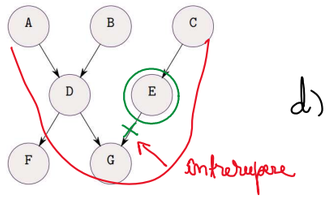
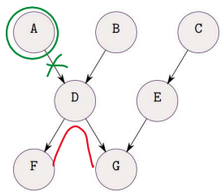
Chiru Cătălin - Mihail

Exercițiul 3:



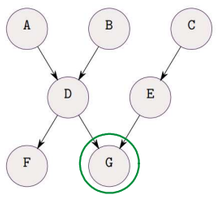
d) $A \perp C \mid E$ ✓



e) $F \perp G \mid A$

F, D, G cauză comună $\Rightarrow$

F și G comunică prin D $\Rightarrow$ ✗



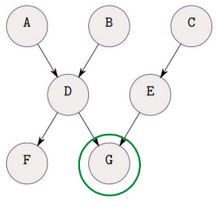
f) $B \perp C \mid G$ ✗

B, D, G - lanț ($B \not\perp G$)

C, E, G - lanț ($C \not\perp G$)

$D \rightarrow G \leftarrow E$ - efect comun ($D \not\perp E \mid G$)

$\Rightarrow B \not\perp C \mid G$



g) $F \perp C \mid G$ ✗

- G efect pentru C $\Rightarrow$ G observat îl activează pe C pt partea din stânga a grafului $C \not\perp D \mid G$
- $F \not\perp D$ (F, D, G cauză comună)

$\Rightarrow F \not\perp C \mid G$ (se influențează datorită lui D)

Exercițiul 5:

b) $p(D \mid G \cap E) = p(D \mid G, E)$

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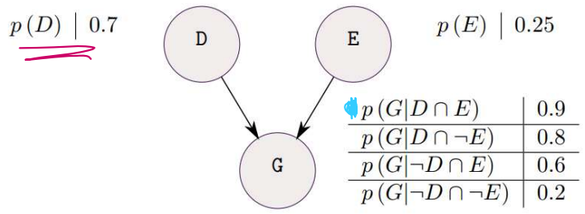


$$p(D \mid G \cap E) \overset{\text{Bayes}}{=} \frac{p(G \cap E \mid D)\, p(D)}{p(G \cap E)} = 0.(7)$$

($p(G \cap E \mid D) = 0.225$, $p(D) = 0.7$, $p(G \cap E) = 0.225$)

$$p(G \cap E \mid D) \overset{\text{Regula prod}}{=} p(E \mid D) \cdot p(G \mid E, D) = 0.225$$

($p(G \mid E, D) = 0.9$)

$$p(E \mid D) \overset{E \perp D}{=} \overset{\text{efect mediu descoperit}}{p(E)} = 0.25$$

Pentru $p(G \cap E) = 0.2025$

VI:

$$p(G \cap E) \overset{\text{R. prod}}{=} p(E)\, p(G \mid E) = 0.25 \cdot 0.81 = 0.2025$$

$$p(G \mid E) \overset{\text{Desc ev sigur } D}{=} p(G \mid E, D)\, p(D) + p(G \mid E, \bar{D})\, p(\bar{D}) =$$

$$= 0.9 \cdot 0.7 + 0.6 \cdot 0.3 =$$

$$= 0.81$$

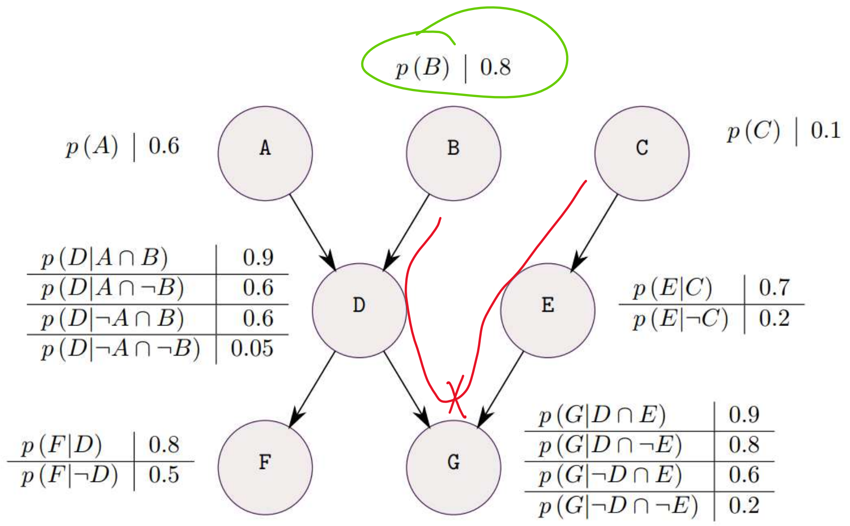
VII:

