```
/* ---------------------- */
/* PART A: DISTRIBUTED LOAD CASE */
/* ---------------------- */

/* 1A: Solve ODE for Segment 1 (distributed load) */
eq: 'diff(u(z), z, 4) = p / (E * I);  /* 4th-order beam equation */
sol_1A: desolve(eq, u(z));            /* General solution for segment 1A */
```

$$\frac{d^4}{dz^4}\,\mathrm{u}(z) = \frac{p}{EI} \qquad \text{(eq)}$$

$$\mathrm{u}(z) = \frac{z^3 \left(\frac{d^3}{dz^3}\,\mathrm{u}(z)\Big|_{z=0}\right)}{6} + \frac{z^2 \left(\frac{d^2}{dz^2}\,\mathrm{u}(z)\Big|_{z=0}\right)}{2} + z\left(\frac{d}{dz}\,\mathrm{u}(z)\Big|_{z=0}\right) + \frac{pz^4}{24EI} + \mathrm{u}(0) \qquad \text{(sol\_ 1A)}$$

```
/* 2A: Solve ODE for Segment 2 (unloaded) */
eq: 'diff(u(z), z, 4) = 0;            /* Homogeneous equation */
sol_2A: desolve(eq, u(z));            /* General solution for segment 2A */
```

$$\frac{d^4}{dz^4}\,\mathrm{u}(z) = 0 \qquad \text{(eq)}$$

$$\mathrm{u}(z) = \frac{z^3 \left(\frac{d^3}{dz^3}\,\mathrm{u}(z)\Big|_{z=0}\right)}{6} + \frac{z^2 \left(\frac{d^2}{dz^2}\,\mathrm{u}(z)\Big|_{z=0}\right)}{2} + z\left(\frac{d}{dz}\,\mathrm{u}(z)\Big|_{z=0}\right) + \mathrm{u}(0) \qquad \text{(sol\_ 2A)}$$

```
/* Apply initial conditions at z=0 */
sol_1A: rhs(subst([
u(0) = c_1,              /* Deflection at z=0 */
'diff(u(z), z, 1) = c_2,  /* Slope at z=0 */
'diff(u(z), z, 2) = c_3,  /* Moment at z=0 */
'diff(u(z), z, 3) = c_4   /* Shear at z=0 */
], sol_1A));

sol_2A: rhs(subst([
u(0) = c_5,              /* Deflection at z=0 */
'diff(u(z), z, 1) = c_6,  /* Slope at z=0 */
'diff(u(z), z, 2) = c_7,  /* Moment at z=0 */
'diff(u(z), z, 3) = c_8   /* Shear at z=0 */
], sol_2A));
```

$$\frac{pz^4}{24EI} + \frac{c_4 z^3}{6} + \frac{c_3 z^2}{2} + c_2 z + c_1 \qquad \text{(sol\_ 1A)}$$

$$\frac{c_8 z^3}{6} + \frac{c_7 z^2}{2} + c_6 z + c_5 \qquad \text{(sol\_ 2A)}$$

```
/* Interface continuity at z=Z */
eq1: subst(Z, z, sol_1A) = subst(Z, z, sol_2A);                    /* Deflection
continuity */
eq2: subst(Z, z, diff(sol_1A, z, 1)) = subst(Z, z, diff(sol_2A, z, 1));  /* Slope
continuity */
eq3: subst(Z, z, diff(sol_1A, z, 2)) = subst(Z, z, diff(sol_2A, z, 2));  /* Moment
continuity */
eq4: subst(Z, z, diff(sol_1A, z, 3)) = subst(Z, z, diff(sol_2A, z, 3));  /* Shear
continuity */

/* Boundary conditions at supports */
eq5: subst(0, z, sol_1A) = 0;              /* Deflection = 0 at left support (z=0)
*/
eq6: subst(L, z, sol_2A) = 0;                 /* Deflection = 0 at right support
(z=L) */
eq7: subst(0, z, diff(sol_1A, z, 2)) = 0;   /* Moment = 0 at left support (z=0)
*/
eq8: subst(L, z, diff(sol_2A, z, 2)) = 0;    /* Moment = 0 at right support
(z=L) */
```

$$\frac{Z^4 p}{24EI} + \frac{Z^3 c_4}{6} + \frac{Z^2 c_3}{2} + Z c_2 + c_1 = \frac{Z^3 c_8}{6} + \frac{Z^2 c_7}{2} + Z c_6 + c_5 \qquad \text{(eq1)}$$

$$\frac{Z^3 p}{6EI} + \frac{Z^2 c_4}{2} + Z c_3 + c_2 = \frac{Z^2 c_8}{2} + Z c_7 + c_6 \qquad \text{(eq2)}$$

