2.11	6.64	14.29	5.73	1.73	5.12	11.99
R^2 (univar)	0.22	0.04	0.21	0.44	0.33	0.07	0.31	0.59	0.27	0.03	0.25	0.56
R^2 (pooled)	0.13				0.22				0.17			

(good and bad) VRPs tends to be higher (lower), consistent with the evidence on the market volatility premium and its components reported by [Kilic and Shaliastovich \(2019\)](#). Finally, the individual bad (total and especially bad) VRPs tend to have a positive (negative) skewness and appear to be more (less) persistent.

We now measure the degree of common variation in firm-level volatility risk premiums using the regression analysis. Following the definition of a common factor in firms' idiosyncratic volatility of [Herskovic et al. \(2016\)](#), we define the common total, good, and bad volatility risk premium factors as the equal-weighted average of individual total, good, and bad time series, respectively. For each individual firm, we then estimate a univariate regression of its volatility risk premium on the common factor. Panel B in Table 1 demonstrates the results of this one-factor regression analysis. The average R^2 statistic from univariate regressions for the total volatility risk premium is 22% and increases to 27% and 33% for the bad and good component cases, respectively. The pooled (OLS) regression reaches lower R^2 statistics ranging from 13% to 22%. In sum, the one-factor model fit is

quite strong given the daily frequency of data and non-overlapping periods used to compute the firm-level VRPs, reinforcing our conclusion that individual time series possess a significant common time variation.

3 Common volatility risk premia and expected stock returns

We now examine the asset pricing implications of common volatility risk premia in the cross-section of stocks. Following the extant literature, we conduct our analysis on a monthly frequency. For this reason, we construct the monthly version of the common total, good, and bad volatility risk premium factors. We first take the average of firms' daily total, good, or bad volatility risk premiums within each month to obtain the monthly observations. Then, we compute the equal-weighted cross-sectional average of the monthly total, good, or bad volatility risk premiums across firms. This procedure follows the construction of the monthly common idiosyncratic volatility of [Herskovic et al. \(2016\)](#).

3.1 Common and market volatility risk premiums

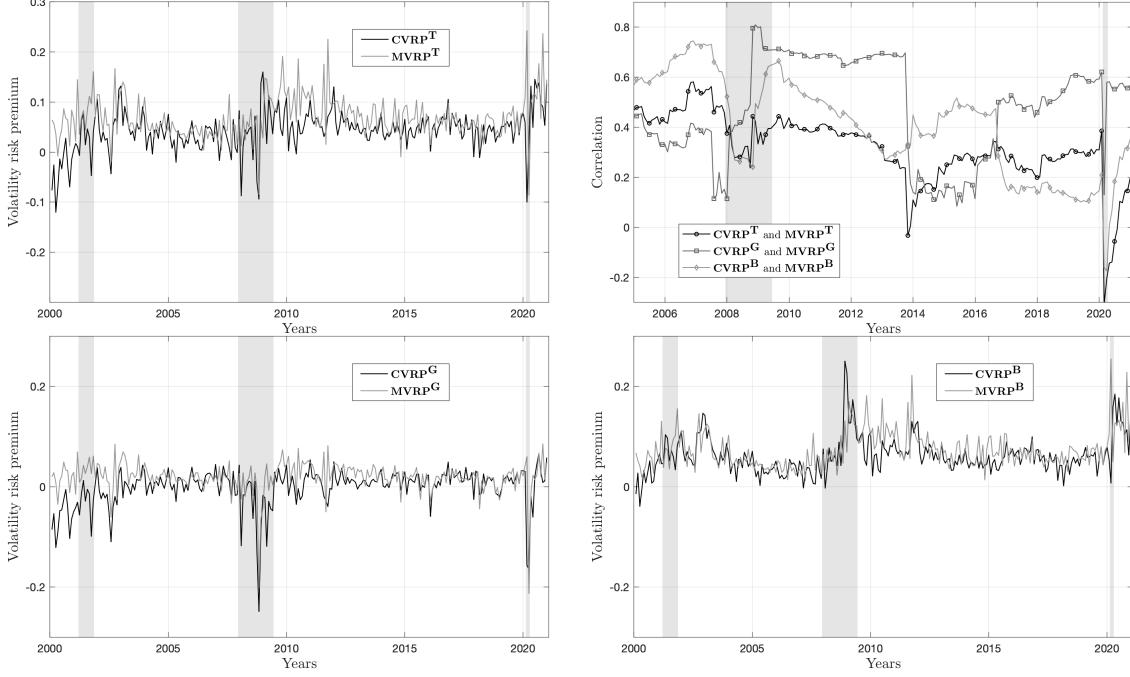
As a natural starts, we compare the common volatility risk premium factors with the market total, good, and bad volatility risk premiums $(MVRP^T, MVRP^G, MVRP^B)$. Figure 2 plots the time series of common and market volatility risk premia as well as the pairwise correlations based on a rolling 60-month window. The corresponding series co-move throughout the sample, however, the commonality is far from perfect. Focusing on the total and bad premiums, correlations are high around 0.5 and 0.6 at the beginning of our sample and decline over time, reaching the values of around 0.2 and 0.3 at the end. The correlation between good components is generally higher and even exceeds 0.6 in some months. Notably, the correlations computed for total and bad volatility risk premiums declined sharply at the onset of the Financial Crisis in 2007 and the COVID recession. The key takeaway is that the common factors based on firm-level volatility risk premia are distinct from the market volatility risk premia. The next sections examine the quantitative asset pricing implications of this incremental information.

3.2 Firms' exposure to the common volatility risk premium factor

We take all common stocks (a share code 10 or 11) listed on the New York Stock Exchange (NYSE), NASDAQ, and AMEX from the Center for Research in Security Prices. At

Figure 2. Common and market volatility risk premia

The figure compares common (\mathcal{CVRP}) and market (\mathcal{MVRP}) volatility risk premia. The top-left, bottom-left, and bottom-right panels illustrate the total, good, and bad components of the volatility risk premia time series, respectively. The top-right panel shows the correlations between the corresponding quantities based on a rolling 60-month window.



the end of each month t , we estimate loadings onto \mathcal{CVRP}^T , \mathcal{CVRP}^G , and \mathcal{CVRP}^B from the following regressions using a 60-month rolling window:

$$r_{i,t} = \beta_i^0 + \beta_i^{\mathcal{CVRP}^X} \mathcal{CVRP}_t^X + \beta_i^{\mathcal{MVRP}^T} \mathcal{MVRP}_t^T + \epsilon_{i,t}, \quad X \in \{\mathcal{T}, \mathcal{G}, \mathcal{B}\} \quad (6)$$

in which $r_{i,t}$ is the stock i 's excess returns and \mathcal{MVRP}^T is the market total volatility risk premium. In the estimation above, we require the stock to have all monthly return observations in the 60-month estimation window and its price at the end of the month t to be higher than \$5 (Bollerslev et al., 2020). We control for the market total volatility premium in the computation of a firm's exposures to common volatility risk premia factors in the spirit of Herskovic et al. (2016). Specifically, the loading $\beta_i^{\mathcal{CVRP}^X}$ from a multivariate regression given by Equation (6) is equivalent to estimating a univariate regression of a stock's excess returns on \mathcal{CVRP}_t^X orthogonalized to \mathcal{MVRP}_t^T . For this reason, we label the obtained loadings as orthogonalizing to the market total volatility risk premium. We also consider replacing the market volatility premium with innovations to the VIX index (Ang et al., 2006), common idiosyncratic volatility (Herskovic et al., 2016), and common

bad implied volatility (Barunik et al., 2023) as well the market bad volatility risk premium (Kilic and Shaliastovich, 2019).

Having estimated the loadings, we implement single-sorted portfolios using equal- and value-weighting schemes. At the end of each month, we sort stocks into quintile portfolios based on their loadings to common volatility premia factors $(\beta_i^{\mathcal{CVRP}^T}, \beta_i^{\mathcal{CVRP}^G}, \beta_i^{\mathcal{CVRP}^B})$, compute returns over the next month, and repeat this process for all months in our sample. Along with the excess expected returns for quintile portfolios, we also report risk-adjusted returns that are the alphas relative to the five factors of Fama and French (2015) or the six factors including momentum of Jegadeesh and Titman (1993).⁸ The risk-adjusted returns allow us to control for other factors known to affect stock returns.

3.3 CVRP-beta sorted portfolios

Table 2 shows results from single portfolio sorts using loadings on common total (Panel A), good (Panel B), and bad (Panel C) volatility risk premiums. In general, quintile portfolios sorted on \mathcal{CVRP}^T -betas and \mathcal{CVRP}^G -betas exhibit monotonically increasing returns; although the spread portfolio excess and risk-adjusted returns are statistically insignificant. Consistent with economic rationale, the average excess returns of \mathcal{CVRP}^B -beta quintiles are monotonically decreasing in both equal- and value-weighted specifications. As a result, spread portfolios formed on \mathcal{CVRP}^B -betas generate statistically and economically significant average returns and alphas. Focusing on a value-weighting scheme, the average excess return on the spread portfolio is -7.32% per annum (t -ratio = -2.93), and annualized five- and six-factor alphas (α_5, α_6) are -9.00% (t -ratio = -2.86) and -9.24% (t -ratio = -3.35). Turning to an equal-weighting scheme, the annualized average excess return, α_5 , and α_6 slightly shrink in absolute values to -6.48% (t -ratio = -2.41), -6.24% (t -ratio = -2.73), and -6.48% (t -ratio = -3.64), respectively, but all remain highly significant in economic and statistical terms.

Table 3 reports analogous portfolio sorts where we obtain loadings by orthogonalizing to innovations to the VIX index (ΔVIX). The key conclusions remain the same. We observe monotonically increasing (decreasing) returns for portfolios we form on \mathcal{CVRP}^G -betas (\mathcal{CVRP}^B -betas). For portfolios formed on exposures to common good volatility risk

⁸We retrieve the factors from Kenneth French's Data Library.

Table 2. Portfolios formed on \mathcal{CVRP} -beta: orthogonalization to $MVRP$

This table presents the average excess returns (RET-RF) and alphas (α_5, α_6) expressed in monthly percentages for equal-weighted and value-weighted quintile portfolios ($\mathcal{P}_i : i = 1, \dots, 5$) and a long-short strategy (\mathcal{P}_{5-1}) formed on loadings to common total (Panel A), good (Panel B), and bad (Panel C) volatility risk premia whilst controlling for the exposure to market total volatility risk premium ($MVRP^T$). Specifically, we estimate common volatility risk premia (\mathcal{CVRP})-betas from bivariate regressions of monthly excess returns on \mathcal{CVRP} and $MVRP^T$ using a rolling 60-month window. The portfolio $\mathcal{P}_1(\mathcal{P}_5)$ comprises stocks with the lowest (highest) \mathcal{CVRP} -betas. The long-short strategy buys \mathcal{P}_5 and sells \mathcal{P}_1 . α_5 is the alpha from the five-factor Fama-French model including the market, size, book-to-market, investment, and profitability factors. α_6 is the alpha relative to the five Fama-French factors and momentum. The numbers in shaded rows are Newey-West adjusted t-statistics of average returns and alphas. The sample is from January 2000 to December 2020.

	Equal-Weighted						Value-Weighted					
	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}
Panel A: \mathcal{CVRP}^T-beta portfolios												
RET-RF	0.67	0.75	0.84	0.83	0.84	0.16	0.71	0.73	0.77	0.80	0.92	0.21
	1.91	2.28	2.27	2.00	1.73	0.93	2.48	2.65	2.41	2.13	1.82	0.74
α_5	0.06	0.13	0.14	0.01	-0.08	-0.15	0.11	0.02	-0.04	-0.06	-0.14	-0.25
	0.75	3.04	2.85	0.18	-1.60	-1.41	0.94	0.40	-0.76	-0.96	-1.00	-1.12
α_6	0.06	0.14	0.14	0.02	-0.07	-0.13	0.11	0.02	-0.04	-0.06	-0.13	-0.25
	0.75	2.97	2.81	0.25	-1.34	-1.49	0.94	0.37	-0.89	-0.92	-0.96	-1.09
Panel B: \mathcal{CVRP}^G-beta portfolios												
RET-RF	0.49	0.68	0.85	0.88	1.02	0.54	0.57	0.66	0.80	1.06	1.07	0.50
	1.42	2.01	2.27	2.11	1.98	1.61	2.14	2.40	2.42	2.70	2.12	1.44
α_5	-0.05	0.08	0.15	0.07	0.01	0.06	0.02	-0.05	-0.03	0.12	0.08	0.06
	-0.41	1.03	2.70	1.37	0.14	0.30	0.20	-0.78	-0.42	2.37	0.62	0.29
α_6	-0.06	0.08	0.15	0.08	0.04	0.10	0.01	-0.06	-0.04	0.13	0.10	0.09
	-0.62	1.04	2.58	1.47	0.53	0.63	0.13	-0.88	-0.44	2.45	0.83	0.42
Panel C: \mathcal{CVRP}^B-beta portfolios												
RET-RF	1.09	0.87	0.74	0.67	0.55	-0.54	1.08	1.05	0.63	0.70	0.46	-0.61
	2.55	2.40	2.01	1.75	1.28	-2.41	2.77	3.26	2.05	1.95	1.08	-2.93
α_5	0.28	0.19	0.04	0.00	-0.24	-0.52	0.28	0.31	-0.17	-0.06	-0.46	-0.75
	2.73	4.59	0.68	0.01	-2.23	-2.73	1.94	4.14	-3.22	-0.61	-3.00	-2.86
α_6	0.30	0.20	0.04	0.00	-0.25	-0.54	0.30	0.32	-0.17	-0.07	-0.48	-0.77
	3.36	4.43	0.68	-0.01	-2.65	-3.64	2.14	4.38	-3.26	-0.72	-3.46	-3.35

premium, excess returns for value-weighted portfolios become now statistically significant and economically meaningful with an annualized excess return of 8.64%. However, the Fama-French factors and momentum subsume this strong performance as shown by insignificant risk-adjusted returns. Turning to portfolios sorted on \mathcal{CVRP}^B -betas, one can observe a significantly negative return spread, though the statistical significance becomes weaker on a risk-adjusted basis. Nevertheless, the magnitudes of the average excess returns and alphas are comparable to those reported in Table 2. In sum, the innovations to the VIX index account for some fraction of the return spread between stocks with the

