Punto 1.

$x(t) = |A\ Cos(2\pi Fot)|^2$

$t \in [-1/2Fo,\ 1/2Fo]$ con $A, Fo \in R^+$

Solución

$t \in [-1/2Fo,\ 1/2Fo] \Rightarrow to = -\frac{1}{2Fo}$

$x(t) = |A\ Cos(2\pi F_o t)|^2$

$T = \frac{1}{2Fo} - \left(-\frac{1}{2Fo}\right) = \frac{2}{2Fo} = \frac{1}{Fo} = To$

$t \in [-To/2,\ To/2]$

$x(t) = A^2\ Cos^2(2\pi Fot) = A^2\left[\frac{1}{2} + \frac{1}{2}\ Cos(2\pi\ 2\ Fot)\right]$

Usando Razones Trigonométricas

$Sen^2(x) = \frac{1 - Cos(2x)}{2}$

$Cos^2(x) = \frac{1 + Cos(2x)}{2}$

Nivel DC → Valor Promedio de una señal en el tiempo

Forma exponencial:

$x(t) = \frac{A^2}{2} + \frac{A^2}{2}\ Cos(2\pi 2\ Fot) \rightarrow \overset{\overset{Co}{\uparrow}}{\underset{\downarrow}{a_o}} + \underset{\downarrow}{a_2}\ Cos(2 \cdot 2\pi Fot)$

Nivel DC $= a_o$ $\quad a_2$

$a_o = \frac{A^2}{2} = C_o = \frac{A^2}{2}$

$a_2 = \frac{A^2}{2} = a_n = \frac{A^2}{2}$

Por otra parte, dado que $x(t)$ es Coseno, este tipo de señales presenta simetría par, esto quiere decir que:

$x(t) = x(-t)$

Por tanto

$bn = 0 \quad \forall n \in \{0, 1, 2 \dots, N\} \qquad N = 50$

Para $n \neq \frac{1}{2Fo}$ $\quad bn \neq 0$

Calculando los coeficientes C_2, C_{-2}, C_n así:

$$C_n = \frac{an - jbn}{2}$$

Y para C_2, C_{-2} sabemos que:

$$A^2 \cos^2(2\pi Fot) = \frac{A^2}{2} + \frac{A^2}{2} \cos(2 \cdot 2\pi Fot)$$

$$\frac{A^2}{2} + \frac{A^2}{2}\left(\frac{e^{j2 \cdot 2\pi Fot} + e^{-j2 \cdot 2\pi Fot}}{2}\right)$$

$$\frac{A^2}{2} + \underbrace{\frac{A^2}{4}}_{C_2} e^{j2 \cdot 2\pi Fot} + \underbrace{\frac{A^2}{4}}_{C_{-2}} e^{-j2 \cdot 2\pi Fot}$$

$$Er[\%] = \left[1 - \frac{1}{Px}\sum_{n=-N}^{N} |C_n|^2\right] \cdot 100\%$$

$$Px = \frac{1}{T}\int_T |x(t)|^2 dt = \frac{1}{1/2Fo - (-1/2Fo)} \int_{-1/2Fo}^{1/2Fo} \left|\frac{A^2}{2} + \frac{A^2}{2}\cos(2\pi 2Fot)\right|^2 dt$$

$$Px = \frac{1}{1/2Fo - (-1/2Fo)}\left(\overset{①}{\int_{-1/2Fo}^{1/2Fo} \frac{A^4}{4} dt} + \overset{②}{\int_{-1/2Fo}^{1/2Fo} \frac{2A^4 \cos(2\pi 2Fot)\, dt}{4}} + \overset{③}{\int_{-1/2Fo}^{1/2Fo} \frac{A^4 \cos(4\pi Fot)\, dt}{4}}\right)$$

$$T = \frac{1}{2Fo} - \left(-\frac{1}{2Fo}\right) = \frac{\cancel{2}}{\cancel{2}Fo} = \frac{1}{Fo} = \frac{1/1}{1/Fo} = Fo$$

