TECHNIQUES OF INTEGRATION

Integration by Parts (Section 3.1; Book 2)

Recall: So far, we have learned how to integrate some basic functions.

e.g.
$$\int x(x+1)dx \rightarrow \text{Expand}$$

e.g. $\int xe^{x^2}dx \rightarrow \text{U-Sub}(x^2)$

Now we will continue to investigate more advanced integration techniques where our previous methods won't work.

e.g.
$$\int x \sin x dx$$

Recall: You may remember that *u*-substitution came about based on undoing the chain rule. Similarly, we can use the product rule for differentiation to derive a useful rule for integration. $\frac{\partial}{\partial x} (x \sin x) = \frac{\partial}{\partial x} (x \sin x) + \frac{\partial}{\partial x} (x \cos x) + \frac{\partial}{\partial x}$

The product rule states that, for f, g differentiable,

Integration by Parts Formula:

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

$$\frac{f(x)g'(x)dx}{f(x)dx} = f(x)g(x) - \int f'(x)g(x)dx \quad u = f(x)$$

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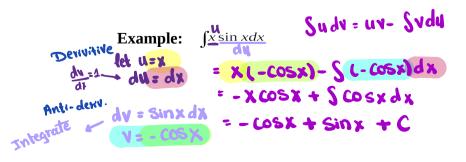
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Question: What if we had chosen u and dv differently in the above example?

let
$$u=\sin x$$
 $dv=xdx$
 $4du=\cos xdx$ $1=\frac{1}{2}x^2$ now it is harder than the original question x
 $3x\sin xdx=(\sin x)(\frac{1}{2}x^2)-5\frac{1}{2}x^2\cos xdx$

Question: How do we choose u and dv?

- The new integral $\int v du$ should be easier than the original
- You have to be able to integrate *dv* to obtain *v*

Examples:
$$\int_{A}^{x} \sec^{2} x dx$$
 $u = \underbrace{x}$
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 $dv = \underbrace{\sec^{2} x} dx$
 $dv = \underbrace{x^{2} \ln x} dx$

Example: $\int_{E}^{x^{2} \ln x} dx$
 $dv = \underbrace{x^{2} \ln x} dx$
 $dv = \underbrace{x^{2$

Question: What about definite integrals?

Integration by Parts for Definite Integrals:

$$\int_{a}^{b} u dv = uv \Big|_{a}^{b} - \int_{a}^{b} v du$$

Example: If the previous question had been $\int_{2}^{5} x^{7} \ln x dx$, in the 1st step we'd get:



Now let's go on to study some more complicated examples of applying integration by parts:

Sometimes, you have to apply integration by parts more than once.

Application: If the rate of change of medication in the bloodstream is $\frac{dA}{dt} = t^2 \cdot e^{-t}$, what is the net change in the amount of medication from time t = 0 to t = 1?

Sometimes, you can apply integration by parts even though you're only integrating a single function.

Example: $\int \frac{1}{\ln x} dx$

=
$$\ln x \cdot x - \xi x \cdot \frac{1}{x} dx$$

Occasionally, you have to "go in circles".

Example: $\int_{0}^{\infty} \int_{0}^{\infty} dx$

=Sinx.ex - Sex cosxdx
not easier nor harder

= Sinx.ex _ Cosx.ex - Sex-sinx dx]

= exsinx - excosx - sersinxdx

Sudv = uv - Svdu $u = In x \quad dv = dx$ $du = \frac{1}{x}dx \quad v = x$

Sudv = uv - Svdu U = Sinx dv = exdx du = cosx dx v = ex

> $U = \cos X$ $dv = e^{x} dx$ $dv = -\sin x dx$ $v = e^{x}$

= Sexsinxdx = exsinx-excosx-Sexsindx

divide by 1/2

Sexsinxdx = 12 exsinx- 1/2 ex cosx + C