Tight Formulation of a Power System

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1 Notations

Variables

$f_{a,b}$	Energy flow between node a and b
$p_{g,t}$	Energy dispatched from unit g at hour t
$\overline{p}_{g,t}$	Maximum energy available from unit g at hour t
$p'_{g,t}$	Energy above the minimum from unit g at time t
$\overline{p}'_{g,t}$	Maximum energy above minimum that is available from unit g at time t
$s_{n,t}$	Energy shortfall at node n and time t
s^r_t	Reserve shortfall at time t
$u_{g,t}$	Commitment status of unit g at time t
$v_{g,t}$	Start-up of unit g at time t or switch variable
$w_{g,t}$	Shutdown of unit g at time t
$\theta_{n,t}$	Volt angle at node n and time t
$\sigma_{g,t}$	Spinning reserve from unit g at time t

Technical parameters

$B_{a,b}$	Line susceptance between node a and b
D_g	Remaining hours that unit g must be off at $t = 1$
$F_{a,b}$	Line capacity between node a and b
F_{N1}	N1-criterion factor, set at 0.75
F_L	Transmission loss as a factor of generation, set at 0.075
G	Set of thermal units
HR_g	Heat rate of unit g
N	Set of nodes. Note that $G \in N$
$Q_{n,t}$	Load at node n and time t
R_t	Reserve requirement of the system at time t
$\overline{P}_g$	Maximum power output of unit g
$\underline{P}_g$	Minimum power output of unit g
RU_g	Ramp-up time of unit g
RD_g	Ramp-down time of unit g
SU_g	Start-up rate (MW/h)
SD_g	Shutdown rate (MW/h)
T	simulation horizon
T^{RD}	Time required to reach shutdown capacity $\lfloor \frac{\bar{P}-SU}{RD} \rfloor$
T^{RU}	Time required to reach maximum capacity $\lfloor \frac{\bar{P}-SU}{RU} \rfloor$
TD_g	Minimum down time of unit g
TU_g	Minimum up time of unit g
U_g	Remaining hours that unit g must be on at $t = 1$

Economic parameters

CF_g	Fuel cost of unit g
CS_g	Switching cost of unit g

2 Rationale for reformulation

A tighter formulation can ease the implementation of Dantzig-Wolfe decomposition. If we can disaggregate an individual generator into its own subproblem, solving the relaxed problem will yield integer solutions. This might help us avoid implementing the branch and bound algorithm. Implementing the Dantzig-Wolfe decomposition with branch and bound is highly inefficient in terms of computation and memory. Furthermore, the literature on unit commitment suggests tighter formulation can decrease a solver's runtime. Here, we explore a candidate formulation for a power system model.

We based our selection on the “Tight formulation” described in Knueven, Ostrowski, Watson (2019). We adopt the 2-bin formulation that models the energy being dispatched at each time step. We exclude the shutdown variable because we do not plan to model the shutdown costs.

3 Description of the Tight formulation

We drop the g notation when discussing the constraints of a single unit to simplify the notation whenver possible.

3.1 Objective function

The power system model minimizes the cost of supplying energy to meet the demand throughout the simulation horizon. Since we focus on the cost of operation, we model the costs of energy dispatched, unit start-up, and energy shortfall.

3.2 Minimum up/down time

The minimum up/down time is a function of the unit status. First, we define the remaining required up/down time at the first time step. This models the continuity of unit commitment from the previous time simulation period.

Minimum up

$$\sum_{i=1}^{\min\{U,T\}} u_i = \min\{U, T\} \tag{1}$$

Minimum down

$$\sum_{i=1}^{\min\{D,T\}} u_i = 0 \tag{2}$$

Malkin (2003) and Rajan & Takriti (2005) independently formulated the minimum up/down time. Although they adopted the 3-bin formulation, we can project out the shutdown variable using the relationship among the unit status variable, the start-up variable, and the shutdown variable.

$$u_t - u_{t-1} = v_t - w_t \tag{3a}$$

$$w_t = u_{t-1} - u_t + v_t \tag{3b}$$

The resulting formulation are still the facets of the convex hull of the minimum up/down polytope. The computation aspect of the 2-bin formulation was investigated by Yang et al. (2017), and Atakan et al. (2018).

In describing the inequalities, we first present the 3-bin formulation, then apply Equation (3b) to project out the shutdown variable.

Minimum up

$$\sum_{i=t-TU+1}^{t} v_i \leq u_t \quad t \in \{TU, ..., T\} \tag{4}$$

Minimum down

$$\sum_{i=t-TD+1}^{t} w_i \leq 1 - u_t \quad t \in \{TD, ..., T\} \tag{5}$$

We can remove the shutdown variable from Equation (5) by using the relationship found in Equation (3b). The result is Equation (6).

$$\sum_{i=t-TD+1}^{t} v_i \leq 1 - u_{t-TD} \quad t \in TD, ..., T \tag{6}$$

Also, Equation (3a) without the shutdown variable becomes Equation (7).

$$u_t - u_{t-1} \leq v_t \quad \forall t \in T \tag{7}$$

In summary, we formulate the minimum up/down duration using Equations (1) (2) (4) (6) (7).

3.3 Generation limits

A thermal unit operates within the maximum/minimum capacities. Instead of using the absolute amount of energy, we formulate the power system using the amount of energy above the minimum capacity p'. This makes our energy variable continuous, instead of semi-continuous when using the absolute value.

