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Abstract

Quantum computers are at the forefront of technology development right now. The race to understand the differences between simulating quantum mechanics on classical computers vs quantum computers is currently being quantified. The uses for quantum computers are still being laid out although there are a few algorithms that work for them, such as Shor's algorithm. The application of these algorithms helps distinguish how and why simulating quantum mechanics on a quantum computer is more beneficial than simulating it on a classical computer. Using qubits we are able to hold multiple states of information in a single process until a highly probabilistic answer is achieved to move on. Using fundamental laws of quantum mechanics such as superposition and entanglement we are able to give qubits a purpose in quantum computers to hold multiple states of information rather than a simple binary state of 0 or 1. An understanding of how quantum mechanics works in these systems using nonrelativistic many-body systems (the Schrödinger equation), relativistic many-fermion systems (the many-body Dirac equation), and Gauge field theories is discussed. Additionally, the exponential return rate of a quantum computer's solution and why its processes are able to handle more complex interactions and calculations than the classical alternative is explored.

Chapter 1

Introduction

Classical Computers are considered single-CPU computers that run on a binary set of rules (1s and 0s) that are known to run algorithms like computers we see in our everyday lives. These computers use operations through brute force that solve for one function at a time rather than simultaneous calculations, resulting in an exponential increase in time to solve. The issue with brute force is that taking a process like decoding a 6-digit passcode could take 72 years. This is because the computer's processor can only process a single bit at a time. When you have nine hundred thousand combinations a computer is going to take 80 milliseconds per guess. A classical supercomputer is a step up from a classical computer, containing several thousand classical processors and graphics processors that are utilized to complete more of these tasks in a shorter amount of time.

Quantum computing is an approach with dimensionalized processing to link different data points resulting in simultaneous calculations. These quantum processors use quantum mechanical system properties like quantum parallelism and quantum interference to solve more complex problems that traditional computers can't handle (*Krantz, P., Kjaergaard, M., Yan, F., Orlando, T. P., Gustavsson, S., & Oliver, W. D. (1970, January 1))*. Dimensionalized processing takes a continuous-variable input that can be set to solve for multiple states at once (*Gschwendtner, M., & Winter, A. (2021, July 13))*. This process of using a single variable in several states at once resolves complex solutions faster than through traditional brute force using qubits (a unit of quantum information). Qubits, or quantum bits, don't run on the same binary system as classical computers. They are able to hold a state that is both on and off at the same time. Unlike a light switch, the value of the bit can be 1 and 0 simultaneously (*Krantz, P., Kjaergaard, M., Yan, F., Orlando, T. P., Gustavsson, S., & Oliver, W. D. (1970, January 1))*.

Quantum computing is used to solve complex problems that need to run through multiple states at once. For instance, traffic control and efficiency for global shipping networks is an example where you need to quickly sort through different possibilities of shipping routes given several hundred box ships. Another example is modeling the interaction of an electron inside a battery. Realistically, we don't know what goes on inside a battery. Chaos theory explains the interactions of electrons through mechanics and mathematics (*Overman, S. (1996, January))*. It reveals the layers of difficulty behind complex and unpredictable interactions of these electrons that with a typical computer we can't model due to the lack of computing power. The benefit of being able to understand these types of interactions and having this power can lead to discoveries and things like more sustainable energy (IBM.). The thought behind quantum computing is that we can resolve complex problems more efficiently using multistate algorithms.

The main point of this physics article review is to discover the physics behind quantum computing, the practical application of physical laws in computers, and what new complex processes will come from it. Given the algorithm, many complex problems are able to be solved and the addition of classical computers, supercomputers, and quantum computers, may help to unlock previously unknown knowledge.

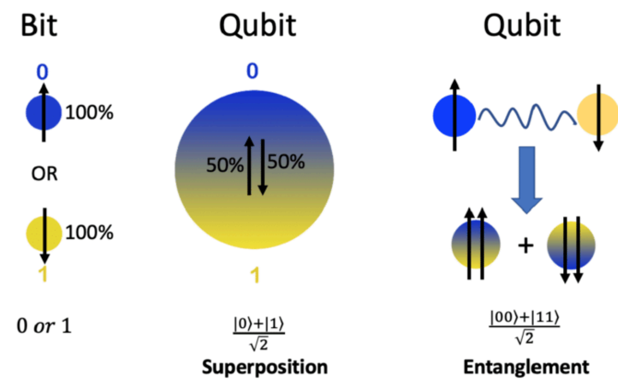
Chapter 2

Physics of Quantum Computers

Quantum Mechanics has opened a door, redefining our reality of what is possible in computing technology. Through the use of quantum mechanics, we can start to build quantum computers. Quantum computers run fundamentally differently compared to traditional computers. Due to their use of quantum operations, quantum computers will run through operations in a series using the amplification of probability to help resolve an answer efficiently. This amplification of probability pushes values that are more likely to be correct to the front of processing to result in an answer sooner in a practical sense compared to classical computers. This chapter will discuss qubits and the two main components of quantum mechanics necessary for quantum computing; entanglement and superposition (*Gamble, S.)*.

