Problem 124: Matrix Math

Difficulty: Medium

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Problem Background

3-dimensional graphics are used in almost every video game. As your character runs across the battlefield or you look around to get a better view of the level, your game console or computer is constantly working to recalculate the view you're seeing. The position and orientation of your viewpoint - the camera - is crucial to the computer's calculations, so it knows what to display. Many systems keep track of the camera's position and the positions of many other 3D models within the view using a matrix.

A matrix is a table of numbers that allows us to represent multi-dimensional values. A computer using a matrix for graphics rendering will frequently alter the values within a matrix to represent motion or rotation of the camera or an object in view. Most often, it will multiply a matrix by a vector (a small matrix with trigonometric values) to determine this new location.

Problem Description

You will need to write a program that can multiply matrices together. Matrix multiplication is a bit more complicated than multiplying two numbers - for one thing, not every pair of matrices can be multiplied.

To multiply two matrices, the first matrix must have the same number of columns as the second matrix has rows. See below:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

Matrix A is a 3x2 matrix; three rows and two columns. Matrix B is a 2x4 matrix; two rows and four columns. The number of columns in A equals the number of rows in B - 2 - so we can multiply matrix A by matrix B to create a new, larger matrix C. C will have the same number of rows as A and the same number of columns as B.

How does this happen? Each value in the product matrix is calculated by multiplying each value in a single column of A by the corresponding value in a row of B, then adding those products together. This is why the numbers of rows and columns in A and B is important - each value needs to be matched up to a value in the other matrix.

To put things another way, consider this formula. When calculating the matrix product AB = C, where...

- A is an *n* x *m* matrix (*n* rows and *m* columns),
- B is an *m* x *p* matrix (*m* rows and *p* columns),
- C is an *n* x *p* matrix (*n* rows and *p* columns),
- *x* is the row of the value being calculated, and
- *y* is the column of the value being calculated:

$$\mathbf{C}_{x,y} = \mathbf{A}_{x,1} * \mathbf{B}_{1,y} + \mathbf{A}_{x,2} * \mathbf{B}_{2,y} + \cdots + \mathbf{A}_{x,p} * \mathbf{B}_{n,y}$$

Let's calculate the product of the A and B matrices above.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$
$$\downarrow$$
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} (1*1)+(2*5) & (1*2)+(2*6) & (1*3)+(2*7) & (1*4)+(2*8) \\ (3*1)+(4*5) & (3*2)+(4*6) & (3*3)+(4*7) & (3*4)+(4*8) \\ (5*1)+(6*5) & (5*2)+(6*6) & (5*3)+(6*7) & (5*4)+(6*8) \end{bmatrix}$$
$$=$$
$$\begin{bmatrix} 11 & 14 & 17 & 20 \\ 23 & 30 & 37 & 44 \\ 35 & 46 & 57 & 68 \end{bmatrix}$$

As you can see, to get each value in the result matrix C, you multiply the first value in that row of A by the first value in that column of B, then multiply the second value in A's row by the second value in B's column. Adding these products together gets you the result for that value.

Note, however, that matrix multiplication is not commutative; the order in which you multiply matrices matters. While AB=C, BA≠C. In the case above, the result of BA is actually undefined. B cannot be multiplied by A because B has more columns than A has rows - you wouldn't be able to match up all of the values correctly to perform the multiplication.

Sample Input

The first line of your program's input, received from the standard input channel, will contain a positive integer representing the number of test cases. Each test case will include the following lines of input:

- A line containing four positive integers, separated by spaces. These integers represent, in order:
  - N, the number of rows in matrix A
  - M, the number of columns in matrix A
  - P, the number of rows in matrix B
  - Q, the number of columns in matrix B
- N lines containing the integer values in matrix A. Each line represents a row; columns are separated by spaces.
- P lines containing the integer values in matrix B. Each line represents a row; columns are separated by spaces.

```
2
3 2 2 4
1 2
3 4
5 6
1 2 3 4
5 6 7 8
2 4 3 2
1 2 3 4
5 6 7 8
1 2
3 4
5 6
```

Sample Output

For each test case, your program must perform the multiplication AB and print the result matrix C. Print each row of the result matrix on a separate line, and separate columns with spaces. If the multiplication AB cannot be performed, print "undefined" instead.

```
11 14 17 20
23 30 37 44
35 46 57 68
undefined
```