$$p(G \cap E) \overset{\text{Desc ev sigur după } D}{=} p(G \cap E \mid D)\, p(D) + p(G \cap E \mid \bar{D})\, p(\bar{D})$$

$= 0.225 \cdot 0.7 + 0.15 \cdot 0.3 = 0.2025$

$$p(G \cap E \mid \bar{D}) = \underbrace{p(E \mid \bar{D})}_{\overset{E \perp D}{=} \, p(E)} \cdot p(G \mid \bar{D}, E) = 0.25 \cdot 0.6 = 0.15$$

Exercițiul 6:



c) $p(B \mid \bar{C}) \overset{B \perp C}{=} p(B) = 0.8$

Singura cale de comunicare ar fi prin efectul comun G, dar G nu este observat pentru

a) $p(B \mid F) \underset{p(\text{părinte} \mid \text{efect})}{\overset{\text{Văd}}{=}} \dfrac{p(F \mid B)\, p(B)}{p(F)} = 0.827$ ✓

$p(F \mid B) = 0.734$, $p(B) = 0.8$, $p(F) = 0.71$

→ aplic Bayes

$$p(F \mid B) \underset{\text{ev sigur}}{\overset{\text{Desc}}{=}} p(F \mid D)\, p(D \mid B) + p(F \mid \bar{D})\, p(\bar{D} \mid B) =$$

$0.8 \cdot 0.78 + 0.5 \cdot 0.22 = 0.734$

sigur
după D $= 0.8 \cdot 0.78 + 0.5 \cdot 0.22 = 0.734$

$$p(D|B) \overset{\text{Desc}}{\underset{\text{ev sigur după } A}{=}} p(D|A,B)\,p(A|B) + p(D|\bar{A},B)\,p(\bar{A}|B) =$$

($p(A|B) = p(A)$ since $A \perp B$; $p(\bar{A}|B) = p(\bar{A})$ since $A \perp B$)

$$= 0.9 \cdot 0.6 + 0.6 \cdot 0.4 = 0.78$$

$$p(\bar{D}|B) \overset{\text{Desc}}{\underset{\text{ev sigur după } A}{=}} p(\bar{D}|A,B)\,p(A|B) + p(\bar{D}|\bar{A},B)\,p(\bar{A}|B) =$$

($p(A|B) = p(A)$ since $A \perp B$; $p(\bar{A}|B) = p(\bar{A})$ since $A \perp B$)

$$= 0.1 \cdot 0.6 + 0.4 \cdot 0.4 = 0.22$$

$$p(F) \overset{\text{Desc}}{\underset{\text{ev. sigur}}{=}} p(F|B)\,p(B) + p(F|\bar{B})\,p(\bar{B}) = 0.734 \cdot 0.8 + 0.614 \cdot 0.2$$

ev. sigur după B (ca să mă folosesc de ce calcule am făcut deja) $= 0.71$

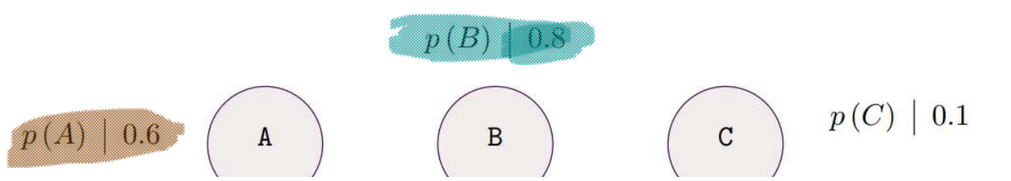
$$p(F|\bar{B}) \overset{\text{Desc}}{=} p(F|\underset{=}{D})\,p(\underset{=}{D}|\bar{B}) + p(F|\underset{=}{\bar{D}})\,p(\underset{=}{\bar{D}}|\bar{B}) =$$