2.1 Qubit

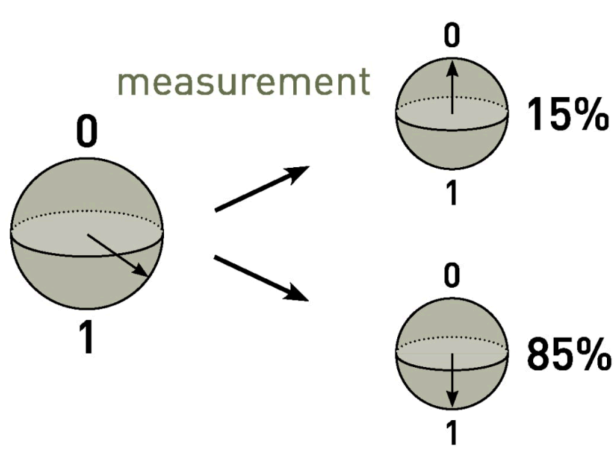
In order to make sense of how information is passed along a quantum system, we must first talk about the unit of information that travels in our quantum system, the qubit, see figure (1). A qubit, like a transistor, has the ability to be off or on like a classical computer bit that operates as a 1 or 0. As mentioned earlier it exists in a state of off and on until we assign a value based on probability, breaking the superposition. Like our electron example, this allows for the qubit to remain in a superposition until quantum operations have been run allowing the state to be assumed after probabilities have decided the best path (*Ladd, T. D., Jelezko, F., Laflamme, R., Nakamura, Y., Monroe, C., & O'Brien, J. L.)*.



(1)

2.2 Quantum Mechanics in Quantum Computers

Superposition in quantum mechanics explains that an object in certain states will have different energy levels. An object at its lowest energy level is known as the ground state. For instance, an electron could have some probability of being in a higher or lower state, and not until we measure the electron will we know what state this seemingly "simultaneous" process is in, see figure (2). The action of measuring the state of the electron is what ultimately results in the breaking of the superposition and outputs a discrete state. (*Steane, A.)*.

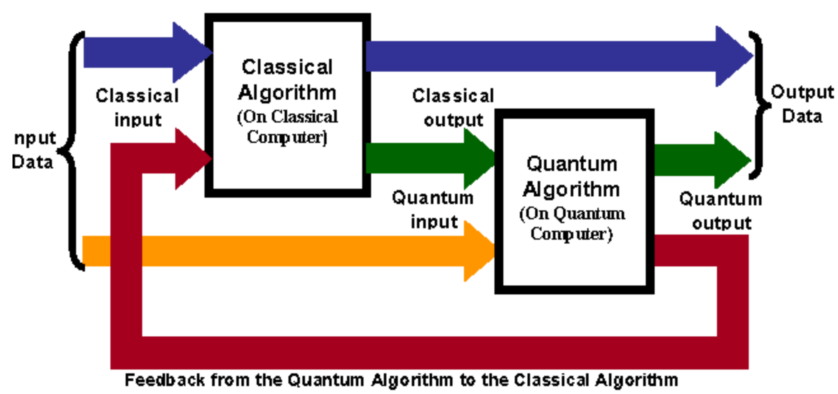


(2)

Quantum entanglement is the other key but "spooky" component of quantum computing. This process of entanglement is thought of as two objects being causally linked. For instance, what we know or measure one object to be would immediately influence the other object despite the distance between the two, see figure (1). (*arXiv, E. T. from the. (2020, April 2))*. In a quantum computer, we abuse the principles of quantum entanglement and the probabilistic nature of superposition to develop a quantum algorithm.

2.3 Quantum Algorithms

Quantum algorithms run using the probabilistic nature of transistors in their superposition and the entanglement of qubits. These together leverage the ability to arrive at a result in a more practical sense, sooner than traditional computers. Answers that have a higher probability of being correct are emphasized in this system while others that appear less likely are suppressed in these computational calculations, resulting in fewer measurements given the correct state. This allows for the most efficient form of solving complex problems in parallel today, see figure (3).



