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Brief paper

Coverage control for heterogeneous mobile sensor networks with bounded position measurement errors☆

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ABSTRACT

The coverage control problem for a network of heterogeneous mobile sensors with bound position measurement errors on a circle is addressed in this paper. A practical measurement error model is considered in which only the upper bounds of the measurement errors are known by the sensors *a priori*. The coverage cost function is defined as the largest arrival time from the mobile sensor network to any point on the circle. Two cases of coverage control with and without order preservation of the sensors respectively, are considered. For each case, the upper bounds on the measurement errors are employed to design an estimation algorithm for each sensor to estimate the difference between neighboring sensors' positions. Then, distributed coverage control laws are developed for the mobile sensors by using the estimated difference. Under the proposed control laws, the sensor network is driven to a neighborhood of the optimal configuration minimizing the cost function and the effect of the measurement errors on the coverage performance is reduced or even eliminated.

1. Introduction

Recently coverage control for heterogeneous mobile sensors with different maximum velocities on a circle is considered in Song, Liu, Feng, and Xu (2016) using accurate position measurements of the sensors. The goal is to minimize a coverage cost function which is defined to be the largest arrival time from the mobile sensor network to any point on the circle. When position measurement errors exist, networked mobile sensors generally cannot be driven to the optimal configuration such that the coverage cost function is minimized. When the accurate measurements are replaced by inaccurate ones, the authors in Dou, Song, Wang, Liu, and Feng (2018) investigate how the upper bounds on the measurement errors affect the coverage cost function. A more challenging problem is how to design coverage control laws to reduce or even eliminate the effect of the measurement errors on the coverage performance by only using the knowledge of these upper bounds.

In coverage control, a coverage cost function is often introduced to characterize how well a mission field is covered by a sensor network (Mesbahi, Abbasi, & Velni, 2019; Schwager, Rus, & Slotine, 2011; Stergiopoulos & Tzes, 2013; Zhai & Hong, 2013). The goal of coverage control is to drive networked mobile agents to the final configuration such that the coverage cost function is optimized. In literature, various coverage cost functions are taken into consideration such as the overall sensing performance of a sensor network (Cortés, Martínez, Karatus, & Bullo, 2004; Miah, Nguyen, Bourque, & Spinello, 2015; Schwager, Rus, & Slotine, 2009; Todescato, Carron, Carli, Pillonetto, & Schenato, 2017; Zuo, Shi, & Yan, 2019), the joint event detection probability (Li & Cassandras, 2005; Sun, Cassandras, & Meng, 2019; Zhong & Cassandras, 2011), and the response time from a sensor network to any point in a mission field (Cortés & Bullo, 2005; Hu & Xu, 2013; Lekien & Leonard, 2009), just to name a few.

Coverage control in a one-dimensional mission space has attracted increasing interest in recent years due to its wide potential applications ranging from environmental boundary monitoring to enemy intrusion detection (Carli & Bullo, 2009; Cheng & Savkin, 2009; Choi & Horowitz, 2010; Flocchini, Prencipe, & Santoro, 2008; Martínez & Bullo, 2006; Martínez, Bullo, Cortés, & Frazzoli, 2007; Song & Fan, 2018; Song, Fan, & Xu, 2020; Susca, Bullo, & Martínez, 2008). In Frasca, Garin, Gerencsér, and Hendrickx (2015), optimal one-dimensional coverage using mobile

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sensors which can fail with a certain probability is addressed. The authors aim at minimizing the expected maximum distance between any point in a one-dimensional environment and the closest active sensor. In Leonard and Olshevsky (2013), distributed coverage control laws are developed to deploy mobile sensors on a line for optimal coverage in a nonuniform field. The work is then extended to the case that each agent can only access noisy samples of the nonuniform field in Davison, Leonard, Olshevsky, and Schwemmer (2014).

It is noted that the proposed coverage control laws in literature are usually based on accurate position information of mobile sensors. However in practice, position measurements of the sensors are often subject to measurement errors. Recently a few works are devoted to the design of distributed coverage algorithms for networked mobile sensors with inaccurate position measurements (Habibi, Mahboubi, & Aghdam, 2016; Mahboubi, Vaezi, & Labeau, 2017; Tzes, Papatheodorou, & Tzes, 2018). Assuming a uniform distribution of the position measurement error, Habibi et al. (2016) investigate the effect of the measurement error on the performance of coverage area maximization. In Tzes et al. (2018), an additively weighted guaranteed Voronoi partitioning is utilized to develop distributed coverage control laws for a group of mobile aerial agents under positioning uncertainty to guarantee monotonic increase of an area coverage metric.

In practice, mobile sensors' position measurement errors often hardly follow any known distributions exactly. Instead, only the upper bounds of the measurement errors are known by the sensors *a priori* (Bauso, Giarré, & Pesenti, 2009; Cortés, 2009; Garulli & Giannitrapani, 2011; He, Duan, Cheng, Shi, & Cai, 2017; He, Zhou, Cheng, Shi, & Chen, 2016). In DeLellis, Garofalo, Iudice, and Mancini (2015), the upper bound on the position measurement errors is employed to develop a recursive algorithm for mobile agents moving along a circle to estimate the relative angular position of the agents. A sufficient condition for the convergence of the algorithm is also provided, which does not require any knowledge of the probability distribution of the bounded measurement errors. This inspires us to reconsider the coverage problem for mobile sensors with bounded measurement errors on a circle and to investigate the design of distributed coverage control laws with guaranteed convergence and coverage performance for the sensors.

The main contributions of this paper can be summarized as follows. Firstly, similar to the work in Dou et al. (2018), two cases of the coverage control problem, that is, coverage control with and without order preservation respectively are considered in this paper. In each case, the upper bounds on the measurement errors are employed to design an estimation algorithm for each sensor to estimate the difference between neighboring sensors' positions. Then, distributed coverage control laws are developed for the mobile sensors by using the estimated difference. It is shown that the effect of the measurement errors on the coverage performance can be reduced or even eliminated under the proposed coverage control laws. Secondly, for the above-mentioned two cases the upper bounds on the coverage cost function as time goes to infinity are given explicitly which only depends on the sensors' maximum velocities and the upper bounds of their measurement errors. In contrast, it is not easy to compute the upper bounds on the coverage cost function provided in Dou et al. (2018) because there exist two unknown constants in those bounds. Finally, unlike the results in Dou et al. (2018) where the sensors' initial positions are required to be close enough to the optimal configuration and their measurement errors are required to be small enough to guarantee order preservation of the sensors, those constraints have been removed in this work. In fact, in the case of coverage control with order preservation it is shown that order preservation is an inherent property of the proposed coverage control law.

The rest of the paper is organized as follows. The problem formulation is given in Section 2. Distributed coverage control laws are developed in Section 3 and their convergence analysis is presented in Section 4. Finally, simulation results and conclusion are provided in Sections 5 and 6, respectively.

2. Problem formulation

Consider a mobile sensor network composed of n mobile sensors with different moving capabilities. The sensors are located on a unit circle initially and are constrained to move on the circle. The position of an arbitrary point q on the circle is defined as the angle measured counterclockwise from the positive horizontal axis. Denote $\mathbb{S}$ as the set of all points on the circle and q_i as the position of sensor i. The distance between sensor i and point $q \in \mathbb{S}$ is given by $d(q_i, q) = \min\{\bar{d}(q_i, q), 2\pi - \bar{d}(q_i, q)\}$, where $\bar{d}(q_i, q) = (q - q_i) \bmod 2\pi$ is the counterclockwise distance from sensor i to point q.