ev sigur după $\underset{=}{D}$ $= 0.8 \cdot 0.38 + 0.5 \cdot 0.62 = 0.614$

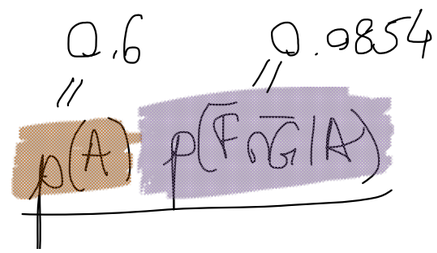
$$p(D|\bar{B}) \overset{\text{Desc}}{=} p(D|\underset{=}{A},\bar{B})\,p(\underset{=}{A}|\bar{B}) + p(D|\underset{=}{\bar{A}},\bar{B})\,p(\underset{=}{\bar{A}}|\bar{B}) =$$

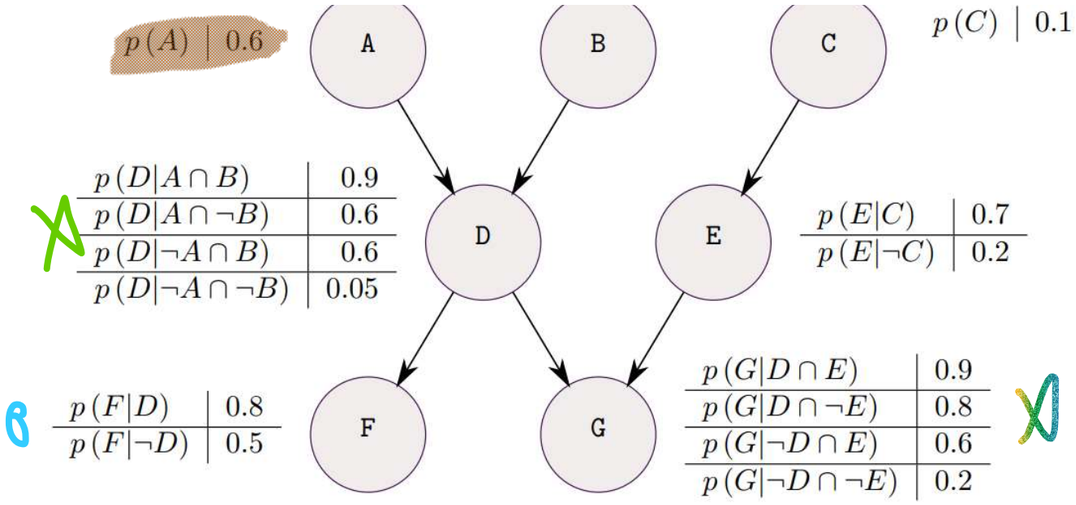
ev sigur după $\underset{=}{A}$ $\quad A \perp B \Rightarrow p(A)$ $\quad A \perp B \Rightarrow p(\bar{A})$

$$= 0.6 \cdot 0.6 + 0.05 \cdot 0.4 = 0.38$$



l) $p(A|\bar{F},\bar{G})$ Permute





$p(A)$	0.6

$p(C)$	0.1

$p(D\|A\cap B)$	0.9
$p(D\|A\cap\neg B)$	0.6
$p(D\|\neg A\cap B)$	0.6
$p(D\|\neg A\cap\neg B)$	0.05

