③ a) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Para o passo base 1

$\sum_{i=1}^{1} i = \frac{1(1+1)}{2} \Rightarrow 1 = 1$ ✓

supondo valido para K

$\sum_{i=1}^{K} i = \frac{K(K+1)}{2} \Rightarrow 1 + 2 + \dots + K = \frac{K(K+1)}{2}$

Provando para K+1

$\sum_{i=1}^{K+1} i = \frac{(K+1)(K+2)}{2}$

$1 + 2 + \dots + K + K+1 = \frac{(K+1)(K+2)}{2}$

$= \frac{K(K+1)}{2} + K+1 =$

$= \frac{K^2 + K + 2K + 2}{2}$

$= \frac{K^2 + 3K + 2}{2}$

$= \frac{(K+1)(K+2)}{2}$

$\Rightarrow (K+1)(K+2)$
$= K^2 + 2K + K + 2$
$= K^2 + 3K + 2$

b) $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Para o passo base ∅

$\sum_{i=0}^{0} i^2 = \frac{n(n+1)(2n+1)}{6} \Rightarrow 0^2 = \frac{0(0+1)(2.0+1)}{6} \Rightarrow 0 = 0$ ✓

supondo Valido para K

$\sum_{i=0}^{K} i^2 = \frac{K(K+1)(2K+1)}{6} \Rightarrow 0 + 1 + 4 + \dots + K^2 = \frac{K(K+1)(2K+1)}{6}$

Provando para K+1

$\sum_{i=0}^{K+1} i^2 = \frac{(K+1)(K+2)(2(K+1)+1)}{6}$

$0 + 1 + 2 + 4 + \dots + K^2 + (K+1)^2 = \frac{(K+1)(K+2)(2(K+1)+1)}{6}$

$= \frac{K(K+1)(2K+1)}{6} + (K+1)^2$

$= \frac{(K^2+K)(2K+1)}{6} + (K+1)(K+1)$

$= \frac{2K^3 + K^2 + 2K^2 + K}{6} + K^2 + 2K + 1$

$= \frac{2K^3 + 3K^2 + K + 6K^2 + 12K + 6}{6}$

$= \frac{2K^3 + 9K^2 + 13K + 6}{6}$

$= \frac{(K+1)(K+2)(2(K+1)+1)}{6}$

$\Rightarrow (K^2 + 2K + K + 2)(2(K+1)+1)$
$= (K^2 + 3K + 2)(2K + 3)$
$= 2K^3 + 3K^2 + 6K^2 + 9K + 4K + 6$
$= 2K^3 + 9K^2 + 13K + 6$

③ c) $\sum_{i=1}^{n} (2i-1) = n^2$

Para o caso base (1)

$\sum_{i=1}^{1} (2.1-1) = 1^2 \Rightarrow 1=1$ ✓

supondo valido para K

$\sum_{i=1}^{K} (2.i-1) = K^2 \Rightarrow 1+3+5+\ldots+(2.K-1) = K^2$

Provando para K+1

$\sum_{i=1}^{K+1} (2.i-1) = (K+1)^2$

$1+3+5+\ldots+(2.K-1)+(2(K+1)-1) = (K+1)^2 \Rightarrow (K+1)(K+1)$

$= K^2+2K+1$

$= K^2+2(K+1)-1$

$= K^2+2K+2-1$

$= K^2+2K+1$

d) $\sum_{i=0}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ $n \geq 1$

Para o caso base (0)

$\sum_{i=0}^{0} 0^3 = \frac{0^2(0+1)^2}{4} \Rightarrow 0=0$

Para o caso base (1)

$\sum_{i=0}^{1} 1^3 = \frac{1^2(1+1)^2}{4} \Rightarrow 1 = \frac{2^2}{4} \Rightarrow 1=1$ ✓

Supondo valido para K

$\sum_{i=0}^{K} i^3 = \frac{K^2(K+1)^2}{4} \Rightarrow 0+1+8+\ldots+K^3 = \frac{K^2(K+1)^2}{4}$

Provando para K+1

$\sum_{i=0}^{K+1} i^3 = \frac{(K+1)^2(K+2)^2}{4}$

$0+1+8+\ldots+K^3+(K+1)^3 = \frac{(K+1)^2(K+2)^2}{4} \Rightarrow \frac{(K+1)(K+1)\ (K+2)(K+2)}{4}$

$= \frac{(K^2+2K+1)(K^2+4K+4)}{4}$

$= \frac{K^4+4K^3+4K^2+2K^3+8K^2+8K+K^2+4K+4}{4}$

$= \frac{K^4+6K^3+13K^2+12K+4}{4}$

$= \frac{K^2(K+1)^2}{4} + (K+1)^3$

$= \frac{K^2.(K+1)(K+1)}{4} + (K+1)(K+1)(K+1)$

$= \frac{K^2(K^2+2K+1)}{4} + (K^2+2K+1)(K+1)$

$= \frac{K^4+2K^3+K^2}{4} + K^3+K^2+2K^2+2K+K+1$

$= \frac{K^4+2K^3+K^2}{4} + K^3+3K^2+3K+1$

$= K^4+2K^3+K^2 + \frac{4K^3+12K^2+12K+4}{4}$

$= \frac{K^4+6K^3+13K^2+12K+4}{4}$