Design of Slepian-Wolf Codes by Channel Code Partitioning

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Abstract

A Slepian-Wolf coding scheme that can achieve arbitrary rate allocation among two encoders was outlined in the work of Pradhan and Ramchandran. Inspired by this work, we start with a detailed solution for general (asymmetric or symmetric) Slepian-Wolf coding based on partitioning a single $systematic$ channel code, and continue with practical code designs using advanced channel codes. By using systematic IRA and turbo codes, we devise a powerful scheme that is capable of approaching any point on the Slepian-Wolf bound. We further study an extension of the technique to multiple sources, and show that for a particular correlation model among the sources, a single practical channel code can be designed for coding all the sources in symmetric and asymmetric scenarios. If the code approaches the capacity of the channel that models the correlation between the sources, then the system will approach the Slepian-Wolf limit. Using systematic IRA and punctured turbo codes for coding two binary sources, each being independent identically distributed, with correlation modeled by a binary symmetric channel, we obtain results which are 0.04 bits away from the theoretical limit in both symmetric and asymmetric Slepian-Wolf settings.

1 Introduction

Compressing two distinct signals by exploiting their correlation can certainly provide a benefit in total rate cost. Moreover, Slepian and Wolf [1] show that lossless compression of two separate sources can be as efficient as if they are compressed together as long as joint decoding is done at the receiver. Recently, several successful attempts of constructing practical coding schemes that exploit the potential of the Slepian-Wolf (SW) theorem have appeared [2–9]. All these schemes, with the exception of [6, 9], are based on *asymmetric codes* [10]; that is, they compress losslessly one source, while the other source is assumed to be perfectly known at the decoder side and is used as side information. Thus, for two discrete, memoryless, identically distributed sources X and Y encoded separately at rates R_1 and R_2, respectively, these codes attempt to reach the two corner points on the SW bound: $(R_1, R_2) = (H(X), H(Y|X))$ and $(R_1, R_2) = (H(Y), H(X|Y))$. But very often, it is desirable to vary the rates of individual encoders while keeping the total sum-rate constant. The first method of achieving this is time sharing. However, time sharing might not be practical because it requires exact synchronization among encoders. The second method is the *source-splitting* approach of Rimoldi and Urbanke [11], which potentially reaches all points on the SW bound by splitting two sources into three subsources of lower entropy. Garcia-Frias and Zhao [6] propose a system consisting of two different turbo codes

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which form a big turbo code with four component codes. In the *symmetric* scenario (the rates of both encoders are the same) suggested, half of the systematic bits from one encoder and half from the other are sent. Further, instead of syndrome bits [2], parity bits are sent.

Recently, Pradhan and Ramchandran [10, 12] outlined a method for constructing a single code based on the syndrome technique [2], which achieves arbitrary rate allocation among the two encoders. The method constructs independent subcodes of the main code and assigns them to different encoders. Each encoder sends only partial information about the source; by combining two received bitstreams a joint decoder should perfectly reconstruct the sources. Since joint decoding is done only on a single code, if this code is approaching the capacity of a channel that models the correlation among the sources, the system will approach the SW limit. Thus, the great advantage of this setup is the need of only one good channel code. It is also shown in [10] that this code does not suffer from any performance loss compared to the corresponding asymmetric code. Moreover, any point on the SW bound can be potentially reached without increasing the encoding/decoding complexity. Further in [10, 12] the method is applied to coding of two noisy observations of a source with scalar quantizer and trellis codes.

Inspired by the work of [10, 12], we first provide a clear and detailed solution based on systematic codes so that advanced channel codes can be employed to yield SW codes that can approach any point on the bound. Following the solution, we propose practical code designs based on powerful systematic channel codes. In [3], it was shown that with low-density parity-check (LDPC) codes it is possible to approach the theoretical limits in the SW asymmetric scenario. Irregular repeat-accumulate (IRA) codes [13] are a special form of LDPC codes which suffer very small performance loss, but can easily be coded in systematic form and have low encoding complexity which make them suitable for multiterminal coding [14]. We choose, therefore, IRA codes in our experiments.

To show how to implement our scheme with convolutional codes, we also treat powerful turbo codes [15]. Turbo codes have already been successfully applied to asymmetric SW coding of two sources. Good results are obtained with both conventional [4] and nonconventional turbo schemes [5–7]. In this paper, we implement symmetric SW coding using conventional punctured turbo codes.