Resolviendo ①

$$= Fo \cdot \frac{A^4}{4}\int_{-1/2Fo}^{1/2Fo} dt \rightarrow Fo \cdot \frac{A^4}{4} \cdot t \Big|_{-1/2Fo}^{1/2Fo} \rightarrow Fo \cdot \frac{A^2}{4} \cdot \left(\frac{1}{2Fo} - \left(-\frac{1}{2Fo}\right)\right) \rightarrow \cancel{Fo} \cdot \frac{A^4}{4} \cdot \frac{1}{\cancel{Fo}}$$

$$= \frac{A^4}{4} = ①$$

Resolviendo ②

$2\pi 2 F_o t = 2 w_o t$
$w_o = \pi 2 F_o$

$$= F_o \cdot \cancel{2} \frac{A^4}{\cancel{4}} \int_{-1/2F_o}^{1/2F_o} \cos(2\pi 2 F_o t)\, dt$$

$$= F_o \cdot \frac{A^4}{2} \int_{-1/2F_o}^{1/2F_o} \cos(2 w_o t)\, dt$$

$$U = 2 w_o t$$

$$du = 2 w_o\, dt$$

$$dt = \frac{du}{2 w_o}$$

$$= F_o \cdot \frac{A^4}{2} \int_{-1/2F_o}^{1/2F_o} \cos(u) \cdot \frac{du}{2 w_o}$$

$$= F_o \cdot \frac{A^4}{2} \cdot \frac{1}{2 w_o} \cdot \operatorname{Sen}(2 w_o t) \Big|_{-1/2F_o}^{1/2F_o}$$

$$F_o \cdot \frac{A^4}{2} \left(\frac{\operatorname{Sen}(2 w_o \cdot 1/2F_o)}{2 w_o} - \frac{\operatorname{Sen}(2 w_o - 1/2F_o)}{2 w_o} \right)$$

Sabiendo que $w_o = \pi 2 F_o$

$$= F_o \cdot \frac{A^4}{2} \left(\frac{\operatorname{Sen}(2\pi \cancel{2F_o} \cdot 1/\cancel{2F_o})}{2\pi 2 F_o} - \frac{\operatorname{Sen}(2\pi \cancel{2F_o} \cdot - 1/\cancel{2F_o})}{2\pi 2 F_o} \right)$$

$$= F_o \cdot \frac{A^4}{2} \cdot \frac{(\operatorname{Sen}(2\pi) + \operatorname{Sen}(2\pi))}{2\pi 2 F_o}$$

$$= F_o \cdot \frac{A^4}{2} \cdot \frac{(2 (\cancel{\operatorname{Sen}(2\pi)})^{\to 0}}{\cancel{2\pi 2 F_o}}$$

$$= \frac{A^4}{2} \cdot 0 = 0 = ②$$

Resolviendo ③

$$= F_0 \cdot \frac{A^4}{4}\int_{-1/2F_0}^{1/2F_0} \cos^2(4\pi F_0 t)\,dt \qquad 4\pi F_0 t = x$$

$$= F_0 \cdot \frac{A^4}{4}\int_{-1/2F_0}^{1/2F_0} \frac{1}{2} + \frac{1}{2}\cos(2 \cdot 4\pi F_0 t)\,dt$$

$$= F_0 \cdot \frac{A^4}{4} \cdot \frac{1}{2}\int_{-1/2F_0}^{1/2F_0} t\,dt + \frac{1}{2}\int_{-1/2F_0}^{1/2F_0} \cos(4\pi 2F_0 t)\,dt$$

$$= F_0 \cdot \frac{A^4}{4} \cdot \frac{1}{2} \cdot t\Big|_{-1/2F_0}^{1/2F_0} + \frac{1}{2}\int_{-1/2F_0}^{1/2F_0} \cos(4w_0 t)\,dt$$

$4\pi 2F_0 t = 4w_0 t$

$w_0 = \pi 2 F_0$

$u = 4w_0 t$

$du = 4w_0\,dt$

$dt = \frac{du}{4w_0}$

$$= \cancel{F_0} \cdot \frac{A^4}{4} \cdot \frac{1}{2} \cdot \frac{1}{\cancel{F_0}} + \frac{1}{2}\int_{-1/2F_0}^{1/2F_0} \cos(u) \cdot \frac{du}{4w_0}$$

$$= \frac{A^4}{8} + \frac{1}{2} \cdot \frac{1}{4w_0} \cdot \operatorname{Sen}\cancel{(4w_0 t)}^{0}\Big|_{-1/2F_0}^{1/2F_0}$$

$$= \frac{A^4}{8} + \frac{1}{2} \cdot 0 = \frac{A^4}{8} = ③$$

Así entonces:

$$P_x = \frac{A^4}{4} + 0 + \frac{A^4}{8}$$

$$P_x = \frac{A^4}{4} + \frac{A^4}{8} \longrightarrow P_x = \frac{3}{8}A^4$$

Así:

$$Er[\%] = \left[\frac{1 - |C_{-2}|^2 + |C_n|^2 + |C_2|^2}{P_x}\right] \times 100\%$$

$$Er\,[\%] = \left[\frac{1 - |A^2/4|^2 + |A^2/2|^2 + |A^2/4|^2}{3/8\ A^4}\right] \times 100\%$$

