



Baysean Inference

<https://www.youtube.com/watch?v=-e8wOcaascM> Thomas Bayes 1701-1761

$$P(\theta | \text{data}) = [P(\text{data} | \theta) \times P(\theta)] / P(\text{data})$$

Z : 確率変数 (random variables)

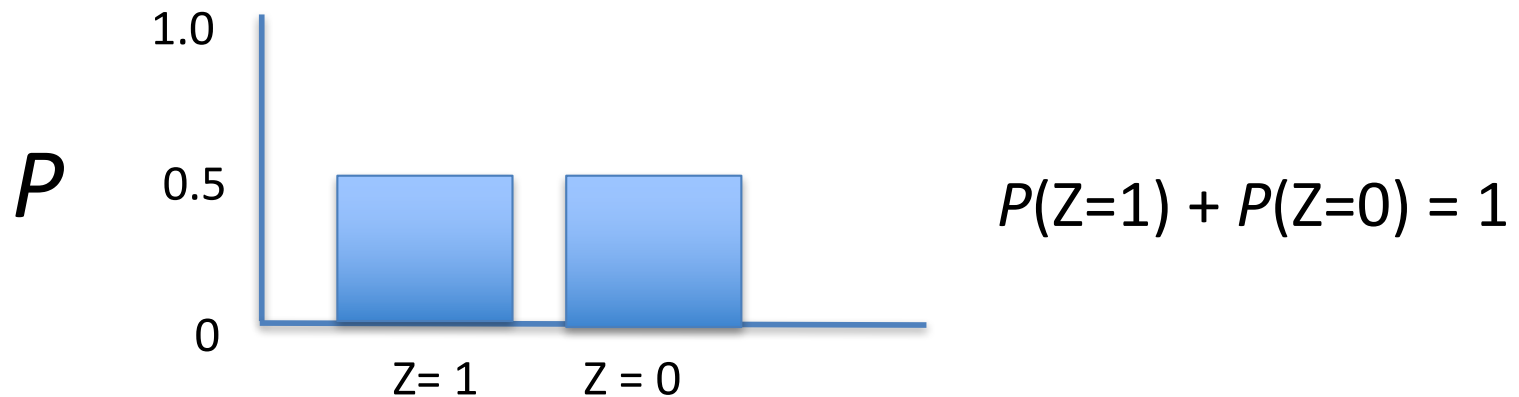
$P(Z)$: 確率質量関数 (probability mass function)

$$\sum_i P(Z_i) = 1$$

Z : 確率変数 (random variables)

