Warm-Ups 06

- Due Feb 25 at 11:59pm
- Points 10
- Questions 5
- Available Feb 24 at 5pm - Feb 25 at 11:59pm
- Time Limit None
- Allowed Attempts Unlimited

Take the Quiz Again

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	14 minutes	8 out of 10

Correct answers will be available on Feb 26 at 12am.

Score for this attempt: 8 out of 10
Submitted Feb 25 at 10pm
This attempt took 14 minutes.

Question 1
2 / 2 pts

Which of the following are true about little-o and big-O?

- [x] If $f \in o(g)$, then $f \in O(g)$
- [x] If $f \in o(g)$, then $g \notin O(f)$
- [x] When $f \sim g$ and $f \notin o(g)$, $f \in O(g)$
- [x] When $f \nsim g$ and $f \in o(g)$, $f \in O(g)$
- [ ] If $f \in O(g)$, then $f \in o(g)$

Choice 1 is true, because f in o(g) means that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 < \infty$, which implies f in O(g).

Choice 2 is true, because if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$, then $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \infty$, so g is not in O(f).

Choices 3 and 4 are true because if f~g or f in o(g), then $\lim_{n\to\infty} \frac{f(n)}{g(n)} < \infty$, so f in O(g).

Choice 5 is not true. There are many counterexamples, e.g. f=g.

Question 2

2 / 2 pts

Which of these relationships apply for $f(n) = \log_3(n)$ and $g(n) = \log_7(n)$?

- [ ] $f \sim g$
- [ ] $f \in o(g)$
- [x] $f \in O(g)$
- [x] $f \in \Theta(g)$

Notice that $\frac{f(n)}{g(n)} = \frac{\ln(n)/\ln(3)}{\ln(n)/\ln(7)} = \frac{\ln(7)}{\ln(3)}$, so f and g are *exactly proportional to each other*! (Changing the base of a log is equivalent to multiplying by a constant factor.) So $f \in \Theta(g)$, which also implies $f \in O(g)$ and $g \in O(f)$.

Question 3

2 / 2 pts

If $f \in \Theta(g)$, then which of the following MUST be true?

- [x] $g \in \Theta(f)$
- [ ] $f \in o(g)$
- [ ] $g \in o(f)$
- [x] $f \in O(g)$
- [x] $g \in O(f)$
- [ ] $f \sim g$

$f \in \Theta(g)$ means that f and g bound each other above and below, to within constant factors. By that definition, it makes sense that the relation is symmetric and $g \in \Theta(f)$, so choice 1 must be true. $f \in \Theta(g)$ iff $f \in O(g)$ and $g \in O(f)$ by definition, so we know choices 4 and 5 must be true as well. The rest of the choices don't necessarily follow from the given statement.

Question 4

2 / 2 pts

If $f \in \Theta(g)$, then which of the following CAN be true?

- [x] $g \in \Theta(f)$
- [ ] $f \in o(g)$

- [ ] $g \in o(f)$
- [x] $f \in O(g)$
- [x] $g \in O(f)$
- [x] $f \sim g$

The same explanation from the previous question applies here for statements 1, 4, 5, which must be true. We also add that if $f \in \Theta(g)$, then $f \sim g$ could also be true. However, we know that statements 2, 3 are false because they denote that one of f, g is asymptotically smaller than the other, which is precluded by Θ.

Incorrect

Question 5

0 / 2 pts

Consider the quantity $\frac{(2n)!}{2^{2n}(n!)^2}$. This will come up later in the course (it is the probability that in flips of a fair coin, exactly will be Heads). Which of the following formulae is asymptotically equal to this? As a reminder, Stirling's Formula says $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$.

- (selected) $\frac{1}{\sqrt{2\pi n}}$
- $\frac{1}{\sqrt{\pi n}}$
- $\sqrt{\frac{2}{\pi n}}$
- $\sqrt{2\pi n}$
- $2^n\sqrt{2\pi n}$

Quiz Score: 8 out of 10