Introduction to Topology

Crosby Collins
crosbyqcollins@gmail.com

Definition 1.1. We say that a collection of subsets $\tau \subseteq \mathcal{P}(X)$ forms a *topology on X* if

(i) $\varnothing, X \in \tau$

(ii) $\{U_i\}_{i=1}^n \in \tau \Rightarrow \bigcap_{i=1}^n U_i \in \tau$

(iii) $\{U_\alpha\}_{\alpha \in A} \in \tau \Rightarrow \bigcup_{\alpha \in A}^n U_\alpha \in \tau$

Definition 1.2. We say that a sequence of points $\{x_i\}_{i \in \mathbb{N}}$ *converges* to x if for any open neighborhood U around x, there exists an $N \in \mathbb{N}$ such that for $i \geq N$, $\{x_i\}_{i \in \mathbb{N}} \in U$.

Definition 1.3. We say that a map $f : X \to Y$ between topological spaces is *continuous* if for every $U \subseteq Y$, $f^{-1}(U)$ is open in X.

Exercise 1.4. Prove that this definition of continuity alligns with the one we already know (solutions may be emailed to me or posted in the comments).

Definition 1.5. We say that a collection $\mathcal{B} \subseteq \tau$ forms a *basis for the topology of* X if every open set of X can be expressed as the finite union of elements of $\mathcal{B}$.