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Report of the application project at the Faculty of AMP

Simulation of a medical therapy method with finite elements

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1. Introduction to radio frequency ablation

Lets talk about:
- Medical Treatment of Tumor
- Radio frequency ablation
- Why RFA Simulation is important
- Motivation
- This project in General

2. Computer-aided simulation of radio frequency ablation

2.1. About Errors in simulations and numerical approaches

- see TUM dissertation
- There are different sources for errors following the simulation from the line from the real problem down to the discrete solution
- Idealization error: discrepancy between reality and the idealized reality and the idealized constitutive laws and boundary conditions -> Systems are often way more complex in reality, every patient is different
- Modeling errors: discrepancy between mathematical formulation and physical model -> e.g. using dimensionally reduced approaches, like linear dependencies or even constant parameters
- Discretization errors: discrepancy between the continous description and discrete discription of the model
- Solution errors: using iterative approximation methods and rounding errors
- It's basically a butterfly effect
- Optimizing one error source often conflicts with another one -> e.g. handling nonlinearity can cause fatal numerical errors (at least that's what Kroeger said ...)

2.2. The physics behind radio frequency ablation

3. Mathematical aspects of discrete simulation

3.1. Theory of finite elements

3.1.1. Elliptical problems

- Elliptical problems in general
- Parabolic / time-dependent problems
- build up system of PDE's to describe problem
- Using the cylindric domain, different domains

3.1.2. FEM for electrical fields

- special domain
- boundary conditions

3.1.3. FEM for temperature fields

- boundary condition (heat source or sink)

3.2. Solving systems of ODE over time

- This is for temperature distribution

3.3. Axial symmetrie

- Using axial symmetrie to simplify computations
- reducing one dimension 3D -> 2D
- significant savings calculations time and complexity
- approach: fourier decomposition in angular direction to reduce dependency on the angular φ
- using static models, only dependency on space
- maybe Torus elements

3.4. FEM in cylindric Coordinates

- Rewrite the equations to cylindric coordinates

Laplace in cartesian coordinates:

$$\nabla^2 := \Delta := \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{1}$$

Laplace in cylindric coordinates:

$$\Delta := \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \tag{2}$$

Laplace in polar coordinates:

$$\Delta := \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} \tag{3}$$

4. Discretization of PDEs

4.1. PDE for Electric potential

4.1.1. Weak formulation of the problem

Three parts are interesting: - Inner domain
- Fixed Potential of electrodes

- Inner domain
- Outer boundary -> Robin

Constant material parameters:

4.1.2. Inner Domain

- The electric potential of the inner domain is described as :

$$-\nabla \cdot (\sigma(x,y,z,t)\nabla \varphi(x,y,z,t)) = 0 \tag{4}$$

- Elliptical boundary problem
- Assuming constant material parameters: $\nabla \sigma = 0$
- Solution is independent from σ so we can cut it out - Equation becomes Laplaces' equation, phi becomes time independent

$$-\Delta \varphi(x,y,z) = 0 \tag{5}$$

- Using a cylindric domain, we can use cylinder coordinates (see ref Laplace in cylinder)

$$-\Delta \varphi(r,\phi,z) = -\frac{1}{r}\frac{\partial \varphi}{\partial r} - \frac{\partial^2 \varphi}{\partial r^2} - \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \phi^2} - \frac{\partial^2 \varphi}{\partial z^2} = 0 \tag{6}$$

- Since the domain has axis symmetry, the solution for φ is independant from the angular ϕ
- So equation simplifies to

$$-\frac{1}{r}\frac{\partial \varphi}{\partial r} - \frac{\partial^2 \varphi}{\partial r^2} - \frac{\partial^2 \varphi}{\partial z^2} = 0 \tag{7}$$

- PDE is now parabolic and no longer elliptic
- We will care about a more complex formulation later

4.1.3. Electrodes

- $\phi = \pm 1$

4.1.4. Outer boundary

- For first try, a simplification with natural boundary conditions

$$n \cdot \nabla \varphi = 0 \tag{8}$$

- In cylindrical coordinates

$$TODO \tag{9}$$

4.2. PDE for temperature Distribution

4.2.1. Weak formulation

Following Kroeger, the temperature distribution is modeled by the heat equation:

$$\partial_t(\rho c T) - \nabla \cdot (\lambda \nabla T) = Q \tag{10}$$

The heat equation is a well known parabolic partial differential equation.

