ex3 Group 17 Lab 3

1. Derive the formulas for (i) number of comparisons, and (ii) average-case number of swaps for bubble sort [0.4 pts]

(i) The number of comparisons is the same for arrays of the same size because the current element is always compared to the next, to check if they should be swapped in the case that the current is bigger than the next. However, because each time we decrease the number of elements we look at (the max element bubbles to the end of the array), we decrease the number of comparisons each time. There are n-1 comparisons in the first pass, so:

no_comparisons = n-1 + n-2 + n-3 + (…) + 1

= $\sum_{i=1}^{n-1} i$

or in other words, the sum of the first n-1 numbers. Thanks to Gauss we know that the closed form of this expression is $\frac{(n-1)(n)}{2}$ or $\frac{n^2-n}{2}$ which corresponds to a complexity of O(n^2)

(ii) The average-case number of swaps is similar to the number of comparisons. This is because in the average case, that is, if the order of the array elements is random, on each iteration there will be half as many swaps as there will be comparisons. Therefore, it approximates to $\frac{1}{2}\frac{n^2-n}{2}$ or just $\frac{n^2-n}{4}$, which also simplifies to O(n^2).

4. Separately plot the results of #comparisons and #swaps by input size, together with appropriate interpolating functions. Discuss your results: do they match your complexity analysis? [0.2 pts]

They do match complexity analysis. Because the number of comparisons is the same in all three cases, we can test our expression of $\frac{n^2-n}{2}$, and it is accurate as for example n = 10 we have that $\frac{n^2-n}{2}$ evaluates to $\frac{10^2-10}{2} = \frac{100-10}{2} = \frac{90}{2} = 45$, exactly matching the comparison count computed by the function that runs the algorithm. This is the same for other input sizes. For number of swaps, the values vary slightly, but on average they match the expected function of $\frac{n^2-n}{4}$. Both comparisons and swaps are supported by the fit function $a\ (x^2 - x)$, which provided an ‘a’ value of 0.5 and about 0.25 respectively.