(3)

Figure 1: The structure link of classical part and quantum part of algorithm.

2.4 Shor's Algorithm

A method that has come from the benefit of quantum computing is breaking the infamous prime number algorithm encryption, also known as, Shor's algorithm. This process involves taking an integer N to find another integer p between 1 and N that would divide by N. The issue we run into with classical computers is that the reduction of a factor can only be simplified to a matter of ordering/sorting. This is where a quantum algorithm comes into play, to solve this problem of "order finding." Peter Shor's method states that some integers have a unique prime number but finding this prime number is considered extremely difficult. The belief is that finding the unique factor of a high prime number given thousands of digits is too difficult for any classical computer to brute force through. With Peter Shor's polynomial-time quantum algorithms, we are able to resolve the issue of factoring without a classical computer that would take two thousand CPU years to complete. Polynomial-time algorithms are given the name due to the operation to solve for the amount of time it takes to complete operations for a finite size of an input. For an input n, it could take to the exponential of any constant c, to solve for the computing time, showing n^c to be a polynomial (*Shor, P. W. (1999)*).

Chapter 3

Simulating Quantum Mechanics

Quantum computers have been in the works for decades and their full potential is still unrealized. The development of scientific fields and ideas takes time and collaboration.

In 1980, Paul Benioff used a collection of Turing machines to model a microscopic mechanical Hamiltonian simulation (Rietsche, R., Dremel, C., Bosch, S., Steinacker, L., Meckel, M., & Leimeister, J.-M. (2022, October 9)). Computation and physics heads alike began working on a connection between their two respective fields. This led to meetings and discussions on the inability of classical computers to simulate physics properly. Quantum behavior could not be modeled on classical computers so the idea of universal quantum simulators to model these complex interactions was looked into but never fully developed (Nature Publishing Group. (2022, January 10)).

Fast forward to 1994, Shor's factorization algorithm was a prime candidate for a quantum algorithm but building the quantum computer for it to run on would prove to be a challenge. No other algorithms had been developed other than Shor's to be used in a quantum computer and because of that, there was little interest in having a product without mass capabilities (Benioff, P. (1980)). A few years after due to theory and experimentation, further insight into the foundations of quantum mechanics was discovered and a new excitement began.

As discussed before, classical computers aren't efficient due to their nature of going through every permutation in a calculation, also known as combinatorics. Quantum computers on the other hand have the ability to more accurately model complex interactions like those of our natural world using more efficient calculations, unlike classical computers. Quantum computers are important in many fields and will continue to grow and be applied.

The difficulty behind a quantum computer is that we don't have many algorithms that work well with a quantum computer. As discussed before, Shor's algorithm was the first to be used successfully in a quantum computer. Due to the nature of quantum mechanics and Schrodinger's equation, quantum computers work best for simulating quantum mechanics and getting a better understanding of things like gauge fields or Dirac fermions (Bruce M. Boghosian, & Washington Taylor IV. (1998, October 21)).

Feynman argued that the ability to model quantum mechanical systems is exponentially faster on a quantum computer than it is for a classical computer due to the issue of factoring. Being able to model quantum mechanics on a quantum computer helps better conceptualize the difference in power and speed between a classical computer. Using three different ideas of quantum theories, Seth Lloyd proposed the idea that these general cases of quantum systems can exponentially speed up the simulation time for quantum mechanics on a quantum computer.

3.1 Non-relativistic Systems (The Schrödinger Equation)

The first of these ideas is using the Schrödinger equation for non-relativistic systems. We use this equation in quantum computing to model the wave function of a qubit over time. This in turn can then be used to measure the probability of observing a qubit in a certain state. Modeling and solving for the Schrödinger equation is a crucial step in decreasing the time to compute. Prediction in a quantum computer makes future system behaviors more accurate, which is important when considering optimization and design of algorithms.