$$|C_{-2}|^2 = |C_2|^2 = \frac{A^4}{16}$$

$$Er\,[\%] = \left[\frac{1 - A^4/16 + A^4/4 + A^4/16}{3/8\ A^4}\right] \times 100\%$$

$$Er\,[\%] = \left[1 - \frac{\cancel{3/8\ A^4}^{\,1}}{\cancel{3/8\ A^4}}\right] \times 100\%$$

$$Er\,[\%] = [1 - 1] \times 100\% = 0\%$$

Por serie trigonometrica

$$x(t) = a_0 + \sum_{n=1}^{N} a_n \cos(n w_0 t) + b_n \operatorname{Sen}(n w_0 t)$$

$$x(t) = a_0 + \sum_{n=1}^{N} a_n \cos(n 2\pi F_0 t) + b_n \operatorname{Sen}(n 2\pi F_0 t)$$

$$C_n = \frac{a_n - j b_n}{2}$$

Donde:

$$a_0 = C_0 = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} x(t)\, dt$$

$$a_0 = \frac{1}{1/2F_0 - (-1/2F_0)} \int_{-1/2F_0}^{1/2F_0} \left(\frac{A^2}{2} + \frac{A^2}{2} \cos(2 w_0 t)\right) dt$$

$$\frac{1}{2F_0} + \frac{1}{2F_0} = \frac{1/\cancel{2}}{\cancel{2}F_0} \rightarrow \frac{1}{1/F_0} = F_0$$

$$a_0 = F_0 \cdot \frac{A^2}{2} \int_{-1/2F_0}^{1/2F_0} dt + \frac{A^2}{2} \int_{-1/2F_0}^{1/2F_0} \cos(2 w_0 t)\, dt$$

$$a_0 = F_0 \cdot \frac{A^2}{2} \cdot t \Big|_{-1/2F_0}^{1/2F_0} + \frac{A^2}{2} \, \text{Sen}(2w_0 t) \Big|_{-1/2F_0}^{1/2F_0}$$

$$a_0 = F_0 \cdot \frac{A^2}{2} \left[\frac{1}{2F_0} - \left(-\frac{1}{2F_0} \right) \right] + \frac{A^2}{2} \, \text{Sen} \left(2w_0 \cdot \frac{1}{2F_0} \cdot 2w_0 \cdot \frac{-1}{2F_0} \right)$$

$$a_0 = F_0 \cdot \frac{A^2}{2} \left[\frac{1}{2F_0} + \frac{1}{2F_0} \right] + \frac{A^2}{2} \, \text{Sen} \left(\frac{\cancel{2}w_0}{\cancel{2}F_0} + \frac{\cancel{2}w_0}{\cancel{2}F_0} \right)$$

$$a_0 = F_0 \cdot \left[\frac{A^2}{2} \cdot \frac{1}{F_0} \left[\frac{1}{2} + \frac{1}{2} \right] + \frac{A^2}{2} \cdot \frac{1}{F_0} \, \text{Sen}(w_0 + w_0) \right.$$

$$a_0 = \cancel{F_0} \cdot \frac{1}{\cancel{F_0}} \left[\frac{A^2}{2} \left[\cancel{\frac{1}{2} + \frac{1}{2}}^{\,1} \right] + \frac{A^2}{2} \, \cancel{\text{Sen}(2w_0)}^{\,0} \right]$$

$$a_0 = C_0 = 1 \cdot \left[\frac{A^2}{2} \right] = \frac{A^2}{2}$$

Por otra parte, dado que $x(t)$ es coseno está tipo de señales presenta simetría par, esto quiere decir que:

$$x(t) = x(-t)$$

Por tanto

$$b_n = 0 \quad \forall n \in \{0, 1, 2 \ldots, N\} \qquad N = 50$$

Para $n \neq \frac{1}{2F_0}$ $\quad b_n \neq 0$

Ahora calculamos a_n

$$a_n = \frac{2}{t_f - t_i} \int_{t_i}^{t_f} x(t) \cos(n w_0 t) \, dt$$

$$a_n = \frac{2}{1/2F_0 - (-1/2F_0)} \int_{-1/2F_0}^{1/2F_0} \frac{A^2}{2} + \frac{A^2}{2} \cos(2\pi 2 F_0 t) \cos(n w_0 t) \, dt$$

$$a_n = 2F_0 \int_{-1/2F_0}^{1/2F_0} \left(\frac{A^2}{2} \cos(n w_0 t) + \frac{A^2}{2} \cos(2 w_0 t) \right) \cos(n w_0 t) \, dt$$