We also study an extension of the method [10, 12] to SW coding of multiple sources [17], which is of special importance in sensor networks. Indeed, after quantization of an observed corrupted version of the source, each distinct sensor encodes its observation by exploiting the correlation between the observations and the source [18]. Thus, to reach the theoretical limits [18], a code for lossless compression capable of trading-off transmission rates among sensors is needed. We argue that as long as the correlation among the sources is such that their sum is a Bernoulli-p process, which corresponds to less general correlation models than in [8], a single channel code can be used to approach the joint entropy limit. In addition, the complexity of encoding/decoding does not exceed that of the asymmetric codes. Furthermore, in contrast to the asymmetric codes, the obtained code has additional error detection capability.

The paper is organized as follows. In Section 2 we present our method for designing a single code for SW coding of multiple sources. In Section 3 we show how this theoretical approach can be applied to practical code constructions using systematic IRA and turbo codes. Experimental results for two sources, conclusions, and suggestions for future work are given in the last two sections.

2 Code Construction

We consider an SW coding system which consists of L encoders and a joint decoder. Let $X_1, \ldots, X_L$ be discrete, memoryless, uniformly distributed, correlated, random sources and let $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_L$ denote their realizations. The i-th encoder compresses X_i at rate R_i independently from the information available at other encoders. The decoder receives the bitstreams from all the encoders and jointly decodes them. It should reconstruct all received source messages with arbitrarily small probability of error. The achievable rate region is then [17]:

$$R_{i_1} + \cdots + R_{i_k} \quad \geq \quad H(X_{i_1} \ldots X_{i_k} | X_{j_1} \ldots X_{j_{L-k}}),$$

where for $k \leq L$, $\{i_1, \ldots, i_k\} \subseteq \{1, \ldots, L\}$, and $\{j_1, \ldots, j_{L-k}\} = \{1, \ldots, L\} \backslash \{i_1, \ldots, i_k\}$.

Our goal is to construct a practical code that can potentially approach the above bound for any achievable rate allocation among the encoders. We treat the binary case and assume that all X_i's are of length n bits. We have the following definition.

Definition 1 *A general SW code is a pair $(\mathcal{C}, \mathcal{M})$, where $\mathcal{C}$ is an (n, k) linear binary channel code given by generator matrix $\boldsymbol{G}_{k \times n}$ and $\mathcal{M}$ is an ordered set of integers $\{m_1, \ldots, m_L\}$ such that $\sum_{j=1}^{L} m_j = k$.*

For each $i = 1, \ldots, L$, we form code $\mathcal{C}_i$ as a subcode of $\mathcal{C}$ with generator matrix $\boldsymbol{G}_{i_{m_i \times n}}$ which consists of m_i rows of $\boldsymbol{G}$ starting from row $m_1 + \cdots + m_{i-1} + 1$. Without loss of generality suppose that the code $\mathcal{C}$ is systematic. Let $m_{i-} = m_1 + \cdots + m_{i-1}$ and $m_{i+} = m_{i+1} + \cdots + m_L$. $\boldsymbol{I}_k$ will designate the $k \times k$ identity matrix, and $\boldsymbol{O}_{k_1 \times k_2}$ is the $k_1 \times k_2$ all-zero matrix. Then, for $\boldsymbol{G} = \begin{bmatrix} \boldsymbol{I}_k & \boldsymbol{P}_{k \times (n-k)} \end{bmatrix}$, the generator matrix of subcode $\mathcal{C}_i$ is

$$\boldsymbol{G}_i = \begin{bmatrix} \boldsymbol{O}_{m_i \times m_{i-}} & \boldsymbol{I}_{m_i} & \boldsymbol{O}_{m_i \times m_{i+}} & \boldsymbol{P}_{i_{m_i \times (n-k)}} \end{bmatrix}, \tag{1}$$

where $\boldsymbol{P}^T = \begin{bmatrix} \boldsymbol{P}_1^T & \ldots & \boldsymbol{P}_L^T \end{bmatrix}$.