$$\frac{Z^2 p}{2EI} + Z c_4 + c_3 = Z c_8 + c_7 \qquad \text{(eq3)}$$

$$\frac{Z p}{EI} + c_4 = c_8 \qquad \text{(eq4)}$$

$$c_1 = 0 \qquad \text{(eq5)}$$

$$\frac{L^3 c_8}{6} + \frac{L^2 c_7}{2} + L c_6 + c_5 = 0 \qquad \text{(eq6)}$$

$$c_3 = 0 \qquad \text{(eq7)}$$

$$L c_8 + c_7 = 0 \qquad \text{(eq8)}$$

```
/* Solve system for integration constants */
eq_list: [eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8]$
var_list: [c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8]$
constants: linsolve(eq_list, var_list);      /* 8x8 linear system solution */
```

$$\left[c_1 = 0\,,\, c_2 = \frac{\left(Z^4 - 4LZ^3 + 4L^2Z^2\right)p}{24EIL}\,,\, c_3 = 0\,,\, c_4 = \frac{\left(Z^2 - 2LZ\right)p}{2EIL}\,,\, c_5 = -\left(\frac{Z^4 p}{24EI}\right)\,,\, c_6 = \frac{\left(Z^4 + 4L^2Z^2\right)p}{24EIL}\,,\, c_7 = -\left(\frac{Z}{2}\right.\right.$$

(constants)

```
(%i1) /* Apply constants to solutions */
sol_1A: subst(constants, sol_1A);          /* Final solution segment 1A */
sol_2A: subst(constants, sol_2A);          /* Final solution segment 2A */

/* ---------------------- */
/* PART B: POINT LOAD CASE */
/* ---------------------- */

/* 1B: Solve ODE for both segments (unloaded) */
eq: 'diff(u(z), z, 4) = 0;          /* Homogeneous equation */
sol_1B: desolve(eq, u(z));          /* General solution (same for both segments)
*/
sol_2B: sol_1B;                     /* Copy solution for segment 2B */
```

$$\frac{pz^4}{24EI} + \frac{\left(Z^2 - 2LZ\right)pz^3}{12EIL} + \frac{\left(Z^4 - 4LZ^3 + 4L^2Z^2\right)pz}{24EIL} \qquad \text{(sol\_ 1A)}$$

$$\frac{Z^2pz^3}{12EIL} - \frac{Z^2pz^2}{4EI} + \frac{\left(Z^4 + 4L^2Z^2\right)pz}{24EIL} - \frac{Z^4p}{24EI} \qquad \text{(sol\_ 2A)}$$

$$\frac{d^4}{dz^4}\,\mathrm{u}(z) = 0 \qquad \text{(eq)}$$

$$\mathrm{u}(z) = \frac{z^3\left(\frac{d^3}{dz^3}\,\mathrm{u}(z)\Big|_{z=0}\right)}{6} + \frac{z^2\left(\frac{d^2}{dz^2}\,\mathrm{u}(z)\Big|_{z=0}\right)}{2} + z\left(\frac{d}{dz}\,\mathrm{u}(z)\Big|_{z=0}\right) + \mathrm{u}(0) \qquad \text{(sol\_ 1B)}$$

$$\mathrm{u}(z) = \frac{z^3\left(\frac{d^3}{dz^3}\,\mathrm{u}(z)\Big|_{z=0}\right)}{6} + \frac{z^2\left(\frac{d^2}{dz^2}\,\mathrm{u}(z)\Big|_{z=0}\right)}{2} + z\left(\frac{d}{dz}\,\mathrm{u}(z)\Big|_{z=0}\right) + \mathrm{u}(0) \qquad \text{(sol\_ 2B)}$$

```
/* Apply initial conditions at z=0 */
sol_1B: rhs(subst([
u(0) = c_1,              /* Deflection at z=0 */
'diff(u(z), z, 1) = c_2,  /* Slope at z=0 */
'diff(u(z), z, 2) = c_3,  /* Moment at z=0 */
'diff(u(z), z, 3) = c_4   /* Shear at z=0 */
], sol_1B));

sol_2B: rhs(subst([
u(0) = c_5,              /* Deflection at z=0 */
'diff(u(z), z, 1) = c_6,  /* Slope at z=0 */
'diff(u(z), z, 2) = c_7,  /* Moment at z=0 */
'diff(u(z), z, 3) = c_8   /* Shear at z=0 */
], sol_2B));
```

$$\frac{c_4 z^3}{6} + \frac{c_3 z^2}{2} + c_2 z + c_1 \qquad \text{(sol\_ 1B)}$$

$$\frac{c_8 z^3}{6} + \frac{c_7 z^2}{2} + c_6 z + c_5 \qquad \text{(sol\_ 2B)}$$

```
/* Interface conditions at z=W (point load location) */
eq1: subst(W, z, sol_1B) = subst(W, z, sol_2B);                    /* Deflection
continuity */
eq2: subst(W, z, diff(sol_1B, z, 1)) = subst(W, z, diff(sol_2B, z, 1));  /* Slope
continuity */
eq3: subst(W, z, diff(sol_1B, z, 2)) = subst(W, z, diff(sol_2B, z, 2));   /*
Moment continuity */
/* Shear force jump due to point load F */
eq4: -E*I*subst(W, z, diff(sol_1B, z, 3)) + E*I*subst(W, z, diff(sol_2B, z, 3))
= F;

/* Boundary conditions at supports (same as Part A) */
eq5: subst(0, z, sol_1B) = 0;              /* Deflection = 0 at left support */
eq6: subst(L, z, sol_2B) = 0;              /* Deflection = 0 at right support */
eq7: subst(0, z, diff(sol_1B, z, 2)) = 0;   /* Moment = 0 at left support */
eq8: subst(L, z, diff(sol_2B, z, 2)) = 0;   /* Moment = 0 at right support */
```

