Exercise 09

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Proof the idempotence of the opening (γ) closing (φ) operation. In a formal way, it can be expressed as:

$$(\varphi\gamma)(\varphi\gamma)(I) = \varphi\gamma(I) \tag{1}$$

It is equivalent to prove the following:

$$(\varphi\gamma)(\varphi\gamma)(I) \leq \varphi\gamma(I) \wedge (\varphi\gamma)(\varphi\gamma)(I) \geq \varphi\gamma(I) \rightarrow (\varphi\gamma)(\varphi\gamma)(I) = \varphi\gamma(I) \tag{2}$$

First we are going to prove that $(\varphi\gamma)(\varphi\gamma)(I) \leq \varphi\gamma(I)$. For doing this, as the closing is an idempotent operation we can suppose that:

$$\begin{aligned} \varphi\gamma(I) &= \varphi\varphi\varphi\gamma(I) \\ \varphi\gamma(I) &\geq \varphi\gamma\varphi\gamma(I) \end{aligned} \tag{3}$$

As the opening is antiextensive and closing is extensive, we can affirm that the closing (φ) is greater or equal than the closing (γ).

$$\varphi \geq \gamma \tag{4}$$

Second, we are going to prove that $(\varphi\gamma)(\varphi\gamma)(I) \geq \varphi\gamma(I)$. For doing this, as the opening is idempotent we can assume that:

$$\begin{aligned} \varphi\gamma(I) &= \varphi\gamma\gamma\gamma(I) \\ \varphi\gamma(I) &\leq \varphi\gamma\varphi\gamma(I) \end{aligned} \tag{5}$$

As the opening is antiextensive, and closing is extensive. We can affirm:

$$\varphi \geq \gamma \tag{6}$$

Finally, as the two inequalities are proved, we have proved the equality of the both assumptions.