Consider that each sensor evolves according to the following discrete-time dynamics

$$q_i(k+1) = q_i(k) + u_i(k), \tag{1}$$

where $u_i(k)$ is the control input of sensor i. Due to the sensors' different moving capabilities, each sensor can move along the circle with a different maximum velocity λ_i which is known *a priori* by itself. Note that $u_i(k)$ also denotes the velocity of sensor i at time step k. Consequently, different saturation constraints are imposed on the sensors' control inputs, that is, $-\lambda_i \le u_i(k) \le \lambda_i,\ \forall k \ge 0,\ \forall i \in \mathcal{I}_n = \{1, \dots, n\}$. Throughout the paper, let $\lambda_{n+i} \equiv \lambda_i$ and $q_{n+i} \equiv q_i + 2\pi$ for all $i \in \{0, \dots, n\}$. Note that points p and $p + 2\pi$ represent the same point on the circle. Therefore, sensors i and $n+i$ refer to the same sensor in this paper.

For our analysis, mobile sensors are labeled counterclockwise according to their initial positions on the circle, that is,

$$0 \le q_1(0) < \cdots < q_i(0) < q_{i+1}(0) < \cdots < q_n(0) < 2\pi. \tag{2}$$

Fixed interaction topology is assumed for the mobile sensor network in this paper. The neighbors of each sensor are defined as its immediate clockwise and counterclockwise sensors on the circle at the initial time. Then, the neighboring set of each sensor is given by $\mathcal{N}_i = \{i-1, i+1\}$.

In practice, mobile sensors often interact with each other via sensing and/or communication. Due to the existence of sensing noise, each sensor i usually cannot acquire the accurate information of $q_j(k) - q_i(k),\ j \in \mathcal{N}_i$. Therefore, we assume that the measured difference between $q_i(k)$ and $q_j(k)$ by sensor i is denoted by $d_{i,j}(k) = q_j(k) - q_i(k) + e_{i,j}(k),\ i \in \mathcal{I}_n,\ j \in \mathcal{N}_i$, where $e_{i,j}(k)$ is the measurement error which satisfies the following assumption.

Assumption 1. Each sensor's measurement error $e_{i,j}(k)$ satisfies $|e_{i,j}(k)| \le \delta_i,\ i \in \mathcal{I}_n,\ j \in \mathcal{N}_i,\ k \ge 0$ with the bound δ_i being known by sensor i *a priori*.

Given these definitions, the coverage cost function $T(q_1, \dots, q_n) = \max_{q\in\mathbb{S}} \min_{i\in\mathcal{I}_n} d(q_i, q)/\lambda_i$ is considered in this paper, which defines the largest arrival time from the mobile sensor network to any point on the circle (Song et al., 2016). Due to the existence of position measurement errors, the coverage cost function generally cannot be minimized and the coverage performance is often degraded. This paper will investigate how to design distributed coverage control laws to reduce the effect of the measurement errors on the coverage cost function T and thus the coverage performance.

3. Distributed coverage control laws

In this paper, two cases of the coverage control problem will be considered. The first case is coverage control without order preservation, where it is only required that the order of the sensors when time goes to infinity is the same as their initial order. The second case is coverage control with order preservation, where the spatial order of the sensors is required to be preserved throughout the coverage task. For the first case, the following coverage control law is considered

$$u_i(k) = \lambda_i \mathrm{sat}(\tilde{u}_i(k)),\ i = 1, \ldots, n, \tag{3}$$

where $\mathrm{sat}(\tilde{u}_i(k)) = \mathrm{sign}(\tilde{u}_i(k))\min\{1, |\tilde{u}_i(k)|\}$ and

$$\tilde{u}_i(k) = \eta_i[(\lambda_{i-1} + \lambda_i)\tilde{d}_{i,i+1}(k) + (\lambda_i + \lambda_{i+1})\tilde{d}_{i,i-1}(k)] \tag{4}$$

with $\eta_i > 0$ being a low control gain and

$$\tilde{d}_{i,j}(k) = \frac{\check{d}_{i,j}(k) + \hat{d}_{i,j}(k)}{2},\ j \in \mathcal{N}_i \tag{5}$$

being used to estimate the true position difference $q_j(k) - q_i(k)$. In the above equation, $\check{d}_{i,j}(k)$ and $\hat{d}_{i,j}(k)$ iterate according to the following equations respectively

$$\check{d}_{i,j}(k+1) = \max\{\bar{d}_{i,j}(k+1) - \bar{\delta}_{i,j},\ \check{d}_{i,j}(k) + u_j(k) - u_i(k)\} \tag{6}$$

and

$$\hat{d}_{i,j}(k+1) = \min\{\bar{d}_{i,j}(k+1) + \bar{\delta}_{i,j},\ \hat{d}_{i,j}(k) + u_j(k) - u_i(k)\} \tag{7}$$

with $\bar{d}_{i,j}(k) = (d_{i,j}(k) - d_{j,i}(k))/2$ and $\bar{\delta}_{i,j} = (\delta_i + \delta_j)/2$. The initial values of $\check{d}_{i,j}(k)$ and $\hat{d}_{i,j}(k)$ are given by $\check{d}_{i,j}(0) = \bar{d}_{i,j}(0) - \bar{\delta}_{i,j}$ and $\hat{d}_{i,j}(0) = \bar{d}_{i,j}(0) + \bar{\delta}_{i,j}$, respectively.

Let $\bar{e}_{i,j}(k) = (e_{i,j}(k) - e_{j,i}(k))/2$. Denote $\check{e}_{i,j}$ and $\hat{e}_{i,j}$ as the minimum and maximum of $\bar{e}_{i,j}(k)$ during the coverage task, that is, $\check{e}_{i,j} = \min_{k\geq 0}\bar{e}_{i,j}(k)$ and $\hat{e}_{i,j} = \max_{k\geq 0}\bar{e}_{i,j}(k)$. Assume that $\bar{e}_{i,j}(k)$ reaches its minimum and maximum for the first time at time steps $k^l_{i,j}$ and $k^h_{i,j}$, respectively. For each $i \in \mathcal{I}_n$, define $k^*_{i,j} = \max\{k^l_{i,j}, k^h_{i,j}\}$. The following lemma gives the estimation error $\tilde{d}_{i,j}(k) - (q_j(k) - q_i(k))$.

Lemma 1. *Under the proposed estimation algorithm (5),*

$$\begin{aligned}&\tilde{d}_{i,j}(k) - (q_j(k) - q_i(k))\\ =&\ \frac{1}{2}(\max\{\bar{e}_{i,j}(0),\ \bar{e}_{i,j}(1), \ldots,\ \bar{e}_{i,j}(k)\} +\\ &\min\{\bar{e}_{i,j}(0),\ \bar{e}_{i,j}(1), \ldots,\ \bar{e}_{i,j}(k)\}),\ i \in \mathcal{I}_n,\ j \in \mathcal{N}_i\end{aligned} \tag{8}$$

holds for all $k \geq 0$. When $k \geq k^*_{i,j}$, $\tilde{d}_{i,j}(k) - (q_j(k) - q_i(k)) = (\check{e}_{i,j} + \hat{e}_{i,j})/2$ always holds.

