Homework 2 Due October 22, 2021

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Problem 2.1

A die is rolled twice. What is the probability that the sum of the faces is greater than 5 (event *E*), given that

a) the first outcome was a 3?

$$m(\omega)=\frac{1}{6},\quad E-first\ outcome\ was\ 3\rightarrow \omega-3{,}4{,}5{,}6\ \ P(F)=\frac{4}{6}=\frac{2}{3}$$

b) the first outcome was greater than 4?

$$m(\omega)=\frac{1}{6},\quad E-first\ outcome\ was\ greater\ than\ 4\rightarrow 5\ or\ 6\rightarrow \omega-1{,}2{,}3{,}4{,}5{,}6\ \ P(F)=\frac{6}{6}=1$$

c) the first outcome was 1?

$$m(\omega)=\frac{1}{6},\quad E-first\ outcome\ was\ 1\rightarrow \omega-5{,}6\ P(F)=\frac{2}{6}=\frac{1}{3}$$

d) the first outcome was less than 5?

$$m(\omega)=\frac{1}{6},\quad E-first\ outcome\ was\ less\ than\ 5\rightarrow 1\ or\ 2\ or\ 3\ or\ 4\rightarrow \omega-1{,}2{,}3{,}4{,}5{,}6\ \ P(F)=\frac{7}{12}$$

Problem 2.2

There are two urns. In urn I there are 5 white balls and 6 black balls, in urn II there are 4 white balls and 3black balls. 2 balls from urn I are picked at random and transferred in urn II. Then, two balls are chosen at random from urn II.

What is the probability that both balls are black?

$$F-Both\ balls\ are\ black\ \ P(F)=\frac{3}{11}\cdot\frac{20}{72}+\frac{3}{11}\cdot\frac{12}{72}+\frac{2}{11}\cdot\frac{6}{72}=\frac{60+36+36+12}{792}=\frac{144}{792}=\frac{2}{11}$$

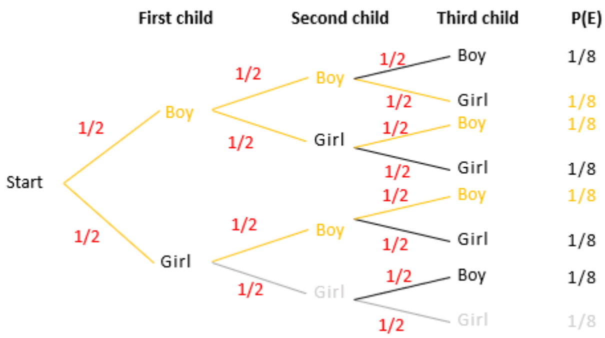
What is the probability that balls ofthe same color were transferred from I to urn II if balls chosen from urn II are of different color? $P(both\ balls\ are\ white\ \ from\ urn\ I)xP(balls\ are\ diferent\ color\ \ from\ urn\ II)$ $P(both\ balls\ are\ black\ \ from\ urn\ I)xP(balls\ are\ diferent\ color\ \ from\ urn\ II)$ $x2\ (as\ we\ can\ choose$ $W-B\ or\ B-W\ balls\ priority$

$$F-balls\ are\ chosen\ from\ urn\ 2\ ,P(F)==\frac{6}{11}\rightarrow P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{8/33}{7/8}=4/9$$

Problem 2.3

What is the probability that a family of three children has

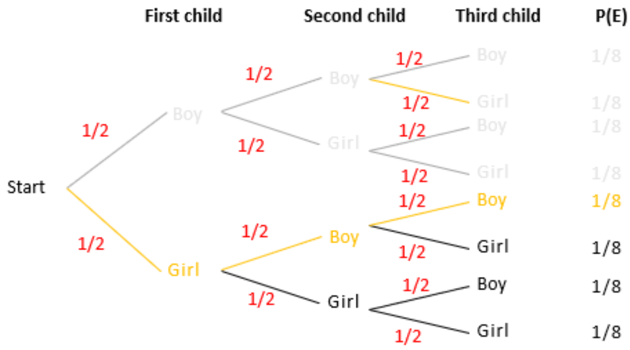
a) two boys given that it has at least one boy?



A = { at last one boy }
B = { two boys }

$$P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{3/8}{7/8}=3/7$$

b) two boys given that the first child is a girl?

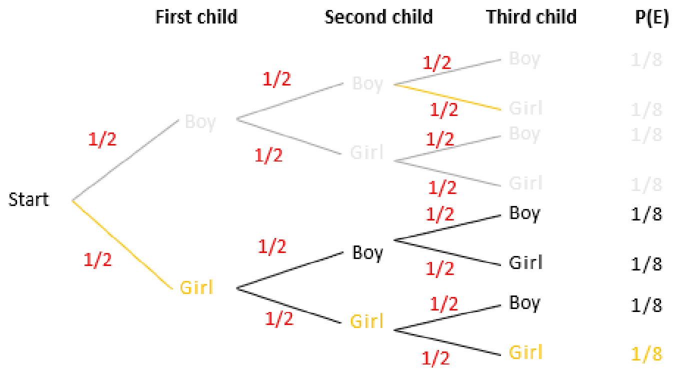


A = { first child is a girl}

B = { two boys }

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{1/8}{4/8} = 1/4$$

c) three girls, given that the first child is a girl?