One choice for the $(n - m_i) \times n$ parity matrix $\boldsymbol{H}_i$ of $\mathcal{C}_i$ is

$$\boldsymbol{H}_i \quad = \quad \begin{bmatrix} \boldsymbol{I}_{m_{i-}} & \boldsymbol{O}_{m_{i-} \times m_i} & \boldsymbol{O}_{m_{i-} \times m_{i+}} & \boldsymbol{O}_{m_{i-} \times (n-k)} \\ \boldsymbol{O}_{m_{i+} \times m_{i-}} & \boldsymbol{O}_{m_{i+} \times m_i} & \boldsymbol{I}_{m_{i+}} & \boldsymbol{O}_{m_{i+} \times (n-k)} \\ \boldsymbol{O}_{(n-k) \times m_{i-}} & \boldsymbol{P}_i^T & \boldsymbol{O}_{(n-k) \times m_{i+}} & \boldsymbol{I}_{n-k} \end{bmatrix}. \tag{2}$$

Encoding is simply the multiplication of the incoming n-length vector $\boldsymbol{x}_i = [\boldsymbol{u}_i \quad \boldsymbol{a}_i \quad \boldsymbol{v}_i \quad \boldsymbol{q}_i]$ (vectors $\boldsymbol{u}_i$, $\boldsymbol{a}_i$, $\boldsymbol{v}_i$, and $\boldsymbol{q}_i$ are of length m_{i-}, m_i, m_{i+}, and $n - k$, respectively) with the parity matrix $\boldsymbol{H}_i$. In this way, the *syndrome vector* $\boldsymbol{s}_i^T = \boldsymbol{H}_i \boldsymbol{x}_i^T$ of length $n - m_i$ is formed as:

$$\boldsymbol{s}_i^T \quad = \quad \begin{bmatrix} \boldsymbol{u}_i^T \\ \boldsymbol{v}_i^T \\ \boldsymbol{q}_i^T \oplus \boldsymbol{P}_i^T \boldsymbol{a}_i^T \end{bmatrix}, \tag{3}$$

where $\oplus$ denotes addition in GF(2).

Let a length n row-vector $\boldsymbol{t}_i$ be defined as

$$\boldsymbol{t}_i^T = \begin{bmatrix} \boldsymbol{u}_i^T \\ \boldsymbol{O}_{m_i \times 1} \\ \boldsymbol{v}_i^T \\ \boldsymbol{q}_i^T \oplus \boldsymbol{P}_i^T \boldsymbol{a}_i^T \end{bmatrix}. \tag{4}$$

Then, $\boldsymbol{x}_i \oplus \boldsymbol{t}_i = \boldsymbol{a}_i \boldsymbol{G}_i$ is a valid codeword of $\mathcal{C}_i$, and thus also of $\mathcal{C}$.

The decoder collects all syndromes $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_L$ and forms the sum $\boldsymbol{t}_1 \oplus \cdots \oplus \boldsymbol{t}_L$. From linearity, it follows that $\boldsymbol{x}_1 \oplus \boldsymbol{t}_1 \oplus \cdots \oplus \boldsymbol{x}_L \oplus \boldsymbol{t}_L$ is a valid codeword of $\mathcal{C}$. The task of the decoder is then to find a codeword $\boldsymbol{c}$ that is closest (in Hamming distance) to the vector $\boldsymbol{t}_1 \oplus \cdots \oplus \boldsymbol{t}_L$. Let the vector $[\hat{\boldsymbol{a}}_1 \ldots \hat{\boldsymbol{a}}_L]$ be the decoded systematic part of the codeword $\boldsymbol{c}$. The sources are recovered as: $\hat{\boldsymbol{x}}_i = \hat{\boldsymbol{a}}_i \boldsymbol{G}_i \oplus \boldsymbol{t}_i$.

Given the length of the messages n, the number of encoders L, and the set of desirable transmission rates $R_1, \ldots, R_L$ (that are achievable [17]), parameters of the SW code are selected in the following way. For $i = 1, \ldots, L$, $m_i = n - R_i$, $k = \sum_{j=1}^{L} m_j$. If the joint distribution of random variables $X_1, \ldots, X_L$ is such that $w(\boldsymbol{x}_1 \oplus \cdots \oplus \boldsymbol{x}_L) \leq t$, where $w(\cdot)$ denotes the Hamming weight, then the code $\mathcal{C}$ should be an (n, k, d_H) code that can correct at least t errors; thus, the Hamming distance of the code is $d_H \geq 2t + 1$, and from the sphere packing bound $n - k \geq \log \sum_{j=0}^{t} \binom{n}{j}$ must hold.