Table 3. Portfolios formed on \mathcal{CVRP} -beta: orthogonalization to ΔVIX

This table presents the average excess returns (RET-RF) and alphas (α_5, α_6) expressed in monthly percentages for equal-weighted and value-weighted quintile portfolios ($\mathcal{P}_i : i = 1, \dots, 5$) and a long-short strategy (\mathcal{P}_{5-1}) formed on loadings to common total (Panel A), good (Panel B), and bad (Panel C) volatility risk premia whilst controlling for the exposure to shocks to the VIX index (ΔVIX) of [Ang et al. \(2006\)](#). Specifically, we estimate common volatility risk premia (\mathcal{CVRP})-betas from bivariate regressions of monthly excess returns on \mathcal{CVRP} and ΔVIX using a rolling 60-month window. The portfolio $\mathcal{P}_1(\mathcal{P}_5)$ comprises stocks with the lowest (highest) \mathcal{CVRP} -betas. The long-short strategy buys \mathcal{P}_5 and sells \mathcal{P}_1 . α_5 is the alpha from the five-factor Fama-French model including the market, size, book-to-market, investment, and profitability factors. α_6 is the alpha relative to the five Fama-French factors and momentum. The numbers in shaded rows are Newey-West adjusted t-statistics of average returns and alphas. The sample is from January 2000 to December 2020.

	Equal-Weighted						Value-Weighted					
	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}
Panel A: \mathcal{CVRP}^T-beta portfolios												
RET-RF	0.82	0.82	0.78	0.80	0.70	-0.13	0.81	0.74	0.89	0.73	0.81	0.00
	2.12	2.25	2.21	2.04	1.54	-0.81	2.36	2.43	2.96	2.13	1.87	0.00
α_5	0.07	0.11	0.10	0.07	-0.09	-0.17	0.15	0.02	0.11	-0.07	-0.16	-0.31
	0.86	2.73	1.84	0.84	-0.93	-1.00	0.84	0.24	2.40	-0.73	-1.08	-1.00
α_6	0.08	0.11	0.11	0.08	-0.09	-0.17	0.16	0.02	0.11	-0.07	-0.16	-0.33
	0.97	2.63	1.79	0.84	-0.90	-1.06	0.95	0.35	2.42	-0.92	-1.14	-1.10
Panel B: \mathcal{CVRP}^G-beta portfolios												
RET-RF	0.49	0.66	0.81	0.94	1.02	0.52	0.54	0.61	0.80	0.98	1.25	0.72
	1.41	1.78	2.24	2.29	2.09	1.88	1.89	1.90	2.82	2.56	2.76	2.44
α_5	-0.11	0.03	0.12	0.15	0.08	0.19	-0.05	-0.11	0.08	0.05	0.25	0.30
	-0.98	0.30	2.38	2.74	0.86	0.98	-0.40	-1.38	1.08	0.56	2.34	1.58
α_6	-0.13	0.02	0.13	0.16	0.10	0.22	-0.06	-0.11	0.07	0.05	0.26	0.33
	-1.31	0.28	2.27	2.71	1.40	1.53	-0.47	-1.39	1.01	0.55	2.72	1.74
Panel C: \mathcal{CVRP}^B-beta portfolios												
RET-RF	1.15	0.89	0.79	0.62	0.47	-0.69	1.10	1.01	0.83	0.65	0.43	-0.67
	2.42	2.44	2.13	1.65	1.09	-2.03	2.48	3.04	2.85	1.81	1.00	-2.22
α_5	0.25	0.16	0.12	-0.03	-0.22	-0.47	0.23	0.24	0.04	-0.11	-0.41	-0.64
	1.64	3.16	1.73	-0.31	-1.47	-1.60	1.35	2.41	0.57	-1.08	-2.18	-1.91
α_6	0.27	0.17	0.12	-0.04	-0.24	-0.50	0.25	0.25	0.04	-0.12	-0.43	-0.68
	2.32	3.44	1.68	-0.37	-1.77	-2.14	1.65	2.97	0.60	-1.28	-2.50	-2.30

strongest and weakest exposure to common downside volatility premiums. However, this effect cannot completely explain a substantial return differential.

Another possibility is that our common factors capture firms' volatility risk stemming not from the global volatility – the VIX index – but the idiosyncratic component of stock return. We test this hypothesis by controlling for innovations in common idiosyncratic volatility (ΔCIV) proposed by [Herskovic et al. \(2016\)](#) in the exposure computations.⁹ Table

⁹We download the time series of common idiosyncratic volatility and respective tradable factors from Bernard Herskovic's website.

Table 4. Portfolios formed on \mathcal{CVRP} -beta: orthogonalization to ΔCIV

This table presents the average excess returns (RET-RF) and alphas (α_5, α_6) expressed in monthly percentages for equal-weighted and value-weighted quintile portfolios ($\mathcal{P}_i : i = 1, \dots, 5$) and a long-short strategy (\mathcal{P}_{5-1}) formed on loadings to common total (Panel A), good (Panel B), and bad (Panel C) volatility risk premia whilst controlling for the exposure to shocks to common idiosyncratic volatility (ΔCIV) of [Herskovic et al. \(2016\)](#). Specifically, we estimate common volatility risk premia (\mathcal{CVRP})-betas from bivariate regressions of monthly excess returns on \mathcal{CVRP} and ΔCIV using a rolling 60-month window. The portfolio $\mathcal{P}_1(\mathcal{P}_5)$ comprises stocks with the lowest (highest) \mathcal{CVRP} -betas. The long-short strategy buys \mathcal{P}_5 and sells \mathcal{P}_1 . α_5 is the alpha from the five-factor Fama-French model including the market, size, book-to-market, investment, and profitability factors. α_6 is the alpha relative to the five Fama-French factors and momentum. The numbers in shaded rows are Newey-West adjusted t-statistics of average returns and alphas. The sample is from January 2000 to December 2020.

	Equal-Weighted						Value-Weighted					
	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}
Panel A: \mathcal{CVRP}^T-beta portfolios												
RET-RF	0.85	0.79	0.80	0.83	0.65	-0.20	0.78	0.82	0.76	0.76	0.71	-0.06
	2.05	2.29	2.30	2.06	1.49	-1.53	2.16	2.94	2.41	2.22	1.63	-0.35
α_5	0.10	0.12	0.12	0.10	-0.17	-0.27	0.06	0.09	0.01	-0.05	-0.23	-0.29
	1.27	2.95	2.38	1.26	-2.27	-2.20	0.51	1.30	0.23	-0.88	-1.58	-1.19
α_6	0.11	0.12	0.13	0.10	-0.17	-0.28	0.06	0.09	0.01	-0.05	-0.23	-0.30
	1.39	2.94	2.32	1.21	-2.04	-2.27	0.58	1.28	0.14	-0.80	-1.63	-1.27
Panel B: \mathcal{CVRP}^G-beta portfolios												
RET-RF	0.57	0.69	0.84	0.89	0.94	0.37	0.58	0.63	0.76	1.07	0.97	0.40
	1.46	1.94	2.33	2.19	1.99	1.45	1.83	1.94	2.63	2.73	2.21	1.55
α_5	-0.09	0.04	0.15	0.12	0.04	0.12	-0.06	-0.08	-0.03	0.16	0.11	0.17
	-0.79	0.52	2.37	2.21	0.38	0.64	-0.57	-1.26	-0.64	1.70	1.02	0.94
α_6	-0.10	0.04	0.15	0.13	0.06	0.16	-0.07	-0.09	-0.03	0.16	0.13	0.21
	-1.13	0.51	2.25	2.37	0.86	1.17	-0.66	-1.46	-0.66	1.73	1.31	1.12
Panel C: \mathcal{CVRP}^B-beta portfolios												
RET-RF	1.08	0.87	0.77	0.64	0.56	-0.51	1.06	0.99	0.75	0.61	0.53	-0.53
	2.37	2.35	2.13	1.71	1.34	-2.05	2.49	2.97	2.63	1.82	1.23	-2.17
α_5	0.23	0.17	0.09	-0.03	-0.20	-0.43	0.23	0.23	-0.02	-0.12	-0.38	-0.60
	1.98	3.68	1.59	-0.38	-1.76	-2.01	1.67	2.57	-0.30	-1.52	-2.21	-2.25
α_6	0.25	0.18	0.10	-0.04	-0.20	-0.46	0.24	0.24	-0.02	-0.13	-0.39	-0.63
	2.67	3.67	1.56	-0.43	-2.08	-2.72	2.02	2.69	-0.28	-1.73	-2.56	-2.81

[4](#) reports the results. The \mathcal{CVRP}^B sort still remains associated with a statistically and economically significant (abnormal) return spread.

We further assess the significance of the \mathcal{CVRP}^B -beta return spread by testing whether this cross-sectional predictive power can be captured by other variables pertaining to volatility risk in negative returns. Specifically, we consider the market bad volatility risk premium and innovations to commonalities in firm-level implied volatilities as alternative control variables in the \mathcal{CVRP}^B -beta estimation. The former allows us to distinguish the information content within co-movement among firm-level and market volatil-

Table 5. Portfolios formed on \mathcal{CVRP}^B -beta: additional single-sorted portfolios

This table presents the average excess returns (RET-RF) and alphas (α_5, α_6) expressed in monthly percentages for equal-weighted and value-weighted quintile portfolios ($\mathcal{P}_i : i = 1, \dots, 5$) and a long-short strategy (\mathcal{P}_{5-1}) formed on loadings to common total (Panel A), good (Panel B), and bad (Panel C) volatility risk premia whilst controlling for the exposure to market bad volatility risk premium (\mathcal{MVRP}^B) of Kilic and Shaliastovich (2019) or shocks to common bad implied volatility ($\Delta\mathcal{CIVOL}^B$) of Barunik et al. (2023). Specifically, we estimate common volatility risk premia (\mathcal{CVRP})-betas from bivariate regressions of monthly excess returns on \mathcal{CVRP} and \mathcal{MVRP}^B ($\Delta\mathcal{CIVOL}^B$) using a rolling 60-month window. The portfolio $\mathcal{P}_1(\mathcal{P}_5)$ comprises stocks with the lowest (highest) \mathcal{CVRP} -betas. The long-short strategy buys \mathcal{P}_5 and sells \mathcal{P}_1 . α_5 is the alpha from the five-factor Fama-French model including the market, size, book-to-market, investment, and profitability factors. α_6 is the alpha relative to the five Fama-French factors and momentum. The numbers in shaded rows are Newey-West adjusted t-statistics of average returns and alphas. The sample is from January 2000 to December 2020.

	Equal-Weighted						Value-Weighted					
	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}
Panel A: Orthogonalization to \mathcal{MVRP}^B												
RET-RF	1.00	0.86	0.77	0.72	0.58	-0.43	1.05	0.94	0.67	0.70	0.52	-0.53
	2.42	2.45	2.14	1.81	1.31	-2.25	2.88	2.88	2.07	1.93	1.14	-2.43
α_5	0.21	0.17	0.09	0.01	-0.22	-0.43	0.24	0.20	-0.13	-0.10	-0.42	-0.66
	2.66	4.52	1.67	0.18	-2.59	-3.16	1.85	2.32	-2.18	-1.01	-3.05	-2.87
α_6	0.22	0.18	0.09	0.01	-0.22	-0.44	0.25	0.20	-0.13	-0.10	-0.42	-0.68
	2.75	4.09	1.66	0.19	-2.65	-3.65	2.00	2.29	-2.17	-1.06	-3.20	-3.12
Panel B: Orthogonalization to $\Delta\mathcal{CIVOL}^B$												
RET-RF	1.07	0.88	0.81	0.64	0.52	-0.55	1.01	1.02	0.79	0.65	0.49	-0.52
	2.40	2.38	2.29	1.67	1.20	-2.13	2.40	3.02	2.87	1.84	1.15	-2.17
α_5	0.22	0.19	0.13	-0.03	-0.24	-0.47	0.20	0.27	0.01	-0.10	-0.40	-0.60
	1.95	4.34	2.11	-0.35	-1.97	-2.07	1.36	3.06	0.13	-1.10	-2.47	-2.13
α_6	0.24	0.20	0.13	-0.04	-0.25	-0.49	0.22	0.27	0.01	-0.11	-0.41	-0.62
	2.65	5.10	2.07	-0.40	-2.23	-2.69	1.57	3.42	0.14	-1.28	-2.80	-2.49

ity risk premiums in negative individual stock and market returns. The latter indicates whether changes to down implied semi-variance measures drive the observed effect. Table 5 presents the results. Panel A shows that average excess and risk-adjusted returns for \mathcal{CVRP}^B -beta portfolios that orthogonalize to \mathcal{MVRP}^B are statistically significant and economically meaningful. Indeed, the t -statistics of return estimates exceed 2 in absolute values with the annualized spread in average returns ranging from -5.16% to -6.60% and the annualized alphas ranging from -5.16% to -8.16% . This suggests that commonalities in firm-level bad volatility risk premiums are not driven by the same information content we take from the market index bad volatility premium or co-movement among innovations in firm-level bad implied volatilities.

