4) a) $T(1)=0$; $T(n) = T(n-1) + c$; c constante e $n > 1$

① Formula original
$T(n) = T(n-1) + c$

② descubra o passo
$T(n)$ esta descrito em função de $T(n-1)$

③ isole as equações para os proximos passos $T(n-1)$ e $T(n-2)$
$T(n-1) = T(n-2) + c$
$T(n-2) = T(n-3) + c$

④ substitua os valores isolados na formula original
$T(n) = T(n-1) + c$
$= (T(n-2) + c) + c$
$= (T(n-3) + c) + c) + c$
$= (T(n-3) + 3c$

⑤ identifique a formula do i-esimo passo
$T(n) = T(n-i) + i.c$

⑥ descubra o valor de (i) de forma a igualar o parametro $T(X)$ ao parametro (valor de n) no caso base

$T(n-i) = T(1)$
$n - i = 1$
$i = n-1$

⑦ substitua o valor de (i) na formula do iesimo caso.

$T(n) = T(n-i) + i.c$
$= T(n-(n-1)) + (n-1).c$
$= T(1) + (n-1).c$
$= (n-1) * c$

⑧ complexidade: $T(n) \in O(n)$

⑨ Provando por indução
- caso base
$T(1) = n-1.c$
$0 = 1-1.c$
$0 = 0$

- passo indutivo: $T(n-1) = ((n-1)-1).c$

- $T(n) = T(n-1) + c$
$= ((n-1)-1).c + c$
$= c(n-1) - \cancel{c} + c$
$\boxed{= (n-1).c}$

④ b) $T(1) = 0; \; T(m) = T(m-1) + 2^m$

① Formula original: $T(m) = T(m-1) + 2^m$

② descubra o passo: $T(m)$ esta descrito em função de $T(m-1)$

③ Isole as equações para os proximos passos

$T(m-1) = T(m-2) + 2^{m-1}$

$T(m-2) = T(m-3) + 2^{m-2}$

④ substitua os valores isolados na formula Original

para $T(m-1)$

$T(m) = (T(m-2) + 2^{m-1}) + 2^m$

para $T(m-2)$

$T(m) = (T(m-3) + 2^{m-2}) + 2^{m-1} + 2^m$

$= T(m-K) + 2^{m-(K-1)} + 2^{m-(K-2)} + 2^m$

$k=m$ $\quad + 2^{m-m-1} + 2^{m-m-2} + 2^{m-K}$

$2^{-1} \quad 2^{-2} \quad + 2^0$

$S = \frac{(1^\circ \text{termo}).(\text{Razão}^m - 1)}{\text{Razão} - 1}$

$= \frac{(1)(2^{(m-1)-1} - 1)}{2-1}$

$= 2^{m-1} - 1$

$= 2^{m+1} - 2^1 - 2^0 - 1$

$= 2^{m+1} - 4$

⑤ identifique a formula do iesimo passo

$T(m) = T(m-i) + 2^{m+1} - 4$

⑥ descubra o valor de i de forma a igualar o parametro de $T(X)$ ao parametro (valor de m) no caso base

