5 Agent-based simulations

Only in a quite limited sense does the single individual create out of himself the mode of speech and of thought we attribute to him. He speaks the language of his group; he thinks in the manner in which his group thinks.

Karl Mannheim (1936)

The term "agent-based" refers to a particular type of simulation. Agent-based simulations have two essential components: agents and environment. An agent's behavior is the result of simple rules based on local interactions. The environment has certain autonomy, i.e. it has a certain level of independence from what the agents do, but it can also be influenced by the agents' behavior. The interaction of agents among each other, as well as the interaction of agents with the surrounding environment is modeled. This is in contrast to more traditional simulations typically based on systems of differential equations. Examples of agent-based simulations are Josh Epstein and Bob Axtell's "Sugarscape" model (Epstein and Axtell 1996), John Casti's "Would-be worlds" (Casti, 1997), Hemelrijk's simulations of artificial monkeys (Hemelrijk, 1998a, 1998b, 1999), and, of course, the NetLogo simulations that were described in Chapter 3 (Resnick, 1997).

5.1 The Sugarscape model

The Sugarscape (SSC) model consists of an environment, a landscape, with a particular distribution of various resources (e.g. sugar) that the agents need for their survival. Some regions are rich in sugar, some poor. Agents have certain traits like vision (how far they can see), or metabolism (how fast they consume resources). Agents can "sense" their surrounding (local) environment in order to decide in which direction to move, and they can "eat" the sugar they find on their way. With every movement an agent burns a particular amount of sugar, equal to its metabolic rate. Agents are considered dead once they have burnt up all their sugar. As we will see, a remarkable number of phenomena emerge from the interaction of these simple agents.

The Sugarscape scenario can be made arbitrarily complex by introducing, for example, seasons, pollution, reproduction, additional resources, trade, markets, legislation (e.g. inheritance), credits (borrowing and lending), diseases, etc. In this way, a kind of laboratory for the social sciences can be constructed. New kinds of questions can be asked and old questions can be answered in new ways. An example is the field of "agent-based economics", which investigates economic questions without making assumptions about being near equilibrium.

We now look at a few examples in more detail.

Sugarscape is a grid world. Figure 1a shows the distribution of sugar in the grid world, Figure 1b has an initial distribution of agent added.

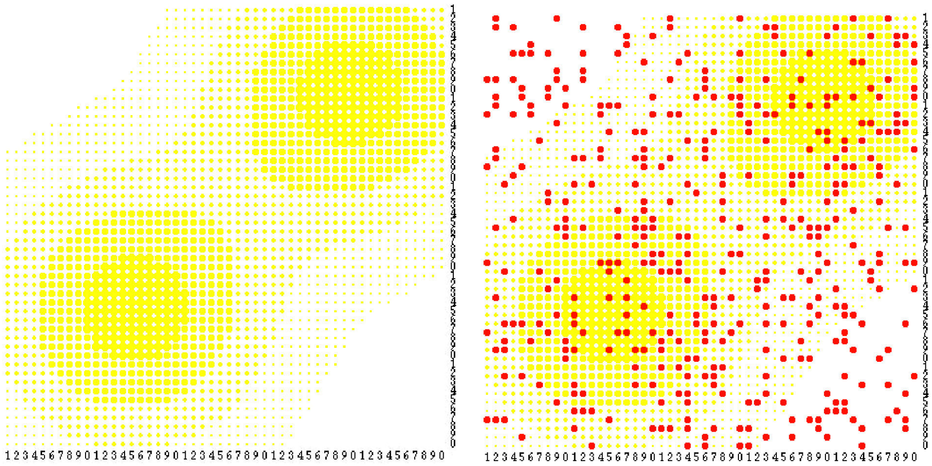

Figure 1: *(a) Distribution of sugar. (b) Initial distribution of agents.*

Rules

There are two kinds of rules, for the environment and for the agents.

Environment rules:

— 2-dimensional grid (lattice) (50x50)

— Specified for each location (x,y):

- current sugar level

- capacity (i.e. the maximum possible amount of sugar in that particular location)

— Distribution: peaks North East and South West — terraces

— Sugar levels: 0 to 4

SSC grow back rule G_α: At each lattice position, sugar grows back at a rate of α units per time interval up to the capacity at that position. If the sugar grows back instantaneously, the rule is called G_∞.

Agent rules:

Agents are characterized by a set of fixed and variable states.

Fixed: Metabolism (amount of sugar used per unit time), field of vision.

Variable: Amount of sugar. Agents are given some initial endowment of sugar, which they carry with them as they move around in the Sugarscape. Sugar, which is collected, but not metabolized, is added to the agent's sugar store. There is no limit to how much sugar an individual agent may accumulate.

Figure 2 illustrates the agent's field of vision. It can only see in the directions of the arrows, it does not see the shaded areas.

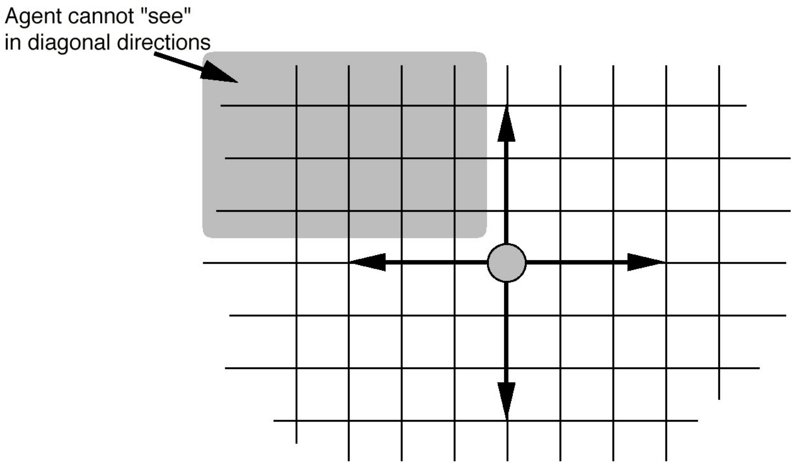

Figure 2: *Agent vision. In this example, the agent can only see north, east, south, and west, but not northeast, etc.*

Agents move according to the following “agent movement rule”:

Agent movement rule M:

— Look as far as vision permits in the four principal lattice directions and identify the unoccupied site(s) having the most sugar.

— If the biggest sugar value appears on multiple sites then select the nearest one.

— Move to this site (the agent can only move in four directions, i.e. North, East, South, West, but not North-East, North-West, etc.).

— Collect all the sugar in this new position.

The sugar level of the agent is incremented by the amount of sugar available on the new grid point and decremented by its metabolic rate.

Figure 3 shows the evolution of the Sugarscape model under rules ($\{G_\infty\}$ $\{M\}$).

