Exercise 09

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Proof the idempotence of the opening (γ) closing (φ) operation.

In a formal way, it can be expressed as:

$$(\varphi\gamma)(\varphi\gamma)(I) = \varphi\gamma(I) \tag{1}$$

It is equivalent to prove the following:

$$(\varphi\gamma)(\varphi\gamma)(I) \leq \varphi\gamma(I) \wedge (\varphi\gamma)(\varphi\gamma)(I) \geq \varphi\gamma(I) \rightarrow (\varphi\gamma)(\varphi\gamma)(I) = \varphi\gamma(I) \tag{2}$$

1 Proof: $(\varphi\gamma)(\varphi\gamma)(I) \leq \varphi\gamma(I)$

First we are going to construct a proof for $(\varphi\gamma)(\varphi\gamma)(I) \leq \varphi\gamma(I)$. For doing this, as the closing is an idempotent operation we can consider that:

$$\varphi\varphi\varphi\gamma(I) = \varphi\gamma(I) \tag{3}$$

As the opening is antiextensive and closing is extensive, we can affirm that the closing is greater or equal than the opening ($\varphi \geq \gamma$). Substituting the second closing operation by an opening we can confirm that:

$$\varphi\gamma\varphi\gamma(I) \leq \varphi\gamma(I) \tag{4}$$

2 Proof: $(\varphi\gamma)(\varphi\gamma)(I) \geq \varphi\gamma(I)$

Second, we are going to prove that $(\varphi\gamma)(\varphi\gamma)(I) \geq \varphi\gamma(I)$. For doing this, as the opening is idempotent we can assume that:

$$\varphi\gamma\gamma\gamma(I) = \varphi\gamma(I) \tag{5}$$

As the opening is antiextensive, and closing is extensive. We can affirm that the opening is greater or equal than the opening ($\varphi \geq \gamma$). Substituting the second opening operation by a closing we can affirm that:

$$\varphi\gamma\varphi\gamma(I) \geq \varphi\gamma(I) \tag{6}$$

3 Conclusions

Finally, as the two parts of the and ($\wedge$) are proved, we have proved the equality of the both assumptions. So we can affirm that the opening-closing operation is idempotent.