The Schrödinger equation for a single free particle of mass m moving in d dimensions is

$$i\frac{\partial}{\partial t}\psi(\mathbf{x},t) = -\frac{1}{2m}\sum_i \frac{\partial^2}{\partial (x^i)^2}\psi(\mathbf{x},t)$$

Given this equation, we are able to write the wave function describing the state of a particle or qubit at any time, t.

$$\psi(t) = \sum_i \psi_i(t)|i\rangle,$$

3.2 Relativistic Systems (Dirac fermions)

The second of these ideas is simulating relativistic systems such as gauge fields and Dirac fermions because the Schrödinger equation isn't an accurate representation for these kinds of physical models. Proposed by Feynman was a simulation of free Dirac fermions in a variable *d* of dimensions. This type of modeling was found to be satisfied by the first-order two-component Dirac equation [1]. What was found was that a free system of many Dirac particles that did not interact in 1 + 1 dimensions could be successfully simulated on a quantum computer(Bruce M. Boghosian, & Washington Taylor IV. (1998, October 21)).

$$\left(i\partial\!\!\!/ - m\right)\psi = 0$$

Figure [1]

The only issue that comes from the simulation of Dirac fermions is their presentation on a lattice in more than one spatial dimension. The best way to describe the issue is to describe it in terms of the fermion doubling problem. This issue occurs due to naively putting fermions in a multi-dimensional lattice, which creates more fermionic states than what we expect or want (Goswami, G., & Bandyopadhyay, P. (1997, September 1)). Another notable issue is that we can’t simulate a single particle Dirac equation in any dimension greater than one. This requires a rule for the placement of fermions in a cubic lattice.

3.3 Gauge Field Theories

Being able to simulate interacting Dirac particles on a lattice leads to the approach of simulating a full second-quantized field theory for these particles using gauge field theories. The goal is to create a way to simulate these interactions but with increased speed output. The best application of this theory is in the use of a Hamiltonian lattice.

The issue of finding a rule for the propagation of a single particle in a lattice that satisfies Maxwell’s equation arises. The solution seems to be to write out the propagation of a photon using spinor notation to describe the model that agrees with Maxwell’s equation. Using a time-step formula you could find the state of the system given by the exponential of the Hamiltonian operator. (Bruce M. Boghosian, & Washington Taylor IV. (1998, October 21)).

3.4 Discussion

Using these theories of quantum mechanics a more understandable and quantifiable difference is made between the computational power and speed of a quantum computer against a classical computer. Being able to simulate even further theories of quantum mechanics should help in the education and progression of these ideas and further increase the power of complex computational processes.

Chapter 4

Summary and Conclusion

Although classical computers are good for most computations, time is the most crucial variable when it comes to a solution. As discussed, the application of quantum mechanics in quantum computers helps to identify a more quantifiable difference between its predecessor. The difference, found from a quantum computer's first use of Shor's algorithm, is an exponential decrease in time to compute versus that of a classical computer. Through this, we discovered a new advantageous way to calculate answers in a more applicable and sensible time frame compared to the human life span.

Simulating quantum physics on a quantum computer has proven useful given some of the fundamental theoretical ideas of quantum mechanics. The first fundamental laws exploited are when using superposition and entanglement to describe the inner machinations of a qubit. Using a few quantum theories such as nonrelativistic many-body systems (the Schrödinger equation), relativistic many-fermion systems (the many-body Dirac equation), and Gauge field theories, we are able to simulate quantum mechanics and quantify the advantage of a quantum computer over a classical computer. Using the probabilistic nature of Schrödinger's equation, quantum computers can hold multiple states of information at one point using qubits like a particle's energy state until measured. Using this helps determine an algorithm whose output is correct to a high degree of probability.

Quantum mechanics will continue to be understood and visualized through quantum computers. With that, a race begins to find out what will break a quantum computer's algorithm process in terms of "time to compute" as it's done to its forerunner, the classical computer.

References

Rieffel, E., & Polak, W. (2012). Quantum Computing: A Gentle Introduction, MIT Press, Cambridge, MA, 2011. (372 pp.). ISBN 978-0-262-01506-6. *Physics Today*, *65*(2). https://doi.org/10.1063/pt.3.1442

Bruce M. Boghosian, & Washington Taylor IV. (1998, October 21). *Simulating quantum mechanics on a quantum computer*. Physica D: Nonlinear Phenomena. Retrieved March 1, 2023, from https://doi.org/10.1016/s0167-2789(98)00042-6

Goswami, G., & Bandyopadhyay, P. (1997, September 1). *Fermion doubling on a lattice and topological aspects of chiral anomaly*. AIP Publishing. Retrieved March 13, 2023, from https://aip.scitation.org/doi/10.1063/1.532136

Rietsche, R., Dremel, C., Bosch, S., Steinacker, L., Meckel, M., & Leimeister, J.-M. (2022, October 9). *Quantum Computing - electronic markets*. SpringerLink. Retrieved February 20, 2023, from https://link.springer.com/article/10.1007/s12525-022-00570-y