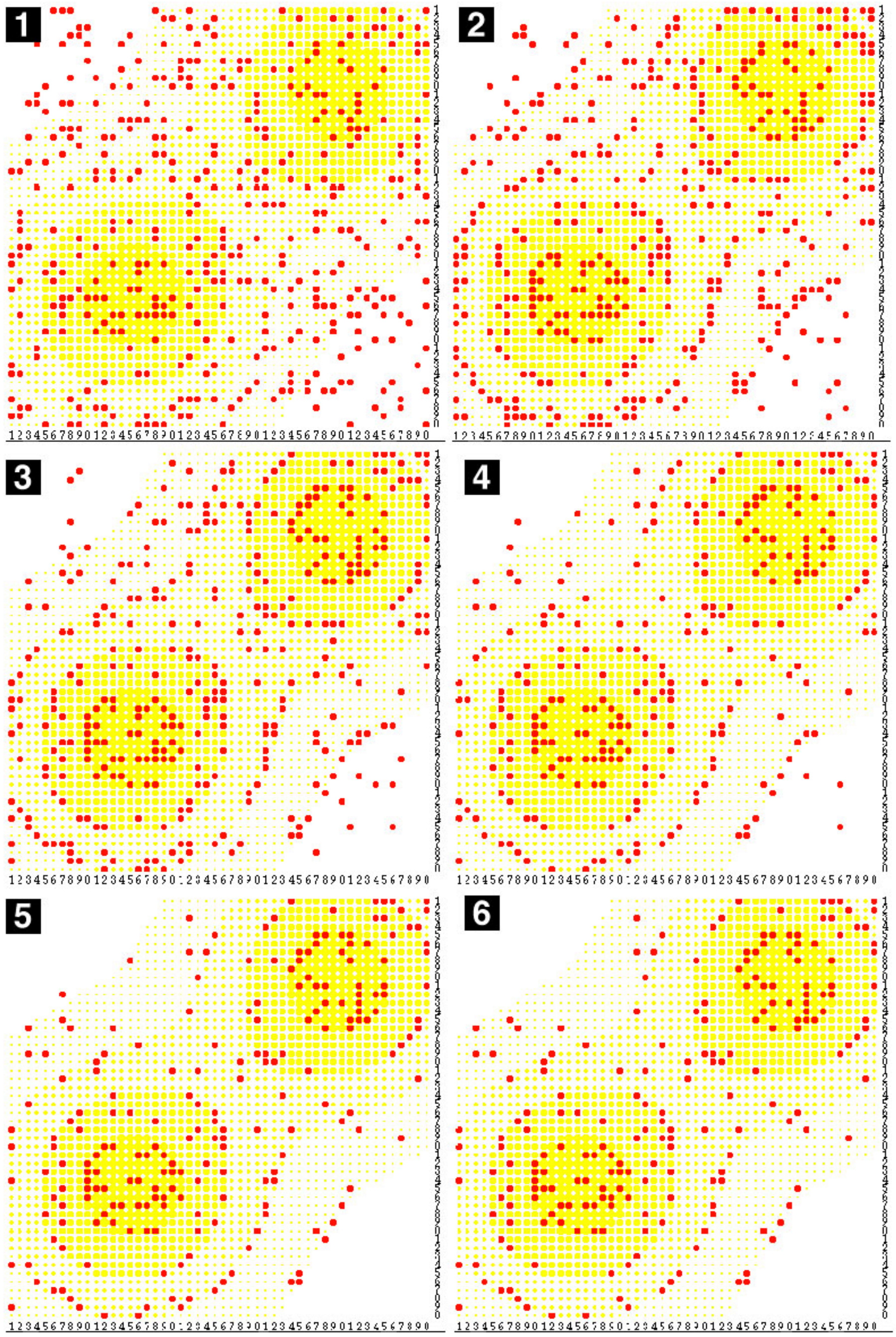

Figure 3: *Evolution under rules ({G$_\infty$} {M}). It can be clearly seen that the agents start forming regular patterns, i.e. they gather along the edges of the terraces.*

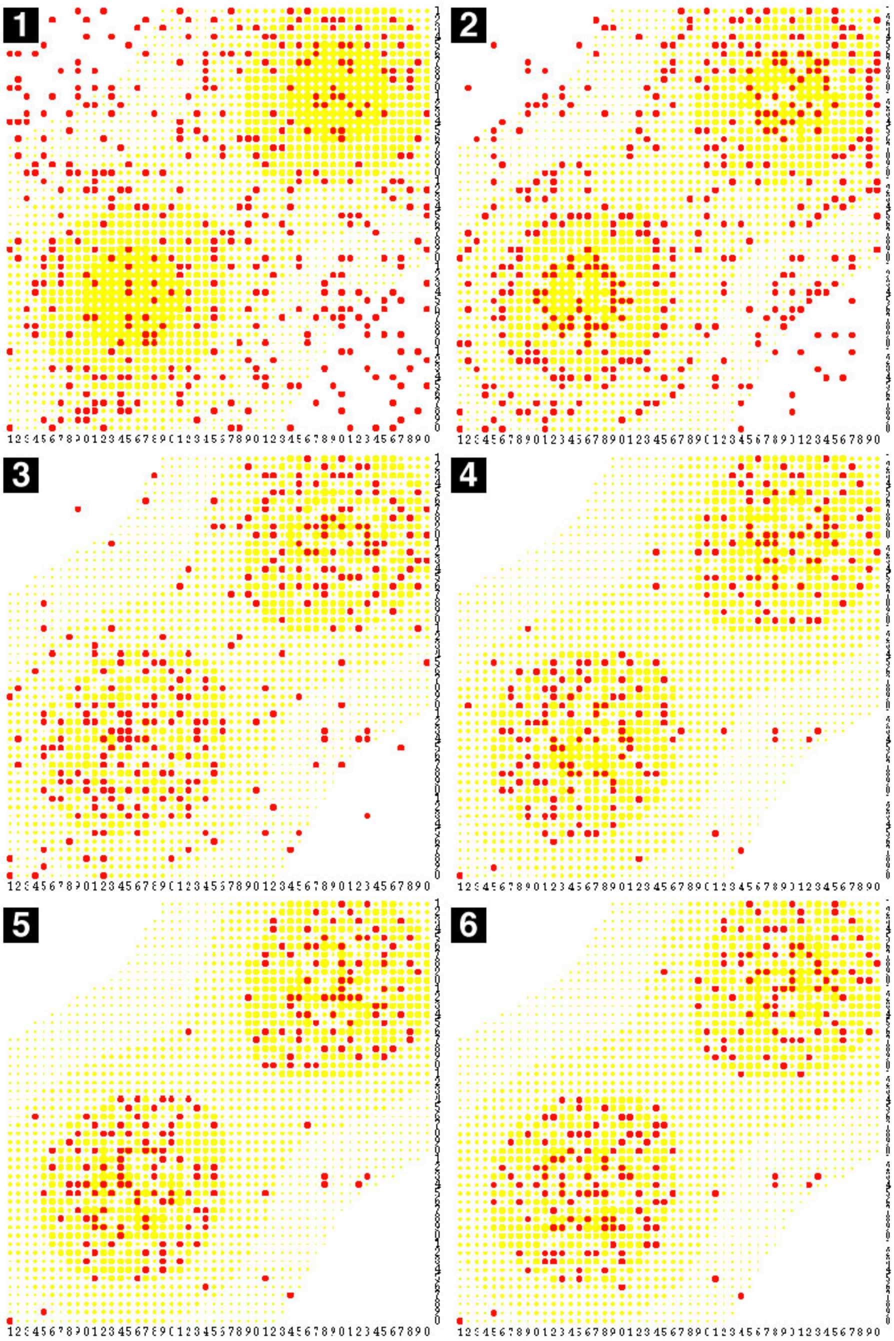

Figure 4: *Development with the grow back rule G_1 that states that everything grows back to capacity within one time step.*

Two relatively separate communities are forming, each sitting on top of one of the peaks in the Sugarscape.