Proof. From the iterative algorithm (6) and initial values of $\check{d}_{i,j}(k)$, one has

$$\begin{aligned}\check{d}_{i,j}(1) &= \max\{\bar{d}_{i,j}(1) - \bar{\delta}_{i,j},\ \check{d}_{i,j}(0) + u_j(0) - u_i(0)\}\\ &= \max\{q_j(1) - q_i(1) + \bar{e}_{i,j}(1) - \bar{\delta}_{i,j},\ q_j(1) - q_i(1)\\ &\quad + \bar{e}_{i,j}(0) - \bar{\delta}_{i,j}\}\\ &= q_j(1) - q_i(1) - \bar{\delta}_{i,j} + \max\{\bar{e}_{i,j}(0),\ \bar{e}_{i,j}(1)\}.\end{aligned}$$

When $k = 2$,

$$\begin{aligned}\check{d}_{i,j}(2) &= \max\{\bar{d}_{i,j}(2) - \bar{\delta}_{i,j},\ \check{d}_{i,j}(1) + u_j(1) - u_i(1)\}\\ &= \max\{q_j(2) - q_i(2) + \bar{e}_{i,j}(2) - \bar{\delta}_{i,j},\ q_j(2) - q_i(2)\\ &\quad + \max\{\bar{e}_{i,j}(0),\ \bar{e}_{i,j}(1)\} - \bar{\delta}_{i,j}\}\\ &= q_j(2) - q_i(2) - \bar{\delta}_{i,j} + \max\{\bar{e}_{i,j}(0),\ \bar{e}_{i,j}(1),\ \bar{e}_{i,j}(2)\}.\end{aligned}$$

Following the similar procedure, one has

$$\begin{aligned}\check{d}_{i,j}(k) =&\ q_j(k) - q_i(k) - \bar{\delta}_{i,j} + \max\{\bar{e}_{i,j}(0),\\ &\bar{e}_{i,j}(1), \ldots,\ \bar{e}_{i,j}(k)\},\ i \in \mathcal{I}_n,\ j \in \mathcal{N}_i.\end{aligned} \tag{9}$$

Similarly, one can show that

$$\begin{aligned}\hat{d}_{i,j}(k) =&\ q_j(k) - q_i(k) + \bar{\delta}_{i,j} + \min\{\bar{e}_{i,j}(0),\\ &\bar{e}_{i,j}(1), \ldots,\ \bar{e}_{i,j}(k)\},\ i \in \mathcal{I}_n,\ j \in \mathcal{N}_i\end{aligned} \tag{10}$$

always holds under the iterative algorithm (7). From (5), (9), and (10), it can be easily verified that $\tilde{d}_{i,j}(k) - (q_j(k) - q_i(k)) = (\max\{\bar{e}_{i,j}(0),\ \bar{e}_{i,j}(1), \ldots,\ \bar{e}_{i,j}(k)\} + \min\{\bar{e}_{i,j}(0),\ \bar{e}_{i,j}(1), \ldots, \bar{e}_{i,j}(k)\})/2$ holds for all $k \geq 0$. Note that $\bar{e}_{i,j}(k)$ has reached its minimum $\check{e}_{i,j}$ and maximum $\hat{e}_{i,j}$ when $k \geq k^*_{i,j}$. As a result, for all $k \geq k^*_{i,j}$ $\min\{\bar{e}_{i,j}(0),\ \bar{e}_{i,j}(1), \ldots,\ \bar{e}_{i,j}(k)\} = \check{e}_{i,j}$and $\max\{\bar{e}_{i,j}(0),\ \bar{e}_{i,j}(1), \ldots,\ \bar{e}_{i,j}(k)\} = \hat{e}_{i,j}$ hold. It follows from Eq. (8) that $\tilde{d}_{i,j}(k) - (q_j(k) - q_i(k)) = (\check{e}_{i,j} + \hat{e}_{i,j})/2$ when $k \geq k^*_{i,j}$. ■

Remark 1. Note that $|\bar{e}_{i,j}(k)| \leq \bar{\delta}_{i,j}, \forall k \geq 0$. It follows from Eqs. (9) and (10) that $\check{d}_{i,j}(k) \leq q_j(k) - q_i(k) \leq \hat{d}_{i,j}(k),\ i \in \mathcal{I}_n,\ j \in \mathcal{N}_i$ holds for all $k \geq 0$. This implies that the true value of $q_j(k) - q_i(k)$ is always located in the estimation interval $[\check{d}_{i,j}(k), \hat{d}_{i,j}(k)]$ whose length is non-increasing with respect to time and the middle value in the estimation interval is used in the proposed algorithm (5) to estimate the true value of $q_j(k) - q_i(k)$. Moreover, after $\bar{e}_{i,j}(k)$ reaches its minimum and maximum the length of the estimation interval is fixed and the estimation error remains to be a constant $(\check{e}_{i,j} + \hat{e}_{i,j})/2$ even if new position measurements are obtained by the sensors.

For the case of coverage control with order preservation, the following coverage control law is proposed

$$u_i(k) = \lambda_i \mathrm{sat}(\sigma(\tilde{u}_i(k))),\ i = 1, \ldots, n, \tag{11}$$

where $\sigma(x) = \max\{x, 0\}$ and

$$\tilde{u}_i(k) = \eta_i[(\lambda_{i-1} + \lambda_i)\check{d}_{i,i+1}(k) + (\lambda_i + \lambda_{i+1})\check{d}_{i,i-1}(k)] \tag{12}$$

with $\check{d}_{i,j}(k)$ iterating according to Eq. (6). The following lemma shows that the spatial order of the sensors is always preserved under the coverage control law (11).

Lemma 2. *Given the initial condition (2), $q_1(k) < \cdots < q_i(k) < q_{i+1}(k) < \cdots < q_n(k) < 2\pi + q_1(k)$ holds for all $k \geq 0$ under the distributed coverage control law (11) with $0 < \eta_i < 1/[\lambda_i(\lambda_{i-1} + \lambda_i)]$.*

Proof. Let $d_i(k) = q_{i+1}(k) - q_i(k)$. To show order preservation of the sensors, it suffices to prove that $d_i(k+1) > 0,\ \forall i \in \mathcal{I}_n$ if $d_i(k) > 0,\ \forall i \in \mathcal{I}_n$. Since $u_i(k) \geq 0$ holds for all $k \geq 0$ under the coverage control law (11), one has $d_i(k+1) = d_i(k) + u_{i+1}(k) - u_i(k) \geq d_i(k) - \lambda_i \mathrm{sat}(\sigma(\tilde{u}_i(k)))$. It follows from Eq. (9) that $\check{d}_{i,j}(k) \leq q_j(k) - q_i(k),\ i \in \mathcal{I}_n,\ j \in \mathcal{N}_i$ holds for all $k \geq 0$ under the estimation algorithm (6). When $d_i(k) > 0,\ \forall i \in \mathcal{I}_n$, one has $\check{d}_{i,i+1}(k) \leq d_i(k)$ and $\check{d}_{i,i-1}(k) \leq -d_{i-1}(k) < 0$. Consequently, $d_i(k+1) \geq d_i(k) - \lambda_i \mathrm{sat}(\sigma(\eta_i(\lambda_{i-1} + \lambda_i)d_i(k)))$. When $0 < \eta_i < 1/[\lambda_i(\lambda_{i-1} + \lambda_i)]$, it can be verified that $d_i(k+1) > 0$ holds for all $i \in \mathcal{I}_n$ if $d_i(k) > 0,\ \forall i \in \mathcal{I}_n$. ■

4. Convergence analysis

In this section, it will be shown that the effect of the measurement errors on the coverage performance can be reduced or even eliminated under the proposed coverage control laws. We first consider the case without order preservation. The following lemma shows the convergence of the proposed coverage control law (3).

Lemma 3. *Let $0 < \eta_i \leq min\{1, \lambda_i/(c(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})), 1/(2\lambda_i max\{\lambda_{i-1}+\lambda_i, \lambda_i+\lambda_{i+1}\})\}$, $\forall i \in \mathcal{I}_n$ with c being a positive constant to be given in the proof. If the sensors' measurement errors satisfy that $\bar{e}_{i,j}(k)$, $j \in \mathcal{N}_i$ reaches its minimum and maximum in finite time steps for all $i \in \mathcal{I}_n$, the control input of each sensor will converge to zero as time goes to infinity under the distributed coverage control law (3).*

