**Homework 4** Due November 24, 2021

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# Problem 4.1

Suppose that you roll the fair, ten-sided die. Let the random variable *X* be the remainder when the number on top is divided by 3, and let the random variable *Y* be the remainder when the number on top is divided by 4.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| N | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| X | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 |
| Y | 1 | 2 | 3 | 0 | 1 | 2 | 3 | 0 | 1 | 2 |

1. Are the random variables *X* and *Y* independent?

b) Compute *P* (*X* = 2*, Y* = 1)*, P* (*X* = 0 *| Y* = 1)*.*

# Problem 4.2

Let *X* be the number of heads after a coin is tossed three times. Let *Y* denote the face that comes up after rolling a die. Let *Z* = *X − Y.* Find the expected value *E*(*Z*)*,* variance *V* (*Z*) and standard deviation *D*(*Z*).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | x | 0 | 1 | 2 | 3 |
| y |  | 1/8 | 3/8 | 3/8 | 1/8 |
| 1 | 1/6 | -1/(1/48) | 0/(1/16) | 1/(1/16) | 2/(1/48) |
| 2 | 1/6 | -2/(1/48) | -1/(1/16) | 0/(1/16) | 1/(1/48) |
| 3 | 1/6 | -3/(1/48) | -2/(1/16) | -1/(1/16) | 0/(1/48) |
| 4 | 1/6 | -4/(1/48) | -3/(1/16) | -2/(1/16) | -1/(1/48) |
| 5 | 1/6 | -5/(1/48) | -4/(1/16) | -3/(1/16) | -2/(1/48) |
| 6 | 1/6 | -6/(1/48) | -5/(1/16) | -4/(1/16) | -3/(1/48) |



, as E(Y)…

V(X - Y) = V(X) + V(Y)= -

# Problem 4.3

I propose you a game! You pick a number between 2 and 12. Then you roll two fair dice. The result is the sum of the tosses.

* If your number is not the sum of the tosses, then you lose a dollar.
* If your number is the sum of the tosses, then you win *k* dollars.

What is the best number to choose initially? What value of *k* will make this game fair? *Explain your answers.*

# Problem 4.4

A baker blends 600 raisins into a dough mix and, from this, makes 500 cookies.

1. Find the probability that a randomly picked cookie will have no raisins.

raisins = 600

cookies = 500

lambda = raisins/cookies

# P (X = 0) meaning 0 raisins on a randomly chosen cookie

prob = **exp**(1)^(-lambda)

1. Find the probability that a randomly picked cookie will have exactly two raisins.

raisins = 600

cookies = 500

lambda = raisins/cookies

# P (X = 2) meaning 2 chocolate chips on a randomly chosen cookie

prob = lambda\*\*2/2 \* **exp**(1)\*\*(-lambda)

1. Find the probability that a randomly chosen cookie will have at least five raisins in it.

raisins = 600

cookies = 500

lambda = raisins/cookies

# P (X = 2) meaning 2 chocolate chips on a randomly chosen cookie

prob = - 1 - **exp**(1)\*\*(-lambda) - lambda \* **exp**(1)\*\*(-lambda) – (lambda\*\*2) \* **exp**(1)\*\*(-lambda) – (lambda\*\*3) \* **exp**(1)\*\*(-lambda) – (lambda\*\*4) \* **exp**(1)\*\*(-lambda)

= 0,0077

# Problem 4.5

A shooter takes 10 shots at a target and has probability 0*.*3 of hitting the target with each shot, independently of all other shots. Let *X* be the number of successful hits.

1. What is the distribution of *X*?

|  |  |
| --- | --- |
| 𝑋 | 𝑚(𝑋) |
| 0 | 0.028248 |
| 1 | 0.12106 |
| 2 | 0.23347 |
| 3 | 0.26683 |
| 4 | 0.20012 |
| 5 | 0.10292 |
| 6 | 0.03676 |
| 7 | 0.009 |
| 8 | 0.00145 |
| 9 | 0.00014 |
| 10 | 0.000006 |

1. What is the probability of scoring no hits?
2. What is the probability of scoring more hits than misses?
3. Find the expectation and the variance of *X*.
4. Suppose the shooter has to pay 3$ to enter the shooting range and he gets 2$ dollars for each hit. Let *Y* be his profit. Find the expectation and the variance of *Y* .
5. Now let’s assume that the shooter enters the shooting range for free and gets the number of dollars that is equal to the square of the number of hits. Let *Z* be his profit. Find the expectation of *Z*.

# Problem 4.6

BONUS PROBLEM.

You are throwing a fair coin until you have two heads consecutively. What is the expected number of throws?

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