We now investigate double-sorted portfolios whilst controlling for other characteristics

Table 6. Portfolios formed on \mathcal{CVRP}^B -beta controlling for other characteristics

At the end of each month, we conditionally double-sort stocks based on common bad volatility risk premium (\mathcal{CVRP}^B)-beta after controlling for various variables. The control variables include market capitalization (MCAP), market total ($MVRP^T$) and bad ($MVRP^B$) volatility risk premia of [Kilic and Shaliastovich \(2019\)](#), shocks to the VIX index (ΔVIX) of [Ang et al. \(2006\)](#), common idiosyncratic volatility (ΔCIV) of [Herskovic et al. \(2016\)](#), and common bad implied volatility ($\Delta CIVOL^B$) of [Barunik et al. \(2023\)](#). In each case, we first sort stocks into quintiles using the control variable, then within each quintile, we sort stocks into quintile portfolios based on \mathcal{CVRP}^B -beta so that \mathcal{P}_1 (\mathcal{P}_5) is the portfolio of stocks with the lowest (highest) \mathcal{CVRP}^B -betas. This table presents average returns across the five control quintiles to produce quintile portfolios with dispersion in \mathcal{CVRP}^B -beta but with similar levels of the control variable. The spread portfolio \mathcal{P}_{5-1} is the difference between \mathcal{P}_5 and \mathcal{P}_1 . This table presents the average excess returns (RET-RF) and alphas (α_5, α_6) expressed in monthly percentages for equal-weighted portfolios. α_5 is the alpha from the five-factor Fama-French model including the market, size, book-to-market, investment, and profitability factors. α_6 is the alpha relative to the five Fama-French factors and momentum. The numbers in shaded rows are Newey-West adjusted t-statistics of average excess and alphas. The sample is from January 2000 to December 2020.

	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}		\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}
MCAP							$MVRP^T$						
RET-RF	1.07	0.86	0.79	0.67	0.54	-0.53		1.06	0.87	0.77	0.68	0.55	-0.51
	2.55	2.37	2.13	1.72	1.21	-2.10		2.52	2.37	2.04	1.78	1.28	-2.27
α_5	0.25	0.17	0.13	-0.03	-0.25	-0.50		0.28	0.17	0.07	-0.03	-0.21	-0.49
	2.11	3.77	1.70	-0.30	-2.11	-2.25		2.83	3.99	1.10	-0.35	-2.05	-2.65
α_6	0.27	0.17	0.14	-0.03	-0.26	-0.53		0.29	0.17	0.07	-0.03	-0.22	-0.51
	2.91	3.52	1.66	-0.36	-2.45	-2.98		3.56	3.82	1.10	-0.38	-2.44	-3.58
ΔVIX							ΔCIV						
RET-RF	1.11	0.89	0.78	0.68	0.47	-0.64		1.07	0.86	0.79	0.69	0.52	-0.55
	2.50	2.36	2.08	1.80	1.10	-2.20		2.49	2.26	2.14	1.79	1.26	-2.42
α_5	0.25	0.16	0.09	-0.02	-0.22	-0.46		0.25	0.14	0.11	-0.02	-0.21	-0.47
	2.10	3.40	1.38	-0.20	-1.48	-1.85		2.54	3.33	1.90	-0.19	-2.09	-2.51
α_6	0.27	0.17	0.10	-0.02	-0.23	-0.49		0.27	0.15	0.11	-0.02	-0.22	-0.49
	2.76	4.26	1.38	-0.26	-1.76	-2.42		3.33	3.30	1.89	-0.21	-2.41	-3.32
$MVRP^B$							$\Delta CIVOL^B$						
RET-RF	1.00	0.88	0.78	0.71	0.56	-0.44		1.06	0.88	0.81	0.67	0.51	-0.55
	2.42	2.45	2.10	1.84	1.29	-2.17		2.50	2.32	2.15	1.75	1.22	-2.23
α_5	0.22	0.17	0.09	-0.01	-0.20	-0.42		0.25	0.17	0.09	-0.03	-0.21	-0.46
	2.91	4.51	1.63	-0.15	-2.26	-2.96		2.53	4.12	1.52	-0.36	-1.77	-2.27
α_6	0.23	0.18	0.09	-0.01	-0.21	-0.44		0.27	0.18	0.09	-0.03	-0.22	-0.49
	3.16	4.19	1.61	-0.15	-2.41	-3.63		3.17	4.66	1.50	-0.42	-2.02	-2.91

following the procedure in [Ang et al. \(2006\)](#) and [Herskovic et al. \(2016\)](#). We first sort stocks into quintiles using the characteristic of interest, then within each quintile, we sort stocks into equal-weighted quintile portfolios based on \mathcal{CVRP}^B -betas. We finally take the equal-weighted average returns across five control groups to generate dispersion in \mathcal{CVRP}^B -betas with similar levels of control variables. This neutralizes the quintile portfolios formed on loadings to the common bad volatility risk premium from the characteristic of interest. We consider a set of controls including market capitalization, the market total and bad

Table 7. Double-sorted portfolios formed on \mathcal{CVRP}^B -beta and other characteristics

At the end of each month, we conditionally double-sort stocks based on common bad volatility risk premium (\mathcal{CVRP}^B)-beta after controlling for various variables. The control variables include market capitalization (MCAP), market total (\mathcal{MVRP}^T) and bad (\mathcal{MVRP}^B) volatility risk premia of [Kilic and Shaliastovich \(2019\)](#), shocks to the VIX index (ΔVIX) of [Ang et al. \(2006\)](#), common idiosyncratic volatility (ΔCIV) of [Herskovic et al. \(2016\)](#), and common bad implied volatility ($\Delta CIVOL^B$) of [Barunik et al. \(2023\)](#). In each case, we first sort stocks into quintiles using the control variable, then within each quintile, we sort stocks into quintile portfolios based on \mathcal{CVRP}^B -beta so that \mathcal{P}_1 (\mathcal{P}_5) is the portfolio of stocks with the lowest (highest) \mathcal{CVRP}^B -betas. This table presents the average excess returns of the twenty-five double-sorted portfolios. The spread portfolio \mathcal{P}_{5-1} is the difference between \mathcal{P}_5 and \mathcal{P}_1 . The numbers in shaded rows and columns are Newey-West adjusted t-statistics of average returns. The sample is from January 2000 to December 2020.

	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}		\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}	
MCAP							\mathcal{MVRP}^T							
\mathcal{P}_1	0.92	0.70	0.75	0.41	0.41	-0.51	-2.02	0.79	0.61	0.56	0.58	0.47	-0.31	-1.21
\mathcal{P}_2	1.11	0.82	0.58	0.73	0.60	-0.51	-1.69	1.13	0.73	0.87	0.68	0.50	-0.63	-2.57
\mathcal{P}_3	1.14	0.95	0.86	0.71	0.63	-0.51	-1.77	1.11	0.84	0.68	0.79	0.65	-0.45	-2.13
\mathcal{P}_4	1.17	0.89	0.90	0.69	0.57	-0.60	-2.10	1.13	1.02	0.86	0.66	0.45	-0.68	-2.43
\mathcal{P}_5	0.99	0.92	0.88	0.80	0.48	-0.51	-2.02	1.14	1.13	0.87	0.70	0.65	-0.49	-1.74
\mathcal{P}_{5-1}	0.07	0.22	0.13	0.38	0.07			0.35	0.53	0.30	0.12	0.18		
	0.30	1.25	0.76	2.80	0.38			1.20	1.81	1.25	0.46	0.97		
ΔVIX							ΔCIV							
\mathcal{P}_1	0.98	0.88	0.70	0.74	0.52	-0.47	-1.21	1.12	1.01	0.89	0.80	0.73	-0.40	-1.25
\mathcal{P}_2	1.25	1.03	0.98	0.69	0.49	-0.75	-2.23	1.31	1.09	0.98	0.80	0.45	-0.85	-2.76
\mathcal{P}_3	1.24	0.89	0.86	0.76	0.60	-0.64	-1.92	1.16	0.86	0.87	0.67	0.63	-0.53	-2.04
\mathcal{P}_4	1.18	0.91	0.72	0.73	0.44	-0.75	-2.32	1.01	0.73	0.69	0.65	0.45	-0.57	-2.40
\mathcal{P}_5	0.89	0.73	0.65	0.47	0.29	-0.61	-2.52	0.73	0.60	0.53	0.52	0.34	-0.39	-2.07
\mathcal{P}_{5-1}	-0.09	-0.15	-0.06	-0.28	-0.23			-0.40	-0.40	-0.36	-0.29	-0.39		
	-0.25	-0.45	-0.21	-0.95	-0.83			-1.07	-1.27	-1.07	-0.93	-1.68		
\mathcal{MVRP}^B							$\Delta CIVOL^B$							
\mathcal{P}_1	1.03	1.05	0.80	0.72	0.64	-0.39	-1.25	1.11	1.09	0.91	0.76	0.71	-0.40	-1.10
\mathcal{P}_2	1.07	0.88	0.86	0.66	0.59	-0.48	-1.92	1.22	0.93	0.98	0.70	0.50	-0.71	-2.06
\mathcal{P}_3	1.03	0.85	0.82	0.79	0.59	-0.44	-2.18	1.28	0.87	0.84	0.71	0.62	-0.66	-2.13
\mathcal{P}_4	1.05	0.76	0.76	0.79	0.50	-0.55	-2.48	0.97	0.92	0.82	0.61	0.48	-0.49	-2.27
\mathcal{P}_5	0.83	0.85	0.68	0.55	0.48	-0.35	-1.74	0.72	0.59	0.49	0.55	0.25	-0.47	-2.11
\mathcal{P}_{5-1}	-0.20	-0.20	-0.12	-0.17	-0.16			-0.39	-0.50	-0.42	-0.20	-0.46		
	-0.61	-0.68	-0.61	-0.80	-0.90			-0.86	-1.28	-1.21	-0.66	-1.50		

volatility risk premium as well as innovations to the VIX index, common idiosyncratic volatility, and common firm-level bad implied volatility.

Table 6 reports the results. In all cases, we observe significant excess returns for spread portfolios that range from annualized values of -7.68% to -5.28%. The alphas are also statistically significant and of similar magnitudes with annualized estimates ranging from -6.36% to -5.04%. Table 7 augments the previous results by reporting the average excess returns for 5×5 double sorts using the same characteristics. For each control, the highest \mathcal{CVRP}^B -beta stocks earn substantially lower returns within each control characteristic's quantile. The majority of these economically substantial spreads (nineteen out of thirty cases) are also statistically significant at the 5% confidence level. In contrast, controlling for

the \mathcal{CVRP}^B -beta spread, all but one return spread based on control variables are abysmal both economically and statistically.

In sum, the commonalities in firm-level volatility risk premia relating to anticipated future downward price movements can predict future returns. Stocks with higher (lower) \mathcal{CVRP}^B -betas generate lower (higher) returns. This holds after accounting for a battery of alternative factors including the market volatility risk premia and various alternative proxies for volatility risks. Intuitively, this finding can be explained by the fact that, in equilibrium, investors are willing to accept lower returns for stocks that load strongly on \mathcal{CVRP}^B as they act as hedging devices for adverse changes to investment opportunities (Campbell et al., 2018; Farago and Tédongap, 2018). Although our results for \mathcal{CVRP}^G -beta portfolios are statistically insignificant, they are consistent with the positive link between good market variance risk premium and future stock returns in Kilic and Shaliastovich (2019).¹⁰

3.4 Pricing CVRP-beta sorted portfolios

Here, we conduct a two-stage Fama and MacBeth estimation to explore the pricing implications of commonalities among firm-level volatility risk premia. We obtain factor betas in the first stage by regressing monthly excess returns of each test asset on constant and asset pricing factors. The second stage is a single cross-sectional regression for the average excess returns for all test assets on the factor betas and a constant. In what follows, we report the risk-premia estimates from the second stage. We use Newey-West t -statistics with 12 lags that adjust for errors in variables as in Shanken (1992). We also report the adjusted R-squared statistics, R_{adj}^2 , from each of the second-stage regressions.

Our test assets stem from the same estimates we use to obtain the monthly portfolio returns in Section 3.3. We proxy the corresponding common volatility risk premium factors using the long-short strategies (\mathcal{P}_{5-1}) for $\mathcal{CVRP}^{T/G/B}$ -beta portfolios from Table 2. We consider various asset pricing factors to price $\mathcal{CVRP}^{T/G/B}$ -beta portfolios. Our baseline utilizes the five factors of Fama and French (2015) and one of the $\mathcal{CVRP}^{T/G/B}$

¹⁰Appendix B reports additional robustness analysis. Table A1 shows portfolio sorts on \mathcal{CVRP}^T , \mathcal{CVRP}^G , and \mathcal{CVRP}^B -beta portfolios with no orthogonalization. These convey the same message as Table 2. Table A2 shows analogous results for portfolios we form on \mathcal{MVRP}^T , \mathcal{MVRP}^G , and \mathcal{MVRP}^B -beta portfolios. These results show no pattern across quintile portfolios and no evidence of significant excess or risk-adjusted returns.