Proposition 1 *If the parameters of a general SW code $(\mathcal{C}, \mathcal{M})$ are selected as above and the correlation of the sources is such that $w(\boldsymbol{x}_1 \oplus \cdots \oplus \boldsymbol{x}_L) \leq t$, then the decoding error equals zero.*

Proof: The proof follows directly from [12] and the discussion above. □

A great advantage of this technique is that only *one* good channel code is needed. Indeed, for $L = 2$, if the binary code $\mathcal{C}$ is approaching the capacity of a binary symmetric channel (BSC), then the general SW code $(\mathcal{C}, \mathcal{M})$ will approach the SW limit as long as the joint correlation between X_1 and X_2 can be modeled with the same BSC. However, in the case $L > 2$, finding a channel that models the correlation among sources becomes more involved. As long as this correlation is such that $X_1 \oplus \cdots \oplus X_L$ is a Bernoulli-p process, a single channel code $\mathcal{C}$ can be efficiently designed. This can be the case in the remote multiterminal setting [14] where an encoder observes only a noisy version of the source. However, this constraint on the correlation model makes this approach less general than [8].

The method also applies to the case when $\mathcal{C}$ is a convolutional code, as will be shown in the next section on the example with punctured turbo codes.

For clarity, we give an example of the code construction for the case $L = 2$ using a *systematic* channel code (a similar example but with a non-systematic code is hinted in [10, 12]). Let X and Y be two discrete, memoryless, uniformly distributed variables of length seven bits such that the Hamming distance between them is at most one. The source messages are separately encoded and sent to a joint decoder. The decoder then attempts to losslessly reconstruct both sources. The SW bound for this case is 10 bits [1]. It can be achieved in the asymmetric scenario by transmitting one source, e.g., X, at rate

$R_1 = H(X) = 7$ bits and by coding the second source, Y, at $R_2 = H(Y|X) = 3$ bits. We show how the same total rate can be achieved with the symmetric approach by using $R_1 = R_2 = 5$ bits.

Since $n = 7$ bits, and we need a code that can correct at least one bit error, for an SW code $\mathcal{C}$ we select the systematic (7,4) Hamming code, defined by the generator matrix:

$$\boldsymbol{G}_{k\times n} = [\boldsymbol{I}_4\ \boldsymbol{P}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

Its parity matrix is:

$$\boldsymbol{H} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Further we construct two subcodes of $\mathcal{C}$, $\mathcal{C}_1$ and $\mathcal{C}_2$, by splitting $\boldsymbol{G}$ into two generator matrices, $\boldsymbol{G}_1$ that contains the first $m = 2$ rows of $\boldsymbol{G}$, and $\boldsymbol{G}_2$ that contains the last two rows. X is coded using $\mathcal{C}_1$ and Y using $\mathcal{C}_2$. Let $\boldsymbol{P}^T = \begin{bmatrix}\boldsymbol{P}_1^T & \boldsymbol{P}_2^T\end{bmatrix}$. Then for the $(n-m)\times n$ parity-check matrices, $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, of $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively, we get from (2):

$$\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{O}_{m\times m} & \boldsymbol{I}_m & \boldsymbol{O}_{m\times(n-k)} \\ \boldsymbol{P}_1^T & \boldsymbol{O}_{(n-k)\times m} & \boldsymbol{I}_{n-k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{O}_{m\times m} & \boldsymbol{I}_{n-m} \\ \boldsymbol{P}_1^T & \end{bmatrix} = \begin{bmatrix} 0\,0\,1\,0\,0\,0\,0 \\ 0\,0\,0\,1\,0\,0\,0 \\ 1\,1\,0\,0\,1\,0\,0 \\ 0\,1\,0\,0\,0\,1\,0 \\ 1\,0\,0\,0\,0\,0\,1 \end{bmatrix},$$

$$\boldsymbol{H}_2 = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{O}_{m\times m} & \boldsymbol{O}_{m\times(n-k)} \\ \boldsymbol{O}_{(n-k)\times m} & \boldsymbol{P}_2^T & \boldsymbol{I}_{n-k} \end{bmatrix} = \begin{bmatrix} 1\,0\,0\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0\,0\,0 \\ 0\,0\,1\,0\,1\,0\,0 \\ 0\,0\,1\,1\,0\,1\,0 \\ 0\,0\,1\,1\,0\,0\,1 \end{bmatrix},$$

since both $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ have rank $n - m$ and $\boldsymbol{H}_1\boldsymbol{G}_1^T = \boldsymbol{H}_2\boldsymbol{G}_2^T = \boldsymbol{O}_{(n-m)\times m}$.