Figure 5 shows the population development over time for rules ({G_1} {M}). It converges to an asymptotic value of a value somewhat above 200. This value is called the carrying capacity, i.e. the number of individuals that this particular environment can support. Figure 6 displays the carrying capacity as a function of mean vision for three different values of metabolism (m=1, 2, 3). Trivially, for small metabolism the carrying capacity is higher. There is also a slight increase as mean vision increases: Because agents can see further, they are, in a sense, more fit to survive.

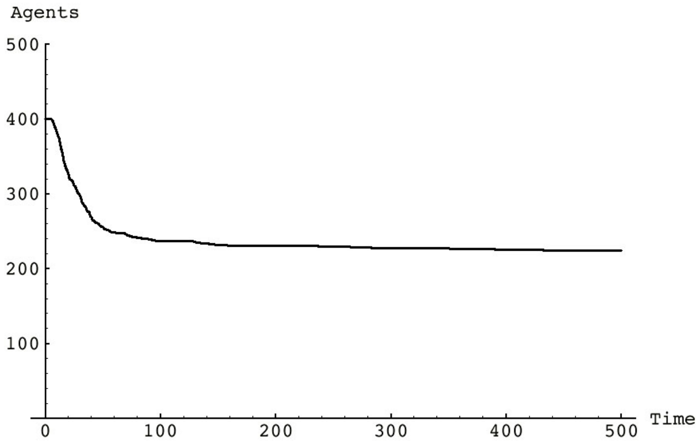

Figure 5: *Population development over time.*

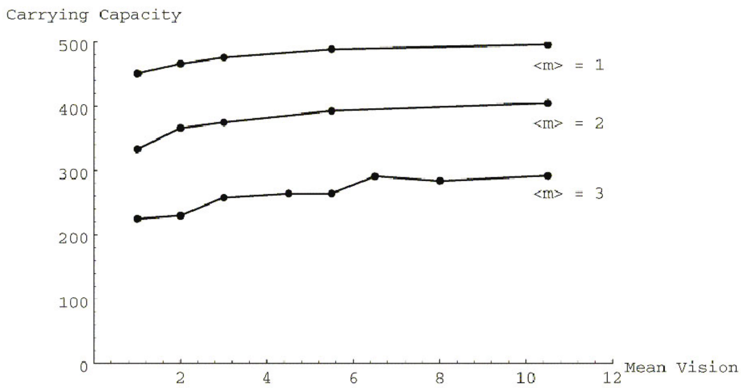

Figure 6: *Carrying capacities as a function of mean agent vision for three different values of metabolism.*

Wealth distribution

So far, agents have had indefinite life span as long as they had sufficient sugar supply. If we limit the age, we have to define an "agent replacement rule" R[a,b]:

Agent replacement rule $R_{[a,b]}$: When an agent dies it is replaced by an agent of age 0 having random genetic attributes, random position on the Sugarscape, random initial endowment, and a maximum age randomly selected from the range [a,b]. Figure 7 shows the development of the wealth distribution over time using agent replacement rule $R_{[60,100]}$. A wealth distribution emerges from the local rules defined earlier: A very small portion of the population owns the better part of the wealth available. This is an example of a macroscopic pattern arising from local interaction of agents. The term *self-organization* is also used for this type of phenomenon. What evolves is in fact a society of economic inequality.

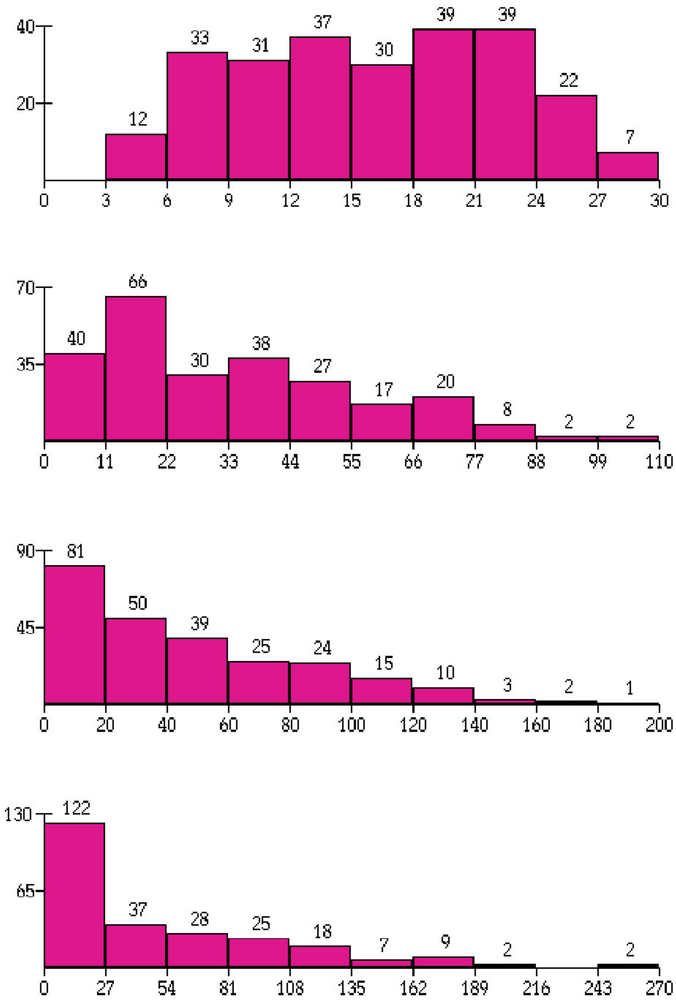

Figure 7: *Wealth histogram under rules ({G_1}, {M, $R_{[60,100]}$}) from a random initial distribution of agents.*

Migration

If we start with a distribution of agents in one corner, as shown in Fig. 8, we get waves propagating through the landscape. It is interesting to note that the waves propagate from lower left to upper right in a diagonal motion even though individual agents can only move north, east, south, and west. Diagonal movement is something not available to individual agents. The diagonal waves are emergent from the local rules. If seasons are introduced, the migration patterns change. Seasons can be simulated by having different grow-back rates for sugar.