$T(m-i) \Leftrightarrow T(1)$

$m - i = 1$

$-i = 1 - m$

$i = m - 1$

⑦ substitua o valor de i na formula do iesimo caso

$T(m) = T(m-i) + 2^{m+1} - 4$

$= T(m-(m-1)) + 2^{m+1} - 4$

$= T(1) + 2^{m+1} - 4$

$= 0 + 2^{m+1} - 4$

$=$

⑧ Identifique a complexidade dessa formula: $T(m) \in \Theta(2^m)$

⑩ Prova por indução

- Passo base para $m=1$

$T(1) = 2^{m+1} - 4$

$0 = 2^{1+1} - 4$

$0 = 2^2 - 4$

$0 = 0$

- Hipotese $T(m-1) = 2^{(m-1)+1} - 4$

- Passo $T(m) = T(m-1) + 2^m$

$= 2^{(m-1)+1} - 4 + 2^m$

$= 2^m + 2^m - 4$

$\boxed{= 2^{m+1} - 4}$

$T(n) = 2^{n+1} - 4$

4) c) $T(1) = K$; $T(n) = c\,T(n-1)$; c, K constantes e $n > 0$

① formula original: $T(n) = c\,T(n-1)$

② descubra o passo: $T(n)$ esta descrito em funcão de $T(n-1)$

③ isole as equações proximos passos

$$T(n-1) = c\,T(n-2)$$
$$T(n-2) = c\,T(n-3)$$

④ substituindo na formula original

$T(n-1)$: $T(n) = c\,(c\,T(n-2))$
$$= c^2\,T(n-2)$$

$T(n-2)$:
$$T(n) = c^2\,(c\,T(n-3))$$
$$= c^3\,T(n-3)$$

⑤ identifique a formula do i-esimo passo

$$T(n) = c^i\,T(n-i)$$

⑥ descobrindo valor de i de forma a igualar o parametro $T(X)$ ao valor de n do caso base

-supondo $K=1$

$$T(1) = T(n-i)$$
$$K = n - i$$
$$i = n - K$$
$$i = n - 1$$

⑦ substituindo valor de i na formula do iesimo caso

$$T(n) = c^{i}\,T(n-i) = c^{n-1}\,T(n-(n-1)) = c^{n-1}\,T(1)$$
$$= c^{n-1} . K$$

⑧ identifique a complexidade: $T(n) \in \Theta(c^n)$

⑨ Prova por indução

para o passo base

$T(n) = c^{n-1} . K$ para $n = 1$, $c = 1$, $K = 1$ o resultado esperado é K

$$= 1^{1-1} . 1$$

$$1 = 1$$

por indução assumimos que a formula esta correta para $(n-1)$

$$T(n) = c^{n-1} . K$$
$$T(n-1) = c^{(n-1)-1} . K$$
$$= c^{n-2} . K$$

temos

$$T(n) = c\,T(n-1)$$
$$= c\,(c^{n-2} . K)$$
$$= c . c^{n-2} . K$$
$$= c^{n-1} . K$$

4) d) $T(1) = 1;\ T(n) = 3T(n/2) + n$; para $n > 1$

① Formula original: $T(n) = 3T(n/2) + n$

② Tamanho do passo: $T(n/2)$

③ Expandindo proximos passos

$$T(n/2) = 3T(n/4) + n/2$$
$$T(n/4) = 3T(n/8) + n/4$$

④ Substituindo na formula original

$T(n/2)$:
$$T(n) = 3(3T(n/4) + n/2) + n$$
$$3^2 T(n/4) + \frac{3n}{2} + n$$

$T(n/4)$:
$$T(n) = 3^2(3T(n/8) + n/4) + \frac{3n}{2} + n$$
$$= 3^3 T(\tfrac{n}{8}) + \frac{3^2 n}{4} + \frac{3n}{2} + n$$
$$= 3^3 T(\tfrac{n}{2^3}) + n \cdot \left[\frac{3^2}{2^2} + \frac{3^1}{2^1} + \frac{3^0}{1^0}\right]$$

$3^0 + 3^1 + 3^2 +$
$1 + 3 + 9 +$
$1 \cdot 3 = 3$
$3 \cdot 3 = 9$ Razão é 3

⑤ i-esimo passo
$$T(n) = 3^i T(\tfrac{n}{2^i}) + n\left[\left(\tfrac{3}{2}\right)^{i-1} + \left(\tfrac{3}{2}\right)^{i-2} + \left(\tfrac{3}{2}\right)^0\right]$$

⑥ encontrando i
$$T(1) = T(\tfrac{n}{2^i})$$
$$1 = \frac{n}{2^i} \qquad \log_a b = x \iff a^x = b$$
$$2^i = n$$
$$i = \log_2 n$$