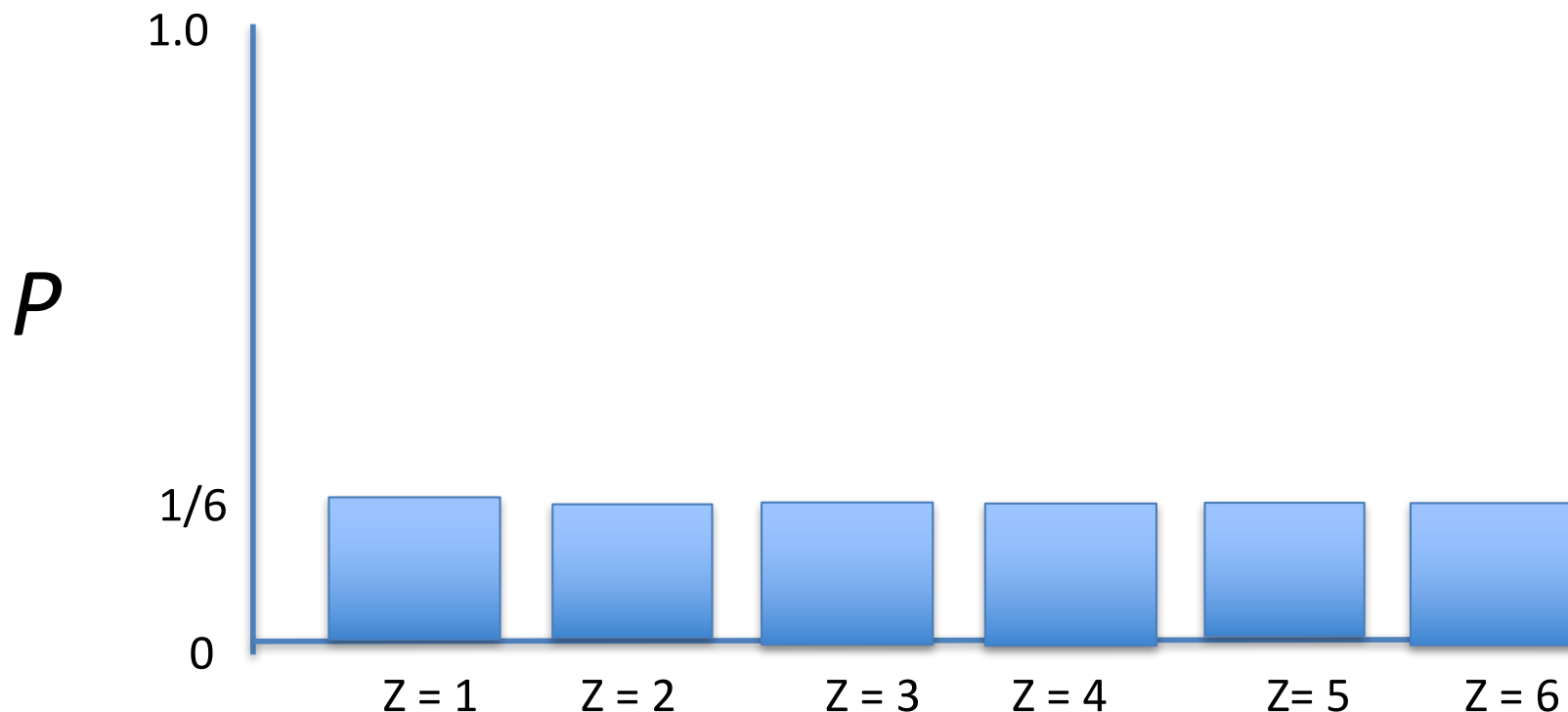
$P(Z)$: 確率質量関数 (probability mass function)

コイン投げ (Coin throw) $Z = \text{おもて: } 1 \text{ or } \text{うら: } 0$
 $P(1) = 0.5, P(0) = 0.5$