Table 8. Fama-MacBeth analysis: decile portfolios formed on \mathcal{CVRP} -beta

This table shows the Fama-MacBeth two-pass regression analysis for decile portfolios formed on loadings to common total (\mathcal{CVRP}^T), good (\mathcal{CVRP}^G), and bad (\mathcal{CVRP}^B) volatility risk premia. For each set of test assets, we first control for a tradable long-short portfolio formed on the \mathcal{CVRP} -beta and the five Fama-French factors – market, size, book-to-market, investment, and profitability. Then, we augment the independent variables by including, one at a time, momentum and tradable long-short portfolios formed on the corresponding market volatility risk premia (\mathcal{MVRP})-beta and loadings on the VIX, common idiosyncratic volatility, and corresponding common implied volatility innovation. The numbers in shaded rows are Newey-West adjusted t-statistics of coefficients. The bottom row reports the adjusted R-squared. The sample is from January 2005 to December 2020.

	\mathcal{CVRP}^T						\mathcal{CVRP}^G						\mathcal{CVRP}^B						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
λ_0	0.27	0.43	1.58	0.50	0.36	0.37	0.45	0.54	0.46	0.42	0.28	0.45	1.80	1.56	1.81	0.38	1.97	0.24	
	0.57	0.90	0.15	0.80	0.76	0.38	0.86	0.75	0.75	0.74	0.20	0.67	0.72	0.63	0.72	0.14	0.78	0.09	
$\lambda_{\mathcal{CVRP}^{T/G/B}}$	0.16	0.16	0.78	0.16	0.15	0.16	0.48	0.48	0.48	0.48	0.48	0.48	-0.48	-0.48	-0.48	-0.50	-0.49	-0.49	
	0.85	0.83	1.37	0.83	0.81	0.85	1.45	1.45	1.45	1.45	1.45	1.45	-2.10	-2.11	-2.09	-2.11	-2.11	-2.10	
λ_{Mkt}	1.01	1.03	1.18	1.48	1.59	1.05	0.75	0.54	0.72	0.75	1.25	0.75	-3.06	-2.50	-3.15	0.24	-2.24	0.16	
	0.90	0.92	-1.67	1.22	1.29	0.88	0.95	0.46	0.71	0.95	0.30	0.89	-0.66	-0.54	-0.68	0.05	-0.49	0.03	
λ_{SMB}	-0.45	-0.76	-1.39	-1.43	-1.34	-0.66	0.39	0.48	0.41	0.44	0.07	0.39	2.58	2.04	2.67	-0.26	0.84	0.12	
	-0.43	-0.69	-1.14	-0.84	-1.09	-0.31	0.79	0.90	0.71	0.68	0.02	0.69	0.88	0.69	0.91	-0.07	0.27	0.04	
λ_{HML}	-0.52	-1.01	-1.11	-0.91	-1.12	-0.57	-3.22	-2.96	-3.22	-3.27	-3.74	-3.22	1.30	1.39	1.49	2.03	1.93	1.96	
	-0.53	-0.93	0.57	-0.77	-1.02	-0.49	-2.97	-1.72	-2.94	-2.72	-0.89	-2.91	1.41	1.59	1.33	2.33	2.28	1.99	
λ_{RMW}	0.78	0.56	1.63	0.43	0.51	0.72	2.46	2.30	2.44	2.48	2.70	2.46	0.17	0.07	0.22	0.40	0.54	0.50	
	2.13	1.58	0.46	0.67	1.36	1.11	3.51	2.63	3.44	3.28	1.20	3.58	0.36	0.17	0.53	0.79	0.93	1.04	
λ_{CMF}	1.09	0.37	0.61	0.68	0.85	0.98	-1.14	-1.08	-1.15	-1.15	-1.17	-1.14	0.98	0.23	0.27	0.37	0.68	0.49	0.74
	1.54	0.42					-1.66	-1.99	-1.96	-1.98	-1.94		0.38	0.43	0.60	1.03	0.77	1.12	
λ_{MOM}		-1.85					2.37						-1.62						
		-1.17					2.70						-0.92						
$\lambda_{\mathcal{MVRP}^{T/G/B}}$		2.00					0.26						-0.64						
		2.27					0.38						-1.97						
$\lambda_{\Delta VIX}$			-0.64					-0.47						-1.27					
			-2.03					-0.72						-0.73					
$\lambda_{\Delta CIV}$				-1.47					0.45						-2.02				
				-1.76					0.24						-1.06				
$\lambda_{\mathcal{CIVOL}^{T/G/B}}$					-0.43					-0.71						-0.92			
					-1.04					-1.30						-0.57			
R^2_{adj}	0.57	0.79	0.97	0.48	0.82	0.43	0.99	0.99	0.99	0.99	0.99	0.99	0.97	0.98	0.96	0.99	0.99	0.98	

factors. Then, we sequentially add momentum (MOM), market volatility risk premium ($\mathcal{MVRP}^{T/G/B}$) and tradable long-short portfolios formed on exposure to ΔVIX , ΔCIV , and $\Delta \mathcal{CIVOL}^{T/G/B}$. We have 6 alternative pricing models for each set of test assets.

Table 8 reports the risk premia estimates for decile portfolios formed on \mathcal{CVRP} -betas. We use decile portfolios here to ensure we have enough of a cross-section to include additional asset pricing factors. Columns 1–6 show results for \mathcal{CVRP}^T -beta decile portfolios, meanwhile columns 7–12, and 13–18 contain estimates for the \mathcal{CVRP}^G -beta and \mathcal{CVRP}^B -beta decile portfolios, respectively. The risk premia estimates for \mathcal{CVRP}^T and \mathcal{CVRP}^G are statistically insignificant. Turning to risk premia estimates for \mathcal{CVRP}^B , however, the annualized risk premia estimates range from -6% to -5.76%. The coefficients are economically meaningful and are consistent with the spread portfolio returns in Table 2. Note also that the estimates are robust across all specifications even after accounting for $\mathcal{MVRP}^{T/G/B}$ and other alternative volatility risk proxies. The adjusted R-squared values are high, par-

Table 9. Fama-MacBeth analysis: double-sorted portfolios formed on \mathcal{CVRP} - and \mathcal{MVRP} -beta

This table shows the Fama-MacBeth two-pass regression analysis for twenty-five conditionally double-sorted portfolios based on common volatility risk premium (\mathcal{CVRP})-beta and market volatility risk premium (\mathcal{MVRP})-beta. In each case, we first sort stocks into quintiles based on \mathcal{MVRP} -beta, then within each quintile, we sort stocks into quintile portfolios based on \mathcal{CVRP} -beta so that \mathcal{P}_1 (\mathcal{P}_5) is the portfolio of stocks with the lowest (highest) \mathcal{CVRP} -betas. For each set of test assets, we first control for a tradable long-short portfolio formed on the \mathcal{CVRP} -beta and the five Fama-French factors – market, size, book-to-market, investment, and profitability. Then, we augment the independent variables by including, one at a time, momentum and tradable long-short portfolios formed on the corresponding market volatility risk premia (\mathcal{MVRP})-beta and loadings on the VIX, common idiosyncratic volatility, and corresponding common implied volatility innovation. The numbers in shaded rows are Newey-West adjusted t-statistics of coefficients. The bottom row reports the adjusted R-squared. The sample is from January 2005 to December 2020.

	25 double-sorted on \mathcal{CVRP}^T and \mathcal{MVRP}^T						25 double-sorted on \mathcal{CVRP}^G and \mathcal{MVRP}^G						25 double-sorted on \mathcal{CVRP}^B and \mathcal{MVRP}^B					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
λ_0	1.01	0.99	1.03	1.34	1.05	1.16	0.77	0.78	0.83	0.67	0.78	0.72	1.34	1.31	1.39	1.38	1.33	1.36
	3.22	3.19	3.54	3.82	3.46	3.82	1.85	1.86	2.00	1.48	1.90	1.43	3.03	3.26	3.44	3.72	3.03	3.14
$\lambda_{\mathcal{CVRP}^T/G/B}$	0.19	0.17	0.18	0.15	0.14	0.15	0.47	0.47	0.43	0.46	0.47	0.48	-0.52	-0.52	-0.52	-0.52	-0.52	-0.52
	0.93	0.82	0.92	0.71	0.74	0.81	1.40	1.38	1.36	1.37	1.43	1.48	-2.33	-2.23	-2.33	-2.33	-2.24	-2.33
λ_{Mkt}	0.04	0.12	-0.02	-0.25	-0.05	-0.15	0.11	0.10	-0.02	0.19	0.10	0.17	-0.56	-0.54	-0.56	-0.59	-0.55	-0.60
	0.07	0.21	-0.04	-0.40	-0.09	-0.32	0.17	0.15	-0.04	0.28	0.17	0.29	-1.10	-0.97	-1.09	-1.15	-1.06	-1.24
λ_{SMB}	-0.30	-0.48	-0.27	-0.44	-0.38	-0.35	-0.18	-0.17	-0.07	-0.13	-0.17	-0.19	-0.06	-0.04	-0.09	-0.06	-0.05	-0.04
	-0.96	-1.51	-0.87	-1.38	-1.16	-1.05	-0.59	-0.55	-0.23	-0.47	-0.61	-0.69	-0.16	-0.13	-0.27	-0.19	-0.14	-0.13
λ_{HML}	-0.50	-0.38	-0.38	-0.28	-0.03	-0.18	0.11	0.11	0.05	0.04	0.09	0.08	0.07	0.11	-0.10	0.10	0.05	0.17
	-0.59	-0.44	-0.66	-0.34	-0.05	-0.32	0.23	0.24	0.10	0.10	0.18	0.15	0.11	0.20	-0.22	0.14	0.08	0.29
λ_{RMW}	0.41	0.37	0.39	0.37	0.35	0.36	0.41	0.41	0.39	0.40	0.41	0.42	-0.01	-0.01	-0.02	0.02	-0.01	0.02
	1.33	1.17	1.35	1.18	1.20	1.24	1.90	1.94	1.66	1.82	1.92	1.84	-0.03	-0.04	-0.07	0.06	-0.05	0.07
λ_{CMA}	-0.44	-0.39	-0.47	-0.39	-0.50	-0.47	-0.17	-0.18	-0.36	-0.17	-0.19	-0.12	0.25	0.27	0.21	0.26	0.25	0.29
	-1.29	-1.17	-1.30	-1.13	-1.44	-1.36	-0.41	-0.36	-0.96	-0.42	-0.59	-0.36	0.72	0.90	0.71	0.72	0.71	0.83
λ_{MOM}	-1.09						-0.43						-0.24					
	-1.17						-0.64						-0.27					
$\lambda_{\mathcal{MVRP}^T/G/B}$		0.28					0.48						-0.08					
		1.25					1.61						-0.22					
$\lambda_{\Delta VIX}$		-0.38						0.01						0.27				
		-0.99						0.02						0.59				
$\lambda_{\Delta CIV}$			-0.51						-0.23						0.00			
			-1.26						-0.60						0.01			
$\lambda_{\mathcal{CTVOP}^T/G/B}$				-0.20						-0.26						0.23		
				-0.47						-0.59						0.47		
R^2_{adj}	0.69	0.77	0.68	0.71	0.77	0.70	0.77	0.76	0.77	0.77	0.76	0.76	0.92	0.91	0.91	0.91	0.91	0.91

ticularly for \mathcal{CVRP}^B -beta decile portfolios.

Table 9 considers three additional groups of test assets: i) 25 double-sorted portfolios formed on \mathcal{CVRP}^T - and \mathcal{MVRP}^T -betas in columns 1–6; ii) 25 double-sorted portfolios formed on \mathcal{CVRP}^G - and \mathcal{MVRP}^G -betas in columns 7–12; and iii) 25 double-sorted portfolios on \mathcal{CVRP}^B - and \mathcal{MVRP}^B -betas in columns 13–18. The \mathcal{CVRP}^T and \mathcal{CVRP}^G risk premium estimates are not significantly different from zero, while \mathcal{CVRP}^B risk premium estimates are economically meaningful and statistically significant with similar magnitudes reported before.

We now focus on additional test assets constructed on \mathcal{CVRP}^B -beta and other controls. Table 10 reports risk premium estimates for 25 portfolios double-sorted on \mathcal{CVRP}^B -betas and exposures to ΔVIX in columns 1–6; 25 portfolios double-sorted on \mathcal{CVRP}^B -beta and exposures to ΔCIV in columns 7–12; and 25 portfolios double-sorted on \mathcal{CVRP}^B -betas

Table 10. Fama-MacBeth analysis: additional double-sorted portfolios

This table shows the Fama-MacBeth two-pass regression analysis for twenty-five conditionally double-sorted portfolios based on common volatility risk premium (\mathcal{CVRP})-beta and loadings on shocks to the VIX index, common idiosyncratic volatility, and common bad implied volatility. In each case, we first sort stocks into quintiles based on control variables, then within each quintile, we sort stocks into quintile portfolios based on \mathcal{CVRP} -beta so that P_1 (P_5) is the portfolio of stocks with the lowest (highest) \mathcal{CVRP} -betas. For each set of test assets, we first control for a tradable long-short portfolio formed on the \mathcal{CVRP} -beta and the five Fama-French factors – market, size, book-to-market, investment, and profitability. Then, we augment the independent variables by including, one at a time, momentum and tradable long-short portfolios formed on the corresponding market volatility risk premia (\mathcal{MVRP})-beta and loadings on the VIX, common idiosyncratic volatility, and corresponding common implied volatility innovation. The numbers in shaded rows are Newey-West adjusted t-statistics of coefficients. The bottom row reports the adjusted R-squared. The sample is from January 2005 to December 2020.

