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Report of the application project at the Faculty of AMP

Simulation of a medical therapy method with finite elements

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Nuremberg, 01. January 1900

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1. Introduction to radio frequency ablation

Lets talk about:

- Medical Treatment of Tumor
- Radio frequency ablation
- Why RFA Simulation is important
- Motivation / This project in General

2. Computer-aided simulation of radio frequency ablation

2.1. Discrete Numerical Simulation

- In real world physics models are often bounded by reality
- Geometrical boundary conditions are often vague
- In most cases there is no reasonable analytical approach to solve these problems
- Modern numerical approaches are very flexible in this regard
- Simulations done right can be easily modified and adapted to different models and boundaries

2.2. About Errors in simulations and numerical approaches

- see TUM dissertation
- There are different sources for errors following the simulation from the line from the real problem down to the discrete solution
- Idealization error: discrepancy between reality and the idealized reality and the idealized constitutive laws and boundary conditions -> Systems are often way more complex in reality, every patient is different
- Modeling errors: discrepancy between mathematical formulation and physical model -> e.g. using dimensionally reduced approaches, like linear dependencies or even constant parameters
- Discretization errors: discrepancy between the continuous description and discrete description of the model
- Solution errors: using iterative approximation methods and rounding errors
- It's basically a butterfly effect

- Optimizing one error source often conflicts with another one -> e.g. handling nonlinearity can cause fatal numerical errors (at least that's what Kroeger said ...)

2.3. The physics behind radio frequency ablation

- Generating electrical energy with a generator
- Generated heat is distributed on the tissue
- Temperature rises do to constant electrical energy input
- Interesting is: - Temperature distribution over time
- What else? TODO
- Electrical energy can be approximated by the potential of the electrodes on the probes

$$\varphi : \text{TODO} \tag{1}$$

$$\text{ElectricalEnergy} : \text{TODO} \tag{2}$$

- No energy is lost
- Electrical Energy becomes heat energy by Tissue resistance
- Heat Energy is distributed by heat equation

$$\text{Heatequation} : \text{TODO} \tag{3}$$

- Discretization of the equation in space and time domain
- Time can be modeled continuously or in discrete intervalls
- Discrete intervalls are is typically more practical in modeling but less efficient or exact
- Discrete intervalls can be refined if necessary

3. Mathematical aspects of discrete simulation

3.1. Theory of finite elements

3.1.1. Elliptical problems

- Elliptical problems in general
- build up system of PDE's to describe problem
- Using the cylindric domain, different domains

3.1.2. Parabolic problems

- Parabolic / time-dependent problems
- Ignoring time dependency first and calculate the resulting elliptical problem - Build system of equations for elliptical problem
- Describe the parabolic PDE as an ODE with matrices from elliptical PDE
- Solving the system of ODE equations over discrete time intervalls
- Runge-Kutta
- For this simulation, backward euler / implicit euler formula does the job

3.1.3. FEM in Electrostatics

- General info
- special domain
- boundary conditions

3.1.4. FEM in Temperature Fields / perhaps Fluid Dynamics

- General info
- boundary condition (heat source or sink)
- In this simulation, heat source is from electrical energy

3.2. Numerical solution of system of ODE's

3.3. Axial symmetrie

- Using axial symmetrie to simplify computations
- reducing one dimension 3D \rightarrow 2D
- problems are qsuivalent
- significant savings calculations time and complexity
- approach: fourier decomposition in angular direction to reduce dependency on the angular φ
- using static models, only dependency on space
- maybe Torus elements

4. Discretization of PDEs

4.1. Computational Domain

- Using one needle, whole geometry domain is axis symmetric around one needle
- Problem can be reduced to 2D problem using ring elements and cylindric coordinates
- Eliminate dependency on angular ϕ from the calculations
- For visualisation, symmetric results can be reconstructed to 3D
- Whole calculation will be in cylindric coordinates

4.2. FEM in cylindric Coordinates

- Rewrite the equations to cylindric coordinates
- Calculations are made on a cross-section with angular $\phi = 0$
- Define boundaries \rightarrow new artificial boundary around the rotation axis to be taken into consideration
- Explain how the new boundary can be treated

Laplace in cartesian coordinates:

$$\nabla^2 := \Delta := \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (4)$$

Laplace in cylindric coordinates:

$$\Delta := \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \quad (5)$$

TODO : Write something

Laplace in polar coordinates:

$$\Delta := \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \quad (6)$$

4.3. PDE for Electric potential

4.3.1. Weak formulation of the problem

4 areas can be distinguished from a mathematical point of view

- Inner domain
- Fixed Potential of electrodes
- Outer boundaries with no fixed potential -> Robin
- Rotation axis, artificial boundary -> Neumann

Constant material parameters:

4.3.2. Inner Domain

- The electric potential of the inner domain is described as :

$$-\nabla \cdot (\sigma(x, y, z, t) \nabla \phi(x, y, z, t)) = 0 \quad (7)$$

- Elliptical boundary problem
- Assuming constant material parameters: $\nabla \sigma = 0$
- Solution is independent from σ so we can cut it out - Equation becomes Laplaces' equation, ϕ becomes time independent

$$-\Delta \phi(x, y, z) = 0 \quad (8)$$

- Using a cylindric domain, we can use cylinder coordinates (see ref Laplace in cylinder)

$$-\Delta \phi(r, \phi, z) = -\frac{1}{r} \frac{\partial \phi}{\partial r} - \frac{\partial^2 \phi}{\partial r^2} - \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \phi^2} - \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (9)$$

- Since the domain has axis symmetry, the solution for ϕ is independent from the angular ϕ
- So equation simplifies to

$$-\frac{1}{r} \frac{\partial \phi}{\partial r} - \frac{\partial^2 \phi}{\partial r^2} - \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (10)$$

4.3.3. Electrodes

- Potential difference on the electrodes is fixed by definition
- For calculations, potential will be defined as ± 1

$$\phi = \pm 1 \quad (11)$$

4.3.4. Outer boundary

- For first try, a simplification with natural boundary conditions

$$n \cdot \nabla \phi = 0 \quad (12)$$

- In cylindrical coordinates

$$TODO \quad (13)$$

4.3.5. Rotation axis

- Axis symmetry, so here apply natural neumann boundary conditions TODO

4.4. Calculation of electrical energy

- ϕ can be calculated on every discrete point
- Calculate power for every point
- Tissue Resistance
- Effective power
- Calculate electric energy from electric power

4.5. PDE for temperature Distribution

4.5.1. Weak formulation

From physics above, the temperature distribution is modeled by the heat equation:

$$\partial_t(\rho c T) - \nabla \cdot (\lambda \nabla T) = Q \quad (14)$$

The heat equation is a well known parabolic partial differential equation.

We are assuming ρ and c are constant

ρ = density

c = specific heat capacity

λ = thermal conductivity, which is depending on T

$T = T(r,z,t)$ = temperature

$Q = Q(r,z,t)$ = heat energy

Cylindrical coordinates: see 'Transient Heat Transfer in a Partially Cooled Cylindrical Rod' from Lawrence Agbezuge

$$\rho c \frac{\partial T}{\partial t} - \frac{d\lambda}{dT} \left[\left(\frac{\partial T}{\partial r} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right] - \lambda \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) = Q \quad (15)$$

For the first run, we assume λ is also constant too, which greatly reduces the complexity of the problem to the form

$$\rho c \frac{\partial T}{\partial t} - \lambda \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) = Q \quad (16)$$

TODO: explain Q her

$$Q_{total} = Q_{rf} + Q_{perf} \quad (17)$$

- Q_{rf} is described above
- Q_{perf} is blood perfusion
- TODO: Maybe explain this in the physics above???

5. Applied FEM technologies

5.1. Weak solutions

5.1.1. Electric potential

Electric potential / Laplace's equation in cylindrical domain:

$$a_w(u, v) := \int_{\Omega} (\partial_r u \partial_r v + \partial_z u \partial_z v) r dr dz = \int_{\Omega} f v r dr dz \quad (18)$$

- $u \in H_r^1(\Omega) \cap \{v|_{\Gamma_0} = 0\}$
- $v \in H_r^1(\Omega) \cap \{v|_{\Gamma_0} = 0\}$

Approximate with linear regression functions

Linear regression functions for reference triangles:

$$\phi_1(\xi, \eta) = 1 - \xi - \eta \quad (19)$$

$$\phi_2(\xi, \eta) = \xi \quad (20)$$

$$\phi_3(\xi, \eta) = \eta \quad (21)$$

Specific PDE for electric potential, inner domain:

$$a_w(u, v) := \int_{\Omega} (\partial_r u \partial_r v + \partial_z u \partial_z v) r dr dz = 0 \quad (22)$$

5.1.2. Temperature Distribution

This is basically the problem above but as a hyperbolic problem

Using semidiscrete solution and iterate solution over time

We are applying method of the discontinuous galerkein fem

For reference see Jung, Langer : Methode der finiten Elemente für Ingenieure, chapter 7.1

Weak formulation for the problem:

We are looking for $u(r, z, t) \in V_{g1}$ with $\dot{u} \in L_2(\Omega)$ for almost every $t \in (0, T)$, so

$$(\dot{u}, v)_0 + a(t; u, v) = \langle F(t), v \rangle \text{ for all } v \in V_0 \quad (23)$$

and for almost every $t \in (0, T)$ is the "Anfangsbedingung -> suche englische Formulierung"

$$(u(r, z, 0), v)_0 = (u_0, v)_0 \text{ for all } v \in V_0 \quad (24)$$

The formal model above is given by

$$\begin{aligned}
(\dot{u}, v)_0 &= \int_{\Omega} \dot{u}(r, z, t) v(r, z) dr dz = \int_{\Omega} \frac{\partial u(r, z, t)}{\partial t} v(r, z) dr dz, \\
a(t; u, v) &= \int_{\Omega} \left[\lambda_1(r, z, t) \frac{\partial u}{\partial r} \frac{\partial v}{\partial r} + \lambda_2(r, z, t) \frac{\partial u}{\partial z} \frac{\partial v}{\partial z} \right] \cdot r \cdot dr dz + \int_{\Gamma_3} \alpha(r, z, t) u(r, z, t) v(r, z) ds, \\
\langle F(t), v \rangle &= \int_{\Omega} f(r, z, t) v(r, z) dr dz + \int_{\Gamma_2} g_2(r, z, t) v(r, z) ds + \int_{\Gamma_3} \alpha(r, z, t) u_A(r, z, t) v(r, z) ds, \\
V_{g1} &= \text{TODO}, \\
V_0 &= \text{TODO}
\end{aligned}$$

Adapted for the temperature distribution, assuming λ and all material parameters are constant:

$$a_w(t; u, v) := \int_{\Omega} \rho c (\partial_t u \cdot v) dr dz + \int_{\Omega} \lambda (\partial_r u \partial_r v + \partial_z u \partial_z v) r dr dz = \int_{\Omega} f v r dr dz \quad (25)$$

5.2. Discretization / Triangulation

5.2.1. Grid generation

5.2.2. Grid refinement

5.3. Assembling system of equation

5.3.1. Assemble elementwise

5.3.2. Add boundary Conditions

5.4. Error estimations

5.4.1. H1-Norm

5.4.2. L2-Norm

5.4.3. Maybe energy norm???

6. Numerical challenges / Numerical aspects in general

6.1. Numerical integration

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6.3. Surface integral

6.4. Grid refinement

6.5. Solving the system of equations

7. Applied simulation

7.1. Generating TestData / Get reference data

- Material parameters see Stein TODO
- Using specification data from electrical generator

7.2. Solving the PDEs

7.3. Combine everything to continuous time dependent simulation

7.4. Interpretation of result numbers

- Interpret numbers
- Compare with data from experiment or other simulations
- TODO compare with other simulations

8. Programming technologies

8.1. Performance Optimization

8.2. MatLab vs C++

- Could have done the whole simulation using only MATLAB
- MATLAB is a scripting language that calls Fortran Subroutines, which are highly efficient in calculating problems of linear algebra
- However, MATLAB has to call these subroutines in an efficient way to take these performance advantages
- It is extremely easy to write bad and inperformant code in MATLAB, if it is used in the wrong way
- Efficient implementation required hardcoding routines and is very stiff
- To me it was important to write flexible code, that can be easily adapted and extended to try out different modification
- This is way easier when using loops and subroutines instead of hard coded implementations, also the code because way more easier to read and fix
- So I was going for a combined implementation of MatLab and C++
- When using flexible code design, C++ allows performance advantages in using loops etc over MATLAB
- However, MATLAB allows easy function hadnling, what makes the algorithms more accessible for the reader
- In the end I combined the advantages of both languages
- MatLab serves as frame for the pre- and postprocessing, like grid generation and graphical output of the numerical results.
- Also the scripts serve as a mathematical documentation of the whole simulation for the reader
- Computation intense subroutines are done in C++

8.3. Graphical output

9. Summary and Outlook

9.1. Project Summary

- One could argue that writing a simulation from scratch is a waste of time
- There are many highly useful numerical software solutions for numerical simulation and numerical problems
- Usually there is no need to write an own detailed implementation
- Creating own scripts and implementations helps to understand numerical problems and error sources
- This approach helps enormously to increase the ability to use these software products effectively and to generate better simulations and is mandatory to improve
- There can also be no software developer without understanding how a computer works numbers

9.1.1. strengths and flaws

- why is it good, why is it bad - good: numerical results do match the general expectation
- bad: model is too simplified to represent real world conditions

9.1.2. future modifications

- Material parameters are dependent on Temperature and potential ->
- Using variable instead of fixed material parameters
- Take the evaporation of water into account
- Different types of perfusion
- Defining more realistic and complex boundary conditions
- Perhaps a second needle in a 3D simulation

9.2. State of the current Research

- Research in the simulation of medical therapy methods
-

9.3. Other FEM projects and software

- FENICS
- COMSOL
- ANSYS

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A. Source code Visual C++

Listing 1: For loop to print numbers from 1 to 10

```
1 // Print numbers from 1 to 10
2 #include <stdio.h>
3 int main() {
4     int i;
5     for (i = 1; i < 11; ++i)
6     {
7         printf("%d_", i);
8     }
9     return 0;
10 }
```

B. Source code MatLab

TODO