Let realizations of the sources be $\boldsymbol{x} = [0\ 0\ 1\ 0\ 1\ 1\ 0]$ and $\boldsymbol{y} = [0\ 1\ 1\ 0\ 1\ 1\ 0]$. Since the Hamming distance between $\boldsymbol{x}$ and $\boldsymbol{y}$ is one, we should be able to decode the messages correctly.

We form syndromes for both $\boldsymbol{x}$ and $\boldsymbol{y}$. To do so, we write $\boldsymbol{x}$ and $\boldsymbol{y}$ in the form

$$\begin{aligned} \boldsymbol{x} &= [\boldsymbol{a}_1 \quad \boldsymbol{v}_1 \quad \boldsymbol{q}_1] = [00 \quad 10 \quad 110], \\ \boldsymbol{y} &= [\boldsymbol{u}_2 \quad \boldsymbol{a}_2 \quad \boldsymbol{q}_2] = [01 \quad 10 \quad 110]. \end{aligned}$$

The length $n - m$ syndromes, $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$, formed by the two subcodes are

$$\begin{aligned} \boldsymbol{s}_1^T &= \boldsymbol{H}_1\boldsymbol{x}^T = \begin{bmatrix} \boldsymbol{v}_1^T \\ \boldsymbol{P}_1^T\boldsymbol{a}_1^T \oplus \boldsymbol{q}_1^T \end{bmatrix} = [1\ 0\ 1\ 1\ 0]^T \\ \boldsymbol{s}_2^T &= \boldsymbol{H}_2\boldsymbol{y}^T = \begin{bmatrix} \boldsymbol{u}_2^T \\ \boldsymbol{P}_2^T\boldsymbol{a}_2^T \oplus \boldsymbol{q}_2^T \end{bmatrix} = [0\ 1\ 0\ 0\ 1]^T. \end{aligned}$$

The length n row-vectors $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$ are then given by

$$\boldsymbol{t}_1^T = \begin{bmatrix} \boldsymbol{O}_{m\times 1} \\ \boldsymbol{v}_1^T \\ \boldsymbol{P}_1^T \boldsymbol{a}_1^T \oplus \boldsymbol{q}_1^T \end{bmatrix} = [0\ 0\ 1\ 0\ 1\ 1\ 0]^T$$
$$\boldsymbol{t}_2^T = \begin{bmatrix} \boldsymbol{u}_2^T \\ \boldsymbol{O}_{m\times 1} \\ \boldsymbol{P}_2^T \boldsymbol{a}_2^T \oplus \boldsymbol{q}_2^T \end{bmatrix} = [0\ 1\ 0\ 0\ 0\ 0\ 1]^T.$$

Then the row-vectors $\boldsymbol{x}\oplus\boldsymbol{t}_1$ and $\boldsymbol{y}\oplus\boldsymbol{t}_2$ are codewords of the codes $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively.

Thus, by sending $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ from the two encoders to the joint decoder, the decoder finds the codeword in $\mathcal{C}$ that is closest to $\boldsymbol{t}_1 \oplus \boldsymbol{t}_2 = [0\ 1\ 1\ 0\ 1\ 1\ 1]$. Since there is no error in decoding, this codeword will be $\boldsymbol{x} \oplus \boldsymbol{t}_1 \oplus \boldsymbol{y} \oplus \boldsymbol{t}_2 = [0\ 0\ 1\ 0\ 1\ 1\ 1]$ because the Hamming distance between $\boldsymbol{x}$ and $\boldsymbol{y}$ is one and the minimal Hamming distance of the code $\mathcal{C}$ is three. The corresponding reconstructions $\hat{\boldsymbol{a}}_1 = \boldsymbol{a}_1$ and $\hat{\boldsymbol{a}}_2 = \boldsymbol{a}_2$ are then obtained as the systematic part of the codeword. Since $\boldsymbol{a}_1\boldsymbol{G}_1 = \boldsymbol{x} \oplus \boldsymbol{t}_1$ and $\boldsymbol{a}_2\boldsymbol{G}_2 = \boldsymbol{y} \oplus \boldsymbol{t}_2$, the sources are reconstructed as $\hat{\boldsymbol{x}} = \hat{\boldsymbol{a}}_1\boldsymbol{G}_1 \oplus \boldsymbol{t}_1 = [0\ 0\ 1\ 0\ 1\ 1\ 0] = \boldsymbol{a}_1\boldsymbol{G}_1 \oplus \boldsymbol{t}_1$, $\hat{\boldsymbol{y}} = \hat{\boldsymbol{a}}_2\boldsymbol{G}_2 \oplus \boldsymbol{t}_2 = [0\ 1\ 1\ 0\ 1\ 1\ 0] = \boldsymbol{a}_2\boldsymbol{G}_2 \oplus \boldsymbol{t}_2$. We thus see that $\boldsymbol{x}$ and $\boldsymbol{y}$ are indeed recovered error-free.

