Problem 147: Playing with Polynomials

Difficulty: Medium

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Originally Published: Code Quest 2021

Problem Background

If you've taken an algebra course, you're probably familiar with polynomial expressions. These expressions usually take a form similar to the one shown below:

$$f(x) = a + bx + cx^2 + dx^3 + \cdots$$

The letters in this expression (except for x) represent the coefficients of each term in the equation. The small numbers appearing above most of the x's are the exponents of each term. Large equations like these are often used in physics; for example, the equation below can be used to find the distance travelled by an accelerating object.

$$d(t) = x + vt + 0.5at^2$$

Occasionally it's necessary to multiply polynomial expressions together to create an even larger expression. Multiplying small polynomials can be done by hand without too much trouble, but very large ones can prove difficult. Let's write a program to do the work for us.

Problem Description

When multiplying polynomial expressions, each term in the first expression needs to be individually multiplied with each term in the second expression. You might have learned the "FOIL" method in school for multiplying two-term polynomials together. To demonstrate, let's multiply the expressions $(2x + 3x^2)$ and $(4x + x^2)$.

The letters in FOIL stand for First, Outer, Inner, and Last, indicating how the terms should be multiplied together. We start with the first term in each expression:

$$2x * 4x = 8x^2$$

Just as in normal multiplication, we multiply the coefficients together. The exponents over the x's (which are both 1, since they're not displayed) get added together, however; remember that $x \times x = x^2$.

We repeat this process with the three other pairs of terms; the outer terms (first term of the first expression and second term of the second expression), the inner terms (second term of the first and first term of the second), and the last terms (of both expressions):

$$2x * x^2 = 2x^3$$

$$3x^2 * 4x = 12x^3$$
$$3x^2 * x^2 = 3x^4$$

Each of these new terms gets added together to form a new polynomial expression with four terms:

$$(2x + 3x^2) * (4x + x^2) = 8x^2 + 2x^3 + 12x^3 + 3x^4$$

We're not quite done yet, however. As you can see, there are two terms with an exponent of 3; $2x^3$ and $12x^3$. Since these terms have the same exponent, we can simplify the expression by simply adding their coefficients together. This results in a final answer of:

$$(2x + 3x^2) * (4x + x^2) = 8x^2 + 14x^3 + 3x^4$$

The FOIL method specifically applies to two-term polynomial expressions, but the same principle applies to polynomials with more terms. Each term in the first expression gets multiplied by each term in the second expression; multiplying coefficients and adding exponents. Once all the terms have been multiplied, simplify the expression by adding together coefficients that share an exponent.

Sample Input

The first line of your program's input, received from the standard input channel, will contain a positive integer representing the number of test cases. Each test case will include two lines representing polynomial expressions. Each line will contain between 2 and 9 (inclusive) integers separated by spaces, representing the coefficients of a polynomial expression. The first integer represents the coefficient paired with the exponent 0; the second, the coefficient paired with the exponent 1, and so on.

```
2
0 2 3
0 4 1
1 2 3 4
0 5 6
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Sample Output

For each test case, your program must print a single line containing the polynomial expression obtained by multiplying the two given expressions together. Each term in the polynomial with a non-zero coefficient should be printed in order of increasing exponent. These terms should be separated from other terms by a plus sign (+) or minus sign (-) (use a minus sign when the following coefficient is negative), and should contain the following:

- The integer coefficient. The coefficient should not be printed if its absolute value is equal to 1 AND the exponent is not equal to 0.
- The lowercase letter x, a caret (^), and the integer exponent. The caret and exponent should not be printed if the exponent is less than or equal to 1. The letter x should not be printed if the exponent is equal to 0.

8x^2+14x^3+3x^4

5x+16x^2+27x^3+38x^4+24x^5