⑦ aplicando i na formula
- primeiro encontrar a formula da soma geometrica

Formula soma:
$$S = \frac{(1^o\ termo).(Razão^n - 1)}{razão - 1} = \frac{(1).\left(\frac{3}{2}^{\log_2 n} - 1\right)}{\frac{3}{2} - 1} = \frac{\frac{3}{2}^{\log_2 n} - 1}{\frac{1}{2}} =$$
$$= \left(\frac{2}{1}\right)\left(\left(\frac{3}{2}\right)^{\log_2 n} - 1\right) = \left(\frac{2}{1}\right)\cdot\left(\frac{3^{\log_2 n}}{2^{\log_2 n}} - 1\right)$$
$$= \left(\frac{2}{1}\right)\cdot\left(\frac{n^{\log_2 3}}{n^{\log_2 2}} - 1\right) = \left(\frac{2}{1}\right)\cdot\left(\frac{n^{\log_2 3}}{n^1} - 1\right) = \frac{2n^{\log_2 3}}{n} - 2$$

aplicando na formula iesimo termo
$$T(n) = 3^i T(\tfrac{n}{2^i}) + \cancel{n}\left(\frac{2n^{\log_2 3}}{\cancel{n}} - 2\right)$$
$$= 3^{\log_2 n} T(\tfrac{n}{2^{\log_2 n}}) + 2n^{\log_2 3} - 2n$$
$$= n^{\log_2 3} T(\tfrac{n}{n^{\log_2 2}}) + 2n^{\log_2 3} - 2n$$
$$= n^{\log_2 3} T(\tfrac{n}{n}) + 2n^{\log_2 3} - 2n$$
$$= n^{\log_2 3} T(1) + 2n^{\log_2 3} - 2n$$
$$\boxed{= 3n^{\log_2 3} - 2n}$$

⑧ complexidade
$$T(n) \in \Theta(n^{\log_2 3})$$

⑨ Provando por indução

- Caso Base:
$$T(1) = 3n^{\log_2 3} - 2$$
$$1 = 3.1^{\log_2 3} - 2$$
$$1 = 3 - 2$$
$$1 = 1$$

- supondo:
$$T(n/2) = 3(\tfrac{n}{2})^{\log_2 3} - 2(\tfrac{n}{2})$$

- temos
$$T(n) = 3T(n/2) + n$$
$$= 3\left(3(\tfrac{n}{2})^{\log_2 3} - 2(\tfrac{n}{2})\right) + n$$
$$= 3\left(3\left(\frac{n^{\log_2 3}}{2^{\log_2 3}}\right) - 2(\tfrac{n}{2})\right) + n$$
$$= 3\left(3\left(\frac{n^{\log_2 3}}{3^{\log_2 2}}\right) - 2(\tfrac{n}{2})\right) + n$$
$$= 3\left(\cancel{3}\left(\frac{n^{\log_2 3}}{\cancel{3}}\right) - \cancel{2}(\tfrac{n}{\cancel{2}})\right) + n$$
$$= 3n^{\log_2 3} - 3n + n$$
$$\boxed{= 3n^{\log_2 3} - 2n}$$

4 e) $T(1) = 1 \quad T(n) = 8T(n/2) + n$

① Formula original: $T(n) = 8T(\frac{n}{2}) + n$

② Tamanho do passo é $T(n/2)$

③ Expandindo proximos passos: $T(n/2)$ e $T(n/4)$

$$T(n/2) = 8T(n/4) + \frac{n}{2}$$

$$T(n/4) = 8T(n/8) + \frac{n}{4}$$

④ substituindo

$T(n/2)$

$$T(n) = 8\left(8T(n/4) + \frac{n}{2}\right) + n$$
$$= 8^2 T(\tfrac{n}{4}) + 4n + n$$

$T(n/4)$

$$T(n) = 8^2\left(8T(\tfrac{n}{8}) + \tfrac{n}{4}\right) + 4n + n$$
$$= 8^3 T(\tfrac{n}{8}) + 16n + 4n + n$$
$$= 8^3 T(\tfrac{n}{2^3}) + 4^2 n + 4^1 n + 4^0 n$$

⑤ iesimo passo

$$T(n) = 8^i T(\tfrac{n}{2^i}) + 4^{i-1} n + 4^{i-2} n + 4^0 n$$
$$= 8^i T(\tfrac{n}{2^i}) + n\left[4^{i-1} + 4^{i-2} + 4^0\right]$$

