Exercise 02 C

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We consider an square image I whose size is $\mathcal{N}x\mathcal{N}$, and three types of structuring elements.

First type is structuring element $\mathcal{B}$, which is a 2-dimensional structuring element of size $\mathcal{M}x\mathcal{M}$.

$$\mathcal{B} = \begin{pmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,m} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m,1} & b_{m,2} & \cdots & b_{m,m} \end{pmatrix} \tag{1}$$

Second type is $\mathcal{C}$ which is a 1 dimensional horizontal structuring element:

$$\mathcal{C} = \begin{pmatrix} c_1 & b_2 & \cdots & c_m \end{pmatrix} \tag{2}$$

Third type is a $\mathcal{D}$ wich is a 1 dimensional vertical structuring element:

$$\mathcal{D} = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ c_m \end{pmatrix} \tag{3}$$

We can assume those two properties of the elements:

$$\mathcal{B} = \delta_{\mathcal{C}}(\mathcal{D}) = \delta_{\mathcal{D}}(\mathcal{C}) \tag{4}$$

Calculate the number of max operations that must be computed in order to process a $\mathcal{N}x\mathcal{N}$ input image using the following alternatives:

- First alternative: $\delta_{\mathcal{B}}(I)$
- Second alternative: $\delta_{\mathcal{C}}\delta_{\mathcal{D}}(I)$

1 Considerations

We are going to compute the number of elementary *max* operations that we have to do in order to compute the max of a list of elements.

Considering only two elements:

$$max(x_1, x_2) \rightarrow 1 operation \tag{5}$$

Considering three elements:

$$max(x_1, x_2, x_3) = max(x_1, max(x_2, x_3)) \rightarrow 2\, operations \tag{6}$$

Considering four elements:

$$max(x_1, x_2, x_3, x_4) = max(x_1, max(x_2, max(x_3, x_4))) \rightarrow 3\, operations \tag{7}$$

Finally if we consider n elements:

$$max(x_1, x_2, \cdots, x_n) = max(x_1, max(x_2, \cdots, max(x_{n-1}, x_n))) \rightarrow n{-}1\, operations \tag{8}$$

2 First alternative: $\delta_{\mathcal{B}}(I)$

For computing the operations of the structuring element, whose size is $\mathcal{M}x\mathcal{M}$, we have M^2 elements. Following the reasoning of previous equation:

$$\mathcal{M}x\mathcal{M} \rightarrow \mathcal{M}^2 - 1\, operations \tag{9}$$

As the image have $\mathcal{N}x\mathcal{N}$ elements, or $\mathcal{N}^2$. We can compute the total number of operations ($NumOp_!$):

$$NumOp_1 = \mathcal{N}^2(\mathcal{M}^2 - 1) \tag{10}$$

3 Second alternative: $\delta_{\mathcal{C}}\delta_{\mathcal{D}}(I)$

For each structuring element as their size is $\mathcal{M}$, we can say that the number of elementary operations is $M - 1$, following the preovious reasoning.

As the image have $\mathcal{N}x\mathcal{N}$ elements, or $\mathcal{N}^2$. We can compute the total number of operations, with only one structuring element:

$$NumOp_2 = \mathcal{N}^2(\mathcal{M} - 1) \tag{11}$$

But, we have two elements, so we have to multiply the number of operations by two:

$$NumOp_2 = 2\mathcal{N}^2(\mathcal{M} - 1) \tag{12}$$

4 Conclusion

The number of operations for each alternative is:

$$\begin{aligned} NumOp_1 &= \mathcal{N}^2(\mathcal{M}^2 - 1) \\ NumOp_2 &= 2\mathcal{N}^2(\mathcal{M} - 1) \end{aligned} \tag{13}$$

We can confirm that the number of operations in the second alternative is lower than the first one.