Proof. When $k \geq k_i^* = \max\{k_{i,i-1}^*, k_{i,i+1}^*\}$, it follows from Lemma 1 that $\tilde{d}_{i,j}(k) = q_j(k) - q_i(k) + \tilde{w}_{i,j}$, $\forall j \in \mathcal{N}_i$ with $\tilde{w}_{i,j} = (\check{e}_{i,j} + \hat{e}_{i,j})/2$. Consequently, the term $\tilde{u}_i(k)$ in the proposed coverage control law (3) becomes $\tilde{u}_i(k) = \eta_i[(\lambda_{i-1}+\lambda_i)(d_i(k) + \tilde{w}_{i,i+1}) + (\lambda_i+\lambda_{i+1})(-d_{i-1}(k) + \tilde{w}_{i,i-1})]$ for all $k \geq k_i^*$. Since $\bar{e}_{i,j}(k) = (e_{i,j}(k) - e_{j,i}(k))/2$, one has $\bar{e}_{i,j}(k) = -\bar{e}_{j,i}(k)$. As a result, $\check{e}_{i,j} = \min_{k\geq 0}\bar{e}_{i,j}(k) = \min_{k\geq 0} -\bar{e}_{j,i}(k) = -\max_{k\geq 0}\bar{e}_{j,i}(k) = -\hat{e}_{j,i}$. Similarly, one can show that $\hat{e}_{i,j} = -\check{e}_{j,i}$. Therefore, $\tilde{w}_{i,j} = -\tilde{w}_{j,i}$, $j \in \mathcal{N}_i$ holds for all $i \in \mathcal{I}_n$. When $k \geq k_i^*$, the term $\tilde{u}_i(k)$ can be rewritten as

$$\tilde{u}_i(k) = \eta_i[(\lambda_{i-1}+\lambda_i)(d_i(k) + \tilde{w}_{i,i+1}) - (\lambda_i+\lambda_{i+1})(d_{i-1}(k) + \tilde{w}_{i-1,i})]. \tag{13}$$

Let $k^* = \max_{i\in\mathcal{I}_n} k_i^*$. Note that k^* is finite if each $\bar{e}_{i,j}(k)$, $i \in \mathcal{I}_n$, $j \in \mathcal{N}_i$ reaches its minimum and maximum in finite time. Consider the following Lyapunov function candidate

$$V(k) = \sum_{i=1}^{n} \frac{\lambda_i \tilde{u}_i^2(k)}{\eta_i(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})} \tag{14}$$

when $k \geq k^*$. Without loss of generality, each low gain η_i is required to be located in $(0, 1]$. Let $c > 0$ be a constant such that $c \geq \sup_{\eta_i\in(0,1],\ q_i(k^*)\in\mathbb{S}} V(k^*)$. Let $L_V(c) = \{q_i \in \mathbb{S} : V(\mathbf{q}) \leq c\}$, where $\mathbf{q} = [q_1, \ldots, q_n]^T$. For $\mathbf{q} \in L_V(c)$, one has $\lambda_i\tilde{u}_i^2(k)/(\eta_i(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})) \leq c$, $\forall i \in \mathcal{I}_n$. When $0 < \eta_i \leq \lambda_i/(c(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1}))$, $\mathbf{q} \in L_V(c)$ implies that $|\tilde{u}_i(k)| \leq 1$ holds for all $i \in \mathcal{I}_n$. Therefore, given $0 < \eta_i \leq \min\{1, \lambda_i/(c(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1}))\}$, when $k \geq k^*$ the proposed coverage control laws for mobile sensors with $\mathbf{q} \in L_V(c)$ can be rewritten as

$$\begin{aligned} u_i &= \lambda_i\tilde{u}_i(k) \\ &= \lambda_i\eta_i[(\lambda_{i-1}+\lambda_i)(d_i(k) + \tilde{w}_{i,i+1}) - (\lambda_i+\lambda_{i+1}) \\ &\quad (d_{i-1}(k) + \tilde{w}_{i-1,i})],\ i \in \mathcal{I}_n. \end{aligned} \tag{15}$$

Next, we will show that $V(k)$ is non-increasing under the coverage control law (15) after time step k^*. Let $\Delta V(k) = V(k+1) - V(k)$. From the definition of $V(k)$, one has

$$\Delta V(k) = \sum_{i=1}^{n} \frac{2\lambda_i\tilde{u}_i(k)(\tilde{u}_i(k+1) - \tilde{u}_i(k))}{\eta_i(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})} + \sum_{i=1}^{n} \frac{\lambda_i(\tilde{u}_i(k+1) - \tilde{u}_i(k))^2}{\eta_i(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})}. \tag{16}$$

The first term on the right-hand side of Eq. (16) can be rewritten as

$$\begin{aligned} V_1(k) &= \sum_{i=1}^{n} \frac{2\lambda_i\tilde{u}_i(k)}{(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})}\big[(\lambda_{i-1}+\lambda_i)(d_i(k+1) \\ &\quad - d_i(k)) - (\lambda_i+\lambda_{i+1})(d_{i-1}(k+1) - d_{i-1}(k))\big] \\ &= \sum_{i=1}^{n} \frac{2\lambda_i\tilde{u}_i(k)}{\lambda_i+\lambda_{i+1}}(d_i(k+1) - d_i(k)) - \\ &\quad \sum_{i=0}^{n-1} \frac{2\lambda_{i+1}\tilde{u}_{i+1}(k)}{\lambda_i+\lambda_{i+1}}(d_i(k+1) - d_i(k)). \end{aligned}$$

Since $q_{n+1}(k) = q_1(k) + 2\pi$ and $\lambda_{n+1} = \lambda_1$, it follows from Eqs. (1) and (15) that $\tilde{u}_{n+1}(k) = \tilde{u}_1(k)$, $k \geq k^*$. Recall that $q_n(k) = q_0(k) + 2\pi$ and $q_{n+1}(k) = q_1(k) + 2\pi$. One has $d_n(k) = d_0(k) = q_1(k) + 2\pi - q_n(k)$, $k \geq 0$. Note also that $\lambda_n = \lambda_0$ and $\lambda_{n+1} = \lambda_1$. For $k \geq k^*$, one has

$$\begin{aligned} V_1(k) &= \sum_{i=1}^{n} \frac{2}{\lambda_i+\lambda_{i+1}}(\lambda_i\tilde{u}_i(k) - \lambda_{i+1}\tilde{u}_{i+1}(k)) \\ &\quad (d_i(k+1) - d_i(k)) \\ &= -\sum_{i=1}^{n} \frac{2}{\lambda_i+\lambda_{i+1}}(u_{i+1}(k) - u_i(k))^2. \end{aligned} \tag{17}$$

Similarly, for $k \geq k^*$ the second term on the right-hand side of (16) satisfies

$$\begin{aligned} V_2(k) &= \sum_{i=1}^{n} \frac{\eta_i\lambda_i}{(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})}\big[(\lambda_{i-1}+\lambda_i)(d_i(k+1) \\ &\quad - d_i(k)) - (\lambda_i+\lambda_{i+1})(d_{i-1}(k+1) - d_{i-1}(k))\big]^2 \\ &\leq \sum_{i=1}^{n} \frac{2\eta_i\lambda_i(\lambda_{i-1}+\lambda_i)}{\lambda_i+\lambda_{i+1}}(d_i(k+1) - d_i(k))^2 + \\ &\quad \sum_{i=1}^{n} \frac{2\eta_i\lambda_i(\lambda_i+\lambda_{i+1})}{\lambda_{i-1}+\lambda_i}(d_{i-1}(k+1) - d_{i-1}(k))^2 \\ &= \sum_{i=1}^{n} \frac{2}{\lambda_i+\lambda_{i+1}}[\eta_i\lambda_i(\lambda_{i-1}+\lambda_i) + \eta_{i+1}\lambda_{i+1} \\ &\quad (\lambda_{i+1}+\lambda_{i+2})](u_{i+1}(k) - u_i(k))^2. \end{aligned} \tag{18}$$

It follows from (17) and (18) that

$$\begin{aligned} \Delta V(k) &\leq -\sum_{i=1}^{n} \frac{2}{\lambda_i+\lambda_{i+1}}[1 - \eta_i\lambda_i(\lambda_{i-1}+\lambda_i) - \eta_{i+1} \\ &\quad \lambda_{i+1}(\lambda_{i+1}+\lambda_{i+2})](u_{i+1}(k) - u_i(k))^2. \end{aligned}$$

Therefore, when $0 < \eta_i \leq \min\{1, \lambda_i/(c(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})), 1/(2\lambda_i\max\{\lambda_{i-1}+\lambda_i, \lambda_i+\lambda_{i+1}\})\}$, $\Delta V(k) \leq 0$, $\forall k \geq k^*$ with equality holding if and only if $u_i(k)$ is identical for all $i \in \mathcal{I}_n$. Since $V(k) \geq 0$ always holds, $V(k)$ will converge to a constant under the coverage control law (3). This implies that $\lim_{k\to\infty}\Delta V(k) = 0$ and $u_i(k)$ will reach a consensus for all sensors as time goes to infinity.