① ②

$$a_n = 2F_o \int_{-1/2F_o}^{1/2F_o} \frac{A^2}{2} \cos(n\omega_o t)\,dt + 2F_o \int_{-1/2F_o}^{1/2F_o} \frac{A^2}{2} \cos(4\pi F_o t)\cos(n\omega_o t)\,dt$$

Resolviendo la primera Integral ①

$$\frac{A^2}{2}\int_{-1/2F_o}^{1/2F_o} \cos(n\omega_o t)\,dt$$

Se sabe que:

$$= \int \cos(\alpha x)\,dx = \frac{\operatorname{Sen}(\alpha x)}{\alpha}$$

Evaluando en los límites

$$= \frac{A^2}{2}\left[\cancel{\frac{\operatorname{Sen}(n\omega_o t)}{n\omega_o}}\right]\Bigg|_{-1/2F_o}^{1/2F_o} = \frac{A^2}{2}$$

Dado que Sen(x) es una función Impar, si los límites son simétricos alrededor de cero, el resultado de esta integral es cero cuando $n \neq 0$

Para $n=0$, la integral resulta en $\frac{A^2}{2}$

Resolviendo la Segunda Integral

$$= \frac{A^2}{2}\int_{-1/2F_o}^{1/2F_o} \cos(4\pi F_o t)\cos(n\omega_o t)\,dt$$

Usando la identidad del producto de cosenos

$$\cos A \cos B = \frac{1}{2}\left(\cos((n\omega_o - 4\pi F_o)t) + \cos((n\omega_o + 4\pi F_o)t)\right)$$

Reescribimos la Integral:

$$2 \cdot F_o \cdot \frac{A^2}{4}\int_{-1/2F_o}^{1/2F_o} \cos((\cancel{n\omega_o - 4\pi F_o}^{\,0})t) + \cos((\cancel{n\omega_o + 4\pi F_o}^{\,0})t))\,dt$$

Cada una de estas integrales será cero a menos que el argumento del coseno sea cero.

Por ende, el resultado final para

$$a_n = \frac{A^2}{2}$$

$$C_n = \frac{a_n - jb_n}{2}$$

$$C_n = \frac{A^2/2 - j0}{2}$$

$$C_n = \frac{A^2}{4}$$

$$\frac{A^2}{2} + \frac{A^2}{2}\cos(2\pi\, 2F_0 t) \rightarrow a_0 + \sum_{n=1}^{N} a_n \cos(n w_0 t) + b_n \operatorname{Sen}(n w_0 t)$$

$$a_0 = \frac{A^2}{2} \rightarrow \text{Nivel DC}$$

$$a_2 = \frac{A^2}{2}$$

Punto 2

$$c(t) = A_c \cos(2\pi F_c t)$$

$$y(t) = \left(1 + \frac{m(t)}{A_c}\right) c(t)$$

Solución:

La transformada de Fourier modulada se puede encontrar

$$Y(w) = F\{y(t)\} = F\left\{\left(1 + \frac{m(t)}{A_c}\right) c(t)\right\} = F\{c(t)\} + \frac{1}{A_c} F\{m(t)\,c(t)\}$$

Utilizando tablas

$$C(w) = F\{c(t)\} = F\{A_c \cos(2\pi F_c t)\} = A_c F\left\{\frac{e^{j2\pi F_c t} + e^{-j2\pi F_c t}}{2}\right\}$$

$$F\{e^{\pm j w_0 t}\} = 2\pi\,\delta(w \mp w_0)$$

Por consiguiente

$$C(w) = A_c \pi\left(\delta(w - 2\pi F_c) + \delta(w + 2\pi F_c)\right)$$

De forma similar:

$$\frac{1}{A_c} F\{m(t)c(t)\} = \frac{1}{A_c} F\{m(t) A_c \cos(2\pi F_c t) = F\{m(t) \cos(2\pi F_c t) = F\left\{\frac{m(t)e^{j2\pi F_c t} + m(t)e^{-j2\pi F_c t}}{2}\right\}$$

$$F\{x(t)e^{\pm j w_0 t}\} = X(\omega \mp \omega_0)$$

Se obtiene:

$$\frac{1}{A_c} F\{m(t)c(t)\} = \frac{1}{2}(M(\omega - 2\pi F_c) + M(\omega + 2\pi F_c))$$

Finalmente el espectro

$$Y(\omega) = A_c \pi(\delta(\omega - 2\pi F_c) + \delta(\omega + 2\pi F_c)) + \frac{1}{2}(M(\omega - 2\pi F_c) + M(\omega + 2\pi F_c))$$