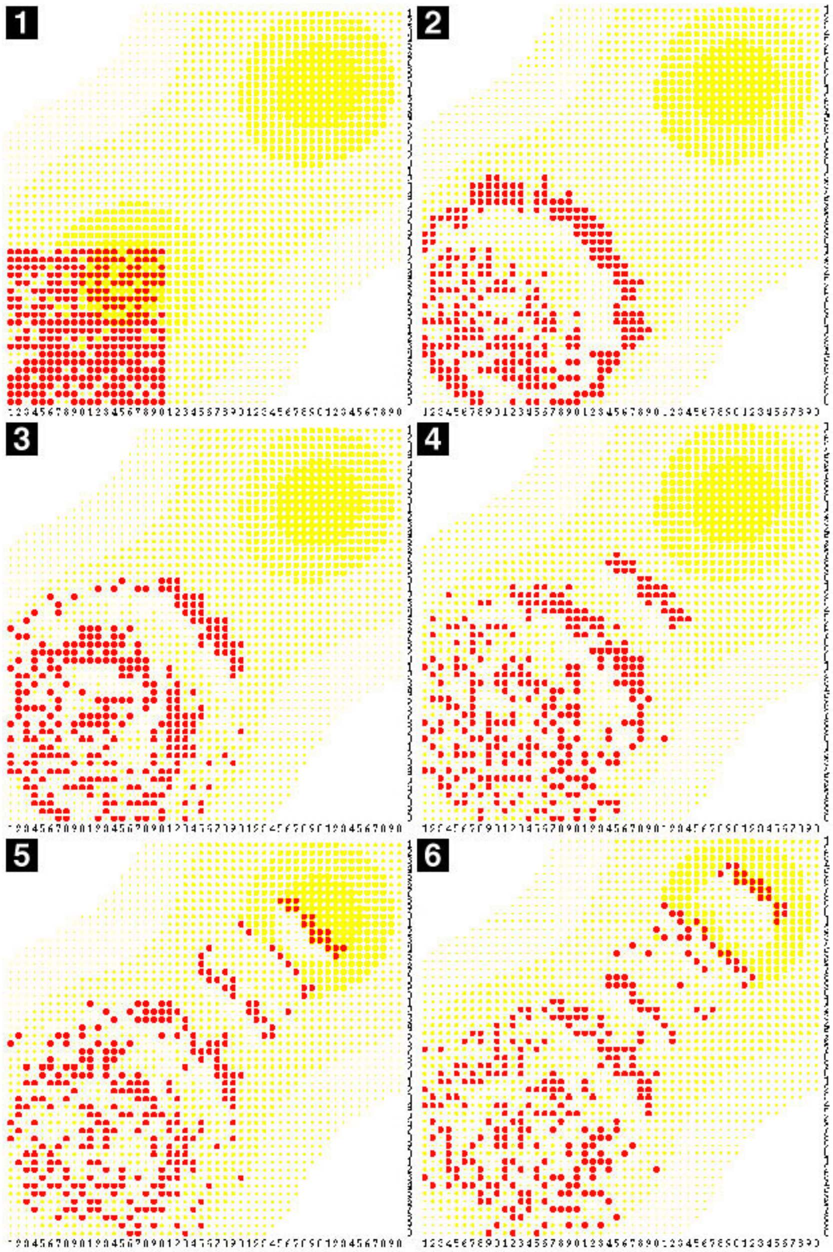

Figure 8: *Emergent diagonal waves of migrators under Rules ({G$_1$}, {M}) from an initial distribution of agents in a square in the lower right corner.*

Pollution can be introduced by having (a) a pollution formation rule, (b) a pollution diffusion rule, and (c) and agent movement rule modified for pollution.

Pollution formation rule $P_{\alpha\beta}$:

When sugar quantity is gathered from the Sugarscape, a certain amount of pollution, collection pollution, is produced, i.e. αs. When sugar amount m is consumed (metabolized), pollution is also produced, consumption pollution, βm. The total pollution on a grid point at time t, p^t, is the sum of the pollution present at the previous tine, plus the pollution resulting from production and consumption activities:

$$p^t = p^{t-1} + \alpha s + \beta m. \quad (1)$$

Agent movement rule M modified for pollution:

— Look as far as vision permits in the four principal lattice directions and identify the unoccupied site(s) having the maximum sugar to pollution ratio.

— If the maximum sugar to pollution ratio appears on multiple sites, then select the nearest one.

— Move to this site.

— Collect all the sugar at this new position.

Pollution diffusion rule D_α:

— After α time periods and at each site, compute the pollution flux - the average pollution level over all von Neumann neighboring sites.

— Each site's flux becomes its new pollution level.

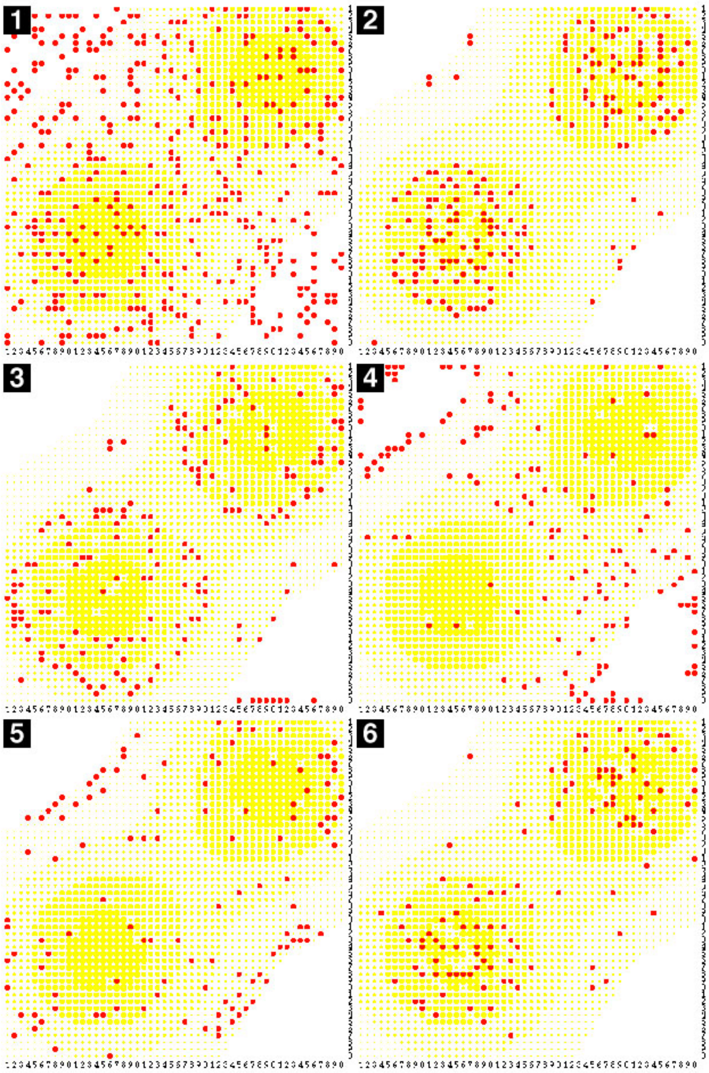

Figure 9: *Agent migration over time under this pollution diffusion rule system ({G_1, D_1}, {M, P_{11}}). Initially, everything is the same. Over time, pollution starts to build up and agents leave the polluted regions. The result is also a reduced carrying capacity.*

In summary, this case study demonstrates the surprising sufficiency of simple local rules to produce emergent structures. The interesting part is, that the rules are local and appear quite far from the social, collective phenomena they entail. Good examples are the migration patterns (the diagonal waves) and the skewed wealth distribution. The Sugarscape model can be used as some sort of laboratory *in silico*, in which we can "grow" fundamental social structures that help us learn more about what type of micro mechanisms are sufficient for generating macrostructures of interest.