	25 double-sorted on \mathcal{CVRP}^B and ΔVIX						25 double-sorted on \mathcal{CVRP}^B and ΔCIV						25 double-sorted on \mathcal{CVRP}^B and \mathcal{CTVOL}^B					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
λ_0	0.75	1.02	0.69	1.02	0.70	0.69	0.44	0.63	0.31	1.07	1.14	1.17	0.48	0.76	0.45	1.70	0.98	1.78
	2.38	2.78	2.03	2.78	1.90	2.03	1.38	1.81	0.79	2.00	2.33	1.99	1.68	2.51	1.23	3.88	2.97	4.41
$\lambda_{\mathcal{CVRP}^B}$	-0.68	-0.68	-0.63	-0.68	-0.68	-0.63	-0.63	-0.60	-0.62	-0.65	-0.54	-0.66	-0.75	-0.70	-0.74	-0.71	-0.60	-0.80
	-2.58	-2.57	-2.58	-2.57	-2.73	-2.58	-2.45	-2.41	-2.43	-2.48	-2.26	-2.48	-2.63	-2.59	-2.88	-2.59	-2.39	-2.70
λ_{Mkt}	-0.02	-0.24	-0.12	-0.24	0.00	-0.12	0.78	0.35	0.98	-0.01	-0.26	-0.02	0.00	-0.34	0.01	-1.04	-0.49	-0.95
	-0.06	-0.46	-0.28	-0.46	0.00	-0.28	1.54	0.66	2.01	-0.02	-0.42	-0.04	0.00	-0.73	0.02	-1.69	-0.94	-1.84
λ_{SMB}	0.13	0.03	0.36	0.03	0.17	0.36	-0.56	-0.43	-0.61	-0.51	-0.23	-0.60	0.40	0.35	0.42	-0.04	0.30	-0.24
	0.33	0.07	0.78	0.07	0.33	0.78	-1.66	-1.31	-1.80	-1.55	-0.69	-1.74	1.17	1.04	1.02	-0.12	0.92	-0.69
λ_{HML}	-0.15	-0.19	-0.45	-0.19	-0.12	-0.45	-0.33	0.12	-0.53	0.12	0.06	-0.02	0.41	0.57	0.41	0.48	0.48	0.31
	-0.32	-0.41	-0.97	-0.41	-0.26	-0.97	-0.57	0.25	-0.99	0.25	0.12	-0.04	1.09	1.47	1.09	1.27	1.22	0.81
λ_{RMW}	0.93	0.85	0.44	0.85	0.93	0.44	0.28	0.32	0.37	0.24	0.23	0.23	0.94	0.85	0.94	0.74	1.12	0.82
	1.85	2.00	1.13	2.00	1.83	1.13	0.81	0.90	0.98	0.71	0.65	0.66	2.59	2.35	2.59	2.02	3.11	2.24
λ_{CMA}	-0.68	-0.68	-0.65	-0.68	-0.68	-0.65	-1.00	-0.41	-1.00	-0.75	-0.56	-0.85	-0.15	0.19	-0.13	-0.08	0.16	-0.26
	-2.08	-2.08	-1.99	-2.08	-2.06	-1.99	-3.26	-1.39	-3.27	-2.93	-2.29	-3.20	-0.42	0.62	-0.40	-0.24	0.53	-0.70
λ_{MOM}	-0.12						-2.06						-1.96					
	-0.43						-2.08						-2.23					
$\lambda_{\mathcal{MVRP}^B}$		-1.20						-0.54					-0.69					
		-1.87						-1.40					-1.25					
$\lambda_{\Delta VIX}$			-0.12						-0.35					-0.37				
			-0.43						-1.20					-1.29				
$\lambda_{\Delta CIV}$				0.12						-0.60					-0.76			
				0.27						-2.07					-1.93			
$\lambda_{\mathcal{CTVOL}^B}$					-1.20						-0.42					-0.38		
					-1.87						-1.24					-1.15		
R^2_{adj}	0.82	0.82	0.85	0.82	0.81	0.85	0.82	0.88	0.81	0.85	0.90	0.84	0.78	0.81	0.76	0.84	0.84	0.83

and exposures to $\Delta \mathcal{CTVOL}^B$ in columns 13–18. All risk premia estimates for \mathcal{CVRP}^B are negative and significant with annualized estimates ranging from -9.60% to -7.20% . The cross-sectional adjusted R-squared statistics are similar to decile \mathcal{CVRP}^B -beta portfolios and 25 portfolios double-sorted on \mathcal{CVRP}^B -betas and \mathcal{MVRP}^B -betas.

Overall, the two-stage Fama-MacBeth regression analysis is consistent with the results based on portfolio sorts. Our findings conform with the economic rationale that stocks with higher sensitivities to common downside volatility risk premium factor tend to appreciate during times of high uncertainty and hence act as hedging instruments, earning lower average returns in the future. In turn, investors are willing to accept lower future average returns of the highest \mathcal{CVRP}^B -beta stocks to hedge against commonalities in bad volatility risk premia.

3.5 Three-pass regression analysis

Two-stage Fama-MacBeth regressions suffer from omitted variable bias. We, therefore, investigate the asset pricing implications of \mathcal{CVRP}^B using the three-pass regression approach in [Giglio and Xiu \(2021\)](#). Under this framework, we need not specify or observe all factors within a pricing model. This is because it relies on principal components analysis (PCA) of test assets to recover the factor space and additional regressions to proxy risk premia. In what follows, we present results from pricing models that contain our \mathcal{CVRP}^B factor and the market risk premium.

We exploit two benefits of the [Giglio and Xiu \(2021\)](#) approach. First, holding test assets constant, the risk-premia estimates and model fit do not change as you start adding additional pricing factors. Thus, we can consider multiple dimensions of robustness to our main results with brevity. Second, we can test the null hypothesis that a factor is a weak pricing factor. This enables us to understand whether the test assets capture well the variation in the pricing factor itself, whilst being able to recover the risk-premia estimates of potentially strong factors, which means we can interpret inference reliably.

Table 11 reports the three-pass regression results using various different test assets using \mathcal{CVRP}^B -beta portfolios. The first column considers decile portfolios formed on \mathcal{CVRP}^B -betas. Columns 2–5 contain results for 25 double-sorted portfolios formed on various alternative proxies of volatility risk. These include double sorts formed on \mathcal{CVRP}^B - and $MVRP^B$ -betas, \mathcal{CVRP}^B - and ΔVIX -betas, \mathcal{CVRP}^B - and ΔCIV -betas, and \mathcal{CVRP}^B - and $CIVOL^B$ -betas. Below the risk premium estimates, we report the Newey-West t -statistics and the p-values for the Wald tests of the null hypothesis that the factor is a weak pricing factor. The bottom two rows report the adjusted R-squared, R_{adj}^2 , and the number of PCA factors required to span the factor space. Across each set of test assets, the risk premium estimates for \mathcal{CVRP}^B are negative and significant ranging from annualized values of -6.60% to -5.40% . We reject the null hypothesis that the \mathcal{CVRP}^B factor is a weak pricing factor. All specifications have large cross-sectional R-squared statistics and the number of PCA factors is either 2 or 3. In conclusion, our estimates for \mathcal{CVRP}^B risk premium are consistent with those obtained from either portfolio sorting exercises or two-pass regressions. This approach reveals that we can reject the null hypothesis that commonalities in bad firm-level volatility risk premium is a weak pricing factor.

Table 11. Three-pass regression analysis

This table shows the three-pass regression analysis of [Giglio and Xiu \(2021\)](#). The test assets are decile portfolios formed on loadings to common bad volatility risk premia (\mathcal{CVRP}^B), twenty-five conditionally double-sorted portfolios based on \mathcal{CVRP}^B -beta and one of four controls: loadings on market bad volatility risk premium (\mathcal{MVRP}^B), shocks to the VIX index (ΔVIX), common idiosyncratic volatility (ΔCIV), and common bad implied volatility ($\Delta CIVOL^B$). For double-sorted portfolios, we first sort stocks into quintiles based on control variables, then within each quintile, we sort stocks into quintile portfolios based on \mathcal{CVRP}^B -beta so that \mathcal{P}_1 (\mathcal{P}_5) is the portfolio of stocks with the lowest (highest) \mathcal{CVRP}^B -betas. For all cases, the independent variables are the tradable long-short portfolio formed on \mathcal{CVRP}^B -beta and the market excess return. We report t-statistics in shaded rows under risk premia estimates. The Wald (p-value) entries are p-values from the null hypothesis that the asset pricing factor is a weak pricing factor. Rejection of the null implies the factor is a strong pricing factor. R^2_{adj} is the adjusted R-squared and No. Factors is the number of PCA factors the model requires to recover the factor space. The sample is from January 2005 to December 2020.

	Deciles	\mathcal{MVRP}^B	ΔVIX	ΔCIV	$\Delta CIVOL^B$
λ_0	-0.27	1.34	1.32	1.82	1.38
	-0.19	13.18	7.81	8.00	6.95
$\lambda_{\mathcal{CVRP}^B}$	-0.49	-0.48	-0.55	-0.45	-0.47
	-2.18	-2.10	-1.94	-2.01	-2.00
Wald p-value	0.00	0.00	0.00	0.01	0.00
λ_{Mkt}	0.85	-0.46	-0.43	-0.77	-0.46
	0.70	-1.37	-1.23	-2.13	-1.30
Wald p-value	0.00	0.00	0.00	0.00	0.00
R^2_{adj}	0.95	0.91	0.75	0.84	0.76
No. factors	3	3	2	2	2

4 Common volatility risk premia and stock market return predictability

We now examine the predictive power of commonalities among firm-level volatility risk premia for market returns. The literature on market return predictability is vast. Some studies document that predictive relationships change over time with key financial ratios being successful in certain periods ([Fama and French, 1988](#)) or at specific periods of business cycles ([Dangl and Halling, 2012](#)). Another observation from earlier work is that well-known predictors contain little information at horizons below 6 months ([Fama and French, 1988](#)). In the context of a second-moment risk premium, [Bollerslev et al. \(2009\)](#) shows that variance risk premium can predict market returns at short horizons. [Kilic and Shaliastovich \(2019\)](#) demonstrate that good and bad variance premia have a long-term predictive power. Subsequent studies show that a variety of variance risk proxies contain information for future stock market returns ([Pyun, 2019; Han and Li, 2021; Lochstoer and Muir, 2022; Fan et al., 2022](#)).

Following [Fama and French \(1988\)](#) and [Han and Li \(2021\)](#), we implement multi-period

univariate and bi-variate predictive regressions of the excess log market return, $r_{t,t+H}^m$, as

$$\sum_{h=1}^H \frac{r_{t+h}^m}{H} \equiv r_{t,t+H}^m = a + bX_t + \epsilon_{t,t+H} \quad (7)$$

$$r_{t,t+H}^m = a + \sum_{i=1}^N b_i X_{i,t} + \epsilon_{t,t+H} \quad (8)$$

where we use the CRSP value-weighted return index to proxy the market return, $r_{t,t+H}^m$. In Equation (7), a is a constant, b is a slope coefficient, X_t is a predictor variable. In Equation (8), a is a constant, b_i are slope coefficients and $X_{i,t}$ are the predictor variables. This allows us to understand the additional information $\mathcal{CVRP}^{\mathcal{T}/\mathcal{G}/\mathcal{B}}$ carries over and above existing predictors. In both cases, $H = \{1, 3, 6, 12, 18, 24\}$ stands for the predictive horizon in months. For each predictive regression, we use Newey West standard errors with $H - 1$ lags. We present coefficient estimates along with t -statistics that test the null hypothesis $H_0 : b_i = 0$ against the alternative $H_1 : b_i \neq 0$, and adjusted R-squared values, R_{adj}^2 .

Table 12 reports the results from univariate predictive regressions. In Panel A, we consider volatility premia type predictors, and in Panel B we show results from the [Welch and Goyal \(2008\)](#) predictors. Panel A provides substantial evidence that $\mathcal{CVRP}^{\mathcal{T}}$ and $\mathcal{CVRP}^{\mathcal{B}}$ contain information regarding future stock returns at all horizons up to 24 months ahead. A one percent increase in $\mathcal{CVRP}^{\mathcal{B}}$ ($\mathcal{CVRP}^{\mathcal{T}}$) predicts an increase in average excess market returns of 0.18% (0.22%) next month, 0.10% (0.11%) next quarter, 0.13% (0.11%) over the next 6 months, 0.16% (0.15%) over the next 12 months, and 0.14% (0.12%) over the next two years. The corresponding t -statistics are 1.69 (3.75), 1.45 (3.44), 4.04 (3.51), 4.51 (5.50), and 4.26 (4.21), respectively. When using $\mathcal{MVRP}^{\mathcal{T}/\mathcal{G}/\mathcal{B}}$, we find similar results in terms of significance and magnitude. With the exception of [Welch and Goyal \(2008\)](#)'s stock variance, svar, other volatility premia-related proxies contain little relevant information for future market returns. Notably, the adjusted R-squared values at long horizons are highest when using $\mathcal{CVRP}^{\mathcal{B}}$.

