Ejercicio 2 sección 1.6.5

a) $\cos(3\alpha) = \cos^3(\alpha) - 3\,sen(\alpha)\,sen^2(\alpha)$

b) $sen(3\alpha) = 3\cos^2(\alpha)\,sen(\alpha) - sen^3(\alpha)$

$$\begin{array}{ccccc} & & 1 & & \\ & 1 & & 1 & \\ 1 & & 2 & & 1 \\ 1 & 3 & & 3 & 1 \end{array}$$

$\cos(3\alpha) + \acute{y}\,sen(3\alpha) = (\cos\alpha + \acute{y}\sin\alpha)^3 \longrightarrow$ de Moivre's theorem

$(\cos\alpha + \acute{y}\sin\alpha)^3 = \cos^3\alpha + 3\cos^2\alpha\,\acute{y}\sin\alpha + 3\cos\alpha\,(\acute{y}\,sen\alpha)^2 + (\acute{y}\,sen\alpha)^3$

$= \cos^3\alpha - 3\cos\alpha\,sen^2\alpha + \acute{y}\,[3\cos^2\alpha\,\sin^2\alpha - sen^3\alpha]$

Real part $\cos(3\alpha) = \cos^3\alpha - 3\cos\,sen^2\alpha$

Img part $sen(3\alpha) = 3\cos^2\alpha\,\sin^2\alpha - sen^3\alpha$

· Encuentre   todos las   raices de la siguiente expresion (Ejercicio S sección 1.6.5)

a) $\sqrt{2\acute{y}}$ b) $\sqrt{1-\sqrt{3\acute{y}}}$ c) $(-1)^{1/3}$ d) $8^{1/6}$ e) $\sqrt[4]{-8-8\sqrt{3}\,\acute{y}}$

$z = \sqrt{2\acute{y}}$

a) $(2\acute{y})^{1/2} \longrightarrow z = 2\left[\cos\left(\frac{\pi}{2}\right) + \acute{y}sen\left(\frac{\pi}{2}\right)\right]$

$w_k = r^{1/n}\left(\cos\left(\frac{\theta + 2\pi k}{n}\right) + \acute{y}\,sen\left(\frac{\theta + 2\pi k}{n}\right)\right)$

$w_0 = 2^{1/2}\left[\cos\left(\frac{\frac{\pi}{2} + 2\pi(0)}{2}\right) + \acute{y}\,sen\left(\frac{\frac{\pi}{2} + 2\pi(0)}{2}\right)\right] = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + \acute{y}\,sen\left(\frac{\pi}{4}\right)\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + \acute{y}\,\frac{\sqrt{2}}{2}\right)$

$= (1 + \acute{y})$

$w_1 = 2^{1/2}\left[\cos\left(\frac{\frac{\pi}{2} + 2\pi}{2}\right) + \acute{y}\,sen\left(\frac{\frac{\pi}{2} + 2\pi}{2}\right)\right] = \sqrt{2}\left(\cos\left(5\frac{\pi}{4}\right) + \acute{y}sen\left(5\frac{\pi}{4}\right)\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} - \acute{y}\,\frac{\sqrt{2}}{2}\right)$

$= (-1 - \acute{y})$

b) $(1 - \sqrt{3}\,\acute{y})^{1/2} \longrightarrow r = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$ $\tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$ $z = 2\left(\cos\left(-\frac{\pi}{3}\right) + \acute{y}\,sen\left(-\frac{\pi}{3}\right)\right)$

$w_k = r^{1/n}\left(\cos\left(\frac{\theta + 2\pi k}{n}\right) + \acute{y}\,sen\left(\frac{\theta + 2\pi k}{n}\right)\right)$

$w_0 = 2^{1/2}\left[\cos\left(\frac{-\frac{\pi}{3} + 2\pi(0)}{2}\right) + sen\left(\frac{-\frac{\pi}{3} + 2\pi(0)}{2}\right)\right] = \sqrt{2}\left(\cos\left(-\frac{\pi}{6}\right) + \acute{y}\,sen\left(-\frac{\pi}{6}\right)\right) = \sqrt{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\acute{y}\right)$

$\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}\acute{y}\right)$

$w_1 = 2^{1/2}\left[\cos\left(\frac{-\frac{\pi}{3} + 2\pi(1)}{2}\right) + sen\left(\frac{-\frac{\pi}{3} + 2\pi(1)}{2}\right)\right] = \sqrt{2}\left(\cos\left(\frac{9\pi}{6}\right) + \acute{y}\,sen\left(\frac{5\pi}{6}\right)\right) = \sqrt{2}\left(\frac{-\sqrt{3}}{2} + \frac{1}{2}\acute{y}\right)$