Increasing the complexity

In principle the Sugarscape model can be made arbitrarily complex. Let us just look at a few examples. We can introduce sexual reproduction and study evolution in our laboratory. In order to be able to reproduce, agents must fulfill two conditions: (1) they must be of old enough and (2) they must have a sufficient amount of sugar, in order that they can share it with their offspring. In addition we need a rule of sexual reproduction.

Agent sex rule S:

— Select a neighboring agent at random.

— If the neighbor is fertile and of the opposite sex and at least one of the agents has an empty neighboring site (for the baby), then a child is born.

— Repeat for all neighbors.

All agents, including babies, use the same agent movement rule M. The sex of each child is determined randomly. The child's genetic makeup (metabolism, vision, maximum age, etc.) is determined from parental genetics through Mendelian rules. A simple example for vision and metabolism is given in table 1. Assume that one parent has (v, m), the other (V, M):

Table 1: *Crossover of genetic attributes in sexual reproduction.*

	Metabolism	
Vision	m	M
v	(m,v)	(M,v)
V	(m,V)	(M,V)

One of the results of these experiments is demonstrated in Fig. 10, showing mean vision and metabolism.

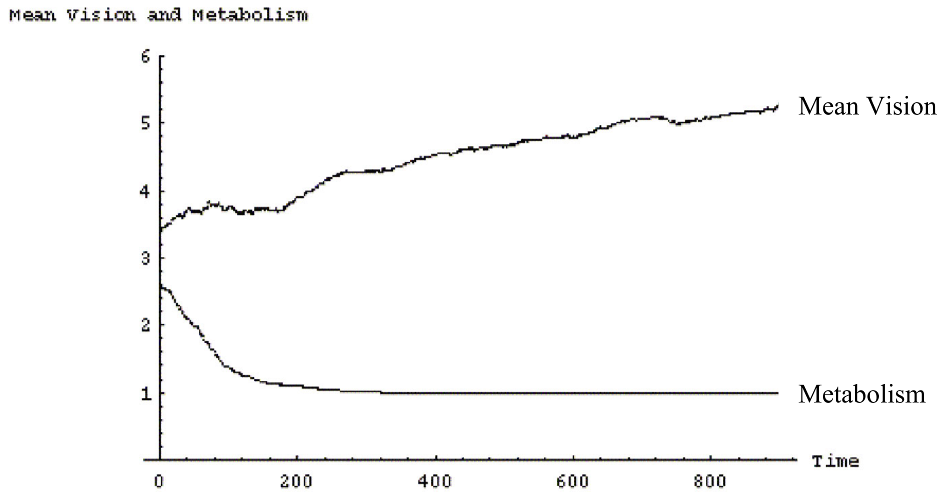

Figure 10: *Evolution of mean agent vision and metabolism under rule set ({G_1}, {M,S}).*

Figure 10 shows that mean vision increases and mean metabolism decreases. One interpretation of this is that the average fitness in the society has increased. But these fitter agents might be the cause of their own extinction (e.g. through overgrazing and explosive reproduction). Then, even though they have high vision, the overall fitness of the society would be low. A sustainable co-evolution with the environment is a necessary condition for "fitness". Experiments with Sugarscape suggest that fitness should be conceived as another emergent property, not as something—such as vision—that can be determined by inspection of isolated individuals.

Let us add inheritance to the social system.

Agent inheritance rule I:

When an agent dies its wealth is equally divided among all its living children.

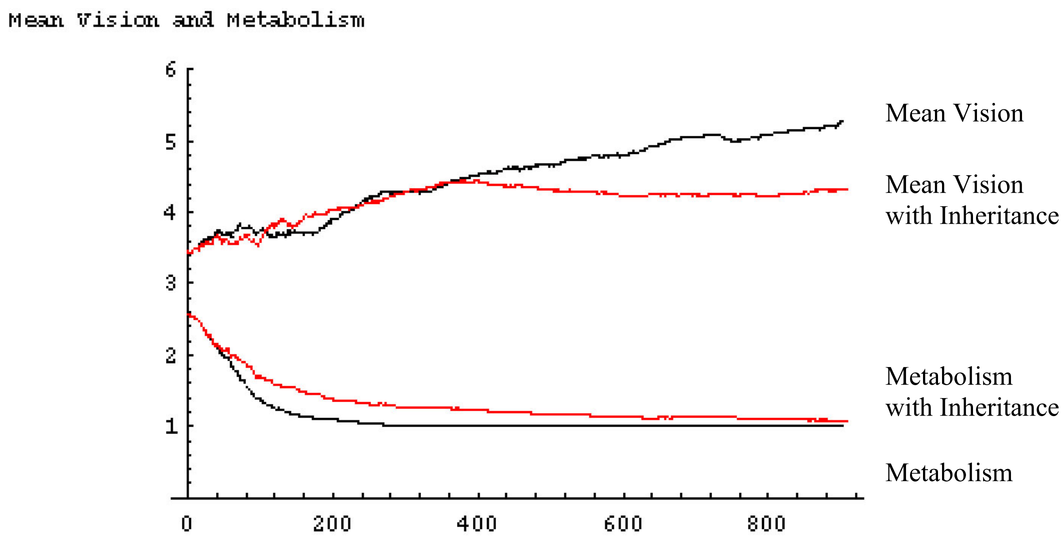

Figure 11: *Evolution of mean vision and metabolism under rule set ({G_1}, {M,S,I}).*

Note that the selective pressure for vision is mitigated by the inheritance rule I! In other words, there is an interaction of social rules (legislation), and biological traits of individuals. To illustrate the explosive content potentially contained in these innocuous simulations, let us quote Epstein and Axtell (1996, p. 68):

> *"Interestingly, some 'Social Darwinists' oppose wealth transfers to the poor on the ground that the undiluted operation of selective pressures is 'best for the species.' Conveniently, they fail to mention that intergenerational transfers of wealth from the rich to their offspring dilute those very pressures."*
>
> *[Social] inequality grows under inheritance.*

Sugar and spice: the beginnings of agent-based economics

Let us now introduce an additional commodity, spice. At each point on the grid there are now two values, one for sugar and one for spice. A similar distribution as for sugar is assumed for spice: a "hill" in the Northwest and Southeast. Each agent not only requires sugar for its metabolism, but also spice. Each agent keeps two separate accumulations, one for sugar and one for spice, and has two distinct metabolisms, one for each good. Agents die if either their sugar or their spice accumulation falls to zero.

The agent welfare function

A "rational" agent having, say, identical metabolic rates for sugar and spice, but with a large accumulation of sugar and small amounts of spice, should look for sites having more spice than sugar. One way to model this is to have the agent compute how "close" it is to starving to death due to the lack of either sugar or spice. Imagine an agent with metabolisms m_1, and m_2 and accumulations w_1, and w_2. The amount of time until death from starvation given no further resource gathering, is simply

$$\tau_1 = w_1 / m_1; \tau_2 = w_2 / m_2 \tag{2}$$

The relative size of these two quantities is a measure of the relative importance of finding sugar or spice. The welfare function is

$$W(w_1, w_2) = w_1^{m_1/m_\tau} w_2^{m_2/m_\tau} \tag{3}$$

where $m_\tau = m_1 + m_2$. This function is important and can be used by the agent for deciding to which field to move.