Similar examples can be easily given for non-asymmetric coding (e.g., with $R_1 = 4$ and $R_2 = 6$ bits).

3 Practical Code Design

In this section, we design practical SW codes using systematic IRA and turbo codes. We use the notation established in the previous section.

3.1 Systematic IRA Codes

In this subsection we apply the proposed methods to systematic IRA codes [13]. Systematic IRA codes are powerful channel codes that combine the advantages of LDPC codes (message-passing iterative decoding, simple analysis and code design) and turbo codes (linear time encoding). Their performance is comparable to that of irregular LDPC codes of the same codeword length. For simplicity, we consider symmetric SW coding of two binary sources X and Y. Code construction for the general case is essentially the same.

At the first encoder, the length n source output $\boldsymbol{x}$ is split into three parts in the form

$$\boldsymbol{x} = [\boldsymbol{a}_1 \quad \boldsymbol{v}_1 \quad \boldsymbol{q}_1], \tag{5}$$

where $\boldsymbol{a}_1$ and $\boldsymbol{v}_1$ are row-vectors of length $m = \frac{k}{2}$ and $\boldsymbol{q}_1$ is a row-vector of length $n-k = n-2m$.

First, $\boldsymbol{a}_1\boldsymbol{P}_1$ is determined by setting the values of the systematic IRA variable nodes to $[\boldsymbol{a}_1 \quad \boldsymbol{O}_{1\times m}]$, that is, half of the systematic part is set to zero.

Next, the length $n-m$ syndrome $\boldsymbol{s}_1$ that is formed by the first encoder is obtained by appending $\boldsymbol{v}_1$ to $\boldsymbol{a}_1\boldsymbol{P}_1 \oplus \boldsymbol{q}_1$. The encoding procedure is presented in Figure 1.

In a similar way, $\boldsymbol{s}_2$ is formed at the second encoder from $\boldsymbol{y} = [\boldsymbol{u}_2 \quad \boldsymbol{a}_2 \quad \boldsymbol{q}_2]$. At the joint decoder, first, vectors $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$ are formed as explained in the previous section; then, a

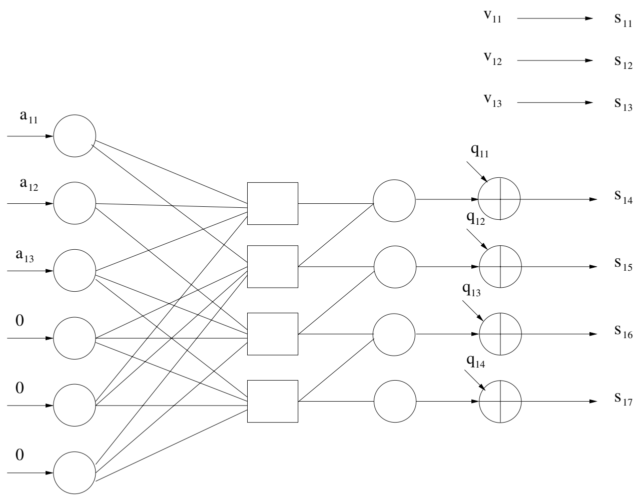

Figure 1: Encoding of source $\boldsymbol{x}$. At each check (square) node all the connected information nodes (cycles on the left) are modulo-2 added and corresponding values of the parity nodes (cycles on the right) are determined. Then, $\boldsymbol{q}_1$ is modulo-2 added. Here $n = 10$, $k = 6$, $m = 3$, $\lambda(x) = 0.25x + 0.75x^2$, and $\rho(x) = x^3$.

