Durata de viață a calculatoarelor e o var. aleatoare continuă cu densitatea

$$f(x) = \begin{cases} K - \frac{x}{50} & 0 \le x \le 10 \\ 0 & , \text{altfel} \end{cases}$$

a) $K = ?$

b) Prob. ca un calculator să se strice în 5 ani?

c) Durata medie de viață a unui calculator $= ?$

Varianța $= ?$

a)

- $f(x) \ge 0 \iff K \ge \frac{x}{50} \Rightarrow K \in [\frac{1}{5}, +\infty)$
- $\int_{-\infty}^{\infty} f(x) = 1$

$$\int_{-\infty}^{\infty} f(x) = \int_0^{10} (K - \frac{x}{50})\, dx = Kx \Big|_0^{10} - \frac{1}{50} \cdot \frac{x^2}{2} \Big|_0^{10}$$

$$= 10K - \frac{100}{100} = 10K - 1$$

$$10K - 1 = 1 \Rightarrow K = \frac{2}{10} = \frac{1}{5} \in [\frac{1}{5}, +\infty) \Rightarrow \boxed{K = \frac{1}{5}}$$

b) $$P(X < 5) = \int_0^5 f(x)\, dx = \int_0^5 \frac{1}{5} - \frac{x}{50}\, dx - \int_0^5 \frac{10-x}{50}\, dx = \frac{1}{50}\left(10x\Big|_0^5 - \frac{x^2}{2}\Big|_0^5\right)$$

$$= \frac{1}{50}\left(10 \cdot 5 - \frac{25}{2}\right) = 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$$

c) Fct. de repartiție:

$x < 0$ avem $F_X(x) = 0$

$x \in [0, 10]$ avem $F_X(x) = \underbrace{\int_{-\infty}^{0} f(x)\, dx} + \int_0^x f(x)\, dx = \int_0^x \frac{10-x}{50}\, dx$

$$F(x) = \frac{1}{50}\left(10x\Big|_0^x - \frac{x^2}{2}\Big|_0^x\right)^0 = \frac{1}{50}\left(10x - \frac{x^2}{2}\right) = \underline{\frac{1}{100}(20x - x^2)}$$

$x > 0$ avem $F_X(x) = 0$.

$$E(X) = \int_0^{10} x \cdot P_X\, dx = \int_0^{10} x \cdot \frac{10-x}{50}\, dx = \frac{1}{50}\left(10 \cdot \frac{x^2}{2}\Big|_0^{10} - \frac{x^3}{3}\Big|_0^{10}\right)$$

$$= \frac{1}{50}\left(10 \cdot \frac{100}{2} - \frac{1000}{3}\right) = 10 - \frac{100}{5 \cdot 3} = \boxed{\frac{5}{3}}$$

$$E(X^2) = \int_0^{10} x^2 \cdot \frac{10-x}{50}\, dx = \frac{1}{50}\left(10 \cdot \frac{x^3}{3}\Big|_0^{10} - \frac{x^4}{4}\Big|_0^{10}\right)$$

$$= \frac{1}{50}\left(10 \cdot \frac{1000}{3} - \frac{10^4}{4}\right) = \frac{10^4}{15 \cdot 10} - \frac{10^4}{2 \cdot 10^2} = \frac{10^3}{15} - \frac{10^2}{2} = \frac{2000 - 15[illegible]}{30}$$

$$= \frac{500}{30} = \boxed{\frac{50}{3}}$$

$$Var(X) = \frac{50}{3} - \frac{25}{9} = \frac{150 - 25}{9} = \boxed{\frac{125}{9}}$$