Similar to the absolute value case, the energy dispatchable above minimum is less than the maximum energy available above minimum.

$$p'_g \leq \overline{p}'_g \tag{8}$$

Knueven et al. (2019) suggests adding constraints to ensure that the capacity is not exceeded during a unit's start-up and shutdown. Their Tight formulation specifies a set of inequalities for the case when there is a thermal unit with a minimum up duration of 1 hour and the shutdown rate (MW/hr) is not equal to the start-up rate. Since we do not expect to encounter this specific case, we turn to a set of inequalities proposed by Morales-Espana et al. (2013).

When a unit has a minimum up time larger than 1 hour, we can strengthen the upper bound with the following inequalities.

$$p'_t + \sigma_t \leq (\overline{P} - \underline{P})u_t - (\overline{P} - SU)v_t - (\overline{P} - SD)w_{t+1} \tag{9}$$

For peaking units with the minimum up/down time of 1 hour, we use the following two constraints.

$$p'_t + \sigma_t \leq (\overline{P} - \underline{P})u_t - (\overline{P} - SU)v_t \tag{10a}$$

$$p'_t + \sigma_t \leq (\overline{P} - \underline{P})u_t - (\overline{P} - SD)w_{t+1} \tag{10b}$$

The upper bound on generation changes during unit ramping. We further introduce inequalities to strengthen the maximum capacity based on Pan & Guan (2016). The inequalities bind the dispatched energy to the unit status and the switch variable. The following inequalities are written as a function of absolute energy p_t, which we can replace with p'_t using $p_t = p'_t + \underline{P_t} u_t$.

$$\overline{p}_t \leq \overline{P} u_t - (\overline{P} - SD) w_{t+1} - \sum_{i=0}^{\min\{TU-2, T^{RU}\}} (\overline{P} - SU - i \times RU) v_{t-i} \tag{11}$$

Since Equation (11) does not cover the start-up trajectory when $TU - 2 < T^{RU}$, we introduce another set of inequalities.

$$\overline{p}_t \leq \overline{P} u_t - \sum_{i=0}^{\min\{TU-1, T^{RU}\}} (\overline{P} - SU - i \times RU) v_{t-i} \tag{12}$$

In parallel to the case of the ramp-up trajectory, we also specify the upper bound on p_t when a unit is shutting down. We first identify the interval of interest based on step t.

$$K_t^{SD} = \min\{T^{RD}, TU - 1, T - t - 1\} \tag{13}$$

$$K_t^{SU} = \min\{T^{RU}, TU - 2 - [K_t^{SD}]^+, t - 1\} \tag{14}$$

Equation (13) specifies the number of future time steps from t that governs ramping down. Given a large K^{SD}, the unit can generate more energy at t because there are future time steps available for ramp down. Similarly, Equation (14) refers to the available period to ramp up.

Given K_t^{SD} and K_t^{SU}, we strengthen the ramp-down trajectory with the following inequalities.

$$\begin{aligned} p_t \leq \overline{P} u_t - \sum_{i=0}^{K_t^{SD}} (\bar{P} - SD - i \times RD) w_{t+1+i} \\ - \sum_{i=0}^{K_t^{SU}} (\overline{P} - SU - i \times RU) v_{t-i} \quad if\ K_t^{SD} > 0 \end{aligned} \tag{15}$$

Note that we need to convert the above equations to be a function of p' and remove the shutdown variable.

3.4 Ramp limits

While the previous section specifies upper bounds for dispatched energy, the ramp limits specifies that the change in dispatched energy should be below a limit. Damci-Kurt et al. (2016) describes inequalities that are facets of the ramping polytope.

Ramp-up
$$\overline{p}'_t - p'_{t-1} \leq (SU - \underline{P} - RU)v_t + RU \times u_t \tag{16}$$

Ramp-down
$$p'_{t-1} - \overline{p}'_t \leq (SD - \underline{P} - RD)w_t + RD \times u_{t-1} \tag{17}$$

3.5 System constraints

When we include the system constraints in the power system model, the constraint polytope might no longer describe the convex hull. Therefore, the constraints are supposed to be in the master problem when implementing the Dantzig-Wolfe decomposition.

Transmission limits
This formulation uses the linearized power flow equation.

$$f_{a,b,t} = B_{a,b}(\theta_{a,t} - \theta_{b,t}) \tag{18a}$$

$$-F_{a,b} \leq f_{a,b,t} \leq F_{a,b} \tag{18b}$$

$$-\pi \leq \theta_{n,t} \leq \pi \tag{18c}$$

$$\theta_{ref,t} = 0 \tag{18d}$$

Flow balance
The energy import, export, and consumption are balanced at each node. Note that renewables are injected directly into a node.

$$\sum_{g \in G} p_{g,t} + p_{RE,t} + \sum_{a \in N} f_{a,b,t} - \sum_{b \in N} f_{a,b,t} + s_{n,t} = Q_{n,t} \tag{19}$$

Since Equation (19) is an equality, the shortfall variable comprises the positive and the negative parts. This allows the slack variable to carry out excess or deficit energy balance.

$$s_{n,t} = s^+_{n,t} - s^-_{n,t} \tag{20}$$

$$s^+_{n,t}, -s^-_{n,t} \geq 0 \tag{21}$$

Reserve requirement

The reserve requirement is based on Carrion & Arroyo (2006) and Ostroski et al. (2012). Knueven et al. (2019) formulates the reserve variable to cover the whole system. We might reformulate this variable to be node-specific to allow for more accurate modeling of the spinning reserve.

$$\sum_{g \in G} \bar{p}_g + s^R_{n,t} \geq \sum_{n \in N} Q_{n,t} + R_t \tag{22}$$