

# Computational Astrophysics

03A. Bayes' Rule. Monty Hall Problem

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# Bayes' Rule

#### Random Variables and Probability Density Functions

A **random**, or **stochastic**, variable is that whose value results from the measurement of a quantity that is subject to random variations. It can take on a set of possible different values, each with an associated probability.

There are discrete and continuos random variables.

The function which ascribes a probability value to each outcome of the random variable is called **probability density function** or **pdf**.

#### Random Variables and Probability Density Functions

Independent Identically Distributed or iid random variables are drawn from the same distribution and are independent. Two random variables, x and y are independent if

$$p(x, y) = p(x)p(y)$$

i.e., the knowledge of the value of x tells nothing about the value of y.

When the variables are not independent, we have

$$p(x, y) = p(x | y)p(y) = p(y | x)p(x).$$

#### Marginal Probability Functions

The Marginal Probability Function is defined as

$$p(x) = \int p(x, y) dy$$

Or equivalently

$$p(x) = \int p(x | y)p(y)dy$$

## Bayes' Rule

From the above results, we obtain the Bayes' rule,

$$p(y | x) = \frac{p(x | y)p(y)}{p(x)} = \frac{p(x | y)p(y)}{\int p(x | y)p(y)dy}.$$

For discrete random variables,  $y_i$ , this expression is writen as

$$p(y_j|x) = \frac{p(x|y_j)p(y_j)}{p(x)} = \frac{p(x|y_j)p(y_j)}{\sum_i p(x|y_i)p(y_i)}$$

## Monty Hall Problem

#### Monty Hall Problem

Doors: A, B and C

#### Assume that:

- We pick door A
- Monty opens door B
- Monty proposes to change door A for door C

We need to calculate two posteriors:

 $P(A \mid B)$ : Probability that A contains the prize if Monty opened B

 $P(C \mid B)$ : Probability that C contains the prize if Monty opened B

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Probability that the prize is at door A given that Monty opens door B

$$P(C \mid B) = \frac{P(B \mid C)P(C)}{P(B)}$$

Probability that the prize is at door C given that Monty opens door B

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

P(A): **Prior**. Probability that A contain the prize before any other event.

P(B|A): Likelihood. Probability that Monty opens B if A contains the prize.

P(B): Normalizing Constant. Probability that Monty opens B. This is usually calculated in terms of the other properties as

$$P(B) = P(B | A)P(A) + P(B | C)P(C)$$

$$P(C \mid B) = \frac{P(B \mid C)P(C)}{P(B)}$$

P(C): **Prior**. Probability that C contain the prize before any other event.

 $P(B \mid C)$ : Likelihood. Probability that Monty opens B if C contains the prize.

P(B): Normalizing Constant. Probability that Monty opens B. This is usually calculated in terms of the other properties as

$$P(B) = P(B | A)P(A) + P(B | C)P(C)$$

#### Priors: P(A) and P(C)

The probability of any door to contain the prize before we pick a door is 1/3. Therefore, the **priors** P(A) and P(C) are

P(A) = 1/3: Prior probability that A contains the prize

P(C) = 1/3: Prior probability that C contains the prize

Likelihood: P(B|A)

If the prize is behind door A, Monty can open B or C. Therefore the probability of opening either (B or C) is 50%. This means that

P(B|A) = 1/2: Likelihood that Monty opens B if A contains the prize

Likelihood: P(B|C)

If the prize is behind door C, Monty can only open B because he cannot open A since that door is picked. This means that

 $P(B \mid C) = 1$ : Likelihood that Monty opens B if C contains the prize

#### **Numerators**

$$P(B|A)P(A) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$$P(B \mid C)P(C) = 1 \times \frac{1}{3} = \frac{1}{3}$$

#### **Normalization Constant**

Since the three events considered cover all possible options and they do not overlap, we can take the sum of the numerators:

$$P(B) = P(B|A)P(A) + P(B|C)P(C) = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

#### Posterior P(A|B)

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} = \frac{1/6}{1/2}$$

$$P(A \mid B) = \frac{1}{3}$$

#### Posterior P(C|B)

$$P(C|B) = \frac{P(B|C)P(C)}{P(B)} = \frac{1/3}{1/2}$$

$$P(C \mid B) = \frac{2}{3}$$

#### Results

$$P(A \mid B) = \frac{1}{3}$$
: Probability that the prize is at door A given that Monty opens door B

$$P(C \mid B) = \frac{2}{3}$$
: Probability that the prize is at door C given that Monty opens door B

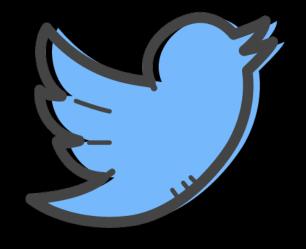
#### THE BEST OPTION IS TO CHANGE THE DOOR!



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