common IRA decoding of $\boldsymbol{t}_1 \oplus \boldsymbol{t}_2$ is performed and $\hat{\boldsymbol{a}}_1$ and $\hat{\boldsymbol{a}}_2$ are obtained as the systematic part of the recovered codeword; finally, $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are reconstructed as:

$$\hat{\boldsymbol{x}} = [\hat{\boldsymbol{a}}_1 \quad \boldsymbol{O}_{1\times m}]\, \boldsymbol{G} \oplus \boldsymbol{t}_1, \tag{6}$$

$$\hat{\boldsymbol{y}} = [\boldsymbol{O}_{1\times m} \quad \hat{\boldsymbol{a}}_2]\, \boldsymbol{G} \oplus \boldsymbol{t}_2. \tag{7}$$

As a result, if the used systematic IRA code can approach the capacity of a channel, then if the same channel models the statistics of $\boldsymbol{x} \oplus \boldsymbol{y}$, the resulting IRA coding scheme based on the above setup will also approach the SW limit for any rate allocation between the encoders. The procedure can be generalized to any asymmetric scenario with any number of sources. However, when more than two sources are used, modeling the exact correlation with a channel is more involved and hence more challenging.

3.2 Turbo Codes

In this subsection we briefly explain the SW code construction with systematic turbo codes [15]. Though turbo codes consist of two convolutional coders, they can be treated as linear block codes. Thus, the technique of Section 2 applies without modification. Indeed, assuming again the symmetric scenario, for the source realization $\boldsymbol{x}$ given by (5), we determine first $\boldsymbol{a}_1\boldsymbol{P}_1$ by coding the k-length vector $[\boldsymbol{a}_1 \quad \boldsymbol{O}_{1\times m}]$ with the first convolutional encoder. The vector $[\boldsymbol{a}_1 \quad \boldsymbol{O}_{1\times m}]$ is also interleaved and fed into the second encoder. The syndrome is formed then as:

$$\boldsymbol{s}_1 \;=\; [\boldsymbol{v}_1 \quad \boldsymbol{a}_1\boldsymbol{P}_1 \oplus \boldsymbol{q}_1]^T .$$

To get $\hat{\boldsymbol{a}}_1$ and $\hat{\boldsymbol{a}}_2$ at the decoder, iterative maximum a posteriori decoding is applied to the vector $\boldsymbol{t}_1 \oplus \boldsymbol{t}_2$ from (4). Then, $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are obtained from (6) and (7), respectively.

4 Results

We simulated SW coding of two independent identically distributed binary discrete sources X and Y whose correlation is modeled as a BSC with crossover probability p. We give experimental results for IRA and turbo codes.

The used systematic (n, k) IRA code is with rate 0.50227 and the degree distribution polynomials [13]: $\lambda(x) = 0.252744x^2 + 0.081476x^{11} + 0.327162x^{12} + 0.184589x^{46} + 0.154029x^{48}$, $\rho(x) = x^8$. The number of iterations in the decoder was limited to 200.

The turbo encoder consists of two identical recursive systematic convolutional encoders from [19] with memory length 4, generators $(31, 27)$ octal, and code rate $1/3$. The parity bits of both encoders were punctured to achieve the code rate of $1/2$. A maximum a posteriori algorithm was used for decoding, with the number of iterations limited to 20.

Obtained results are shown in Figure 2. The SW bound is 1.5 bits. The information block length was $k = 10^4$ and $k = 10^5$ bits. For each point at least 10^8 codeword bits were simulated. The results are given as residual bit error rate (BER) averaged over the two sources as a function of the joint entropy $H(X, Y) = H(X) + H(X|Y) = 1 + H(p)$.

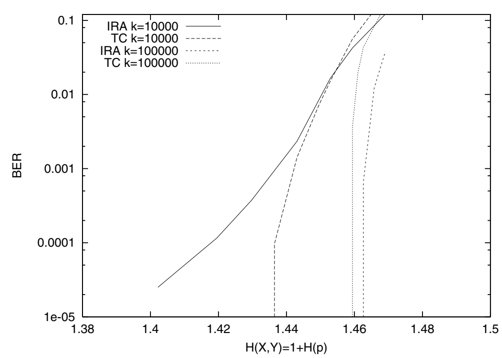

Figure 2: BER averaged over the two sources as a function of the joint entropy $H(X, Y) = 1 + H(p)$ for two different information block lengths k and two different channel coders.