Multi-commodity agent movement rule M:

— Look as far a vision permits in each of the four lattice directions.

— Find the nearest position producing maximum welfare, considering only unoccupied lattice positions.

— Move to the new position.

— Collect all the resources at that location.

Rules of trade

Assume now that an agent is low in sugar but has got a lot of spice, and another nearby agent has a lot of sugar but a small amount of spice. They can both benefit by trading. This immediately raises a number of issues. When will agents trade? How much will they trade? And at what price will exchange occur? There are a variety of ways in which to proceed, depending on the particular economic theory one favors. Here, we only give a very crude characterization. For more details, the interested reader should refer to the excellent book by Epstein and Axtell (1996).

Let us just briefly mention the trade rule. The two key quantities in the trade rule are the MRS, the marginal rate of substitution, and the price. An agent's MRS of spice for sugar is the amount of spice the agent considers to be as valuable as one unit of sugar, that is, the value of sugar in units of spice. It can be shown that this MRS for the welfare function (3) is:

$$MRS = \frac{dw_1}{dw_2} = \frac{\dfrac{\partial W(w_1, w_2)}{\partial w_1}}{\dfrac{\partial W(w_1, w_2)}{\partial w_2}} = \frac{m_1 w_2}{m_2 w_1} = \frac{\tau_2}{\tau_1} \tag{4}$$

If MRS<1, for example, then the agent thinks of itself as being relatively poor in spice. If $MRS_A > MRS_B$, then agent A considers sugar to be relatively more valuable than does agent B, and so A is a sugar buyer and a spice seller, while agent B is the opposite. As long as the MRSs are not the same there is potential for trade. The directions of trade are summarized in table 2:

Table 2: *Relative MRSs (marginal ate of substitution) and the directions of resource exchange.*

	MRS_A>MRS_B		***MRS_A<MRS_B***	
Action	***A***	***B***	***A***	***B***
Buys	***Sugar***	***Spice***	***Spice***	***Sugar***
Sells	***Spice***	***Sugar***	***Sugar***	***Spice***

The bargaining rule to determine the local price is:

$$p(MRS_A, MRS_B) = \sqrt{MRS_A MRS_B} \tag{5}$$

Agent trade rule T:

— Agent and neighbor compute their MRSs; if these are equal then end, else continue.

— The direction of exchange is as follows: spice flows from the agent with the higher MRS to the agent with the lower MRS while sugar goes in the opposite direction.

— Calculate price according to (5).

— The quantities to be exchanged are as follows: if p>1 then p units of spice for 1 unit of sugar; if p<1 then 1/p units of sugar for 1 unit of spice.

— If this trade will (a) make both agents better off (increases the welfare of both agents), and (b) not cause the agents' MRSs to cross over one another, then the trade is made and return to start, else end.

Markets of bilateral traders

Assume that we start with a population of 200 immortal agents, with the welfare function (3), behavioral rules M and T, uniform distributions of metabolism for sugar and spice (between 1 and 5), and initial endowments randomly distributed between 25 and 50, for both sugar and spice. Figure 12 shows the time series for the average trade price with vision set to 1; Figure 13 the logarithm of the standard deviation (SD) for vision set to 1; and Figure 14 the SD for vision randomly distributed between 1 and 15. As we can see, though there is still a lot of variation in price, the standard deviation reaches a relatively low value. As one would expect, with higher mean vision, this value gets lower.

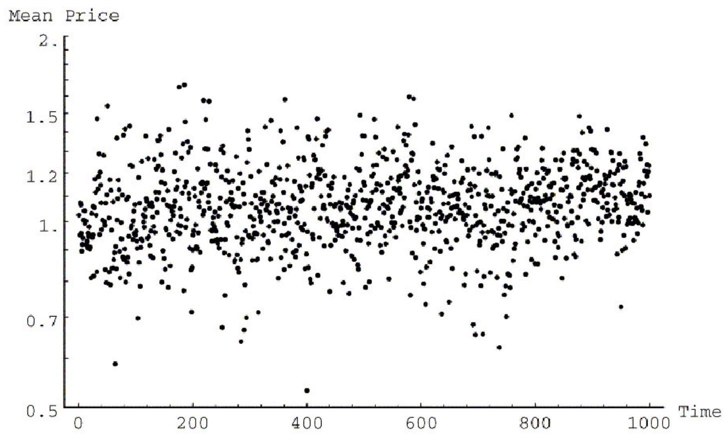

Figure 12: *Typical time series for average trade price with agent vision set to 1.*

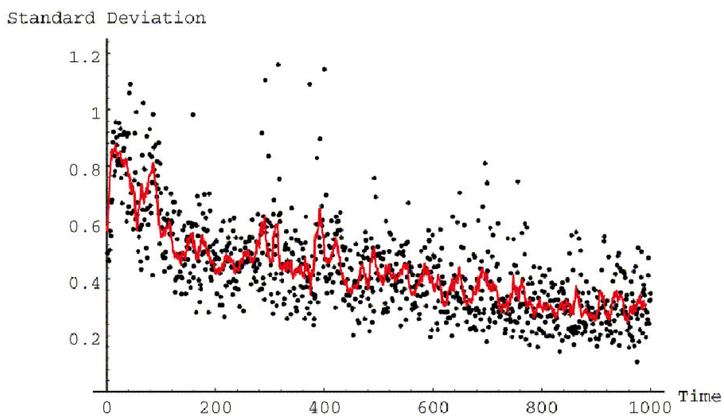

Figure 13: *Typical time series for the standard deviation of the logarithm of average trade price with vision set to 1.*

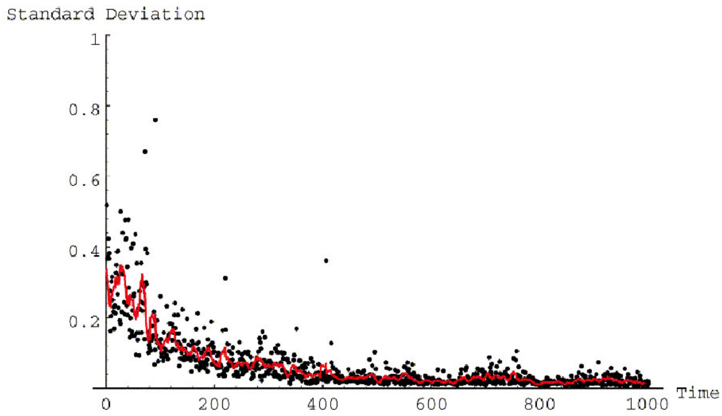

***Figure 14**: Typical time series for the standard deviation of the logarithm of average trade price with vision randomly distributed between 1 and 15.*

If we now decide to have mortal agents, $R_{[a,b]}$, where the age of the agent is uniformly distributed between a and b, we get the time series of Figures 15 and 16.