From Eq. (15), one has $\sum_{i=1}^{n} u_i(k)/[\lambda_i\eta_i(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})] = \sum_{i=1}^{n}(d_i(k)+\tilde{w}_{i,i+1})/(\lambda_i+\lambda_{i+1}) - \sum_{i=1}^{n}(d_{i-1}(k)+\tilde{w}_{i-1,i})/(\lambda_{i-1}+\lambda_i)$. Recall that $d_n(k) = d_0(k) = q_1(k) + 2\pi - q_n(k)$. Note also that $\tilde{w}_{n,n+1}(k) = \tilde{w}_{0,1}(k)$, $\lambda_n = \lambda_0$, and $\lambda_{n+1} = \lambda_1$. As a result, $(d_n(k) + \tilde{w}_{n,n+1})/(\lambda_n+\lambda_{n+1}) = (d_0(k) + \tilde{w}_{0,1})/(\lambda_0+\lambda_1)$, $\forall k \geq k^*$. Therefore, $\sum_{i=1}^{n} u_i(k)/[\lambda_i\eta_i(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})] = 0$, $\forall k \geq k^*$. It has been shown that $u_i(k)$, $i \in \mathcal{I}_n$ will reach a consensus as time goes to infinity. These two facts lead to the conclusion that $\lim_{k\to\infty} u_i(k) = 0$, $\forall i \in \mathcal{I}_n$. ■

Before we proceed, a necessary and sufficient condition for the minimization of the coverage cost function is given.

Lemma 4 (Song et al., 2016)**.** *The coverage cost function $T(q_1, \ldots, q_n)$ reaches its minimum $T^* = \pi/\sum_{i=1}^{n}\lambda_i$ if and only if*

$$\frac{\bar{d}(q_i, q_{i+1})}{\lambda_i+\lambda_{i+1}} = \frac{\bar{d}(q_j, q_{j+1})}{\lambda_j+\lambda_{j+1}},\ \forall i, j \in \mathcal{I}_n. \tag{19}$$

Moreover, $T(q_1, \ldots, q_n) \leq max_{i\in\mathcal{I}_n}\bar{d}(q_i, q_{i+1})/(\lambda_i+\lambda_{i+1})$.

Theorem 1. *Assume that $\sum_{i=1}^{n}\delta_i < 2\pi\, min_{i\in\mathcal{I}_n}(\lambda_i+\lambda_{i+1})/(2\lambda_M + \sum_{i=1}^{n}\lambda_i)$ with $\lambda_M = max_{i\in\mathcal{I}_n}\lambda_i$. If the sensors' measurement errors satisfy that $\bar{e}_{i,j}(k)$, $j \in \mathcal{N}_i$ reaches its minimum and maximum in finite time steps for all $i \in \mathcal{I}_n$, the heterogeneous mobile sensor* *network subject to bounded measurement errors will be driven to a static configuration such that*

$$0 \le T(q_1,\ldots,q_n) - T^* \le \frac{\sum_{i=1}^n \delta_i}{2\sum_{i=1}^n \lambda_i} + \max_{i\in\mathcal{I}_n} \frac{\delta_i+\delta_{i+1}}{2(\lambda_i+\lambda_{i+1})}$$

under the proposed coverage control law (3) *with* $0 < \eta_i \le \min\{1, \lambda_i/(c(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})), 1/(2\lambda_i \max\{\lambda_{i-1}+\lambda_i, \lambda_i+\lambda_{i+1}\})\}$.

Proof. It follows from Lemma 3 that $\lim_{k\to\infty} u_i(k)/[\lambda_i\eta_i(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})] = 0,\ \forall i \in \mathcal{I}_n$ if each $\bar{e}_{i,j}(k),\ i\in\mathcal{I}_n,\ j\in\mathcal{N}_i$ reaches its minimum and maximum in finite time steps. From Eq. (15), one has that $(d_i(k)+\tilde{w}_{i,i+1})/(\lambda_i+\lambda_{i+1})$ reach a consensus for all sensors as time goes to infinity. Let $h = (\lim_{k\to\infty} d_i(k)+\tilde{w}_{i,i+1})/(\lambda_i+\lambda_{i+1})$. Since $\sum_{i=1}^n d_i(k) = 2\pi$ always holds, $h\sum_{i=1}^n \lambda_i+\lambda_{i+1} = 2\pi + \sum_{i=1}^n \tilde{w}_{i,i+1}$. Therefore, $h = (2\pi+\sum_{i=1}^n \tilde{w}_{i,i+1})/2\sum_{i=1}^n \lambda_i$ and

$$\lim_{k\to\infty} d_i(k) = \frac{\lambda_i+\lambda_{i+1}}{2\sum_{i=1}^n \lambda_i}(2\pi + \sum_{i=1}^n \tilde{w}_{i,i+1}) - \tilde{w}_{i,i+1}. \tag{20}$$

Recall that $|\tilde{w}_{i,i+1}| = |\check{e}_{i,i+1}+\hat{e}_{i,i+1}|/2 \le (\delta_i+\delta_{i+1})/2,\ \forall i\in\mathcal{I}_n$. One has

$$\begin{aligned}
&\lim_{k\to\infty} d_i(k)\\
&\ge \frac{\pi(\lambda_i+\lambda_{i+1})}{\sum_{i=1}^n \lambda_i} - \frac{\delta_i+\delta_{i+1}}{2} - \frac{\lambda_i+\lambda_{i+1}}{2\sum_{i=1}^n \lambda_i}\sum_{i=1}^n \frac{\delta_i+\delta_{i+1}}{2}\\
&\ge \frac{\pi(\lambda_i+\lambda_{i+1})}{\sum_{i=1}^n \lambda_i} - \frac{1}{2}\sum_{i=1}^n \delta_i - \frac{\lambda_i+\lambda_{i+1}}{2\sum_{i=1}^n \lambda_i}\sum_{i=1}^n \delta_i\\
&\ge \frac{\pi(\lambda_i+\lambda_{i+1})}{\sum_{i=1}^n \lambda_i} - \frac{2\lambda_M+\sum_{i=1}^n \lambda_i}{2\sum_{i=1}^n \lambda_i}\sum_{i=1}^n \delta_i.
\end{aligned}$$

When $\sum_{i=1}^n \delta_i < 2\pi \min_{i\in\mathcal{I}_n}(\lambda_i+\lambda_{i+1})/(2\lambda_M+\sum_{i=1}^n \lambda_i)$, $\lim_{k\to\infty} d_i(k) > 0$ holds for all $i\in\mathcal{I}_n$. As a result, $\lim_{k\to\infty} d(q_i(k), q_{i+1}(k)) = \lim_{k\to\infty} d_i(k),\ \forall i\in\mathcal{I}_n$. It follows from Lemma 4 that

$$\lim_{k\to\infty} T(k) \le \max_{i\in\mathcal{I}_n} \frac{\lim_{k\to\infty} d_i(k)}{\lambda_i+\lambda_{i+1}}. \tag{21}$$