Nature Publishing Group. (2022, January 10). *40 years of quantum computing*. Nature News. Retrieved February 21, 2023, from https://www.nature.com/articles/s42254-021-00410-6

40 years of quantum computing. *Nat Rev Phys* 4, 1 (2022). https://doi.org/10.1038/s42254-021-00410-6

Benioff, P. (1980). *The computer as a physical system: A microscopic quantum mechanical hamiltonian model of computers as represented by Turing Machines - Journal of*

Statistical Physics. SpringerLink. Retrieved February 22, 2023, from https://link.springer.com/article/10.1007/BF01011339

Bova, F., Goldfarb, A., & Melko, R. (2021, July 23). *Quantum computing is coming. what can it do?* Harvard Business Review. Retrieved February 21, 2023, from https://hbr.org/2021/07/quantum-computing-is-coming-what-can-it-do

Ruane, J., McAfee, A., & Oliver, W. D. (2021, December 14). *Quantum computing for Business Leaders*. Harvard Business Review. Retrieved February 22, 2023, from https://hbr.org/2022/01/quantum-computing-for-business-leaders?ab=at_art_art_1x4_s01

Wong, Y. K., Kanamori, Y., Forcer, T. M., Roy, R., Williams, C. P., Gyongyosi, L., & Amundson, J. (2022, October 31). An extensive review on quantum computers. Retrieved February 23, 2023, from https://www.sciencedirect.com/science/article/pii/S0965997822002381#!

Shor, P. W. (1999). Polynomial-time algorithms for prime factorization and discrete ... Retrieved January 31, 2023, from https://iuuk.mff.cuni.cz/~andrew/EAP/shor1999-1-5.pdf

Overman, S. (1996, January). The New Science of Management: Chaos and quantum theory and method - JSTOR. Retrieved January 31, 2023, from https://www.jstor.org/stable/1181623

Krantz, P., Kjaergaard, M., Yan, F., Orlando, T. P., Gustavsson, S., & Oliver, W. D. (1970, January 1). A Quantum Engineer's Guide to superconducting qubits. AIP Publishing. Retrieved January 30, 2023, from https://aip.scitation.org/doi/10.1063/1.5089550

Gschwendtner, M., & Winter, A. (2021, July 13). Infinite-dimensional programmable quantum processors. PRX Quantum. Retrieved January 30, 2023, from https://journals.aps.org/prxquantum/abstract/10.1103/PRXQuantum.2.030308

Reimer, C., Sciara, S., Roztocki, P., Islam, M., Romero Cortés, L., Zhang, Y., Fischer, B., Loranger, S., Kashyap, R., Cino, A., Chu, S. T., Little, B. E., Moss, D. J., Caspani, L., Munro, W. J., Azaña, J., Kues, M., & Morandotti, R. (2018, December 3). High-dimensional one-way quantum processing implemented on D-level cluster states. Nature News. Retrieved January 30, 2023, from https://www.nature.com/articles/s41567-018-0347-x

Gamble, S. (n.d.). Quantum Computing: What it is, why we want it, and how ... - NCBI bookshelf. Retrieved January 30, 2023, from https://www.ncbi.nlm.nih.gov/books/NBK538701/

Steane, A. (n.d.). Quantum Computing - IOPscience - Institute of Physics. Retrieved January 30, 2023, from https://iopscience.iop.org/article/10.1088/0034-4885/61/2/002/meta

Ladd, T. D., Jelezko, F., Laflamme, R., Nakamura, Y., Monroe, C., & O'Brien, J. L. (n.d.). Quantum Computers. Nature News. Retrieved January 30, 2023, from https://www.nature.com/articles/nature08812

What is quantum computing? IBM. (n.d.). Retrieved January 25, 2023, from https://www.ibm.com/topics/quantum-computing#:~:text=Quantum%20computing%20is%20a%20rapidly,hundreds%20of%20thousands%20of%20developers.

Roberts, J., & Roberts, J. (2019, June 4). *'quantum computers will soon outperform classical machines'*. Horizon Magazine. Retrieved January 25, 2023, from https://ec.europa.eu/research-and-innovation/en/horizon-magazine/quantum-computers-will-soon-outperform-classical-machines#:~:text='The%20big%20difference%20compared%20to,work%20with%20something%20called%20qubits.

Shor's algorithm. IBM Quantum. (n.d.). Retrieved January 25, 2023, from https://quantum-computing.ibm.com/composer/docs/iqx/guide/shors-algorithm