Ejercicio 2 sección 1.6.5

a) $\cos(3\alpha) = \cos^3(\alpha) - 3\,\text{sen}(\alpha)\,\text{sen}^2(\alpha)$

b) $\text{sen}(3\alpha) = 3\cos^2(\alpha)\,\text{sen}(\alpha) - \text{sen}^3(\alpha)$

1
1 1
1 2 1
1 3 3 1

$\cos(3\alpha) + i\,\text{sen}(3\alpha) = (\cos\alpha + i\,\text{sin}\alpha)^3 \longrightarrow$ de moivre's theorem

$(\cos\alpha + i\,\text{sin}\alpha)^3 = \cos^3\alpha + 3\cos^2\alpha\, i\,\text{sin}\alpha + 3\cos\alpha(i\,\text{sen}\alpha)^2 + (i\,\text{sen}\alpha)^3$

$= \cos^3\alpha - 3\cos\alpha\,\text{sen}^2\alpha + i\,[3\cos^2\alpha\,\text{sin}^2\alpha - \text{sen}^3\alpha]$

Real part $\cos(3\alpha) = \cos^3\alpha - 3\cos\,\text{sen}^2\alpha$

Img part $\text{sen}(3\alpha) = 3\cos^2\alpha\,\text{sin}^2\alpha - \text{sen}^3\alpha$

- Encuentre todas las raices de la siguiente expresion (Ejercicio 5 sección 1.6.5)

a) $\sqrt{2i}$ b) $\sqrt{1-\sqrt{3}i}$ c) $(-1)^{1/3}$ d) $8^{1/6}$ e) $\sqrt[4]{-8-8\sqrt{3}\,i}$

$z = \sqrt{2i}$

a) $(2i)^{1/2} \longrightarrow z = 2\left[\cos\left(\frac{\pi}{2}\right) + i\,\text{sen}\left(\frac{\pi}{2}\right)\right]$

$w_K = r^{1/n}\left(\cos\left(\frac{\theta+2\pi k}{n}\right) + i\,\text{sen}\left(\frac{\theta+2\pi k}{n}\right)\right.$

$w_0 = 2^{1/2}\left[\cos\left(\frac{\frac{\pi}{2}+2\pi(0)}{2}\right) + i\,\text{sen}\left(\frac{\frac{\pi}{2}+2\pi(0)}{2}\right)\right] = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\,\text{sen}\left(\frac{\pi}{4}\right)\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\,\frac{\sqrt{2}}{2}\right)$

$= (1+i)$

$w_1 = 2^{1/2}\left[\cos\left(\frac{\frac{\pi}{2}+2\pi}{2}\right) + i\,\text{sen}\left(\frac{\frac{\pi}{2}+2\pi}{2}\right)\right] = \sqrt{2}\left(\cos\left(5\frac{\pi}{4}\right) + i\,\text{sen}\left(5\frac{\pi}{4}\right)\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} - i\,\frac{\sqrt{2}}{2}\right)$

$= (-1-i)$

b) $(1-\sqrt{3}\,i)^{1/2} \longrightarrow r = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{4} = 2$ $\quad \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$ $\quad z = 2\left(\cos\left(-\frac{\pi}{3}\right) + i\,\text{sen}\left(-\frac{\pi}{3}\right)\right.$

$w_K = r^{1/n}\left(\cos\left(\frac{\theta+2\pi k}{n}\right) + i\,\text{sen}\left(\frac{\theta+2\pi k}{n}\right)\right.$

$w_0 = 2^{1/2}\left[\cos\left(\frac{-\frac{\pi}{3}+2\pi(0)}{2}\right) + \text{sen}\left(\frac{-\frac{\pi}{3}+2\pi(0)}{2}\right)\right] = \sqrt{2}\left(\cos\left(-\frac{\pi}{6}\right) + i\,\text{sen}\left(-\frac{\pi}{6}\right)\right) = \sqrt{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$\left(\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i\right)$