It can be seen that similar performance was obtained with both coders. With the length of $k = 10^5$ the gap to the SW limit was about 0.04 bits, which is comparable to the results of the asymmetric approach with LDPC reported in [3]. Note that due to our coding procedure, usually either both sources are recovered error-free or both are corrupted. Also, because of the additional encodings at the decoder side, the errors propagate. Thus, either the whole messages are perfectly reconstructed or they are heavily damaged. (This is the reason why the drop for $k = 10^4$ with IRA codes was not sharp as expected.) Therefore, the decoder can detect errors with high certainty by comparing the two reconstructions.

Next, we simulated three different rate allocations among the encoders by changing the number of rows (m_1 and m_2) in the generator matrices of subcodes assigned to two encoders. Beside the symmetric scenario used in Figure 2, where we set $m_1 = m_2 = k/2$ and obtained equal rates of both encoders, $R_1 = R_2 = \frac{n-k/2}{n}$, we treated two asymmetric cases. In the first one, we set $m_1 = k/3$ and $m_2 = 2k/3$, and thus obtained $R_1 = \frac{n-k/3}{n}$ and $R_2 = \frac{n-2k/3}{n}$. Finally, in the totally asymmetric scenario, m_1 is set to zero, and m_2 to

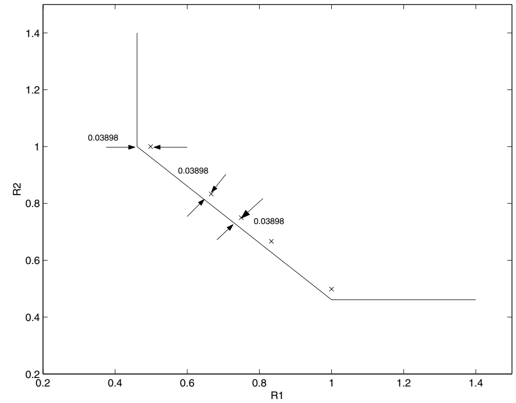

Figure 3: Results with IRA codes and $k = 10^5$ together with the SW bound.

k, which implies $R_1 = H(X) = 1$ and $R_2 = H(Y|X) = \frac{n-k}{n}$. Results obtained with the IRA based scheme and $k = 10^5$ together with the SW bound are shown in Figure 3. Error-free transmission was assumed if BER was lower than 10^{-6}. As expected, all three cases resulted in the same gap of 0.039 bits to the bound. Thus, the different rate allocation did not affect the performance. Similar results were obtained with the punctured turbo coder.

5 Conclusions

Pradhan and Ramchandran [10, 12] outlined a method for constructing a single channel code that achieves arbitrary rate allocation among two encoders in the SW coding problem. We have given a precise and detailed interpretation of what is suggested in [10,12] based on the systematic setup. This enabled us to provide coding designs using advanced systematic IRA and turbo codes that are capable of approaching any point on the SW bound.

We further discussed the extension of the results of [10, 12] to SW coding of multiple sources [17]. We showed that for a particular correlation model among sources, a *single code* can be designed. This observation, made for the first time here, is perhaps the real advantage of this method, as a single code can be used to approach the joint entropy limit. Indeed, if the designed code approaches the capacity of the channel that models correlation, then the system will approach the theoretical limit. Even when the number of sources is high, since all the sources are decoded by a single code, only one (good) code is needed. In addition to this, the inherent error detection capability make our method desirable for both direct and remote multiterminal problems [14].

In the case of more than two sources, the assumed correlation seems restrictive. Future work will investigate the code design for different correlation models. Extensions to general lossless multiterminal settings [20] and nonsystematic codes is a part of ongoing research [21].

To approach the theoretical limits in multiterminal coding with a fidelity criterion, after quantization of the sources, lossless coding is needed to further decrease the rate [2, 16, 22]. Hence, the method proposed here can be applied in this second compression step. Therefore, the design of a single practical code for the whole sensor network that can reach the theoretical limits seems realistic.

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