$p(E\|C)$	0.7
$p(E\|\neg C)$	0.2

$p(F\|D)$	0.8
$p(F\|\neg D)$	0.5

$p(G\|D\cap E)$	0.9
$p(G\|D\cap\neg E)$	0.8
$p(G\|\neg D\cap E)$	0.6
$p(G\|\neg D\cap\neg E)$	0.2

b) $p(H \mid F, G)$ = în funcție de sucesori → Bayes

$= \frac{0.6 \cdot 0.0854}{0.1295} = 0.39567$ ✓

$p(\bar{F}, \bar{G}) = 0.1295$

Dm 3e, știm că $F \perp G \mid A$ → fac descomp eveniment sigur după D, pt că $F \perp G \mid D$

$p(\bar{F} \cap \bar{G} \mid D) = p(\bar{F} \mid D) \cdot p(\bar{G} \mid D)$ (*) (Analog pt $\bar{D}$)

$p(\bar{F} \cap \bar{G}) = p(\bar{F} \cap \bar{G} \mid D)\, p(D) + p(\bar{F} \cap \bar{G} \mid \bar{D})\, p(\bar{D}) =$

(*) ⇓ (* pt $\bar{D}$) ⇓

$= p(\bar{F} \mid D) \cdot p(\bar{G} \mid D) p(D) + p(\bar{F} \mid \bar{D}) \cdot p(\bar{G} \mid \bar{D}) p(\bar{D}) =$

$= 0.2 \cdot 0.175 \cdot 0.7 + 0.5 \cdot 0.7 \cdot 0.3 = 0.1295$

$p(D) \overset{\text{Desc.}}{=} p(D|A,B)\,p(A)\,p(B) + p(D|\bar{A},B)\,p(\bar{A})\,p(B) +$

ev. sigur după A și B

$p(D|A,\bar{B})\,p(A)\,p(\bar{B}) + p(D|\bar{A},\bar{B})\,p(\bar{A})\,p(\bar{B}) =$

cu $A \perp B$

$= 0.9 \cdot 0.6 \cdot 0.8 + 0.6 \cdot 0.4 \cdot 0.8 + 0.6 \cdot 0.6 \cdot 0.2 +$

$+ 0.05 \cdot 0.4 \cdot 0.2 = 0.7$

$p(\bar{G}|\bar{D}) \overset{\text{Desc}}{=} p(\bar{G}|\bar{D},\bar{E})\,p(\bar{E}) + p(\bar{G}|\bar{D},\bar{E})\,p(\bar{E}) =$

ev sigur după $E, E \perp D$

$= 0.4 \cdot 0.25 + 0.8 \cdot 0.75 = 0.7$

Total

$p(E) \overset{\text{Total}}{=} p(E|C)\,p(C) + p(E|\bar{C})\,p(\bar{C}) = 0.7 \cdot 0.1 + 0.2 \cdot 0.9 = 0.25$

$p(\bar{G}|D) \overset{\text{Desc}}{=} p(\bar{G}|D,E)\,p(E) + p(\bar{G}|D,\bar{E})\,p(\bar{E}) =$

ev sigur după $E, E \perp D$

$= 0.1 \cdot 0.25 + 0.2 \cdot 0.75 = 0.175$

$p(\bar{F} \cap \bar{G}|A) \overset{\text{Dm}}{=} p(\bar{F} \cap \bar{G}|D)\,p(D|A) + p(\bar{F} \cap \bar{G}|\bar{D})\,p(\bar{D}|A) =$

(*) ⇓ ⇘ (* pt $\bar{D}$)

nou totală după D; $D \not\perp A$

$= p(\bar{F}|D) \cdot p(\bar{G}|D)\,p(D|A) + p(\bar{F}|\bar{D}) \cdot p(\bar{G}|\bar{D})\,p(\bar{D}|A) =$

$= 0.2 \cdot 0.175 \cdot 0.84 + 0.5 \cdot 0.7 \cdot 0.16 =$

$= 0.0854$

$p(A|A) \overset{\text{Desc}}{=}$ ✗ ✗

$$p(D|A) \overset{\text{DeSC}}{=\!=} p(D|A,B)\,p(B) + p(D|A,\bar{B})\,p(\bar{B}) =$$

e4 sigur după B

$A \perp B$

$$= 0.9 \cdot 0.8 + 0.6 \cdot 0.2 = 0.84$$