$= \left(-\frac{\sqrt{6}}{2} + \frac{1}{2}\acute{y}\right)$

c) $= (-1)^{1/3}$ $\longrightarrow$ $z = -1$ $\longrightarrow$ $z = \cos\pi + \acute{y}\,\text{sen}\,\pi$

$$w_k = r^{1/n}\left(\cos\left(\frac{\theta + 2\pi k}{n}\right) + \acute{y}\,\text{sen}\left(\frac{\theta + 2\pi k}{n}\right)\right)$$

$$w_0 = 1^{1/3}\left[\cos\left(\frac{\pi + 2\pi(0)}{3}\right) + \text{Sen}\left(\frac{\pi + 2\pi(0)}{3}\right)\right] = 1^{1/3}\left(\cos\left(\frac{\pi}{3}\right) + \acute{y}\,\text{sen}\left(\frac{\pi}{3}\right)\right) = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\acute{y}\right)$$

$$w_1 = 1^{1/3}\left[\cos\left(\frac{\pi + 2\pi(1)}{3}\right) + \text{Sen}\left(\frac{\pi + 2\pi(1)}{3}\right)\right] = 1^{1/3}\left(\cos(\pi) + \acute{y}\,\text{sen}(\pi)\right) = \left(-1 + 0\acute{y}\right) = -1$$

$$w_2 = 1^{1/3}\left[\cos\left(\frac{\pi + 2\pi(1)}{3}\right) + \text{Sen}\left(\frac{\pi + 2\pi(1)}{3}\right)\right] = 1^{1/3}\left(\cos\left(\frac{5\pi}{3}\right) + \acute{y}\,\text{sen}\left(\frac{5\pi}{3}\right)\right) = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\acute{y}\right)$$

d) $8^{1/6}$ $\qquad z = 8(\cos(0) + \acute{y}\,\text{sen}(0))$

$$w_0 = 8^{1/6}\left[\cos\left(\frac{0 + 2\pi(0)}{6}\right) + \text{Sen}\left(\frac{0 + 2\pi(0)}{6}\right)\right] = 2^{3/6}\left(\cos(0) + \acute{y}\,\text{sen}(0)\right) = \sqrt{2}\left(1 + 0\acute{y}\right) = \sqrt{2}$$

$$w_1 = 8^{1/6}\left[\cos\left(\frac{0 + 2\pi(1)}{6}\right) + \text{Sen}\left(\frac{0 + 2\pi(1)}{6}\right)\right] = 2^{3/6}\left(\cos\left(\frac{\pi}{3}\right) + \acute{y}\,\text{sen}\left(\frac{\pi}{3}\right)\right) = \sqrt{2}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{2} + \acute{y}\frac{\sqrt{6}}{2}$$

$$w_2 = 8^{1/6}\left[\cos\left(\frac{0 + 2\pi(2)}{6}\right) + \text{Sen}\left(\frac{0 + 2\pi(2)}{6}\right)\right] = 2^{3/6}\left(\cos\left(\frac{2\pi}{3}\right) + \acute{y}\,\text{sen}\left(\frac{2\pi}{3}\right)\right) = \sqrt{2}\left(-\frac{1}{2} + \acute{y}\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}}{2} + \acute{y}\frac{\sqrt{6}}{2}$$

$$w_3 = 8^{1/6}\left[\cos\left(\frac{0 + 2\pi(3)}{6}\right) + \text{Sen}\left(\frac{0 + 2\pi(3)}{6}\right)\right] = 2^{3/6}\left(\cos(\pi) + \acute{y}\,\text{sen}(\pi)\right) = \sqrt{2}\left(-1 + (0)\acute{y}\right) = -\sqrt{2}$$

$$w_4 = 8^{1/6}\left[\cos\left(\frac{0 + 2\pi(4)}{6}\right) + \text{Sen}\left(\frac{0 + 2\pi(4)}{6}\right)\right] = 2^{3/6}\left(\cos\left(\frac{4\pi}{3}\right) + \acute{y}\,\text{sen}\left(\frac{4\pi}{3}\right)\right) = \sqrt{2}\left(-\frac{1}{2} - \acute{y}\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}}{2} - \acute{y}\frac{\sqrt{6}}{2}$$

