s_n = șir de n biți independenți și aleatori

eveniment A_n - 1111 (patru biți 1 consecutivi) într-un șir de n biți

$s_1 = 0$ sau 1 => $P(A_1) = 0$

$s_2 = 0$ sau $1, s_1$ => $P(A_2) = 0$

$s_3 = 0$ sau $1, s_2$ => $P(A_3) = 0$

$s_4 = 0$ sau $1, s_3$ => $2^4 = 16$ posibilități => $P(A_4) = \frac{1}{16}$

$s_5 = 0$ sau $1, s_4$ => $P(A_5) = \frac{1}{2}\frac{1}{16} + \frac{1}{2}\frac{2}{16} = \frac{3}{32}$

- -> Pt $0\, s_4$ => $P(A_4) = \frac{1}{16}$
- -> Pt $1\, s_4$ => 2^4 posibilități cu 2 cazuri favorabile => $\frac{2}{16}$
  - -> $1\,0\, s_3 \Rightarrow 0$
  - -> $1\,1\, s_3$ - { 11110, 11111 }

$s_6 = 0$ sau $1, s_5$ => $P(A_6) = \frac{1}{2} \cdot \frac{3}{32} + \left(\frac{1}{2} \cdot \frac{1}{16} + \frac{1}{8}\right)\frac{1}{2} = \frac{3}{64} + \frac{5}{64} = \frac{1}{8}$

- -> Pt $0\, s_5$ => $P(A_5) = \frac{3}{32}$
- -> Pt $1\, s_5$ => $\frac{1}{2}P(A_4) + \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{2} \cdot \frac{1}{16} + \frac{1}{4} \cdot \frac{1}{2}$
  - -> $1\,0\, s_4$ -> $P(A_4)$
  - -> $1\,1\, s_4$ -> $\frac{4}{2^4} = \frac{1}{2^2} = \frac{1}{4}$
    - -> $1\,1\,0\, s_3$ -> 0
    - -> $1\,1\,1\, s_3$ - { 111100, 111101, 111110, 111111 }

$s_7 = 0$ sau $1\, s_6$ => $P(s_7) = \frac{1}{2} \cdot \frac{1}{8} + \frac{1}{2}\left(\frac{1}{2} \cdot \frac{3}{32} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{9}{16}\right) = \frac{1}{8} \cdot \frac{1}{2} + \frac{1}{2^2}\frac{3}{32} + \frac{1}{2^3} \cdot \frac{1}{16} + \frac{1}{2^5} \cdot \frac{8}{16} = \frac{1}{3\ldots}$

- -> Pt $0\, s_6$ => $P(A_6) = \frac{1}{8}$
- -> Pt $1\, s_6$ => $\frac{1}{2} \cdot \frac{3}{32} + \frac{1}{2} \cdot \frac{9}{32}$
  - -> $1\,0\, s_5$
    - -> $1\,0\,0\, s_4$ -> $P(A_4)$
    - -> $1\,0\,1\, s_4$ -> $\frac{2}{16}$

    $\left(\frac{1}{16} + \frac{2}{16}\right) = \frac{3}{16} \cdot \frac{1}{2} = \frac{3}{32}$
  - -> $1\,1\, s_5$ -> $\frac{9}{32}$
    - -> $1\,1\,0\, s_4$ -> $P(A_4)$
    - -> $1\,1\,1\, s_3$ -> $\frac{1}{2}$

    $\frac{1}{16} + \frac{1}{2} = \frac{9}{16}$

$s_8 = 0$ sau $1\, s_7 \Rightarrow P(A_8) = \frac{1}{2} \cdot \frac{5}{32} + \frac{1}{4} \cdot \frac{1}{8} + \frac{1}{8} \cdot \frac{1}{8} + \frac{1}{16} \cdot \frac{3}{32} + \frac{1}{16} = \frac{3}{16}$

$\to$ Pt $0\, s_7 \to P(A_7) = \frac{5}{32}$

$\to$ Pt $1\, s_7 \Rightarrow \frac{1}{2} \cdot \frac{1}{8} + \frac{1}{2}\left(\frac{3}{64} + \frac{1}{4}\right)$

$\to 10\, s_6 \to P(A_6)$

$\to 11\, s_6 \to \frac{1}{2} P(A_5) + \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{2} \cdot \frac{3}{32} + \frac{1}{4}$

$\to 110\, s_5 \to P(A_5)$

$\to 111\, s_5 \to \frac{1}{2}$

Obs relația de recurență:

$$P(A_n) = \frac{1}{2} P(A_{n-1}) + \frac{1}{2^2} P(A_{n-2}) + \frac{1}{2^3} P(A_{n-3}) + \frac{1}{2^4} P(A_{n-4}) + \frac{1}{2^4}$$