$w_1 = 2^{1/2}\left[\cos\left(\frac{-\frac{\pi}{3}+2\pi(1)}{2}\right) + \text{sen}\left(\frac{-\frac{\pi}{3}+2\pi(1)}{2}\right)\right] = \sqrt{2}\left(\cos\left(\frac{5\pi}{6}\right) + i\,\text{sen}\left(\frac{5\pi}{6}\right)\right) = \sqrt{2}\left(\frac{-\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= \left(-\frac{\sqrt{6}}{2} + \frac{1}{2}i\right)$

c) $= (-1)^{1/3} \longrightarrow z = -1 \longrightarrow z = \cos\pi + i\,\text{sen}\,\pi$

$w_k = r^{1/n}\left(\cos\left(\frac{\theta+2\pi k}{n}\right) + i\,\text{sen}\left(\frac{\theta+2\pi k}{n}\right)\right)$

$w_0 = 1^{1/3}\left[\cos\left(\frac{\pi+2\pi(0)}{3}\right) + \text{Sen}\left(\frac{\pi+2\pi(0)}{3}\right)\right] = 1^{1/3}\left(\cos\left(\frac{\pi}{3}\right) + i\,\text{sen}\left(\frac{\pi}{3}\right)\right) = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\,i\right)$

$w_1 = 1^{1/3}\left[\cos\left(\frac{\pi+2\pi(1)}{3}\right) + \text{Sen}\left(\frac{\pi+2\pi(1)}{3}\right)\right] = 1^{1/3}\left(\cos(\pi) + i\,\text{sen}(\pi)\right) = (-1 + 0\,i) = -1$

$w_2 = 1^{1/3}\left[\cos\left(\frac{\pi+2\pi(1)}{3}\right) + \text{Sen}\left(\frac{\pi+2\pi(1)}{3}\right)\right] = 1^{1/3}\left(\cos\left(\frac{5\pi}{3}\right) + i\,\text{sen}\left(\frac{5\pi}{3}\right)\right) = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\,i\right)$

d) $8^{1/6} \quad z = 8(\cos(0) + i\,\text{sen}(0))$

$w_0 = 8^{1/6}\left[\cos\left(\frac{0+2\pi(0)}{6}\right) + \text{Sen}\left(\frac{0+2\pi(0)}{6}\right)\right] = 2^{3/6}\left(\cos(0) + i\,\text{sen}(0)\right) = \sqrt{2}(1 + 0\,i) = \sqrt{2}$

$w_1 = 8^{1/6}\left[\cos\left(\frac{0+2\pi(1)}{6}\right) + \text{Sen}\left(\frac{0+2\pi(1)}{6}\right)\right] = 2^{3/6}\left(\cos\left(\frac{\pi}{3}\right) + i\,\text{sen}\left(\frac{\pi}{3}\right)\right) = \sqrt{2}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{2} + i\frac{\sqrt{6}}{2}$

$w_2 = 8^{1/6}\left[\cos\left(\frac{0+2\pi(2)}{6}\right) + \text{Sen}\left(\frac{0+2\pi(2)}{6}\right)\right] = 2^{3/6}\left(\cos\left(\frac{2\pi}{3}\right) + i\,\text{sen}\left(\frac{2\pi}{3}\right)\right) = \sqrt{2}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{6}}{2}$

$w_3 = 8^{1/6}\left[\cos\left(\frac{0+2\pi(3)}{6}\right) + \text{Sen}\left(\frac{0+2\pi(3)}{6}\right)\right] = 2^{3/6}\left(\cos(\pi) + i\,\text{sen}(\pi)\right) = \sqrt{2}(-1 + (0)i) = -\sqrt{2}$

