Homework 4

Due April 15, 19:00

Problem 4.1

Recall **The gamma function** from previous homework:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt.$$

Write a program to compute the value of this function from the definition using each of the following approaches:

a) Truncate the infinite interval of integration and use a composite Simpson's rule. You will need to do some experimentation or analysis to determine where to truncate the interval, based on the usual trade-off between efficiency and accuracy.

b) Truncate the interval and use a built-in adaptive quadrature routine of your choice available in MATLAB/GNU Octave or Python. Again, explore the trade-off between accuracy and efficiency.

c) For each method, compute the approximate value of the integral for several values of x in the interval $[1, 10]$. Compare your results with the values given by the built-in function `gamma` . How do the various methods compare in efficiency for a given level of accuracy?

Problem 4.2

It can be easily verified that

$$\pi = \int_0^1 \frac{4}{1+x^2}\, dx.$$

Thus, an approximate value for π can be computed by using numerical integration of the above integral.

a) Write your own implementation of midpoint, trapezoidal, and Simpson's composite rules to compute the approximation of π for various values of stepsize h. What is the order of the error as a function of h for each rule? Compare the accuracy of the rules with each other (based on the known value of π). Is there any point beyond which decreasing h yields no further improvement, and if yes, why?

b) Compute approximation of π again by using any adaptive quadrature routine available in MATLAB/GNU Octave or Python and try various error tolerances.

c) Compare the computational work required (elapsed time) for parts a) and b). To measure elapsed time you might use MATLAB/GNU Octave commands `tic` and `toc` .

Problem 4.3

The intensity of diffracted light near a straight edge is determined by the values of Fresnel integrals

$$C(x) = \int_0^x \cos\left(\frac{\pi t^2}{2}\right) dt,$$
$$S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt,$$

Evaluate these integrals for enough values of x to draw a smooth plot of functions $C(x)$ and $S(x)$ over the interval $[0, 5]$ by:

a) Using composite Simpson's rule.

b) Using an adaptive quadrature routine of your choice available in MATLAB/GNU Octave or Python.

c) Compare your results by computing Fresnel integrals using built-in functions `fresnelc` and `fresnels` in MATLAB/GNU Octave or `scipy.special.fresnel` in Python.

Problem 4.4

Apply Simpson's rule (with $n = 4, 8, 16, 32, 64$) and Gaussian quadrature (with $n = 4, 8, 16, 32, 64$) to compute the integral

$$\int_1^4 \frac{1}{(x-1)^{5/2}}\, dx.$$

Use your own written code. To generate Gaussian nodes and weights you may use built-in functions. Explain your results. Make conclusions!