Primeiro termo = 1

$1 \cdot 4 = 4$

$4 \cdot 4 = 16$ Razão é 4

$1 \cdot 4^1 = 4$

$1 \cdot 4^2 = 16$

⑥ descobrindo valor de i

$$T(\tfrac{n}{2^i}) = T(1)$$

Logaritmo: $\log_a b = x \Leftrightarrow a^x = b$

$$\frac{n}{2^i} = 1$$
$$n = 2^i$$
$$i = \log_2 n$$

⑦ substituindo valor de i na formula

Antes ache o valor da serie geometrica

soma finita

$$S = \frac{(1^o \text{termo})(\text{Razão}^n - 1)}{\text{Razão} - 1}$$

$$= \frac{(1)(4^{(\log_2 n)}) - 1}{4-1} = \frac{4^{\log_2 n} - 1}{3}$$

$$= \frac{n^{\log_2 4^2} - 1}{3} = \frac{n^2 - 1}{3}$$

$$T(n) = 8^{\log_2 n}\, T(\tfrac{n}{2^i}) + n\left(\frac{n^2-1}{3}\right)$$
$$= n^{\log_2 8}\, T(1) + \frac{n^3 - n}{3}$$
$$= n^3 + \frac{n^3 - n}{3}$$
$$= \frac{3n^3 + n^3 - n}{3}$$
$$= \frac{4n^3 - n}{3}$$

⑧ complexidade

$$T(n) = 8T(\tfrac{n}{2}) + n \in \Theta(n^3)$$

⑨ Prova por indução

- para o caso base

$$T(1) = \frac{4 \cdot 1^3 - 1}{3}$$
$$1 = \frac{4-1}{3}$$
$$1 = 1$$

- supondo:

$$T(n/2) = \frac{4(\frac{n}{2})^3 - (\frac{n}{2})}{3}$$

$$T(n) = 8T(\tfrac{n}{2}) + n$$
$$= 8\left(\frac{4(\frac{n}{2})^3 - (\frac{n}{2})}{3}\right) + n$$
$$= \frac{32(\frac{n}{2})^3 - 8(\frac{n}{2})}{3} + n$$
$$= \frac{32\frac{n^3}{2^3} - \frac{8n}{2}}{3} + n$$
$$= \frac{\frac{32n^3}{2^3} - \frac{8n}{2}}{3} + n$$
$$= \frac{4n^3 - 4n + 3n}{3}$$
$$= \frac{4n^3 - n}{3}$$

4 f) $T(1) = 1; \quad T(n) = T(n/3) + n$

① formula original: $T(n) = T(\frac{n}{3}) + n$

② tamanho do passo: $T(n/3)$

③ expandindo proximos passos: $T(n/3)$ e $T(n/6)$

$$T(n/3) = T(n/9) + n/3$$

$$T(n/9) = T(n/27) + n/9$$

④ substituindo em $T(n)$

$$T(n) = \underbrace{(T(n/9) + \tfrac{n}{3})}_{T(n/3)} + n$$

$$T(n) = \underbrace{(T(n/27) + \tfrac{n}{9})}_{T(n/9)} + \frac{n}{3} + n$$

$$= T(\frac{n}{3^3}) + \frac{n}{3^2} + \frac{n}{3^1} + \frac{n}{3^0}$$

$$= T(\frac{n}{3^3}) + n\left[(\frac{1}{3})^2 + (\frac{1}{3})^1 + (\frac{1}{3})^0\right]$$

Razão é $(\frac{1}{3})$

⑥ iesimo passo

$$T(n) = T(\frac{n}{3^i}) + n\left[(\frac{1}{3})^{i-1} + (\frac{1}{3})^{i-2} + (\frac{1}{3})^0\right]$$