$$\frac{W^3 c_4}{6} + \frac{W^2 c_3}{2} + W c_2 + c_1 = \frac{W^3 c_8}{6} + \frac{W^2 c_7}{2} + W c_6 + c_5 \qquad \text{(eq1)}$$

$$\frac{W^2 c_4}{2} + W c_3 + c_2 = \frac{W^2 c_8}{2} + W c_7 + c_6 \qquad \text{(eq2)}$$

$$W c_4 + c_3 = W c_8 + c_7 \tag{eq3}$$

$$EI c_8 \text{ - } EI c_4 = F \tag{eq4}$$

$$c_1 = 0 \tag{eq5}$$

$$\frac{L^3 c_8}{6} + \frac{L^2 c_7}{2} + L c_6 + c_5 = 0 \tag{eq6}$$

$$c_3 = 0 \tag{eq7}$$

$$L c_8 + c_7 = 0 \tag{eq8}$$

```
->      /* Solve system for integration constants */
        eq_list: [eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8]$
        var_list: [c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8]$
        constants: linsolve(eq_list, var_list);      /* 8x8 linear system solution */
```

$$\left[c_1 = 0\,,\, c_2 = \frac{FW^3 \text{ - } 3FLW^2 + 2FL^2W}{6EIL}\,,\, c_3 = 0\,,\, c_4 = \frac{FW \text{ - } FL}{EIL}\,,\, c_5 = \text{-}\left(\frac{FW^3}{6EI}\right)\,,\, c_6 = \frac{FW^3 + 2FL^2W}{6EIL}\,,\, c_7 = \text{-}\left(\right.\right. \tag{constants}$$

```
->      /* Apply constants to solutions */
        sol_1B: subst(constants, sol_1B);          /* Final solution segment 1B */
        sol_2B: subst(constants, sol_2B);          /* Final solution segment 2B */

        /* ---------------------- */
        /* NUMERICAL EVALUATION */
        /* ---------------------- */

        /* Define physical parameters */
```

$$\frac{(FW \text{ - } FL)\, z^3}{6EIL} + \frac{\left(FW^3 \text{ - } 3FLW^2 + 2FL^2W\right) z}{6EIL} \tag{sol_ 1B}$$

$$\frac{FWz^3}{6EIL} - \frac{FWz^2}{2EI} + \frac{\left(FW^3+2FL^2W\right)z}{6EIL} - \frac{FW^3}{6EI} \qquad \text{(sol\_ 2B)}$$

```
(->)  parameters: [p = 3000, F = 50000, Z = 4, W = 7, L = 10];

      /* Substitute numerical values */
      sol_1A: ev(sol_1A, parameters);      /* Numerical solution segment 1A */
      sol_1B: ev(sol_1B, parameters);      /* Numerical solution segment 1B */
      sol_2A: ev(sol_2A, parameters);      /* Numerical solution segment 2A */
      sol_2B: ev(sol_2B, parameters);      /* Numerical solution segment 2B */
```

$$[p=3000\,,F=50000\,,Z=4\,,W=7\,,L=10] \qquad \text{(parameters)}$$

$$\frac{125z^4}{EI} - \frac{1600z^3}{EI} + \frac{51200z}{EI} \qquad \text{(sol\_ 1A)}$$

$$\frac{227500z}{EI} - \frac{2500z^3}{EI} \qquad \text{(sol\_ 1B)}$$

$$\frac{400z^3}{EI} - \frac{12000z^2}{EI} + \frac{83200z}{EI} - \frac{32000}{EI} \qquad \text{(sol\_ 2A)}$$

$$\frac{17500z^3}{3EI} - \frac{175000z^2}{EI} + \frac{1452500z}{EI} - \frac{8575000}{3EI} \qquad \text{(sol\_ 2B)}$$

```
(->)  /* Post-processing */
      deflection_at_7_1: ev(sol_1B + sol_2A, z = 7);          /* Total deflection
      at z=7 */
      deflection_at_7_2: ev(sol_2A + sol_2B, z = 7);          /* Total deflection
      at z=7 */
      slope_total: subst(0, z, -diff(sol_1A + sol_1B, z, 1)); /*  Slope at z=0 */
```

$$\frac{834600}{EI} \qquad \text{(deflection\_ at\_ 7\_ 1)}$$

$$\frac{834600}{EI} \qquad \text{(deflection\_ at\_ 7\_ 2)}$$

$$-\left(\frac{278700}{EI}\right) \qquad \text{(slope\_ total)}$$