サイコロ投げ (Dice throw) $Z = 1, 2, 3, 4, 5, 6$

$P(1) = 1/6, P(2) = 1/6, P(1) = 1/6, P(2) = 1/6, P(1) = 1/6, P(2) = 1/6,$

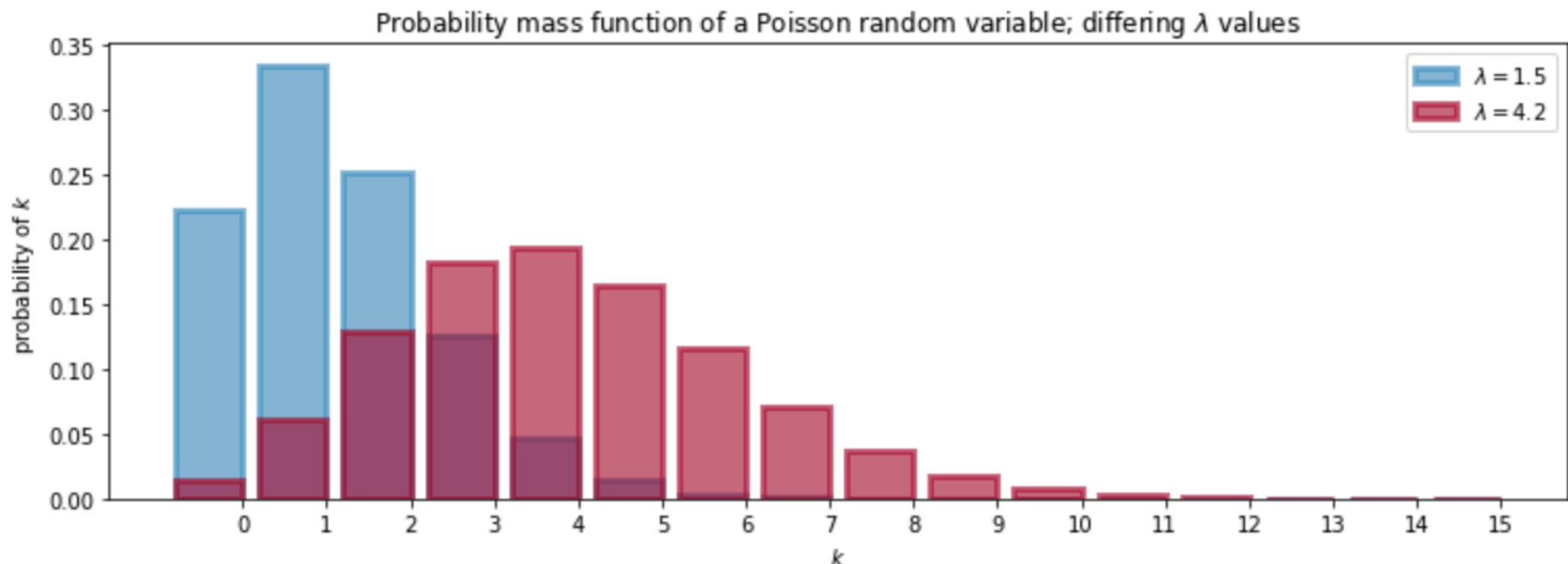


$$P(Z=1) + P(Z=2) + P(Z=3) + P(Z=4) + P(Z=5) + P(Z=6) = \sum_i P(Z = i) = 1$$

ポアソン分布 (Poisson Distribution)

10Kgのパン生地を400粒のレーズンをいれてこねた後、
0.1Kgずつのパンに分けて焼いた時、パン一個あたりの
レーズンの数の分布は？ $\lambda = 4$

$$P(Z = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \dots, \quad \lambda \in \mathbb{R}_{>0}$$



競馬の例 (Horse Race): 馬A と 馬B の勝敗の割合

		A	
		0 負 (lose)	1 勝 (win)
B	0 負 (lose)	30/100	10/100
	1 勝 (win)	10/100	50/100

$$\sum_{i=0}^1 \sum_{j=0}^1 P(Z_A = i, Z_B = j) = 30/100 + 10/100 + 10/100 + 50/100 = 1$$

競馬の例 (Horse Race) : 周辺分布 (Marginal Distribution)

		A		
		0 負 (lose)	1 勝 (win)	$P(Z_B)$
B	0 負 (lose)	30/100	10/100	40/100
	1 勝 (win)	10/100	50/100	60/100
$P(Z_A)$		40/100	60/100	周辺確率

$$P(Z_A=0) = P(Z_A=0, Z_B=0) + P(Z_A=0, Z_B=1) = 30/100 + 10/100 = 40/100$$

条件付き確率 (Conditional Probability)

$$P(A|B) = P(A,B)/P(B)$$

条件付確率 = 同時確率/周辺確率

Aが勝った場合に、Bも勝つ確率

$$P(Z_B=1 | Z_A=1) = P(Z_A=1, Z_B=1) / P(Z_A=1)$$

$$= P(Z_A=1, Z_B=1) / [P(Z_A=1, Z_B=0) + P(Z_A=1, Z_B=1)]$$

$$= (50/100) / [(10/100) + (50/100)] = 5/6$$

		A	
		0 負 (lose)	1 勝 (win)
B	0 負 (lose)	30/100	10/100
	1 勝 (win)	10/100	50/100

→

Bが勝った場合に
Aが勝つ確率は？

ベイズの定理 (Bayes's rule)

$$P(X|Y) = [P(Y|X) \times P(X)] / P(Y)$$

$$1/ \quad P(X|Y) = P(X,Y)/P(Y)$$

$$2/ \quad P(Y|X) = P(X,Y)/P(X)$$

$$3/ \quad P(X|Y) \times P(Y) = P(X,Y) = P(Y|X) \times P(X)$$

$$\rightarrow \quad P(X|Y) = [P(Y|X) \times P(X)] / P(Y)$$

ベイズ推定 (Bayes inference)

$$P(\theta | \text{data}) = [P(\text{data} | \theta) \times P(\theta)] / P(\text{data})$$

θ : 確率分布のパラメータ

(例: Poisson分布の λ)