Recall that $T^* = \pi/\sum_{i=1}^n \lambda_i$. From (20) and (21), one has

$$\begin{aligned}
\lim_{k\to\infty} T(k) - T^* &\le \frac{\sum_{i=1}^n \tilde{w}_{i,i+1}}{2\sum_{i=1}^n \lambda_i} + \max_{i\in\mathcal{I}_n}\frac{-\tilde{w}_{i,i+1}}{\lambda_i+\lambda_{i+1}}\\
&\le \frac{\sum_{i=1}^n \delta_i}{2\sum_{i=1}^n \lambda_i} + \max_{i\in\mathcal{I}_n}\frac{\delta_i+\delta_{i+1}}{2(\lambda_i+\lambda_{i+1})}.
\end{aligned} \tag{22}$$

This completes the proof. ■

Remark 2. Note that $\bar{e}_{i,j}(k)$ can reach its minimum $\check{e}_{i,j}$ and/or maximum $\hat{e}_{i,j}$ in finite time or in infinite time in general. To show the convergence of the coverage control law (3), it is required that each $\bar{e}_{i,j}(k)$ reaches its minimum and maximum in finite time. When this condition is not satisfied, the control law (3) may not converge to zero. However, the effect of the measurement errors can be still reduced under the proposed control law. In fact, when a more extreme measurement error arises, the estimation interval $[\check{d}_{i,j}(k), \hat{d}_{i,j}(k)]$ further reduces, and the control input of the control law (3) is capable of responding and exploiting this reduction. Let Ω be $[\tilde{w}_{i,j}, \tilde{w}_{i,j}+\epsilon]$ when $\tilde{w}_{i,j} = -\bar{\delta}_{i,j}$, $[\tilde{w}_{i,j}-\epsilon, \tilde{w}_{i,j}]$ when $\tilde{w}_{i,j} = \bar{\delta}_{i,j}$, and $[\tilde{w}_{i,j}-\epsilon, \tilde{w}_{i,j}+\epsilon]$ otherwise, where ϵ is an arbitrarily small positive number. It can be concluded that the estimation error $\tilde{d}_{i,j}(k) - (q_j(k)-q_i(k))$ converges to the set Ω after a finite time step no matter whether $\bar{e}_{i,j}(k)$ reaches its minimum and maximum in finite or infinite time. If the estimation algorithm (5) is not employed, $\bar{d}_{i,j}(k)$ instead of $\tilde{d}_{i,j}(k)$ is used in the coverage control law (3), and in this case, $\bar{d}_{i,j}(k) - (q_j(k)-q_i(k))$ is equal to $\bar{e}_{i,j}(k)$ which varies in the set $[-\bar{\delta}_{i,j}, \bar{\delta}_{i,j}]$ throughout the coverage task. Note that Ω is a subset of $[-\bar{\delta}_{i,j}, \bar{\delta}_{i,j}]$, and one thus concludes that even when $\bar{e}_{i,j}(k)$ reaches its minimum and/or maximum as time goes to infinity the effect of the measurement errors on the coverage performance can still be reduced under the proposed coverage control law (3).

Next, we consider the case when the sensors' order is required to be preserved throughout the coverage task. The following lemma shows the convergence of the proposed coverage control law (11) with order preservation.

Lemma 5. *Let* $0 < \eta_i \le \min\{1/[2\pi(\lambda_{i-1}+\lambda_i)], 1/(2\lambda_i \max\{\lambda_{i-1}+\lambda_i, \lambda_i+\lambda_{i+1}\})\},\ \forall i\in\mathcal{I}_n$. *If the sensors' measurement errors satisfy that* $\bar{e}_{i,j}(k),\ j\in\mathcal{N}_i$ *reaches its maximum in finite time step for all* $i\in\mathcal{I}_n$, *the control input of each sensor will converge to zero as time goes to infinity under the distributed coverage control law* (11).

Proof. Note that $k_{i,j}^h$ is the first time step that $\bar{e}_{i,j}(k)$ reaches its maximum during the coverage task. When $k \ge k_i^h = \max\{k_{i,i-1}^h, k_{i,i+1}^h\}$, it follows from Eq. (9) that $\check{d}_{i,j}(k) = q_j(k)-q_i(k)+\hat{w}_{i,j},\ \forall j\in\mathcal{N}_i$ with $\hat{w}_{i,j} = \hat{e}_{i,j} - \bar{\delta}_{i,j}$. Consequently, the term $\tilde{u}_i(k)$ in the proposed coverage control law (11) becomes

$$\begin{aligned}
\tilde{u}_i(k) = {}& \eta_i[(\lambda_{i-1}+\lambda_i)(d_i(k)+\hat{w}_{i,i+1}) + (\lambda_i+\lambda_{i+1})\\
&(-d_{i-1}(k)+\hat{w}_{i,i-1})]
\end{aligned} \tag{23}$$

when $k \ge k_i^h$. Under the proposed coverage control law (11) with $0 < \eta_i < 1/[\lambda_i(\lambda_{i-1}+\lambda_i)]$, the mobile sensors' spatial order is preserved throughout the coverage task as shown in Lemma 2, which implies that $0 < d_i(k) < 2\pi$ always holds for all $i\in\mathcal{I}_n$. Note also that $-2\bar{\delta}_{i,j} \le \hat{w}_{i,j} \le 0$. Therefore, one has $0 \le \sigma(\tilde{u}_i(k)) < 2\pi\eta_i(\lambda_{i-1}+\lambda_i),\ \forall i\in\mathcal{I}_n$. If the low control gain η_i is chosen as $0 < \eta_i < 1/[\max\{\lambda_i, 2\pi\}(\lambda_{i-1}+\lambda_i)]$, when $k \ge k_i^h$ the proposed coverage control law (11) can be rewritten as

$$\begin{aligned}
u_i &= \lambda_i\sigma(\tilde{u}_i(k))\\
&= \lambda_i\sigma(\eta_i[(\lambda_{i-1}+\lambda_i)(d_i(k)+\hat{w}_{i,i+1}) + (\lambda_i+\lambda_{i+1})\\
&\quad(-d_{i-1}(k)+\hat{w}_{i,i-1})]).
\end{aligned} \tag{24}$$

Let $k^h = \max_{i\in\mathcal{I}_n} k_i^h$. Consider the Lyapunov function candidate (14) when $k \ge k^h$. Denote $\lambda_i^\eta = \eta_i\lambda_i(\lambda_{i-1}+\lambda_i) + \eta_{i+1}\lambda_{i+1}(\lambda_{i+1}+\lambda_{i+2})$. Note that $0 < \lambda_i^\eta \le 1$ when $0 < \eta_i \le 1/(2\lambda_i\max\{\lambda_{i-1}+\lambda_i, \lambda_i+\lambda_{i+1}\})$. Following the similar procedure for the proof of Lemma 3, one has

$$\begin{aligned}
\Delta V(k) \le{}& \sum_{i=1}^n \frac{2}{\lambda_i+\lambda_{i+1}}[(\lambda_i\tilde{u}_i(k) - \lambda_{i+1}\tilde{u}_{i+1}(k))(u_{i+1}(k)\\
&- u_i(k)) + \lambda_i^\eta(u_{i+1}(k)-u_i(k))^2]\\
\le{}& -\sum_{i=1}^n \frac{2}{\lambda_i+\lambda_{i+1}}[(\lambda_{i+1}\tilde{u}_{i+1}(k) - \lambda_i\tilde{u}_i(k))\\
&(\lambda_{i+1}\sigma(\tilde{u}_{i+1}(k)) - \lambda_i\sigma(\tilde{u}_i(k))) -\\
&(\lambda_{i+1}\sigma(\tilde{u}_{i+1}(k)) - \lambda_i\sigma(\tilde{u}_i(k)))^2]
\end{aligned}$$

when $0 < \eta_i \le \min\{1/[2\pi(\lambda_{i-1}+\lambda_i)], 1/(2\lambda_i\max\{\lambda_{i-1}+\lambda_i, \lambda_i+\lambda_{i+1}\})\}$. Recall that $\sigma(x) = \max\{x, 0\}$. It can be easily verified that $\Delta V(k) \le 0,\ \forall k \ge k^h$ with equality holding if and only if $\lambda_i\sigma(\tilde{u}_i(k))$ is identical for all $i\in\mathcal{I}_n$. Therefore, $\lim_{k\to\infty}\Delta V(k) = 0$ which implies that $u_i(k) = \lambda_i\sigma(\tilde{u}_i(k)),\ i\in\mathcal{I}_n$ reach a consensus for all sensors as time goes to infinity.