⑦ encontrando i

$$T(\frac{n}{3^i}) = T(1)$$

$$\frac{n}{3^i} = 1 \qquad \log_a b = x \Leftrightarrow a^x = b$$

$$n = 3^i$$

$$i = \log_3 n$$

Formula da soma Geometrica

$$S = \frac{(1^{\circ}\text{termo})(\text{Razão}^n - 1)}{\text{Razão} - 1} = \frac{(1).(\frac{1}{3})^{\log_3 n} - 1}{\frac{1}{3} - 1}$$

$$= \frac{(\frac{1}{3})^{\log_3 n} - 1}{\frac{1-3}{3}} = \frac{(\frac{1}{3})^{\log_3 n} - 1}{\frac{-2}{3}} = (-\frac{3}{2}) \cdot ((\frac{1}{3})^{\log_3 n} - 1)$$

$$\frac{-2}{+3} = -$$

$$= (-\frac{3}{2}) \cdot (\frac{n^{\log_3 1}}{n^{\log_3 3}} - 1) = (-\frac{3}{2}) \cdot (\frac{n^0}{n^1} - 1)$$

$$= \frac{-3}{2n} + \frac{3}{2}$$

⑧ substituindo na formula

$$T(n) = T(1) + \frac{n}{1}\left[-\frac{3}{2n} + \frac{3}{2}\right]$$

$$= 1 + \left[-\frac{3\cancel{n}}{2\cancel{n}} + \frac{3n}{2}\right]$$

$$= \frac{3n}{2} - \frac{3}{2} + 1$$

$$= \frac{3n - 3 + 2}{2}$$

$$\boxed{= \frac{3n - 1}{2}}$$

⑧ complexidade: $T(n) = T(n/3) + n \in \Theta(n)$

⑨ Prova por indução

Base: $T(1) = \frac{3.1 - 1}{2}$

$$1 = \frac{2}{2}$$

$$1 = 1 \checkmark$$

supondo:

$$T(n/3) = \frac{3(\frac{n}{3}) - 1}{2}$$

Portanto: $T(n) = T(n/3) + n$

$$= \frac{3(n/3) - 1}{2} + n$$

$$= \frac{\frac{3}{1}(\frac{n}{3}) - 1}{2} + n = \frac{\frac{3n}{3} - 1}{2} + n = \frac{n - 1}{2} + n$$

$$= \frac{n - 1 + 2n}{2} = \boxed{\frac{3n - 1}{2}}$$

④ G) $T(1) = 1; \quad T(n) = 7T(n/4) + n$

① Formula original: $T(n) = 7T(n/4) + n$

② tamanho do passo: $T(n/4)$

③ Expandindo proximos passos $T(n/4)$ e $T(n/16)$

$$T(n/4) = 7T(n/16) + (n/4)$$
$$T(n/16) = 7T(n/64) + (n/16)$$

④ substituindo na formula:

$T(n/4)$:
$$T(n) = 7T(n/4) + n$$
$$= 7(7T(n/16) + n/4) + n$$
$$= 7^2 T(n/16) + \frac{7n}{4} + n$$

$T(n/16)$
$$= 7^2(7T(n/64) + (n/16)) + \frac{7n}{4} + n$$
$$= 7^3 T(n/64) + \frac{7^2 n}{16} + \frac{7n}{4} + n$$
$$= 7^3 T\left(\frac{n}{4^3}\right) + n\left[\left(\frac{7}{4}\right)^{i-2} + \left(\frac{7}{4}\right)^{i-1} + \left(\frac{7}{4}\right)^0\right]$$

⑤ iesimo passo

$$T(n) = 7^i T\left(\frac{n}{4^i}\right) + n\left[\left(\frac{7}{4}\right)^{i-1} + \left(\frac{7}{4}\right)^{i-2} + \left(\frac{7}{4}\right)^0\right]$$

⑥ Encontrando valor de i

$$T(1) = T\left(\frac{n}{4^i}\right)$$
$$1 = \frac{n}{4^i}$$
$$4^i = n$$
$$i = \log_4 n$$