A = {first child is a girl}

B = {three girls}

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{1/8}{4/8} = 1/4$$

Problem 2.4

Examphobia is a rare disease in which the victim has the delusion that he or she is being subjected to anintense mathematical examination. A person selected uniformly at random has examphobia with probability $1/50$. A person with examphobia has shaky hands with probability $7/10$. A person without examphobia has shaky hands with probability $3/20$.

What is the probability that a person selected uniformly at random has examphobia, given that he or she has shaky hands?

Let D = "you have the disease", and let E = "you have shaky hands "

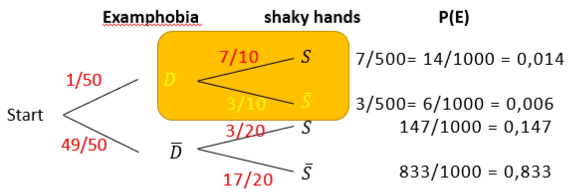
$$P(D|E) = \frac{P(D \cap E)}{P(E)} = 2/23$$

$$P(D|E) = \frac{P(D \cap E)}{P(E)} = 2/23$$

What is probability that a person doesn't have shaky hands given that he or she has examphobia?

$A = person\ doesn't\ have\ shaky\ hands\ given\ that\ he\ or\ she\ has\ examphobia$

Let D = "you have the disease", and let E = "you have shaky hands "



$P(A) = 3/10$

Problem 2.5

Find the probability that in a poker hand (5 cards) you will have

a) royal flush (ten, jack, queen, king ace in one suit)

$$\textit{There are four possible suits for the ace} \rightarrow \textit{Thus, there are 4 possible royal flushes:}$$

$$royal\ flushes\ =\ 4C1\ =\ 4 \rightarrow \textit{Dividing by the number of possible hands gives the probability:}$$

$$P(royal\ flush)\ =\ 4$$

$$P(a)royal\ flush) = \frac{4}{2598960} = 1.539\ \cdot 10^{-6} = \ or\ 1\ in\ 649{,}740$$

b) straight flush (five cards in a sequence in a single suit, but no royal flush)

$$\textit{There are 9 possibilities. If we order the 5 – card hand from highest number to lowest, the first card may be one of the following: king, queen, jack, 10, 9, 8, 7, 6, or 5.}$$

$$\textit{After the first card, whose suit we may choose in 4 ways, the remaining cards are completely determined:}$$

$$\#\ straight\ flushes\ =\ 9\ *C_1^4\ =\ 9\cdot 4\ =\ 36$$

$$\textit{Subtracting the number of royal flushes and dividing by the number of possible hands gives the probability:}$$

$$P(straight\ -\ flush) = \frac{36}{2598960} = \ 1.385\ \cdot 10^{-5} = \ or\ 1\ in\ 72{,}193\frac{1}{3}$$

c) flush (five cards in a single suit, but not a straight or royal flush)

$$\textit{If we pick the suit first, we have } C_1^4\ =\ 4 \textit{ choices. For that suit, there are 13 cards from which we choose 5. Thus, we have } C_5^{13}\ =\ 1287 \textit{ choices.}$$

$$\#\ flushes\ (not\ straight)\ =\ C_1^4\cdot(C_5^{13}-10)\ =\ 4\cdot 1277\ =\ 5108$$

$$P(flush\ not\ straight\ or\ royal\ flush)\ = \frac{5108}{2598960} = \ 1.965\ \cdot 10^{-3} = \ or\ 1\ in\ 508.8.$$

d) one pair?

$$\textit{One – pair is two cards showing the same numbers and another three cards all showing different numbers.}$$

$$\textit{a If we order the 5 – card hand with the pair first, we have } C_1^{13} \textit{ choices for the number showing on the pair. r. The pair will have two out of four suits.}$$

$$\textit{Thus, we have } 4C2\ =\ 6 \textit{ ways to choose the suit}$$

$$\#\ 1-pair\ =\ C_1^{13}*C_2^4*C_3^{12}*C_1^4*C_1^4*C_1^4\ =\ 13\cdot 6\cdot 220\cdot 4\cdot 4\cdot 4\ =\ 1{,}098{,}240$$

$$P(1-\ pair)\ = \frac{4}{2598960} = \ 4.226\ \cdot 10^{-1} = \ or\ 1\ in\ 2.37$$

Problem 2.6

Each of the four engines of an airplane are functioning correctly on a given flight with probability of 0.99, andthe engines function independently of each other. Assume that the plane can make a safe landing if at leasttwo of its engines are functioning properly. What is the probability that the engines will allow a safe landing?

$$n\ =\ 4\ (four\ engines\ )\ and\ p\ =\ 0.99 \rightarrow P(2 <= x\ <= 4)\ =\ 1\ -\ P(0 <= x <= 1)$$
$$=\ 1\ -\ 0.00000397 = \ 0.99999603.$$

Problem 2.7

BONUS PROBLEM.

Two cowboys A and B decide to solve a dispute with a duel. Cowboy A hits his target 1/3 of the time. Cowboy B hits the target 2/3 of the time. It is decided that A will take the first shot, cowboy B will take the second shot (if still alive). This will continue until there is only one left alive. Also, a cowboy can not shoot two timesin a row. What are cowboy A chances of winning the duel?

$$\text{Let } p \text{ be the probability that } A \text{ wins. With probability } 13 \text{ he wins on his first shot. With probability } 23 \text{ he misses.}$$
$$\text{When } B \text{ hits in his first shot, } A \text{ has lost. When } B \text{ misses, } A \text{ again has a chance of } p. \text{ It follows that}$$
$$p = \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{3} \cdot p$$

$$\text{This implies } p = \frac{3}{7}$$