Turning to Panel B, the financial variables – book to market ratio (bm), dividend price ratio (dp), and dividend payout ratio (de); government bonds and bills: short-term treasury bills (tbl), long-term yields (lty), and the term spread (tms); corporate bond spreads: dfr and dfy; and inflation – contain information about future stock returns. For variables that predict stock returns, the adjusted R-squared values are highest at longer horizons.

Table 12. Univariate predictive regressions

This table reports summary statistics of univariate predictive regressions. The dependent variable is the average monthly market excess return over various predictive horizons (1, 3, 6, 12, 18, and 24 months). We use the CRSP value-weighted return as a proxy for the market return. The independent variables include volatility risk premia-related predictors (Panel A) and the common predictors of [Welch and Goyal \(2008\)](#) (Panel B). We report Newey-West adjusted t-statistics in shaded rows under the slope coefficients. R^2_{adj} is the adjusted R-squared. The sample is from January 2000 to December 2020.

Panel A: Volatility risk premia-related predictors													
	1	3	6	12	18	24		1	3	6	12	18	24
\mathcal{CVRP}^T	0.22	0.11	0.11	0.15	0.13	0.12	\mathcal{MVRP}^T	0.12	0.15	0.11	0.08	0.08	0.08
	3.75	3.44	3.51	5.50	4.19	4.21		0.99	2.33	2.84	2.40	2.97	3.47
R^2_{adj}	0.04	0.03	0.06	0.16	0.17	0.20	R^2_{adj}	0.01	0.05	0.06	0.05	0.07	0.09
\mathcal{CVRP}^G	0.15	0.07	0.03	0.07	0.06	0.04	\mathcal{MVRP}^G	0.12	0.11	0.03	0.03	0.03	0.01
	2.06	0.83	0.55	1.24	0.98	0.74		0.57	0.88	0.54	0.62	0.71	0.48
R^2_{adj}	0.02	0.01	0.00	0.04	0.03	0.02	R^2_{adj}	0.01	0.02	0.00	0.00	0.00	0.00
\mathcal{CVRP}^B	0.18	0.10	0.13	0.16	0.14	0.14	\mathcal{MVRP}^B	0.13	0.17	0.16	0.12	0.12	0.12
	1.69	1.45	4.04	4.51	3.61	4.26		1.24	3.85	3.48	2.68	3.61	3.90
R^2_{adj}	0.02	0.02	0.06	0.15	0.17	0.24	R^2_{adj}	0.01	0.06	0.09	0.08	0.11	0.16
ΔVIX	-0.11	-0.02	0.00	-0.01	0.00	-0.01	ΔCIV	0.03	0.00	0.00	-0.01	0.00	0.00
	-1.68	-0.57	0.05	-0.65	-0.43	-1.01		1.12	-0.27	0.20	-1.25	-0.49	-0.62
R^2_{adj}	0.02	0.00	0.00	0.00	0.00	0.00	R^2_{adj}	0.01	0.00	0.00	0.00	0.00	0.00
\mathcal{CTVOL}^B	-0.30	0.05	0.01	0.00	0.02	-0.04	svar	0.02	-0.05	0.23	0.20	0.18	0.24
	-0.76	0.21	0.13	-0.02	0.25	-0.92		0.02	-0.08	0.82	1.12	1.32	1.92
R^2_{adj}	0.01	0.00	0.00	0.00	0.00	0.00	R^2_{adj}	0.00	0.00	0.01	0.01	0.01	0.02
Panel B: The Welch and Goyal (2008) predictors													
	1	3	6	12	18	24		1	3	6	12	18	24
dp	0.04	0.04	0.04	0.04	0.04	0.03	lty	-0.64	-0.60	-0.61	-0.56	-0.56	-0.52
	1.67	2.19	3.42	5.44	5.62	5.91		-3.96	-3.94	-3.95	-4.00	-4.21	-4.25
R^2_{adj}	0.03	0.09	0.19	0.33	0.41	0.46	R^2_{adj}	0.04	0.10	0.18	0.25	0.35	0.39
ep	0.00	0.00	0.00	0.00	0.00	0.00	ltr	0.08	0.02	0.04	0.01	0.00	0.00
	0.31	0.12	0.07	0.27	-0.08	-0.32		1.44	0.23	1.23	0.39	0.13	0.06
R^2_{adj}	0.00	0.00	0.00	0.00	0.00	0.00	R^2_{adj}	0.00	0.00	0.01	0.00	0.00	0.00
de	0.00	0.01	0.01	0.01	0.01	0.01	tms	-0.02	0.03	0.09	0.25	0.37	0.45
	0.56	1.00	1.46	1.84	2.93	2.80		-0.12	0.14	0.53	1.77	2.60	3.56
R^2_{adj}	0.00	0.01	0.03	0.04	0.08	0.11	R^2_{adj}	0.00	0.00	0.00	0.06	0.18	0.34
bm	0.13	0.14	0.15	0.13	0.12	0.10	dfy	-0.13	0.18	0.61	0.78	0.88	0.92
	4.86	6.28	7.15	7.91	7.48	5.75		-0.13	0.22	1.04	2.48	3.06	3.55
R^2_{adj}	0.04	0.13	0.26	0.44	0.50	0.48	R^2_{adj}	0.00	0.00	0.02	0.06	0.11	0.16
ntis	0.12	0.15	0.14	0.09	0.00	-0.01	dfr	0.12	0.01	0.04	0.06	0.05	0.04
	0.49	0.62	0.65	0.53	0.01	-0.08		0.80	0.06	0.39	1.30	1.14	1.23
R^2_{adj}	0.00	0.01	0.02	0.01	0.00	0.00	R^2_{adj}	0.00	0.00	0.00	0.01	0.01	0.01
tbl	-0.38	-0.38	-0.40	-0.43	-0.47	-0.47	infl	0.08	0.27	-0.60	-0.75	-0.44	-0.46
	-2.76	-3.09	-3.56	-4.39	-5.28	-7.97		0.09	0.31	-1.52	-2.38	-2.54	-2.68
R^2_{adj}	0.02	0.07	0.14	0.30	0.52	0.70	R^2_{adj}	0.00	0.00	0.01	0.04	0.02	0.03

Tables 13 and 14 show results from bi-variate predictive regressions that always contain \mathcal{CVRP}^T and \mathcal{CVRP}^B , respectively. In each regression, we add a volatility-related premium or [Welch and Goyal \(2008\)](#) predictor. The top left panel of each table reports the results from corresponding univariate predictive regressions for ease of comparison. The

results provide convincing evidence that \mathcal{CVRP}^T and \mathcal{CVRP}^B contain information over and above existing predictors the literature commonly uses and other volatility premia-type proxies. It is clear that the coefficient estimates are robust when controlling for other predictors and the statistical significance of the \mathcal{CVRP}^T (\mathcal{CVRP}^B) coefficient is always present. In general, the adjusted R-squared statistics increase, particularly from bi-variate predictive regressions with two significant predictors.

Our final set of analyses considers stock market return predictability in an out-of-sample context. We consider univariate regressions using an expanding window. The initial estimation spans January 2000 to December 2004, while the out-of-sample period is January 2005 to December 2020. For each month in the out-of-sample period, we estimate a univariate regression for a specific horizon and make a one-period ahead prediction. Table 15 reports the out-of-sample R-squared statistics for volatility premium-related predictors in Panel A and the variables in [Welch and Goyal \(2008\)](#) in Panel B.

Focusing on Panel A, the out-of-sample predictive power of \mathcal{CVRP}^T and \mathcal{CVRP}^B in predicting market returns is monotonically increasing with the predictive horizon. Our results also suggest that market total and bad volatility risk premia also yield out-of-sample predictive benefits. However, at each horizon, \mathcal{CVRP}^T (\mathcal{CVRP}^B) has higher out-of-sample R-squared statistics than the corresponding measure from index options, \mathcal{MVRP}^T (\mathcal{MVRP}^B). All other volatility risk premia-related variables have a zero or negative out-of-sample R-squared.

Turning to Panel B, we can see that the only variables containing consistent out-of-sample predictive power at all horizons are the book-to-market ratio (bm) and the short-term Treasury bill rate (tbl). Out-of-sample R-squared for bm remains stable at around 0.29 from a 12- to 24-month horizon, whereas we observe a monotonically increasing out-of-sample R-squared for tbl. We attribute the high values at predictive horizons of 12 months or more to the fact that short-term interest rates are near zero with very little variation for 7 years of our out-of-sample period. With the exception of bm and tbl, however, \mathcal{CVRP}^T and \mathcal{CVRP}^B have higher out-of-sample R-squared statistics than all other [Welch and Goyal \(2008\)](#) variables.

Our findings indicate that \mathcal{CVRP}^T and \mathcal{CVRP}^B contain information regarding future

Table 13. Bivariate predictive regressions: \mathcal{CVRP}^T and other predictors

This table reports summary statistics of bivariate predictive regressions for common total volatility risk premium (\mathcal{CVRP}^T) while controlling for other predictors, one at a time. The dependent variable is the average monthly market excess return over various predictive horizons (1, 3, 6, 12, 18, and 24 months). We report Newey-West adjusted t-statistics in shaded rows under the slope coefficients. R_{adj}^2 is the adjusted R-squared. The sample is from January 2000 to December 2020.

	1	3	6	12	18	24		1	3	6	12	18	24
\mathcal{CVRP}^T	0.22	0.11	0.11	0.15	0.13	0.12	\mathcal{CVRP}^T	0.19	0.12	0.14	0.17	0.15	0.14
	3.75	3.44	3.51	5.50	4.19	4.21		3.11	3.79	5.07	6.41	5.61	5.13
R_{adj}^2	0.04	0.03	0.06	0.16	0.17	0.20	ΔVIX	-0.05	0.02	0.05	0.04	0.04	0.03
								-0.84	0.71	1.89	3.72	3.29	3.12
							R_{adj}^2	0.04	0.03	0.07	0.18	0.20	0.22
\mathcal{CVRP}^T	0.20	0.07	0.09	0.14	0.11	0.10	\mathcal{CVRP}^T	0.29	0.12	0.14	0.17	0.16	0.14
	2.18	2.12	3.14	4.67	3.50	3.34		3.95	4.00	4.41	6.53	6.22	5.75
\mathcal{MVRP}^T	0.06	0.13	0.09	0.03	0.04	0.04	ΔCIV	0.05	0.01	0.02	0.02	0.02	0.02
	0.44	1.96	2.46	0.89	1.39	1.67		2.63	0.53	2.87	3.47	4.36	4.76
R_{adj}^2	0.04	0.06	0.09	0.17	0.19	0.22	R_{adj}^2	0.07	0.03	0.07	0.18	0.21	0.24
\mathcal{CVRP}^T	0.21	0.09	0.12	0.16	0.14	0.13	\mathcal{CVRP}^T	0.22	0.11	0.12	0.15	0.13	0.12
	2.41	2.82	3.79	5.93	4.60	4.53		3.47	3.45	3.81	5.75	4.54	4.51
\mathcal{MVRP}^g	0.05	0.08	0.00	-0.04	-0.03	-0.04	\mathcal{CTVOL}^B	0.00	-0.01	0.01	0.01	0.00	0.01
	0.22	0.63	-0.07	-1.05	-0.92	-1.61		-0.05	-0.34	0.34	0.29	0.25	0.62
R_{adj}^2	0.04	0.04	0.06	0.17	0.18	0.21	R_{adj}^2	0.04	0.03	0.06	0.17	0.18	0.21
\mathcal{CVRP}^T	0.20	0.07	0.08	0.13	0.11	0.09	\mathcal{CVRP}^T	0.22	0.11	0.12	0.16	0.14	0.13
	2.56	2.39	2.99	4.26	3.16	2.99		3.29	3.55	3.92	5.85	4.59	4.59
\mathcal{MVRP}^B	0.06	0.15	0.14	0.07	0.07	0.08	svar	0.17	0.02	0.30	0.37	0.32	0.37
	0.56	3.44	3.03	1.56	2.34	2.66		0.16	0.03	1.30	3.55	3.41	3.67
R_{adj}^2	0.04	0.07	0.11	0.18	0.21	0.27	R_{adj}^2	0.04	0.03	0.07	0.18	0.20	0.24
\mathcal{CVRP}^T	0.17	0.09	0.09	0.08	0.06	0.06	\mathcal{CVRP}^T	0.16	0.04	0.05	0.10	0.07	0.07
	2.01	1.79	1.77	2.05	2.78	3.88		2.22	0.92	1.27	3.59	3.32	4.14
dp	0.03	0.04	0.04	0.04	0.03	0.03	lty	-0.47	-0.56	-0.56	-0.46	-0.48	-0.45
	1.00	1.73	2.50	3.80	5.08	6.02		-2.36	-3.37	-3.42	-3.31	-3.78	-4.10
R_{adj}^2	0.05	0.10	0.19	0.37	0.45	0.50	R_{adj}^2	0.06	0.10	0.19	0.31	0.40	0.45
\mathcal{CVRP}^T	0.22	0.11	0.12	0.15	0.13	0.12	\mathcal{CVRP}^T	0.22	0.11	0.12	0.15	0.13	0.12
	3.88	3.49	3.70	5.88	4.11	3.92		3.73	3.52	3.83	5.44	4.18	4.21
ep	0.01	0.00	0.00	0.00	0.00	0.00	ltr	0.09	0.02	0.05	0.01	0.00	0.00
	0.60	0.25	0.18	0.52	0.12	-0.16		1.62	0.26	1.06	0.35	0.07	0.03
R_{adj}^2	0.04	0.03	0.06	0.17	0.18	0.20	R_{adj}^2	0.04	0.03	0.06	0.16	0.18	0.20
\mathcal{CVRP}^T	0.22	0.10	0.10	0.14	0.11	0.10	\mathcal{CVRP}^T	0.23	0.11	0.11	0.13	0.10	0.08
	3.52	2.46	2.97	4.76	3.50	3.41		4.01	3.38	3.72	5.96	4.21	4.33
de	0.00	0.00	0.01	0.00	0.00	0.01	tms	-0.16	-0.04	0.01	0.15	0.28	0.38
	-0.07	0.62	1.02	1.05	2.54	2.43		-0.90	-0.24	0.08	1.36	2.25	3.15
R_{adj}^2	0.04	0.03	0.07	0.17	0.21	0.26	R_{adj}^2	0.04	0.03	0.06	0.19	0.27	0.42
\mathcal{CVRP}^T	0.17	0.08	0.08	0.07	0.06	0.06	\mathcal{CVRP}^T	0.24	0.11	0.10	0.14	0.11	0.10
	2.37	2.13	2.17	3.16	3.33	4.25		3.87	2.75	2.73	4.29	3.72	3.67
bm	0.10	0.13	0.14	0.12	0.11	0.09	dfy	-0.73	-0.10	0.34	0.47	0.62	0.69
	3.14	5.49	7.38	7.22	7.29	5.91		-0.78	-0.11	0.58	1.59	3.34	4.23
R_{adj}^2	0.06	0.13	0.27	0.47	0.53	0.52	R_{adj}^2	0.04	0.03	0.06	0.18	0.22	0.28
\mathcal{CVRP}^T	0.22	0.11	0.12	0.15	0.13	0.12	\mathcal{CVRP}^T	0.22	0.12	0.12	0.16	0.14	0.13
	3.75	3.48	3.48	5.12	4.13	4.21		3.43	3.75	4.32	5.69	4.24	4.06
ntis	0.15	0.17	0.16	0.11	0.01	0.00	dfr	-0.03	-0.08	-0.04	-0.04	-0.04	-0.04
	0.69	0.74	0.79	0.75	0.18	0.06		-0.16	-0.53	-0.45	-0.85	-1.07	-1.12
R_{adj}^2	0.04	0.04	0.08	0.18	0.18	0.20	R_{adj}^2	0.04	0.03	0.06	0.17	0.18	0.21
\mathcal{CVRP}^T	0.18	0.04	0.04	0.07	0.06	0.05	\mathcal{CVRP}^T	0.23	0.12	0.11	0.14	0.13	0.12
	2.39	0.91	0.98	2.87	2.86	2.35		3.86	3.15	2.91	5.06	3.93	3.92
tbl	-0.19	-0.33	-0.36	-0.36	-0.44	-0.45	infl	0.64	0.56	-0.33	-0.40	-0.14	-0.17
	-1.18	-2.31	-2.82	-3.56	-4.44	-6.85		0.71	0.62	-0.76	-1.25	-0.78	-1.00
R_{adj}^2	0.04	0.07	0.14	0.33	0.53	0.71	R_{adj}^2	0.04	0.04	0.06	0.17	0.18	0.21

