4. B: are covid − + = Test pozitiv

$P(B) = \frac{1}{1000}$ $P(+|B) = 0,98$ $P(+|B^c) = 0,05$

$$P(B|+) = \frac{P(+|B)\cdot P(B)}{P(+)} = \frac{P(+|B)\cdot P(B)}{P(+|B)P(B) + P(+|B^c)P(B^c)} =$$

$$= \frac{0,98 \cdot \frac{1}{1000}}{0,98 \cdot \frac{1}{1000} + 0,05 \cdot \frac{999}{1000}} \simeq 0,019$$

$$P(B|+_1+_2) = P(+_1+_2|B)\frac{P(B)}{P(+_1+_2)}$$

$$P(+_1+_2|B) = P(+_1|B)\cdot P(+_2|B) = \left(\frac{98}{100}\right)^2$$

$$P(+_1+_2) = P(+_1+_2|B)P(B) + P(+_1+_2|B^c)\cdot P(B^c)$$

$$P(B|+_1+_2) = \left(\frac{98}{100}\right)^2 \frac{\frac{1}{1000}}{\left(\frac{98}{100}\right)^2 \frac{1}{1000} + \left(\frac{5}{100}\right)^2 \cdot \frac{999}{1000}} \simeq 0,27$$

$$P(B|+_1+_2^c) = P(+_1+_2^c|B)\cdot\frac{P(B)}{P(+_1+_2^c)}$$

$+_1 \perp\!\!\!\perp +_2^c | B$

$$P(+_1+_2^c|B) = P(+_1|B)\,P(+_2^c|B) = \frac{98}{100}\cdot\frac{2}{100}$$

$$P(+_1+_2^c|B^c) = P(+_1|B^c)\,P(+_2^c|B^c) = \frac{5}{100}\cdot\frac{95}{100}$$

$$P(+_1+_2^c) = P(+_1+_2^c|B)\cdot P(B) + P(+_1+_2|B^c)\cdot P(B^c) =$$

$$= \frac{98}{100}\cdot\frac{2}{100}\cdot\frac{1}{1000} + \frac{5}{100}\cdot\frac{95}{100}\cdot\frac{999}{1000} \simeq 0,0474$$

$$P(B|+_1+_2^c) = \frac{98}{100}\cdot\frac{2}{100}\cdot\frac{\frac{1}{1000}}{0,0474} \simeq 0,000413$$

$$P(B|+_1^c+_2) = P(+_1^c+_2|B)\cdot\frac{P(B)}{P(+_1^c+_2)}$$

$+_1^c \perp\!\!\!\perp +_2 | B$

$$P(+_1^c+_2|B) = P(+_1^c|B)\,P(+_2|B) = \frac{2}{100}\cdot\frac{98}{100}$$

$$P(+_1^c+_2|B^c) = P(+_1^c|B^c)\,P(+_2|B^c) = \frac{95}{100}\cdot\frac{5}{100}$$

$$P(+_1^c+_2^c) = P(+_1^c+_2|B)P(B) + P(+_1^c+_2|B^c)P(B^c) =$$

$$= \frac{2}{100}\cdot\frac{98}{100}\cdot\frac{1}{1000} + \frac{95}{100}\cdot\frac{5}{100}\cdot\frac{999}{1000} \simeq 0,0474$$

$$P(B|+_1^c+_2) = \frac{2}{100}\cdot\frac{98}{100}\cdot\frac{\frac{1}{1000}}{0,0474} \simeq 0,000413$$

$P(B|+_1^c+_2) + P(B|+_1^c+_2^c) = 0,00082$

$$P(B^c|+_1^c+_2^c) = P(+_1^c+_2^c|B^c) \cdot \frac{P(B^c)}{P(+_1^c+_2^c)}$$

$X_1^c \perp X_2^c | B$

$$P(+_1^c+_2^c|B^c) = P(+_1^c|B^c) \cdot P(+_2^c|B^c) = \left(\frac{95}{100}\right)^2$$

$$P(+_1^c+_2^c|B) = P(+_1^c|B) \cdot P(+_2^c|B) = \left(\frac{2}{100}\right)^2$$

$$P(+_1^c+_2^c) = P(+_1^c+_2^c|B^c) \cdot P(B^c) + P(+_1^c+_2^c|B) \cdot P(B) =$$

$$= \left(\frac{95}{100}\right)^2 \cdot \frac{999}{1000} + \left(\frac{2}{100}\right)^2 \cdot \frac{1}{1000}$$

$$P(B^c|+_1^c+_2^c) = \left(\frac{95}{100}\right)^2 \cdot \frac{\frac{999}{1000}}{P(+_1^c+_2^c)} = 0,9989$$