$\log_a b = x \Leftrightarrow a^x = b$

Encontrando soma da serie Geometrica

$$S = \frac{(1^o termo) \cdot (Razão^n - 1)}{Razão - 1}$$
$$= \frac{(1) \cdot \left(\frac{7}{4}\right)^{\log_4 n} - 1}{\left(\frac{7}{4}\right) - 1}$$
$$= \frac{\left(\frac{7}{4}\right)^{\log_4 n} - 1}{\frac{7}{4} - \frac{4}{4}} = \frac{\left(\frac{7}{4}\right)^{\log_4 n} - 1}{\left(\frac{3}{4}\right)}$$
$$= \frac{4}{3} \cdot \left(\frac{7}{4}\right)^{\log_4 n} - 1$$
$$= \frac{4}{3} \cdot \left(\frac{7^{\log_4 n}}{4^{\log_4 n}} - 1\right)$$
$$= \frac{4}{3} \cdot \left(\frac{n^{\log_4 7}}{n^{\log_4 4}} - 1\right)$$
$$= \frac{4}{3} \cdot \left(\frac{n^{\log_4 7}}{n} - 1\right)$$
$$= \frac{4n^{\log_4 7}}{3n} - \frac{4}{3}$$

⑦ Aplicando na formula de i-esimo passo

$$T(n) = 7^{\log_4 n} T\left(\frac{n}{4^{\log_4 n}}\right) + n\left(\frac{4n^{\log_4 7}}{3n} - \frac{4}{3}\right)$$
$$= n^{\log_4 7} T\left(\frac{n}{n^{\log_4 4}}\right) + \frac{\cancel{n}}{1}\left(\frac{4n^{\log_4 7}}{3\cancel{n}} - \frac{4}{3}\right)$$
$$= n^{\log_4 7} T(1) + \frac{4n^{\log_4 7}}{3} - \frac{4n}{3}$$
$$= n^{\log_4 7} + \frac{4n^{\log_4 7} - 4n}{3}$$
$$\boxed{= \frac{7n^{\log_4 7} - 4n}{3}}$$

⑧ complexidade

$$T(n) \in \Theta(n^{\log_4 7})$$

⑨ Prova por indução

① caso base: $T(1) = \frac{7 \cdot 1^{\log_4 7} - 4 \cdot 1}{3}$

$1 = 1$ ✓

② supondo

$$T\left(\frac{n}{4}\right) = \frac{7\left(\frac{n}{4}\right)^{\log_4 7} - 4\left(\frac{n}{4}\right)}{3}$$

③ tendo

$$T(n) = 7T\left(\frac{n}{4}\right) + n$$
$$= 7\left(\frac{7\left(\frac{n}{4}\right)^{\log_4 7} - 4\left(\frac{n}{4}\right)}{3}\right) + n$$
$$= 7\left(\frac{7\left(\frac{n}{4}\right)^{\log_4 7} - n}{3}\right) + n$$
$$= 7\left(\frac{7\left(\frac{n^{\log_4 7}}{4^{\log_4 7}}\right) - n}{3}\right) + n$$
$$= 7\left(\frac{\cancel{7}\left(\frac{n^{\log_4 7}}{\cancel{7}}\right) - n}{3}\right) + n$$
$$= 7\left(\frac{n^{\log_4 7} - n}{3}\right) + n$$
$$= \frac{7n^{\log_4 7} - 7n + 3n}{3}$$
$$\boxed{= \frac{7n^{\log_4 7} - 4n}{3}}$$

4) h) $T(1) = 1$; $T(n) = 3T(n/4) + n^2$

① Formula original: $T(n) = 3T(\frac{n}{4}) + n^2$

② Tamanho do passo: $T(n/4)$

③ Expandindo proximos passos: $T(n/4)$ e $T(n/16)$

$$T(n/4) = 3T(n/16) + (\tfrac{n}{4})^2$$
$$T(n/16) = 3T(\tfrac{n}{64}) + (\tfrac{n}{16})^2$$

