1. $t \in \left[-\frac{1}{2f_0}, \frac{1}{2f_0}\right] \rightarrow t_0 = -\frac{1}{2f_0}$

$x(t) = |A\cos(2\pi F_0 t)|^2$

$T = \frac{1}{2f_0} - \left(-\frac{1}{f_0}\right) = \frac{2}{2f_0} = \frac{1}{f_0} = T_0$

$t \in \left[-\frac{T_0}{2}, \frac{T_0}{2}\right]$

$x(t) = A^2\cos^2(2\pi F_0 t)$

$= A^2\left[\frac{1}{2} + \frac{1}{2}\cos(2\pi\, 2F_0 t)\right]$

forma exponencial

$x(t) = \frac{A^2}{2} + \frac{A^2}{2}\cos(2\pi 2F_0 t)$

Por Trigonometrica

$x(t) = |A\cos(2\pi F_0 t)|^2$

$= A^2\cos^2(2\pi F_0 t)$

$= A^2\left(\frac{1 + \cos(2\cdot 2\pi F_0 t)}{2}\right)$

$= \frac{A^2}{2} + \frac{A^2}{2}\cos(2\omega_0 t)$

$x(t) = A^2 + \frac{A^2}{4}e^{-j\omega_0 t} + \frac{A^2}{4}e^{j\omega_0 t}$

$= \frac{A^2}{2}\left[\frac{e^{-2j\omega_0 t} + e^{-2j\omega_0 t}}{2}\right]$

$= \frac{A^2}{2}\cos(2\omega_0 t) + A^2$

$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(n\omega_0 t) + \sum_{n=0}^{\infty} b_n\,\mathrm{Sen}(n\omega_0 t)$

$C_n : \frac{a_n + ib_n}{2} \to 0 = \frac{A^2}{4}$

$b_n = 0 \qquad \frac{a_0}{2} = \frac{A^2}{2} \therefore a_0 = A^2$

$A_n = \frac{A^2}{2}$ si solo si $n = 2$

2. $Y(\omega) = F\{y(t)\} = F\left\{\left(1 + \frac{m(t)}{Ac}\right) c(t)\right\} = F\{c(t)\} + \frac{1}{Ac} F\{m(t)c(t)\}$

Utilizando tablas de Fourier

$C(\omega) = F\{c(t)\} = F\{Ac \cos(2\pi Fc t)\} = Ac\, F\left\{\frac{e^{j2\pi Fc t} + e^{-j2\pi Fc t}}{2}\right\}$

$F\{e^{\pm j\omega_0 t}\} = 2\pi\delta(\omega \mp \omega_0)$

Por consiguiente

$C(\omega) = Ac\,\pi(\delta(\omega - 2\pi Fc) + \delta(\omega + 2\pi Fc))$

De forma similar

$\frac{1}{Ac} F\{m(t)c(t)\} = \frac{1}{Ac} F\{m(t) Ac \cos(2\pi Fc t)\} = F\left\{\frac{m(t) e^{j2\pi Fc t} + m(t) e^{-j2\pi Fc t}}{2}\right\}$

$F\{x(t) e^{\mp j\omega_0 t}\} = X(\omega \mp \omega_0)$

$\frac{1}{Ac} F\{m(t)c(t)\} = \frac{1}{2}(M(\omega - 2\pi Fc) + M(\omega + 2\pi Fc))$

Finalmente

$Y(\omega) = Ac\,\pi(\delta(\omega - 2\pi Fc) + \delta(\omega + 2\pi Fc)) + \frac{1}{2}(M(\omega - 2\pi Fc) + M(\omega + 2\pi Fc))$