We are assuming ρ and c are constant
ρ = density
c = specific heat capacity
λ = thermal conductivity, which is depending on T
T = T(r,z,t) = temperature
Q = Q(r,z,t) = heat energy

Cylindrical coordinates: see 'Transient Heat Transfer in a Partially Cooled Cylindrical Rod' from Lawrence Agbezuge

$$\rho c \frac{\partial T}{\partial t} - \frac{d\lambda}{dT}\left[\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right] - \lambda\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) = Q \tag{11}$$

For the first run, we assume *lambda* is also constant too, which greatly reduces the complexity of the problem to the form

$$\rho c \frac{\partial T}{\partial t} - \lambda\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) = Q \tag{12}$$

TODO: explain Q her

$Q = Q_(rf) + Q_{perf}$

5. Applied FEM technologies

5.1. Weak solutions

5.1.1. Electric potential

Electric potential / Laplace's equation in cylindrical domain:

$$a_w(u,v) := \int_\Omega (\partial_r u \partial_r v + \partial_z u \partial_z v) r dr dz = \int_\Omega f v r dr dz \tag{13}$$

- $u \in H_r^1(\Omega) \cap \{v|_{\Gamma_0} = 0\}$
- $v \in H_r^1(\Omega) \cap \{v|_{\Gamma_0} = 0\}$

Approximate with linear regression functions
Linear regression functions for reference triangles:

$$\phi_1(\xi,\eta) = 1 - \xi - \eta \tag{14}$$
$$\phi_2(\xi,\eta) = \xi \tag{15}$$
$$\phi_2(\xi,\eta) = \eta \tag{16}$$

Specific PDE for electric potential, inner domain:

$$a_w(u,v) := \int_\Omega (\partial_r u \partial_r v + \partial_z u \partial_z v) r dr dz = 0 \tag{17}$$

5.1.2. Temperature Distribution

This is basically the problem above but as a hyperbolic problem
Using semidiscrete solution and iterate solution over time
We are applying method of the discontinuous galerkein fem
For reference see Jung, Langer : Methode der finiten Elemente für Ingenieure, chapter 7.1

Weak formulation for the problem:

We are looking for $u(r,z,t) \in V_{g1}$ with $\dot{u} \in L_2(\Omega)$ for almost every $t \in (0,T)$, so

$$(\dot{u},v)_0 + a(t;u,v) = \langle F(t),v\rangle \; for \; all \; v \in V_0 \tag{18}$$

and for amost every $t \in (0,T)$ is the "Anfangsbedingung -> suche englische Formulierung"

$$(u(r,z,0),v)_0 = (u_0,v)_0 \; for \; all \; v \in V_0 \tag{19}$$

The formal model above is given by

$$\begin{aligned}
(\dot{u},v)_0 &= \int_\Omega \dot{u}(r,z,t)v(r,z)drdz = \int_\Omega \frac{\partial u(r,z,t)}{\partial t} v(r,z)drdz, \\
a(t;u,v) &= \int_\Omega \left[\lambda_1(r,z,t)\frac{\partial u}{\partial r}\frac{\partial v}{\partial r} + \lambda_2(r,z,t)\frac{\partial u}{\partial z}\frac{\partial v}{\partial z}\right] \cdot r \cdot drdz + \int_{\Gamma_3} \alpha(r,z,t)u(r,z,t)v(r,z)ds, \\
\langle F(t),v\rangle &= \int_\Omega f(r,z,t)v(r,z)drdz + \int_{\Gamma_2} g_2(r,z,t)v(r,z)ds + \int_{\Gamma_3} \alpha(r,z,t)u_A(r,z,t)v(r,z)ds, \\
V_{g_1} &= TODO, \\
V_0 &= TODO
\end{aligned}$$

Adapted for the temperature distribution, assuming λ and all material parameters are constant:

$$a_w(t;u,v) := \int_\Omega \rho c(\partial_t u \cdot v)drdz + \int_\Omega \lambda(\partial_r u \partial_r v + \partial_z u \partial_z v) r dr dz = \int_\Omega f v r dr dz \tag{20}$$

5.2. Discretization / Triangulation

5.2.1. Grid generation

5.2.2. Grid refinement

5.3. Assembling system of equation

5.3.1. Assemble elementwise

5.3.2. Add boundary Conditions

5.4. Error estimations

5.4.1. H1-Norm

5.4.2. L2-Norm

5.4.3. Evtl energy norm

6. Numerical challenges / Numerical aspects in general

6.0.1. Numerical integration

6.0.2. Numerical gradient on discrete points

6.0.3. Surface integral

6.0.4. Solving the system of equations

7. Applied simulation

7.1. Generating TestData / Get reference data

7.2. Solving the PDEs

7.3. Combine everything to continous time dependent simulation

7.4. Interpretation of result numbers

8. Programming technologies

8.1. Performance Optimization

8.2. MatLab vs C++

- Basically the performance advantages of using C++
- Combine advantages if both languages
- MatLab as documentation of the simulation

8.3. Graphical output

9. Summary and Outlook

9.1. Project Summary

9.1.1. strengths and flaws

- why is it good, why is it bad

9.1.2. future modifications

9.2. State of the current Research

- Research in the simulation of medical therapy methods

9.3. Other FEM projects and software

- FENICS
- COMSOL
- ANSYS

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[14] Physicists like to think that all you have to do is say 'These are the conditions now what happens next?'. *Richard Feynman*. The Character of Physical Law, 1965.

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Werner Heisenberg. Across the Frontiers, 1972.

A. Source code Visual C++

Listing 1: For loop to print numbers from 1 to 10

```
// Print numbers from 1 to 10
#include <stdio.h>
int main() {
  int i;
  for (i = 1; i < 11; ++i)
  {
    printf("%d␣", i);
  }
  return 0;
}
```

B. Source code MatLab

TODO