④ Aplicando na formula $T(n/4)$:

$$T(n) = 3(3T(\tfrac{n}{16}) + (\tfrac{n}{4})^2) + n^2$$
$$= 3^2 T(\tfrac{n}{16}) + 3(\tfrac{n}{4})^2 + n^2$$
$$= 3^2(3T(\tfrac{n}{64}) + (\tfrac{n}{16})^2) + 3(\tfrac{n}{4})^2 + n^2$$
$$= 3^3 T(\tfrac{n}{4^3}) + 3^2(\tfrac{n}{16})^2 + 3(\tfrac{n}{4})^2 + n^2$$
$$= 3^3 T(\tfrac{n}{4^3}) + (\tfrac{3}{4^2})^2 n^2 + (\tfrac{3}{4^2}) n^2 + n^2$$
$$= 3^3 T(\tfrac{n}{4^3}) + n^2\left[(\tfrac{3}{4^2})^2 + (\tfrac{3}{4^2})^1 + (\tfrac{3}{4^2})^0\right]$$

⑤ iesimo passo:

$$T(n) = 3^i T(\tfrac{n}{4^i}) + n^2\left[(\tfrac{3}{4^2})^{i-1} + (\tfrac{3}{4^2})^{i-2} + (\tfrac{3}{4^2})^0\right]$$

Formula da Soma Geometrica

$$S = \frac{(1^\circ \text{ termo})(\text{razão}^n - 1)}{\text{Razão} - 1}$$
$$= \frac{(1)((\frac{3}{4^2})^{\log_4 n} - 1)}{\frac{3}{4^2} - 1}$$
$$= \frac{\frac{3^{\log_4 n}}{4^{\log_4 n}} - 1}{-\frac{13}{16}}$$
$$= \frac{\frac{n^{\log_4 3}}{n^{\log_4 16}} - 1}{-\frac{13}{16}}$$
$$= (-\tfrac{16}{13})(\tfrac{n^{\log_4 3}}{n^2} - 1)$$
$$= -\frac{16 n^{\log_4 3}}{13 n^2} + \frac{16}{13}$$

⑥ encontrando valor de (i)

$$T(1) = T(\tfrac{n}{4^i})$$
$$1 = \tfrac{n}{4^i} \qquad \log_a b = x \Leftrightarrow b = a^x$$
$$4^i = n$$
$$i = \log_4 n$$

⑦ aplicando na formula do

$$T(n) = 3^{\log_4 n} T(\tfrac{n}{4^{\log_4 n}}) + n^2\left((-\tfrac{16}{13})(\tfrac{n^{\log_4 3}}{n^2} - 1)\right)$$
$$= 3^{\log_4 n} T(\tfrac{n}{n}) + (-\tfrac{16}{13})(n^{\log_4 3} - n^2)$$
$$= n^{\log_4 3} T(1) - \tfrac{16}{13} n^{\log_4 3} + (\tfrac{16}{13}) n^2$$
$$= \frac{13 n^{\log_4 3} - 16 n^{\log_4 3}}{13} + (\tfrac{16}{13}) n^2$$
$$\boxed{= \frac{-3 n^{\log_4 3}}{13} + (\tfrac{16}{13}) n^2}$$

- indução

- base: $T(1) = -\frac{3(1)^{\log_4 3}}{13} + (\frac{16}{13}) 1^2$

$$1 = \tfrac{13}{13}$$
$$1 = 1$$

- hipotese: $T(\frac{n}{4}) = -\frac{3(\frac{n}{4})^{\log_4 3}}{13} + (\frac{16}{13})(\frac{n}{4})^2$

$$= -\frac{3(\frac{n^{\log_4 3}}{4^{\log_4 3}})}{13} + (\tfrac{16}{13})(\tfrac{n^2}{16})$$
$$= -\frac{3(\frac{n^{\log_4 3}}{3})}{13} + \frac{n^2}{13}$$
$$= -\frac{n^{\log_4 3}}{13} + \frac{n^2}{13}$$

- Passo indução

$$T(n) = 3T(\tfrac{n}{4}) + n^2$$
$$= 3(-\tfrac{n^{\log_4 3}}{13} + \tfrac{n^2}{13}) + n^2$$
$$= -\frac{3 n^{\log_4 3}}{13} + \frac{3}{13} n^2 + \frac{13}{13} n^2$$
$$\boxed{= -\frac{3 n^{\log_4 3}}{13} + \frac{16 n^2}{13}}$$