$$w_5 = 8^{1/6}\left[\cos\left(\frac{0 + 2\pi(5)}{6}\right) + \text{Sen}\left(\frac{0 + 2\pi(5)}{6}\right)\right] = 2^{3/6}\left(\cos\left(\frac{5\pi}{3}\right) + \acute{y}\,\text{sen}\left(\frac{5\pi}{3}\right)\right) = \sqrt{2}\left(\frac{1}{2} - \acute{y}\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{2} - \acute{y}\frac{\sqrt{6}}{2}$$

e) $\sqrt[4]{-8 - 8\sqrt{3}\,\acute{y}}$ $\qquad z = 16\left(\cos\left(\frac{4\pi}{3}\right) + \acute{y}\sin\left(\frac{4\pi}{3}\right)\right)$

$$w_0 = 16^{1/6}\left[\cos\left(\frac{\frac{4\pi}{3} + 2\pi(0)}{4}\right) + \text{Sen}\left(\frac{\frac{4\pi}{3} + 2\pi(0)}{4}\right)\right] = 2\left(\cos\left(\frac{\pi}{3}\right) + \acute{y}\,\text{sen}\left(\frac{\pi}{3}\right)\right) = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\acute{y}\right) = 1 + \sqrt{3}\,\acute{y}$$

$$w_1 = 16^{1/6}\left[\cos\left(\frac{\frac{4\pi}{3} + 2\pi(1)}{4}\right) + \text{Sen}\left(\frac{\frac{4\pi}{3} + 2\pi(1)}{4}\right)\right] = 2\left(\cos\left(\frac{5\pi}{6}\right) + \acute{y}\,\text{sen}\left(\frac{5\pi}{6}\right)\right) = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}\acute{y}\right) = -\sqrt{3} + \acute{y}$$

$$w_2 = 16^{1/6}\left[\cos\left(\frac{\frac{4\pi}{3} + 2\pi(2)}{4}\right) + \text{Sen}\left(\frac{\frac{4\pi}{3} + 2\pi(2)}{4}\right)\right] = 2\left(\cos\left(\frac{4\pi}{3}\right) + \acute{y}\,\text{sen}\left(\frac{4\pi}{3}\right)\right) = 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\acute{y}\right) = -1 + \sqrt{3}\,\acute{y}$$

$$w_3 = 16^{1/6}\left[\cos\left(\frac{\frac{4\pi}{3} + 2\pi(3)}{4}\right) + \text{Sen}\left(\frac{\frac{4\pi}{3} + 2\pi(3)}{4}\right)\right] = 2\left(\cos\left(\frac{11\pi}{6}\right) + \acute{y}\,\text{sen}\left(\frac{11\pi}{6}\right)\right) = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\acute{y}\right) = \sqrt{3} - \acute{y}$$

• Ejercicio 6 seccion 1.6.5

a) $\log(-\acute{y}\,e) = 1 - \frac{\pi}{2}\acute{y}$ b) $\log(1-\acute{y}) = \frac{1}{2}\ln(2) - \frac{\pi}{4}\acute{y}$ c) $\log(e) = 1 + 2n\pi\acute{y}$ d) $\log(\acute{y}) = \left(2n + \frac{1}{2}\right)\pi\acute{y}$

• $\log(-\acute{y}\,e)$ • $\log(1-\acute{y})$ $\ln|z| = \sqrt{2}$ $\text{Arg}(z) = -\frac{\pi}{4}$

$\ln|z| + \acute{y}\,\text{Arg}(z)$ $z = -\acute{y}\,e$ $|z| = e$ $\text{Arg}(z) = -\frac{\pi}{2}$ $\ln|z| + \acute{y}\,\text{Arg}(z)$

$\ln|-\acute{y}e| + \acute{y}\,\text{Arg}(-\acute{y}e) = 1 - \frac{\pi}{2}\acute{y}$ $\ln\sqrt{2} - \acute{y}\frac{\pi}{4} \longrightarrow \ln 2^{1/2} - \frac{\pi}{4}\acute{y}$

$$\longrightarrow \frac{1}{2}\ln 2 - \frac{\pi}{4}\acute{y}$$

• $\ln(e)$ $\ln|e| = 1$ $\text{Arg}(e) = 0 + 2\pi n$

$= 1 + 2\pi n i$ ———— $\ln|z| + \text{Arg}(z)$

• $\ln(i)$ $|i| = 1$ $\text{Arg}(i) = \frac{\pi}{2} + 2\pi n$

$\ln|i| + i\,\text{Arg}(i) = 0 + \left(\frac{\pi}{2} + 2\pi n\right) i \Rightarrow \left(2n + \frac{1}{2}\right)\pi i$