Table 14. Bivariate predictive regressions: \mathcal{CVRP}^B and other predictors

This table reports summary statistics of bivariate predictive regressions for common bad volatility risk premium (\mathcal{CVRP}^B) while controlling for other predictors, one at a time. The dependent variable is the average monthly market excess return over various predictive horizons (1, 3, 6, 12, 18, and 24 months). We report Newey-West adjusted t-statistics in shaded rows under the slope coefficients. R^2_{adj} is the adjusted R-squared. The sample is from January 2000 to December 2020.

	1	3	6	12	18	24		1	3	6	12	18	24
\mathcal{CVRP}^B	0.18	0.10	0.13	0.16	0.14	0.14	\mathcal{CVRP}^B	0.15	0.10	0.15	0.17	0.15	0.15
	1.69	1.45	4.04	4.51	3.61	4.26		1.20	1.23	3.79	4.61	3.78	4.37
R^2_{adj}	0.02	0.02	0.06	0.15	0.17	0.24	ΔVIX	-0.08	0.00	0.03	0.02	0.03	0.02
								-1.12	0.04	0.93	2.49	3.26	3.01
							R^2_{adj}	0.03	0.02	0.07	0.15	0.19	0.25
\mathcal{CVRP}^B	0.15	0.06	0.11	0.14	0.13	0.13	\mathcal{CVRP}^B	0.20	0.10	0.14	0.16	0.15	0.15
	1.04	0.65	3.05	4.75	3.54	4.26		1.85	1.31	4.14	4.67	3.76	4.29
\mathcal{MVRP}^T	0.08	0.13	0.09	0.04	0.04	0.04	ΔCIV	0.03	0.00	0.01	0.01	0.01	0.01
	0.55	1.66	2.25	1.18	1.77	2.21		1.43	-0.04	0.89	1.19	2.43	2.11
R^2_{adj}	0.03	0.06	0.09	0.15	0.19	0.26	R^2_{adj}	0.03	0.02	0.07	0.15	0.18	0.25
\mathcal{CVRP}^B	0.19	0.11	0.14	0.16	0.14	0.14	\mathcal{CVRP}^B	0.26	0.16	0.17	0.20	0.19	0.18
	1.86	1.73	3.94	4.58	3.76	4.33		3.07	4.05	4.54	5.61	4.66	4.90
\mathcal{MVRP}^g	0.13	0.12	0.05	0.02	0.02	0.01	\mathcal{CTVOL}^B	-0.06	-0.04	-0.03	-0.04	-0.03	-0.03
	0.68	0.99	0.86	0.62	0.80	0.47		-1.27	-1.29	-1.17	-1.74	-1.75	-1.55
R^2_{adj}	0.03	0.04	0.07	0.15	0.18	0.24	R^2_{adj}	0.04	0.04	0.08	0.20	0.24	0.30
\mathcal{CVRP}^B	0.15	0.02	0.08	0.13	0.12	0.11	\mathcal{CVRP}^B	0.21	0.12	0.14	0.17	0.16	0.15
	0.96	0.26	2.35	4.18	2.90	3.68		3.12	2.90	3.50	4.28	3.56	4.31
\mathcal{MVRP}^B	0.05	0.16	0.12	0.05	0.05	0.06	svar	-0.45	-0.32	-0.08	-0.23	-0.22	-0.16
	0.35	2.19	2.78	1.27	2.02	2.64		-0.42	-0.47	-0.23	-1.33	-1.21	-1.33
R^2_{adj}	0.02	0.06	0.10	0.15	0.19	0.27	R^2_{adj}	0.03	0.03	0.06	0.15	0.18	0.25
\mathcal{CVRP}^B	0.11	0.00	0.05	0.09	0.08	0.09	\mathcal{CVRP}^B	0.12	0.04	0.08	0.12	0.11	0.11
	0.92	0.06	1.71	2.10	1.96	2.60		1.19	0.59	2.08	3.71	3.53	5.04
dp	0.03	0.04	0.04	0.04	0.03	0.03	lty	-0.55	-0.58	-0.55	-0.50	-0.51	-0.46
	1.12	2.24	2.54	3.17	4.09	4.21		-2.57	-3.17	-3.33	-3.70	-4.24	-4.69
R^2_{adj}	0.04	0.09	0.19	0.34	0.43	0.49	R^2_{adj}	0.05	0.10	0.20	0.34	0.45	0.54
\mathcal{CVRP}^B	0.22	0.12	0.16	0.19	0.16	0.16	\mathcal{CVRP}^B	0.17	0.10	0.13	0.16	0.14	0.15
	2.02	1.47	5.12	5.86	4.02	4.43		1.55	1.52	3.79	4.41	3.75	4.48
ep	0.01	0.01	0.01	0.01	0.00	0.00	ltr	0.06	0.00	0.03	-0.01	-0.02	-0.02
	0.90	0.45	0.58	1.23	0.90	0.87		0.99	0.05	0.58	-0.50	-0.79	-0.99
R^2_{adj}	0.03	0.03	0.08	0.19	0.20	0.26	R^2_{adj}	0.02	0.02	0.07	0.15	0.18	0.24
\mathcal{CVRP}^B	0.20	0.08	0.12	0.16	0.13	0.13	\mathcal{CVRP}^B	0.20	0.10	0.13	0.14	0.10	0.08
	1.68	0.87	2.81	4.16	2.76	3.32		1.92	1.50	4.22	4.58	3.25	4.07
de	0.00	0.00	0.00	0.00	0.00	0.00	tms	-0.16	-0.05	-0.02	0.12	0.26	0.35
	-0.40	0.36	0.40	-0.09	0.99	1.19		-0.82	-0.27	-0.11	0.90	1.99	2.84
R^2_{adj}	0.02	0.02	0.07	0.15	0.18	0.25	R^2_{adj}	0.03	0.02	0.06	0.16	0.25	0.41
\mathcal{CVRP}^B	0.13	0.04	0.06	0.08	0.07	0.09	\mathcal{CVRP}^B	0.32	0.14	0.15	0.16	0.12	0.12
	1.05	0.41	1.75	3.09	2.22	2.99		2.77	2.08	2.41	2.50	2.13	2.62
bm	0.11	0.13	0.14	0.12	0.11	0.09	dfy	-1.89	-0.62	-0.22	-0.08	0.23	0.29
	3.58	4.90	6.49	7.24	7.32	5.93		-1.48	-0.59	-0.26	-0.14	0.61	0.99
R^2_{adj}	0.05	0.13	0.28	0.47	0.54	0.56	R^2_{adj}	0.04	0.03	0.06	0.15	0.18	0.25
\mathcal{CVRP}^B	0.19	0.11	0.14	0.16	0.14	0.14	\mathcal{CVRP}^B	0.17	0.10	0.13	0.15	0.14	0.14
	1.86	1.83	3.48	4.07	3.58	4.27		1.53	1.30	3.80	4.61	3.65	4.16
ntis	0.17	0.18	0.18	0.13	0.03	0.02	dfr	0.07	-0.02	0.00	0.01	0.00	-0.01
	0.65	0.73	0.80	0.79	0.35	0.37		0.45	-0.15	0.02	0.29	-0.13	-0.38
R^2_{adj}	0.03	0.03	0.09	0.17	0.18	0.24	R^2_{adj}	0.02	0.02	0.06	0.15	0.17	0.24
\mathcal{CVRP}^B	0.12	0.02	0.06	0.07	0.06	0.04	\mathcal{CVRP}^B	0.21	0.12	0.13	0.15	0.14	0.14
	1.02	0.30	1.49	2.11	2.71	2.48		2.16	2.30	2.92	4.13	3.73	4.38
tbl	-0.26	-0.36	-0.35	-0.37	-0.43	-0.43	infl	0.82	0.71	-0.14	-0.26	0.04	0.04
	-1.55	-2.50	-2.80	-3.57	-4.70	-7.13		0.83	0.75	-0.26	-0.82	0.25	0.20
R^2_{adj}	0.03	0.07	0.15	0.33	0.53	0.73	R^2_{adj}	0.03	0.03	0.06	0.15	0.18	0.24

Table 15. Univariate predictive regressions: out-of-sample performance

This table reports out-of-sample performance statistics of univariate predictive regressions. The dependent variable is the average monthly market excess return over various predictive horizons (1, 3, 6, 12, 18, and 24 months). We use the CRSP value-weighted return as a proxy for the market return. The independent variables include volatility risk premia-related predictors (Panel A) and the common predictors of [Welch and Goyal \(2008\)](#) (Panel B). The in-sample sample is from January 2000 to December 2004. The out-of-sample sample is from January 2005 to December 2020. For each month in the out-of-sample period, we estimate a univariate predictive regression for a specific horizon using the historical data and make a one-period-ahead prediction. The reported numbers are monthly out-of-sample R^2 statistics of this expanding window procedure.

Panel A: Volatility risk premia-related predictors													
	1	3	6	12	18	24		1	3	6	12	18	24
\mathcal{CVRP}^T	0.04	0.02	0.04	0.14	0.14	0.17	\mathcal{MVRP}	-0.01	0.04	0.03	0.02	0.04	0.09
\mathcal{CVRP}^G	0.00	-0.03	-0.04	-0.08	-0.15	-0.24	\mathcal{MVRP}^G	-0.04	-0.03	-0.03	-0.02	0.00	-0.02
\mathcal{CVRP}^B	0.02	0.02	0.02	0.09	0.12	0.19	\mathcal{MVRP}^B	-0.01	0.03	0.05	0.01	0.03	0.12
ΔVIX	-0.02	-0.03	-0.02	0.00	0.00	0.00	ΔCIX	-0.04	-0.02	-0.02	0.00	0.00	0.00
\mathcal{CTVOL}^B	-0.13	-0.29	-0.12	-0.10	-0.07	-0.02	svar	-0.12	-0.08	-0.14	-0.14	-0.12	-0.13
Panel B: The Welch and Goyal (2008) predictors													
	1	3	6	12	18	24		1	3	6	12	18	24
dp	-0.01	-0.09	-0.11	-0.15	-0.16	-0.19	lty	0.01	-0.01	-0.01	-0.06	0.06	0.02
ep	-0.05	-0.22	-0.43	-0.46	-0.33	-0.52	ltr	-0.01	-0.02	-0.01	-0.01	0.00	-0.01
de	-0.04	-0.30	-0.57	-0.20	-0.14	-0.47	tms	-0.01	-0.04	-0.09	-0.14	-0.07	0.16
bm	0.03	0.11	0.22	0.29	0.29	0.28	dfy	-0.04	-0.23	-0.62	-0.15	-0.37	-0.31
ntis	-0.01	-0.06	-0.15	-0.31	-0.47	-0.82	dfr	-0.05	-0.06	-0.04	-0.02	0.02	-0.01
tbl	0.01	0.02	0.04	0.20	0.46	0.69	infl	-0.01	-0.03	0.01	0.03	0.02	0.01

excess market returns. Our analysis shows that including various other proxies that relate to volatility risk does not subsume the predictive ability of the common information within firm-level volatility risk (total and bad).¹¹ Consistent with the literature on the predictability of volatility risk, we show that \mathcal{CVRP}^T and \mathcal{CVRP}^B positively predict future excess market returns. Our results also resonate well with the long-horizon predictive power of the correlation risk premium (CRP) for aggregate excess returns in [Hollstein and Simen \(2020\)](#). Although their CRP may capture similar information to our \mathcal{CVRP}^T measure, we show that commonalities in firm-level VRPs that stem from put options and negative returns contain relevant information for future market returns both in-sample and out-of-sample.

