1.

$F_S(t) + F_F(t) + F_I(t) = F_E(t)$

$F_S(t) = K y(t)$

$F_F(t) = c\frac{dy(t)}{dt}$

$F_I(t) = m\frac{d^2y(t)}{dt^2}$

$m\frac{d^2y(t)}{dt^2} + c\frac{dy(t)}{dt} + Ky(t) = F_E(t) = x(t)$

$\mathcal{L}\left\{\frac{d^n x(t)}{dt}\right\} = s^n X$

$ms^2Y(s) + csY(s) + KY(s) = X(s)$

$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{ms^2+cs+K}$

$V_i(s) = LsI_1(s) + (I_1(s) - I_2(s)) = \frac{1}{Cs}$

$(I_2(s) - I_1(s))\frac{1}{Cs} + I_2(s)R = 0$

$V_o(s) = RI_2(s)$

Despejando $I_1(s)$ con respecto $I_2(s)$

$\frac{1}{Cs}I_2(s) - \frac{1}{Cs}I_1(s) + I_2(s)R = 0$

$I_1(s) = I_2(s)(1 + CRs)$

Reemplazando en la ecuación

$V_i(s) = LsI_2(s)(1 + CRs) + (I_2(s)(1 + CRs) - I_2(s))\frac{1}{Cs}$

$V_i(s) = LsI_2(s) + CRLs^2I_2(s) + I_2(s)\frac{1}{Cs} + I_2(s)R - I_2(s)\frac{1}{Cs}$

$V_i(s) = I_2(s)(CRLs^2 + Ls + R)$

$$\frac{I_2(s)}{V_i(s)} = \frac{1}{CRLs^2 + Ls + R}$$

$$\frac{R\,I_2(s)}{V_i(s)} = \frac{V_o(s)}{V_i(s)} = \frac{R}{CRLs^2 + Ls + R}$$

Factorizando

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{CLs^2 + \frac{L}{R}s + 1}$$