Finally, we show that the control input of each sensor converges to zero by contradiction. Assume that the control inputs of the sensors converge to a positive constant, that is, $\lim_{k\to\infty} u_i(k) = u^* > 0,\ \forall i\in\mathcal{I}_n$. It follows from (24) that

$\lim_{k\to\infty}\tilde{u}_i(k) = u^*/\lambda_i > 0,\ \forall i \in \mathcal{I}_n$. Note that $\hat{w}_{i,j} = \hat{e}_{i,j} - \bar{\delta}_{i,j} \le 0$. From Eq. (23), one has

$$\begin{aligned}
&\sum_{i=1}^{n}\frac{\lim_{k\to\infty}\tilde{u}_i(k)}{\eta_i(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})}\\
&= \sum_{i=1}^{n}\frac{d_i(k)+\hat{w}_{i,i+1}}{\lambda_i+\lambda_{i+1}} + \sum_{i=1}^{n}\frac{-d_{i-1}(k)+\hat{w}_{i,i-1}}{\lambda_{i-1}+\lambda_i}\\
&= \sum_{i=1}^{n}\frac{\hat{w}_{i,i+1}}{\lambda_i+\lambda_{i+1}} + \sum_{i=1}^{n}\frac{\hat{w}_{i,i-1}}{\lambda_{i-1}+\lambda_i}\\
&\le 0.
\end{aligned} \tag{25}$$

Note that the above inequality contradicts the fact that $\lim_{k\to\infty}\tilde{u}_i(k) = u^*/\lambda_i > 0,\ \forall i \in \mathcal{I}_n$. Therefore, all sensors' control input will converge to zero under the coverage control law (11) with $0 < \eta_i \le \min\{1/[2\pi(\lambda_{i-1}+\lambda_i)],\ 1/(2\lambda_i\max\{\lambda_{i-1}+\lambda_i,\ \lambda_i+\lambda_{i+1}\})\}$. ■

Theorem 2. *If the sensors' measurement errors satisfy that $\bar{e}_{i,j}(k),\ j \in \mathcal{N}_i$ reaches its maximum in finite time step for all $i \in \mathcal{I}_n$, the heterogeneous mobile sensor network subject to bounded measurement errors will be driven to a static configuration with order preservation such that*

$$0 \le T(q_1,\ldots,q_n) - T^* \le \frac{n(n-1)\lambda_M}{\sum_{i=1}^{n}\lambda_i}\max_{i\in\mathcal{I}_n}\frac{\delta_i+\delta_{i+1}}{\lambda_i+\lambda_{i+1}}$$

under the proposed coverage control law (11) with $0 < \eta_i \le \min\{1/[2\pi(\lambda_{i-1}+\lambda_i)],\ 1/(2\lambda_i\max\{\lambda_{i-1}+\lambda_i,\ \lambda_i+\lambda_{i+1}\})\}$.

Proof. Note that the mobile sensors' spatial order is always preserved under the proposed coverage control law (11) with $0 < \eta_i \le \min\{1/[2\pi(\lambda_{i-1}+\lambda_i)],\ 1/(2\lambda_i\max\{\lambda_{i-1}+\lambda_i,\ \lambda_i+\lambda_{i+1}\})\}$ as shown in Lemma 2. Recall that $\sigma(\tilde{u}_i(k)) = \max\{\tilde{u}_i(k), 0\}$. It follows from Lemma 5 that $\lim_{k\to\infty}\tilde{u}_i(k)/[\eta_i(\lambda_{i-1}+\lambda_i)(\lambda_i+\lambda_{i+1})] \le 0,\ \forall i \in \mathcal{I}_n$ if each $\bar{e}_{i,j}(k),\ i \in \mathcal{I}_n,\ j \in \mathcal{N}_i$ reaches its maximum in finite time step. It then follows from (23) that

$$\frac{\lim_{k\to\infty}d_i(k)+\hat{w}_{i,i+1}}{\lambda_i+\lambda_{i+1}} - \frac{\lim_{k\to\infty}d_{i-1}(k)-\hat{w}_{i,i-1}}{\lambda_{i-1}+\lambda_i} \le 0 \tag{26}$$

holds for all $i \in \mathcal{I}_n$. Denote $T_i(k) = d_i(k)/(\lambda_i+\lambda_{i+1})$. It can be verified that for all $i \in \mathcal{I}_n$

$$\begin{aligned}
\lim_{k\to\infty} T_i(k) - T_{i-1}(k) &\le -\frac{\hat{w}_{i,i+1}}{\lambda_i+\lambda_{i+1}} - \frac{\hat{w}_{i,i-1}}{\lambda_{i-1}+\lambda_i}\\
&= \frac{\bar{\delta}_{i,i+1}-\hat{e}_{i,i+1}}{\lambda_i+\lambda_{i+1}} + \frac{\bar{\delta}_{i,i-1}-\hat{e}_{i,i-1}}{\lambda_{i-1}+\lambda_i}\\
&\le 2\max_{i\in\mathcal{I}_n}\frac{\delta_i+\delta_{i+1}}{\lambda_i+\lambda_{i+1}}.
\end{aligned} \tag{27}$$

Let $T_M = \max_{i\in\mathcal{I}_n}\lim_{k\to\infty}T_i(k)$ and denote $j = \arg\max_{i\in\mathcal{I}_n}\lim_{k\to\infty}T_i(k)$. It follows from the inequality (27) that $\lim_{k\to\infty}T_{j-1}(k) \ge T_M - 2\max_{i\in\mathcal{I}_n}(\delta_i+\delta_{i+1})/(\lambda_i+\lambda_{i+1})$. As a result, $\lim_{k\to\infty}T_{j-2}(k) \ge \lim_{k\to\infty}T_{j-1}(k) - 2\max_{i\in\mathcal{I}_n}(\delta_i+\delta_{i+1})/(\lambda_i+\lambda_{i+1}) \ge T_M - 4\max_{i\in\mathcal{I}_n}(\delta_i+\delta_{i+1})/(\lambda_i+\lambda_{i+1})$. Following the similar procedure, one has $\lim_{k\to\infty}T_{j+1}(k) \ge T_M - 2(n-1)\max_{i\in\mathcal{I}_n}(\delta_i+\delta_{i+1})/(\lambda_i+\lambda_{i+1})$. Recall that $\sum_{i=1}^{n}d_i(k) = 2\pi$ holds for all $k \ge 0$. Therefore, $\sum_{i=1}^{n}(\lambda_i+\lambda_{i+1})T_M - 2n(n-1)\lambda_M\max_{i\in\mathcal{I}_n}(\delta_i+\delta_{i+1})/(\lambda_i+\lambda_{i+1}) \le \sum_{i=1}^{n}(\lambda_i+\lambda_{i+1})\lim_{k\to\infty}T_i(k) = 2\pi$. Since $\sum_{i=1}^{n}\lambda_i+\lambda_{i+1} = 2\sum_{i=1}^{n}\lambda_i$, $T_M \le [\pi + n(n-1)\lambda_M\max_{i\in\mathcal{I}_n}(\delta_i+\delta_{i+1})/(\lambda_i+\lambda_{i+1})]/\sum_{i=1}^{n}\lambda_i$.

