Examen Parcial I

In this model the internal cellular state is described by the vector:

$$\mathbf{x} = (x_1, x_2, \cdots, x_n)$$

where x_i is the concentration inside of the cell of the chemical specie *i.* Some of these species are nutrients and can diffuse from the environment into the cell, through the cell membrane, with the help of some other species which act as nutrients carriers. The cell metabolism is described here by a number of catalytic reactions which ultimately converts the nutrients into some other species. If this process leads to the increase of most of the concentrations, then the cell volume also increases, eventually doubling its value. When this happen the species evolution is stopped and it is considered that the cell binary fission takes place.

Since we are dealing with concentrations, for simplicity it is assumed that:

$$\sum_i x_i(t) = 1$$

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The evolution of the cell state is given by the ODE system:

$$\frac{dx_i}{dt} = R_i - x_i \sum_j R_j$$

where

$$R_i(\mathbf{x}) \equiv \sum_{j,l} \sigma_{j,i,l}\ x_j x_l - \sum_{j',l'} \sigma_{i,j',l'}\ x_i x_{l'} \Big(+\ D x_{m_i} (\mathrm{X}_i - x_i) \Big),$$

the first two terms stand for the reactions involving the species *i* as product or substrate, catalyzed by the species *l* and *l'*. The factor $\sigma_{i,j,l}$ is equal to 1 if reaction *i* + *l* --> *j* + *l* takes place and 0 otherwise. The third term is written between parenthesis to denote that it is added only to the equations of nutrient species. D is the diffusion coefficient, X_i and x_{mi} are, correspondingly, the concentration of external nutrients and their transporters into the cell.

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The evolution of each cell stops when it divides into two daughter cells. In this simulation this is considered to happen when the volume of the mother cell doubles its initial value. Then, the time t_d when the cellular fission takes places is given here by the following condition:

$$\int_{t_0}^{t=t_d} \sum_j R_j(\mathbf{x})\, dt \; = ln\ 2.$$

Problem:

The R_i of ten cells, each one with 250 chemical species, are provided as ANSI C source files, together with a set of initial conditions. In this case, D=4 and X_i=X=0.2.

Solve the corresponding ODE systems and find which cell divides faster.