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341

Tema 3

I. $\min\limits_{x \in \mathbb{R}^2} \quad f(x) = x_1^4 + x_1 x_2 + (1 + x_2)^2$

2 it. Metoda Gradient Proiectat cu pas = 1 constant

a) $Q = \{ x \in \mathbb{R}^2 \mid 2x_1 + 3x_2 = 1 \}$

b) $Q = \{ x \in \mathbb{R}^2 \mid 2^{\|x\|} \leq 2 \} \rightarrow Q = \{ x \in \mathbb{R}^2 \mid \|x\| \leq 1 \}$

c) Este o pb. convexă când $Q = \{ x \in \mathbb{R}^2 \mid \min \{x_1^2, x_2^2\} \geq 1 \}$?

a)

$$\nabla f(x) = \begin{bmatrix} x_1^3 + x_2 \\ x_1 + 2(1+x_2) \end{bmatrix}$$

$k = 0 \qquad x^0 = [-1, 1]^T$

$$\nabla f(x_0) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$x^1 = \pi_Q (x_0 - 1 \cdot \nabla f x_0) = \pi_Q \left(\begin{bmatrix} -1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) = \pi_Q \left(\begin{bmatrix} -1 \\ -2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0.385 \\ 0.077 \end{bmatrix} \Rightarrow x^1 = \begin{bmatrix} 0.385 \\ 0.077 \end{bmatrix}$$

$k = 1$

$$\nabla f(x_1) = \begin{bmatrix} (0.385)^3 + 0.077 \\ 0.385 + 2 \cdot (1 + 0.077) \end{bmatrix} = \begin{bmatrix} 0.134 \\ 2.539 \end{bmatrix}$$

$$x_2 = \pi_Q \left(\begin{bmatrix} 0.385 \\ 0.077 \end{bmatrix} - 1 \cdot \begin{bmatrix} 0.134 \\ 2.539 \end{bmatrix} \right) = \pi_Q \left(\begin{bmatrix} 0.251 \\ -2.462 \end{bmatrix} \right) = \begin{bmatrix} 1.465 \\ -0.643 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1.465 \\ -0.643 \end{bmatrix}$$

b) $Q = \{ x \in \mathbb{R} \mid 2^{\|x\|} \le 2^1 \}$

$$2^{\|x\|} \le 2^1 \, /_{\log_2} \Rightarrow \|x\| \le 1 \Rightarrow \sqrt{x_1^2 + x_2^2} \le 1$$

$$\nabla f(x) = \begin{bmatrix} x_1^3 + x_2 \\ x_1 + 2(1 + x_2) \end{bmatrix}$$

• $k = 0$ $\quad x^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\nabla f(x^0) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$x^1 = \pi_Q(x^0 - 1 \cdot \nabla f(x^0)) = \pi_Q\left(\begin{bmatrix} 0 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \Rightarrow x^1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

• $k = 1$ $\quad x^1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$$\nabla f(x^1) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$x^2 = \pi_Q(x^1 - 1 \cdot \nabla f(x^1)) = \pi_Q\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) =$$

$$= \pi_Q\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) \Rightarrow x^2 = \begin{bmatrix} -0.7 \\ 0.7 \end{bmatrix}$$

c) $Q = \{ x \in \mathbb{R}^2 \mid \min\{x_1^2, x_2^2\} \ge 1 \} \Rightarrow x_1^2 \ge 1$ și $x_1^2 \ge 1 \Rightarrow x_1, x_2 \in [-\infty, -1] \cup [1, -\infty]$

$\Rightarrow Q = [-\infty, -1] \cup [1, \infty] \times [-\infty, -1] \cup [1, \infty]$

luăm $x^1 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ și $x^2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \in Q$

Segmentul de dreaptă definit de $x_1, x_2 \in Q$ pe puncte ce nu se află în Q $(ex: [0,2]^T) \Rightarrow$ **Nu** este o pb convexă!