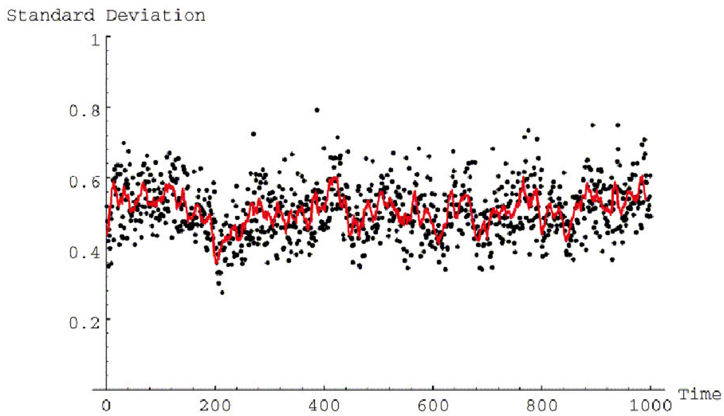

***Figure 15:** Typical time series for the standard deviation in the logarithm of average trade price under rule system ({G1}, {M, $R_{[60,100]}$, T) (with vision randomly distributed between 1 and 5).*

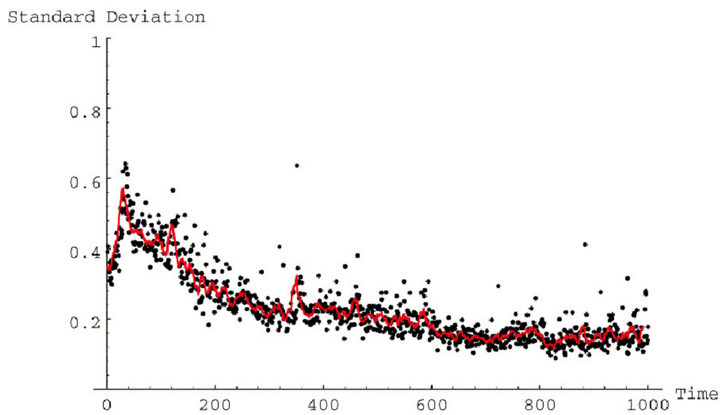

***Figure 16:** Typical time series for the standard deviation in the logarithm of average trade price under rule system ({G1}, {M, $R_{[960,1000]}$, T).*

Figure 15 demonstrates that the variance does not decrease over time. In other words, where trade price is concerned, this economy is *far from equilibrium*. The longer the lifetime the more the economy approaches equilibrium, with maximum values for immortal agents. These considerations suggest that the assumption of an economy near equilibrium may not always be justified.

Conclusion

The Sugarscape model illustrates the idea of a virtual laboratory in which virtual societies can be created and examined. Although we have to be careful with our claims about the relation of such virtual worlds to real worlds, we can conveniently investigate the effects of assumptions that are made. Examples are assumptions about vision, metabolism, welfare function, and lifetime of agents. This type of computational study opens up new ways of doing science, which makes it possible to study new issues. An example is the relation between biological evolution and social factors, like legislation. Remember the effect of inheritance on vision.

5.2 Emergence of structure in societies of artificial animals

Another illustration of virtual laboratories is inspired by primatology. Charlotte Hemelrijk has investigated the emergence of structure in societies of primates in the real world and in simulation. In her simulations she has been able to show that spatial distributions and hierarchical structures can emerge from the local interactions of agents. There is no need to postulate a representation of the hierarchical structure in the individuals' brains (Hemelrijk, 1998a, 1998b, 1999).

The emergence of hierarchies in societies of artificial chimps

Primates are known for their high cognitive capacities, which are thought being manifested in their social behavior, in particular their formation of coalitions. Coalitions are a part of their dominance interactions. Dominance interactions consist of threats and attacks that usually take place between two individuals only. Sometimes, however, a third individual intervenes by attacking one of the partners, thereby supporting the other. This is called coalition formation. The assumption is that primates are highly strategic in their decisions, for example, when they form coalitions and with whom they form them. They are even thought to repay received support. In order to be able to do so, they are presumed to keep records of the frequency of support received from every partner. Yet, in her individual-based computer simulations, Hemelrijk (1998a, 1998b) made a first step towards showing how complex patterns of coalition formation may emerge in the absence of sophisticated cognitive reflections. Inspired by a simulation by Hogeweg (1988) and Hogeweg and Hesper (1979), she implemented a world in which creatures—artificial chimps—dwelled. These creatures were able to move and to see each other. Furthermore, if creatures perceived someone nearby, they engaged into dominance interactions, otherwise they followed rules of moving and turning (Figure 5.17) that kept them aggregated (because real primates are group-living).

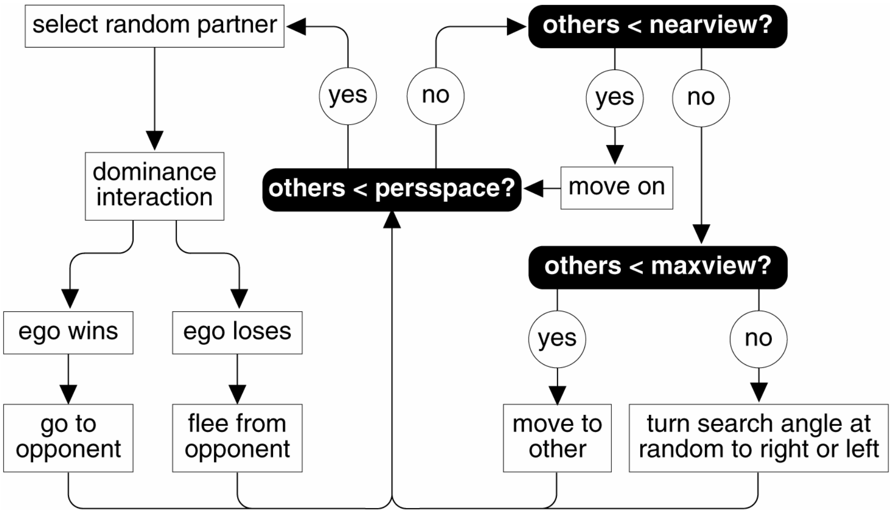

Figure 5.17: *Flow chart for the behavioral rules of the artificial chimps designed by Hemelrijk (1998a, 1998b). The left side of the figure contains dominance rules: after winning, Ego approaches the opponent, after losing it flees from it. The right side concerns aggregation rules: creatures look for others at increasingly larger distances. If they see nobody at all, they turn over a search angle to search for others.*

Note that interactions among these artificial chimps are just triggered by the proximity of others not by record keeping or other strategic considerations. Creatures were not even endowed with rules to support others in fights. Yet, support was recorded as an emergent event. It occurred if creatures happened to attack others that appeared to be already involved in a dominance interaction with someone else. Dominance interactions in the model incorporated the so-called 'loser- winner effect'. This effect has been established in many animal species, such as insects, reptiles, birds, mammals and humans. It implies that the effects of losing (and winning) are self-reinforcing. This means that after losing a fight the chance to loose the next fight is larger (even if the opponent is weak). The winner effect is the converse. By running the model, several forms of emergent social behavior were noted. This is shown in Figure 5.18.

