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Report of the application project at the Faculty of AMP

Simulation of a medical therapy method with finite elements

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Contents

1. Introduction to radio frequency ablation

Lets talk about:
- Medical Treatment of Tumor
- Radio frequency ablation
- Why RFA Simulation is important
- Motivation / This project in General

2. Computer-aided simulation of radio frequency ablation

2.1. Discrete Numerical Simulation

- In real world physics models are often bounded by reality
- Geometrical boundary conditions are often vague
- In most cases there is no reasonable analytical approach to solve these problems
- Modern numerical approaches are very flexible in this regard
- Simulations done right can be easily modified and adapted to different models and boundaries

2.2. About Errors in simulations and numerical approaches

- see TUM dissertation
- There are different sources for errors following the simulation from the line from the real problem down to the discrete solution
- Idealization error: discrepancy between reality and the idealized reality and the idealized constitutive laws and boundary conditions -> Systems are often way more complex in reality, every patient is different
- Modeling errors: discrepancy between mathematical formulation and physical model -> e.g. using dimensionally reduced approaches, like linear dependencies or even constant parameters
- Discretization errors: discrepancy between the continous description and discrete discription of the model
- Solution errors: using iterative approximation methods and rounding errors
- It's basically a butterfly effect

- Optimizing one error source often conflicts with another one -> e.g. handling nonlinearity can cause fatal numerical errors (at least that's what Kroeger said ...)

2.3. The physics behind radio frequency ablation

- Generating electrical energy with a generator
- Generated heat is distributed on the tissue
- Temperature rises do to constant electrical energy input
- Interesting is: - Temperature distribution over time
- What else? TODO
- Electrical energy can be approximated by the potential of the electrodes on the probes

$$\varphi : TODO \tag{1}$$

$$ElectricalEnergy : TODO \tag{2}$$

- No energy is lost
- Electrical Energy becomes heat energy by Tissue resistance
- Heat Energy is distributed by heat equation

$$Heatequation : TODO \tag{3}$$

- Discretization of the equation in space and time domain
- Time can be modeled continuously or in discrete intervalls
- Discrete intervalls are is typically more practical in modeling but less efficient or exact
- Discrete intervalls can be refined if necessary

3. Mathematical aspects of discrete simulation

3.1. Theory of finite elements

3.1.1. Elliptical problems

- Elliptical problems in general
- Parabolic / time-dependent problems
- build up system of PDE's to describe problem
- Using the cylindric domain, different domains

3.1.2. Parabolic problems

- Solving systems of ODE over discrete time intervalls

3.1.3. FEM in Electrostatics

- special domain
- boundary conditions

3.1.4. FEM in Temperature Fields / perhaps Fluid Dynamics

- boundary condition (heat source or sink)

3.2. Numerical solution of system of ODE's

3.3. Axial symmetrie

- Using axial symmetrie to simplify computations
- reducing one dimension 3D -> 2D
- problems are qsuivalent
- significant savings calculations time and complexity

- approach: fourier decomposition in angular direction to reduce dependency on the angular φ
- using static models, only dependency on space
- maybe Torus elements

4. Discretization of PDEs

4.1. Computational Domain

- Using one needle, whole geometry domain is axis symmetric around one needle
- Problem can be reduced to 2D problem using ring elements and cylindric coordinates
- Eliminate dependency on angular ϕ from the calculations
- For visualisation, symmetric results can be reconstructed to 3D
- Whole calculation will be in cylindric coordinates

4.2. FEM in cylindric Coordinates

- Rewrite the equations to cylindric coordinates
- Calculations are made on a cross-section with angular phi = 0
- Define boundaries -> new artificial boundary around the rotation axis to be taken into consideration
- Explain how the new boundary can be treated

Laplace in cartesian coordinates:

$$\nabla^2 := \Delta := \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{4}$$

Laplace in cylindric coordinates:

$$\Delta := \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \tag{5}$$

TODO : Write something

Laplace in polar coordinates:

$$\Delta := \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} \tag{6}$$

4.3. PDE for Electric potential

4.3.1. Weak formulation of the problem

4 areas can be distinguished from a mathematical point of view
- Inner domain
- Fixed Potential of electrodes
- Outer boundaries with no fixed potential -> Robin
- Rotation axis, artificial boundary -> Neumann

Constant material parameters:

4.3.2. Inner Domain

- The electric potential of the inner domain is described as :

$$-\nabla \cdot (\sigma(x,y,z,t)\nabla\varphi(x,y,z,t)) = 0 \tag{7}$$

- Elliptical boundary problem
- Assuming constant material parameters: $\nabla\sigma = 0$
- Solution is independent from σ so we can cut it out - Equation becomes Laplaces' equation, phi becomes time independent

$$-\Delta\varphi(x,y,z) = 0 \tag{8}$$

- Using a cylindric domain, we can use cylinder coordinates (see ref Laplace in cylinder)

$$-\Delta\varphi(r,\phi,z) = -\frac{1}{r}\frac{\partial\varphi}{\partial r} - \frac{\partial^2\varphi}{\partial r^2} - \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\phi^2} - \frac{\partial^2\varphi}{\partial z^2} = 0 \tag{9}$$

- Since the domain has axis symmetry, the solution for φ is independent from the angular ϕ
- So equation simplifies to

$$-\frac{1}{r}\frac{\partial\varphi}{\partial r}-\frac{\partial^2\varphi}{\partial r^2}-\frac{\partial^2\varphi}{\partial z^2}=0 \tag{10}$$

4.3.3. Electrodes

- Potential difference on the electrodes is fixed bx definition
- For calculations, potential will be defined as ± 1

$$\varphi = \pm 1 \tag{11}$$

4.3.4. Outer boundary

- For first try, a simplification with natural boundary conditions

$$n \cdot \nabla\varphi = 0 \tag{12}$$

- In cylindrical coordinates

$$TODO \tag{13}$$

4.3.5. Rotation axis

- Axis symmetry, so here apply natural neumann bundary conditions TODO

4.4. Calculation of electrical energy

- φ can be calculated on every discrete point
- Calculate power for every point
- Tissue Resistance
- Effective power

- Calculate electric energy from electric power

4.5. PDE for temperature Distribution

4.5.1. Weak formulation

From physics above, the temperature distribution is modeled by the heat equation:

$$\partial_t(\rho c T) - \nabla \cdot (\lambda \nabla T) = Q \tag{14}$$

The heat equation is a well known parabolic partial differential equation.

We are assuming ρ and c are constant
ρ = density
c = specific heat capacity
λ = thermal conductivity, which is depending on T
T = T(r,z,t) = temperature
Q = Q(r,z,t) = heat energy

Cylindrical coordinates: see 'Transient Heat Transfer in a Partially Cooled Cylindrical Rod' from Lawrence Agbezuge

$$\rho c \frac{\partial T}{\partial t} - \frac{d\lambda}{dT}\left[\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right] - \lambda\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) = Q \tag{15}$$

For the first run, we assume *lambda* is also constant too, which greatly reduces the complexity of the problem to the form

$$\rho c \frac{\partial T}{\partial t} - \lambda\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) = Q \tag{16}$$

TODO: explain Q her

$$Q_{total} = Q_{rf} + Q_{perf} \tag{17}$$

- Q_{rf} is descripted above
- Q_{perf} is blood perfusion
- TODO: Maybe explain this in the physics above???

5. Applied FEM technologies

5.1. Weak solutions

5.1.1. Electric potential

Electric potential / Laplace's equation in cylindrical domain:

$$a_w(u,v) := \int_\Omega (\partial_r u \partial_r v + \partial_z u \partial_z v) r dr dz = \int_\Omega f v r dr dz \tag{18}$$

- $u \in H_r^1(\Omega) \cap \{v|_{\Gamma_0} = 0\}$
- $v \in H_r^1(\Omega) \cap \{v|_{\Gamma_0} = 0\}$

Approximate with linear regression functions
Linear regression functions for reference triangles:

$$\phi_1(\xi,\eta) = 1 - \xi - \eta \tag{19}$$

$$\phi_2(\xi,\eta) = \xi \tag{20}$$

$$\phi_2(\xi,\eta) = \eta \tag{21}$$

Specific PDE for electric potential, inner domain:

$$a_w(u,v) := \int_\Omega (\partial_r u \partial_r v + \partial_z u \partial_z v) r dr dz = 0 \tag{22}$$

5.1.2. Temperature Distribution

This is basically the problem above but as a hyperbolic problem
Using semidiscrete solution and iterate solution over time
We are applying method of the discontinuous galerkein fem
For reference see Jung, Langer : Methode der finiten Elemente für Ingenieure, chapter 7.1

Weak formulation for the problem:

We are looking for $u(r,z,t) \in V_{g1}$ with $\dot{u} \in L_2(\Omega)$ for almost every $t \in (0,T)$, so

$$(\dot{u}, v)_0 + a(t; u, v) = \langle F(t), v \rangle \; for\; all\; v \in V_0 \tag{23}$$

and for amost every $t \in (0,T)$ is the "Anfangsbedingung -> suche englische Formulierung"

$$(u(r,z,0), v)_0 = (u_0, v)_0 \; for\; all\; v \in V_0 \tag{24}$$

The formal model above is given by

$$\begin{aligned}
(\dot{u}, v)_0 &= \int_\Omega \dot{u}(r,z,t) v(r,z) drdz = \int_\Omega \frac{\partial u(r,z,t)}{\partial t} v(r,z) drdz, \\
a(t; u, v) &= \int_\Omega \left[\lambda_1(r,z,t) \frac{\partial u}{\partial r} \frac{\partial v}{\partial r} + \lambda_2(r,z,t) \frac{\partial u}{\partial z} \frac{\partial v}{\partial z} \right] \cdot r \cdot drdz + \int_{\Gamma_3} \alpha(r,z,t) u(r,z,t) v(r,z) ds, \\
\langle F(t), v \rangle &= \int_\Omega f(r,z,t) v(r,z) drdz + \int_{\Gamma_2} g_2(r,z,t) v(r,z) ds + \int_{\Gamma_3} \alpha(r,z,t) u_A(r,z,t) v(r,z) ds, \\
V_{g_1} &= TODO, \\
V_0 &= TODO
\end{aligned}$$

Adapted for the temperature distribution, assuming λ and all material parameters are constant:

$$a_w(t; u, v) := \int_\Omega \rho c (\partial_t u \cdot v) drdz + \int_\Omega \lambda (\partial_r u \partial_r v + \partial_z u \partial_z v) r drdz = \int_\Omega f v r drdz \tag{25}$$

5.2. Discretization / Triangulation

5.2.1. Grid generation

5.2.2. Grid refinement

5.3. Assembling system of equation

5.3.1. Assemble elementwise

5.3.2. Add boundary Conditions

5.4. Error estimations

5.4.1. H1-Norm

5.4.2. L2-Norm

5.4.3. Evtl energy norm

6. Numerical challenges / Numerical aspects in general

6.1. Numerical integration

6.2. Numerical gradient on discrete points

6.3. Surface integral

6.4. Grid refinement

6.5. Solving the system of equations

7. Applied simulation

7.1. Generating TestData / Get reference data

7.2. Solving the PDEs

7.3. Combine everything to continous time dependent simulation

7.4. Interpretation of result numbers

- Interprete numbers
- Compare with data from experiment or other simulations

8. Programming technologies

8.1. Performance Optimization

8.2. MatLab vs C++

- Could have done the whole simulation using only MATLAB
- MATLAB is a scripting language that calls Fortran Subroutines, which are highly efficient in calculating problems of linear algebra
- However, MATLAB has to call these subroutines in an efficient way to take these performance advantages
- It is extremely easy to write bad and inperformant code in MATLAB, if it is used in the wrong way
- Efficient implementation required hardcoding routines and is very stiff
- To me it was important to write flexible code, that can be easily adapted and extended to try out different modification
- This is way easier when using loops and subroutines instead of hard coded implementations, also the code because way more easier to read and fix
- So I was going for a combined implementation of MatLab and C++
- When using flexible code design, C++ allows performance advantages in using loops etc over MATLAB
- However, MATLAB allows easy function hadnling, what makes the algorithms more accessable for the reader
- In the end I combined the advantages of both languages
- MatLab serves as frame for the pre- and postprocessing, like grid generation and graphical output of the numerical results.
- Also the scripts serve as a mathematical documentation of the whole simulation for the reader
- Computation intense subroutines are done in C++

8.3. Graphical output

9. Summary and Outlook

9.1. Project Summary

- One could argue that writing a simulation from scratch is a waste of time
- There are many highly useful numerical software solutions for numerical simulation and numerical problems
- Usually there is no need to write an own detailed implementation
- Creating own scripts and implementations helps to understand numerical problems and error sources
- This approach helps enormously to increase the ability to use these software products effectively and to generate better simulations and is mandatory to improve
- There can also be no software developer without understanding how a computer works numbers

9.1.1. strengths and flaws

- why is it good, why is it bad - good: numerical results do match the general expectation
- bad: model is to simplified to represent real world conditions

9.1.2. future modifications

- Material parameters are dependent on Temperature and potential ->
- Using variable instead of fixed material parameters
- Take the evaporation of water into account
- Different types of perfusion
- Defining more realistic and complex boundary conditions
- Perhaps a second needle in a 3D simulation

9.2. State of the current Research

- Research in the simulation of medical therapy methods
-

9.3. Other FEM projects and software

- FENICS
- COMSOL
- ANSYS

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A. Source code Visual C++

Listing 1: For loop to print numbers from 1 to 10

```
// Print numbers from 1 to 10
#include <stdio.h>
int main() {
  int i;
  for (i = 1; i < 11; ++i)
  {
    printf("%d␣", i);
  }
  return 0;
}
```

B. Source code MatLab

TODO