¹¹As an additional robust check we conduct the above analysis where we remove all recessions and the COVID-19 pandemic. These results yield the same conclusions as those we present here and are available upon request.

A possible explanation for the positive predictive power of our volatility risk proxies is mistakes in market participants' beliefs ([Drechsler and Yaron, 2011](#)). Such beliefs show up in volatility and equity claims which indicates a volatility premium positively forecasts market returns. [Lochstoer and Muir \(2022\)](#) formalizes this mechanism by incorporating slow-moving beliefs within an asset pricing model to reconcile this phenomenon.

5 Conclusion

This paper shows that total, good, and bad firm-level volatility risk premia obey a strong factor structure. We define the "total" volatility risk premium at the stock level as the difference between the physical and risk-neutral expectations of return volatility. We compute the "good" and "bad" components of volatility risk premia that capture the compensation for the realized volatility in positive and negative returns. We explore the implications of this behavior in both cross-sectional asset pricing and market return predictability exercises. The portfolio-sort analysis shows that stocks in the lowest \mathcal{CVRP}^B -beta quintile earn 7.32% higher average annual returns than those in the highest \mathcal{CVRP}^B -beta quintile. This result is robust to controlling for several other factors related to volatility premia and firm characteristics. Estimating risk premia using Fama-MacBeth two-pass regressions and [Giglio and Xiu \(2021\)](#) three-pass regressions further supports this result and estimates risk premia of similar magnitude and statistical and economic significance.

Our market return predictability exercises suggest that total and bad commonality in firm-level volatility risk premia contain relevant information for market returns up to 24 months ahead. We show that \mathcal{CVRP}^T and \mathcal{CVRP}^B provide incremental predictive power over and above a number of key predictors and other volatility premia-related variables. Out-of-sample R-squared statistics show that the predictive power of \mathcal{CVRP}^T and \mathcal{CVRP}^B increases monotonically with the horizon up to 17% and 19% for 24 months ahead. This is in stark contrast to the predictors used by [Welch and Goyal \(2008\)](#) and others, which show good in-sample performance but no out-of-sample predictive power.

Several implications emerge from this paper. First, common firm volatility risk premia differ from market volatility risk premia. Second, common bad volatility risk premia represent a priced source of risk: the high \mathcal{CVRP}^B -beta stocks produce significantly lower average and risk-adjusted returns relative to the low \mathcal{CVRP}^B -beta stocks. Intuitively, such

assets can act as intertemporal hedges against adverse changes in investment opportunities proxied by higher level of common bad volatility premia. In turn, investors are willing to accept lower returns for such stocks for hedging purposes. Third, the information content of total and bad firm-level volatility risk premia is relevant to future market returns. Thus, we document new market return predictors that practitioners can use in forecasting.

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Appendix for

The Common Factor in Volatility Risk Premia

Abstract

This appendix contains the description of the discretization procedure of model-free implied variance and additional results not included in the main body of the paper.

A Discretization Procedure of Model-Free Implied Variance

Considering total implied variance in Equation (1), the discretization is

$$\mathcal{IV}_{i,t}^T = \frac{2}{T} \sum_{i=1}^n \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2,$$

where T is time to expiration, F is the forward index level derived from the put-call parity as $F = e^{r^f T} [C(K, T) - P(K, T)] + K$ with the risk-free rate r^f , K_0 is the reference price, the first exercise price less or equal to the forward level $F(K_0 \leq F)$, and K_i is the i -th OTM strike price available on a specific date (call if $K_i > K_0$, put if $K_i < K_0$, and both call and put if $K_i = K_0$). $Q(K_i)$ is the average bid-ask of OTM options with an exercise price equal to K_i . If $K_i = K_0$, it will be equal to the average of the at-the-money (ATM) call and put price, relative to the strike price, and $\Delta(K_i)$ is the sum divided by two of the two nearest prices to the exercise price K_0 , namely, $\frac{(K_{i+1}-K_{i-1})}{2}$ for $2 \leq i \leq n-1$. For further details see the CBOE VIX white paper available at https://cdn.cboe.com/api/global/us_indices/governance/Volatility_Index_Methodology_Cboe_Volatility_Index.pdf. Note the standard CBOE methodology considers an interpolation between the two closest to 30-days expiration dates. In our data construction, we take into account only one expiration date closest to 30 days due to options data availability with respect to firm-level stocks.

B Additional Results

Table A1 reports summary statistics of portfolios formed on exposures to common volatility risk premia without orthogonalization to additional factors. Table A2 reports summary statistics of portfolios formed on exposures to market volatility risk premia without orthogonalization to additional factors.

Table A1. Portfolios formed on \mathcal{CVRP} -beta: no orthogonalization

This table presents the average excess returns (RET-RF) and alphas (α_5, α_6) expressed in monthly percentages for equal-weighted and value-weighted quintile portfolios ($\mathcal{P}_i : i = 1, \dots, 5$) and a long-short strategy (\mathcal{P}_{5-1}) formed on loadings to common total (Panel A), good (Panel B), and bad (Panel C) volatility risk premia without controlling for the exposure to other variables. Specifically, we estimate common volatility risk premia (\mathcal{CVRP})-betas from univariate regressions of monthly excess returns on \mathcal{CVRP} using a rolling 60-month window. The portfolio $\mathcal{P}_1(\mathcal{P}_5)$ comprises stocks with the lowest (highest) \mathcal{CVRP} -betas. The long-short strategy buys \mathcal{P}_5 and sells \mathcal{P}_1 . α_5 is the alpha from the five-factor Fama-French model including the market, size, book-to-market, investment, and profitability factors. α_6 is the alpha relative to the five Fama-French factors and momentum. The numbers in shaded rows are Newey-West adjusted t-statistics of average returns and alphas. The sample is from January 2000 to December 2020.

	Equal-Weighted						Value-Weighted					
	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}
Panel A: \mathcal{CVRP}^T-beta portfolios												
RET-RF	0.67	0.77	0.77	0.89	0.82	0.14	0.73	0.73	0.74	0.88	0.89	0.16
	1.96	2.33	2.11	2.12	1.67	0.76	2.50	2.88	2.24	2.36	1.80	0.59
α_5	0.07	0.16	0.07	0.08	-0.11	-0.18	0.16	0.03	-0.06	-0.01	-0.17	-0.34
	0.76	3.20	1.19	1.24	-1.89	-1.57	1.22	0.52	-1.05	-0.09	-1.21	-1.33
α_6	0.06	0.16	0.07	0.09	-0.10	-0.17	0.16	0.02	-0.06	0.00	-0.17	-0.33
	0.76	3.20	1.15	1.29	-1.53	-1.54	1.24	0.43	-1.29	-0.06	-1.16	-1.31
Panel B: \mathcal{CVRP}^G-beta portfolios												
RET-RF	0.48	0.68	0.87	0.94	0.96	0.48	0.57	0.66	0.84	1.08	1.00	0.43
	1.40	2.05	2.31	2.21	1.86	1.45	2.10	2.41	2.47	2.75	2.07	1.30
α_5	-0.05	0.08	0.17	0.11	-0.05	0.00	0.03	-0.05	0.02	0.12	0.02	-0.01
	-0.39	1.11	3.05	1.92	-0.48	0.01	0.30	-0.63	0.28	1.72	0.16	-0.05
α_6	-0.06	0.08	0.17	0.12	-0.03	0.03	0.02	-0.05	0.02	0.13	0.04	0.01
	-0.58	1.11	2.94	1.87	-0.38	0.22	0.22	-0.72	0.26	1.80	0.29	0.07
Panel C: \mathcal{CVRP}^B-beta portfolios												
RET-RF	1.04	0.87	0.82	0.64	0.55	-0.49	1.05	0.98	0.77	0.59	0.50	-0.55
	2.46	2.45	2.29	1.60	1.25	-2.12	2.76	3.11	2.54	1.61	1.13	-2.28
α_5	0.25	0.18	0.14	-0.05	-0.24	-0.49	0.33	0.21	-0.04	-0.20	-0.44	-0.77
	2.39	3.94	2.25	-0.60	-2.19	-2.47	2.03	2.55	-0.62	-1.79	-2.87	-2.61
α_6	0.27	0.19	0.14	-0.06	-0.25	-0.52	0.34	0.21	-0.04	-0.22	-0.45	-0.79
	3.20	3.78	2.18	-0.67	-2.58	-3.36	2.39	2.64	-0.61	-2.08	-3.16	-3.04

Table A2. Portfolios formed on \mathcal{MVRP} -beta: no orthogonalization

This table presents the average excess returns (RET-RF) and alphas (α_5, α_6) expressed in monthly percentages for equal-weighted and value-weighted quintile portfolios ($\mathcal{P}_i : i = 1, \dots, 5$) and a long-short strategy (\mathcal{P}_{5-1}) formed on loadings to market total (Panel A), good (Panel B), and bad (Panel C) volatility risk premia without controlling for the exposure to other variables. Specifically, we estimate market volatility risk premia (\mathcal{MVRP})-betas from univariate regressions of monthly excess returns on \mathcal{MVRP} using a rolling 60-month window. The portfolio $\mathcal{P}_1(\mathcal{P}_5)$ comprises stocks with the lowest (highest) \mathcal{MVRP} -betas. The long-short strategy buys \mathcal{P}_5 and sells \mathcal{P}_1 . α_5 is the alpha from the five-factor Fama-French model including the market, size, book-to-market, investment, and profitability factors. α_6 is the alpha relative to the five Fama-French factors and momentum. The numbers in shaded rows are Newey-West adjusted t-statistics of average returns and alphas. The sample is from January 2000 to December 2020.

	Equal-Weighted						Value-Weighted					
	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}	\mathcal{P}_1	\mathcal{P}_2	\mathcal{P}_3	\mathcal{P}_4	\mathcal{P}_5	\mathcal{P}_{5-1}
Panel A: \mathcal{MVRP}^T-beta portfolios												
RET-RF	0.60	0.78	0.82	0.83	0.90	0.30	0.69	0.71	0.80	0.88	1.13	0.44
	1.59	2.21	2.25	2.08	1.91	1.30	1.97	2.13	2.63	2.71	2.26	1.19
α_5	-0.05	0.11	0.10	0.08	0.03	0.08	0.01	-0.03	0.08	0.05	0.00	-0.01
	-0.56	2.05	2.59	1.42	0.29	0.49	0.10	-0.54	1.13	0.60	0.03	-0.04
α_6	-0.06	0.11	0.11	0.09	0.03	0.09	0.02	-0.03	0.08	0.04	0.00	-0.02
	-0.57	2.14	2.33	1.35	0.33	0.52	0.14	-0.53	1.07	0.58	0.02	-0.06
Panel B: \mathcal{MVRP}^G-beta portfolios												
RET-RF	0.49	0.74	0.80	0.86	1.02	0.53	0.59	0.66	0.98	1.01	1.10	0.50
	1.45	2.19	2.17	2.04	2.02	1.74	2.07	2.14	2.93	2.76	2.22	1.52
α_5	-0.08	0.11	0.10	0.08	0.07	0.15	-0.03	-0.10	0.20	0.10	0.07	0.09
	-0.72	1.67	1.83	1.32	0.73	0.78	-0.23	-1.49	3.00	1.02	0.45	0.38
α_6	-0.09	0.11	0.10	0.09	0.09	0.18	-0.04	-0.10	0.20	0.10	0.08	0.12
	-0.93	1.70	1.77	1.29	1.03	1.11	-0.31	-1.63	3.12	1.05	0.50	0.44
Panel C: \mathcal{MVRP}^B-beta portfolios												
RET-RF	0.85	0.81	0.82	0.77	0.68	-0.17	0.76	0.86	0.81	0.84	0.78	0.02
	1.95	2.15	2.27	2.12	1.55	-0.81	1.68	2.49	2.70	2.78	1.85	0.06
α_5	0.04	0.12	0.12	0.06	-0.07	-0.11	-0.01	0.15	0.08	0.07	-0.22	-0.20
	0.38	1.95	2.20	0.71	-0.56	-0.51	-0.09	1.66	1.34	0.83	-1.71	-0.82
α_6	0.06	0.13	0.12	0.06	-0.08	-0.14	0.01	0.16	0.08	0.06	-0.23	-0.23
	0.62	2.35	2.15	0.71	-0.64	-0.71	0.04	2.02	1.37	0.81	-2.00	-1.12