Due to order preservation of the mobile sensors, $d_i(k) \ge 0,\ i \in \mathcal{I}_n$ holds for all $k \ge 0$. As a result, $\lim_{k\to\infty}\bar{d}(q_i(k), q_{i+1}(k)) = \lim_{k\to\infty}d_i(k),\ \ \forall i \in \mathcal{I}_n$. It follows from Lemma 4 that $\lim_{k\to\infty}T(q_1(k),\ldots,q_n(k)) \le T_M$. Recall that $T^* = \pi/\sum_{i=1}^{n}\lambda_i$ is the minimum of the coverage cost function. Therefore, one has

$$\lim_{k\to\infty}T(k) - T^* \le \frac{n(n-1)\lambda_M}{\sum_{i=1}^{n}\lambda_i}\max_{i\in\mathcal{I}_n}\frac{\delta_i+\delta_{i+1}}{\lambda_i+\lambda_{i+1}}.$$

This completes the proof. ■

Remark 3. When $\tilde{w}_{i,i+1} = 0$ holds for all $i \in \mathcal{I}_n$, it follows from the inequality (22) that $\lim_{k\to\infty}T(k) = T^*$. Similarly, when $\hat{w}_{i,i+1} = 0$ and $\hat{w}_{i,i-1} = 0$ hold for all $i \in \mathcal{I}_n$, it follows from the inequality (26) that $\lim_{k\to\infty}d_i(k)/(\lambda_i+\lambda_{i+1}),\ i \in \mathcal{I}_n$ will reach a consensus as time goes to infinity. As a result, $\lim_{k\to\infty}T(k) = T^*$ as stated by Lemma 4. These two facts indicate that in certain situations the sensor network can be driven to the optimal configuration minimizing the coverage cost function under the proposed coverage control laws (3) and (11) even if only the upper bounds on the measurement errors are known by the sensors.

Remark 4. Note that mobile sensors may overlap or pass by with each other in the case of coverage control without order preservation, which could result in collision between the sensors. Consequently, when collision avoidance needs to be taken into consideration, the proposed coverage control law (11) with order preservation can be used. Since $\sum_{i=1}^{n}\delta_i/\sum_{i=1}^{n}\lambda_i \le \max_{i\in\mathcal{I}_n}(\delta_i+\delta_{i+1})/(\lambda_i+\lambda_{i+1})$, the upper bound on $T - T^*$ in Theorem 2 is larger than that given in Theorem 1. The inherent reason is that due to introduction of the nonlinear function $\sigma(\cdot)$ to guarantee order preservation of the sensors, one can only obtain $\lim_{k\to\infty}\tilde{u}_i(k) \le 0$ instead of $\lim_{k\to\infty}\tilde{u}_i(k) = 0$ in the case of Theorem 2. Thus, it can be regarded as a trade off between the coverage performance and order preservation of the sensors in this paper.

5. Simulations

In this section, a numerical example is given to illustrate the performance of the proposed coverage control laws. Firstly, consider the case of a sensor network with bounded measurement errors under the coverage control law (3). The sensors are randomly deployed on a circle initially. The maximum velocities of the sensors are arbitrarily given by [2.4, 2.8, 3.0, 4.2, 3.6]. Note that the constraint on the upper bounds of the sensors' measurement errors in Theorem 1 is to guarantee that the final order of the sensors is consistent with their initial order. Therefore, this constraint can be removed if the sensors' initial order and final order are identical during a coverage task. Motivated by this observation, the upper bounds on the sensors' measurement errors are set as [1.2, 4.0, 1.6, 3.0, 1.8]. The low gains in control law (3) are chosen as [0.005, 0.006, 0.002, 0.004, 0.003]. Fig. 1 shows time evolution of the sensors' positions $q_i(k)$, control inputs $u_i(k)$, and the function $T(k) - T^*$ when the sensors' measurement errors are given by $e_{i,j}(k) = (1 + 3\sin(\omega_i k + 10(j-i)\phi_i))\delta_i/4$, where ω_i and ϕ_i are selected as [1.5, 12, 8, 0.5, 21] and $[\pi/6, \pi/3, \pi/2, \pi/4, \pi/5]$, respectively. It can be seen that all sensors reach static finally and the coverage cost function T is driven to a neighborhood of its minimum T^*.

Next, consider the case when the coverage control law (11) is used. Each sensor's measurement error is the same as the first case except its upper bound being [0.32, 0.24, 0.45, 0.18, 0.36]. The low gains of the sensors are chosen as [0.010, 0.007, 0.008, 0.006, 0.009]. It can be seen that the sensors' order is preserved throughout the coverage task and the coverage cost function T is driven to a neighborhood of its minimum as shown in Fig. 2. Note that in the final configuration the function $T(k) - T^*$ in Fig. 2 is larger than that in Fig. 1 although the measurement errors in this case are much less than that in the first case. This implies that the proposed coverage control law (3) can reduce the effect of bounded measurement errors on the coverage performance more significantly than the coverage control law (11).

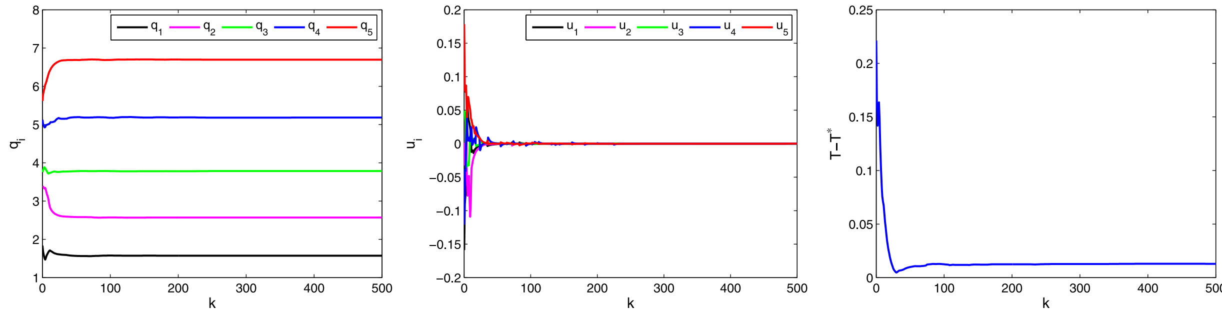

Fig. 1. Time evolution of the sensors' positions $q_i(k)$, control inputs $u_i(k)$, and the function $T(k) - T^*$ under the coverage control law (3).

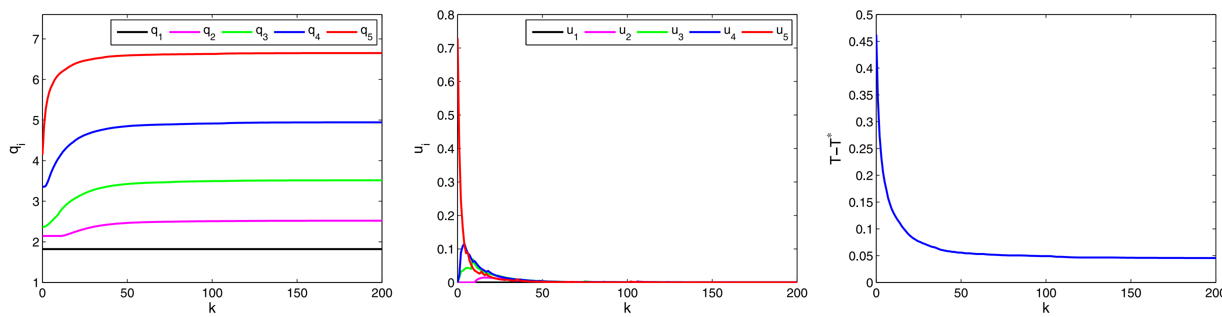

Fig. 2. Time evolution of the sensors' positions $q_i(k)$, control inputs $u_i(k)$, and the function $T(k) - T^*$ under the coverage control law (11).

6. Conclusion

In this paper the coverage control problem for a network of mobile sensors with bounded position measurement errors is addressed. By only using the upper bounds of the sensors' measurement errors, distributed coverage control laws are developed to reduce or even eliminate the effect of the measurement errors on the coverage performance. In this paper, fixed interaction topology is considered for the mobile sensor network. Our ongoing work is focused on extending the current work to the case of mobile sensor networks with time-varying interaction topology.

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