$w_4 = 8^{1/6}\left[\cos\left(\frac{0+2\pi(4)}{6}\right) + \text{Sen}\left(\frac{0+2\pi(4)}{6}\right)\right] = 2^{3/6}\left(\cos\left(\frac{4\pi}{3}\right) + i\,\text{sen}\left(\frac{4\pi}{3}\right)\right) = \sqrt{2}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{6}}{2}$

$w_5 = 8^{1/6}\left[\cos\left(\frac{0+2\pi(5)}{6}\right) + \text{Sen}\left(\frac{0+2\pi(5)}{6}\right)\right] = 2^{3/6}\left(\cos\left(\frac{5\pi}{3}\right) + i\,\text{sen}\left(\frac{5\pi}{3}\right)\right) = \sqrt{2}\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{2} - i\frac{\sqrt{6}}{2}$

e) $\sqrt[4]{-8 - 8\sqrt{3}\,i} \quad z = 16\left(\cos\left(\frac{4\pi}{3}\right) + i\,\text{sin}\left(\frac{4\pi}{3}\right)\right)$

$w_0 = 16^{1/6}\left[\cos\left(\frac{\frac{4\pi}{3}+2\pi(0)}{4}\right) + \text{Sen}\left(\frac{\frac{4\pi}{3}+2\pi(0)}{4}\right)\right] = 2\left(\cos\left(\frac{\pi}{3}\right) + i\,\text{sen}\left(\frac{\pi}{3}\right)\right) = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\,i\right) = 1 + \sqrt{3}\,i$

$w_1 = 16^{1/6}\left[\cos\left(\frac{\frac{4\pi}{3}+2\pi(1)}{4}\right) + \text{Sen}\left(\frac{\frac{4\pi}{3}+2\pi(1)}{4}\right)\right] = 2\left(\cos\left(\frac{5\pi}{6}\right) + i\,\text{sen}\left(\frac{5\pi}{6}\right)\right) = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}\,i\right) = -\sqrt{3} + i$

$w_2 = 16^{1/6}\left[\cos\left(\frac{\frac{4\pi}{3}+2\pi(2)}{4}\right) + \text{Sen}\left(\frac{\frac{4\pi}{3}+2\pi(2)}{4}\right)\right] = 2\left(\cos\left(\frac{4\pi}{3}\right) + i\,\text{sen}\left(\frac{4\pi}{3}\right)\right) = 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\,i\right) = -1 + \sqrt{3}\,i$

$w_3 = 16^{1/6}\left[\cos\left(\frac{\frac{4\pi}{3}+2\pi(3)}{4}\right) + \text{Sen}\left(\frac{\frac{4\pi}{3}+2\pi(3)}{4}\right)\right] = 2\left(\cos\left(\frac{11\pi}{6}\right) + i\,\text{sen}\left(\frac{11\pi}{6}\right)\right) = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\,i\right) = \sqrt{3} - i$

- Ejercicio 6 seccion 1.6.5

a) $\log(-ie) = 1 - \frac{\pi}{2}i$ b) $\log(1-i) = \frac{1}{2}\ln(2) - \frac{\pi}{4}i$ c) $\log(e) = 1 + 2n\pi i$ d) $\log(i) = \left(2n + \frac{1}{2}\right)\pi i$

- $\log(-ie)$

$\ln|z| + i\,\text{Arg}(z) \qquad z = -ie \qquad |z| = e \qquad \text{Arg}(z) = -\frac{\pi}{2}$

$\ln|-ie| + i\,\text{Arg}(-ie) = 1 - \frac{\pi}{2}i$

- $\log(1-i) \qquad \ln|z| = \sqrt{2} \qquad \text{Arg}(z) = -\frac{\pi}{4}$

$\ln|z| + i\,\text{Arg}(z)$

$\ln\sqrt{2} - i\frac{\pi}{4} \longrightarrow \ln 2^{1/2} - \frac{\pi}{4}i$

$\longrightarrow \frac{1}{2}\ln 2 - \frac{\pi}{4}i$