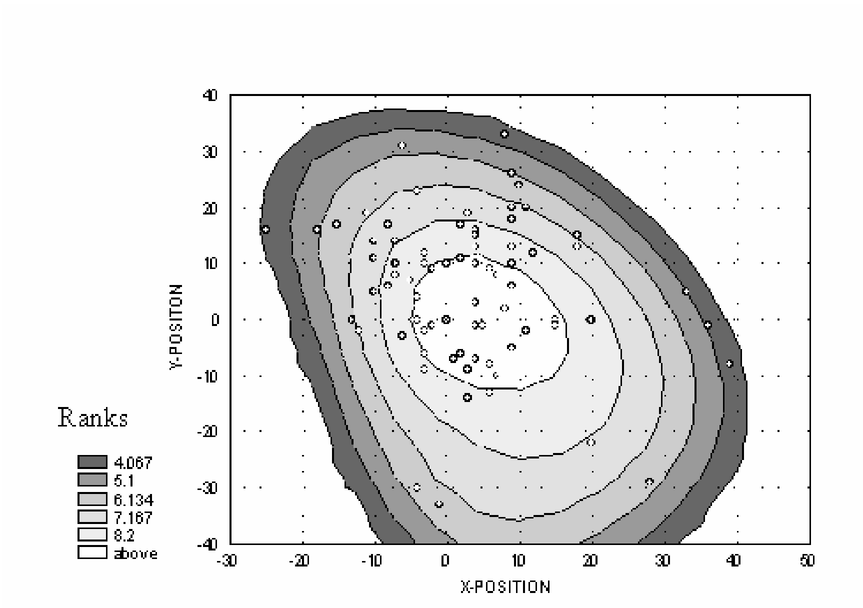

Figure 5.18: *Emergent hierarchies in artificial chimps. Spatial-social structure with concentric rings of chimps of different rank categories. The outer rings are occupied by lower ranking creatures.*

A dominance hierarchy arose, and a social-spatial structure, with dominants in the center and subordinates at the periphery (Fig. 5.18). Remarkably, exactly this same social-spatial structure has been described for several primate species. Furthermore, support in fights appeared to be repaid, despite the absence of a motivation to support or keep records of them. This was a consequence of the occurrence of a series of cooperation that consisted of two creatures alternatively supporting each other to chase away a third. These originated because by fleeing from the attack range of one opponent the victim ended up in the attack range of the other opponent. This typically ended when the spatial structure had changed such that one of both cooperators attacked the other. In particular these series were observed in loose groups, because entities were less disturbed and distracted by others. Additionally, chimps that were more aggressive appeared to co-operate more, due to their longer attack range. These were more easily trotted on by others and more difficult to get away from, resulting in series of repeated attacks.

Thus, the model shows how complex social interaction patterns may arise from local interactions only. It follows that this may also apply to real animals and that interaction patterns need not be genetically or cognitively predefined in the individuals' brains. Furthermore, the model points to some new questions concerning real primates, which would not be asked, if social behavior would be approached from a cognitive perspective only. An example would be: Is cooperation (such as repayment of support) more general in loose than cohesive groups and more prevalent among strongly than mildly aggressive animals? For additional questions, see Hemelrijk's publications.

A note on terminology

Recall the examples discussed in chapter 3. The kinds of phenomena demonstrated in the Didabot experiments and the ants are examples of *self-organization without structural changes*: the Didabots, for example, did not change during the heap building process. If we start a new experiment with the blocks randomly distributed in the environment, the Didabots would behave exactly the same way as in the previous trial. They would try to avoid obstacles. What we observe in Hemelrijk's simulations is *self-organization with structural changes:* The internal state of the individuals, i.e. their dominance value, changes over time, which in turn changes how the agents interact with other agents. Such processes of self-organization with structural changes are especially relevant, for example, during brain development.

5.3 Schelling's segregation model

In the models described above, emergent phenomena are the result of agent and environmental rules (in the case of Sugarscape) or a consequence of dominance interactions (in Hemelrijk's model). In Hemelrijk's models, the agents displayed certain clustering patterns. Such clustering phenomena may also arise due to social avoidance of and preference for certain others, i.e. if the agent rules include factors pertaining to social preference. The Harvard economist, Thomas Schelling (Schelling 1969), developed a similar sociological model in the late '70s. But again, what is observed does not appear to reflect the preferences of a single individual. Using a model of individuals that prefer to be surrounded by a certain minimum percentage of similar individuals, Schelling was able to show, how social avoidance of a minority status, even if slight, appears to be amplified at the level of the group and community structure. He notes that "micro motives" of individuals may lead to unexpected "macro patterns" that are not necessarily desired by any of the individuals, such as in ghetto formation and segregation of the sexes at parties.

Epstein and Axtell present the following somewhat modified version of Schelling's model. They use a 50 x 50 lattice world in the form of a torus. A torus in this context means that the points on the left and right edges of the lattice are considered neighbors (the same holds for the points on the top and on the bottom). 500 cells remain empty and 2000 cells are filled with an about equal number of red and of blue agents.

Agents are steered by the following behavioral rule, *Schelling's agent movement rule*:

— Agents perceive a von Neumann neighborhood of 4 cells.

— At each time step, an agent computes the fraction of neighbors of its own color.

— If this fraction greater than or equal to its own preference, the agent remains where it is.

— If the fraction is below its preference, the agent chooses an acceptable site in a random location.

At the start of each simulation run, agents are placed in random locations. During a run this simulation shows how at every time-step the unsatisfied individuals move. This goes on until everybody is happy. This appears to arise only when the segregation is much more extreme than reflected by the individual's preferences: Most individuals appear to be surrounded by many more individuals of the same type than they had wished for. This comes about because the movement of discontent agents may dissatisfy formerly content agents that just reached their preference limit. Consequently, the latter move too, and most agents will end up being surrounded by more of the same type than they strictly required. These results are compatible with Schelling's original model although he used a finite boundary, a Moore neighborhood of 8 cells and discontent agents selected the nearest acceptable site instead of a random one.

An entertaining description of this model is given also by Mitch Resnick (see pages 81-88, Resnick, 1997), where instead of males and females, the agents are turtles and frogs that dwell on a pond, sitting on pads of water lilies, much like Schelling's agents lived in grid points on a lattice. Resnick also shows how the model can be implemented in NetLogo.

5.4 Conclusion

We have seen that the term "agent-based" refers to a particular type of simulation model, which includes two essential components, agents and environment. An agent's behavior is determined by simple rules based on local interactions. The environment has certain autonomy, i.e. it has a certain level of independence from what the agents do, but it can also be influenced by the agents' behavior. The interaction of the agents among each other, as well as the interaction of the agents with their environment is modeled separately and independently from each other (contrary to more traditional kinds of simulation where often systems of differential equations are used). Agent-based models can be used in different areas of science. They are often easier to realize than time-consuming and expensive field studies and the results can be used in different research fields.

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