Current and Resistance

So far, only electrostatics – the Physics of stationary charges

Now: Electric Currents – charges in motion

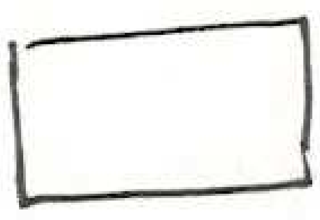
Electric Current

there must be a net flow of charge – movement of charges is not enough

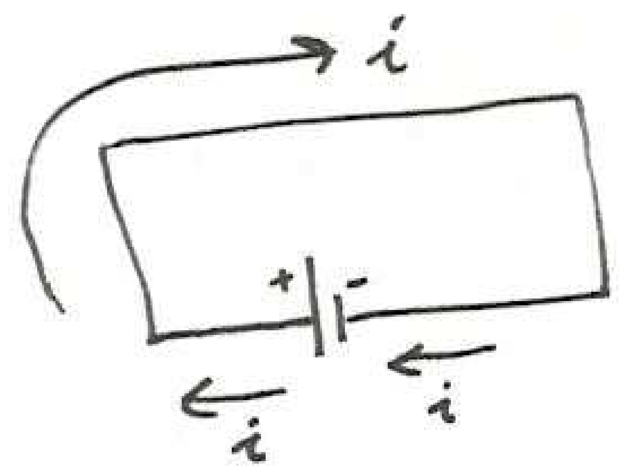
1) Charges in an isolated copper wire
   vs. the wire connected to a Battery.

2) Flow of water through a garden hose
   $\longrightarrow$ no net charge: protons move along with the electrons in the same direction.

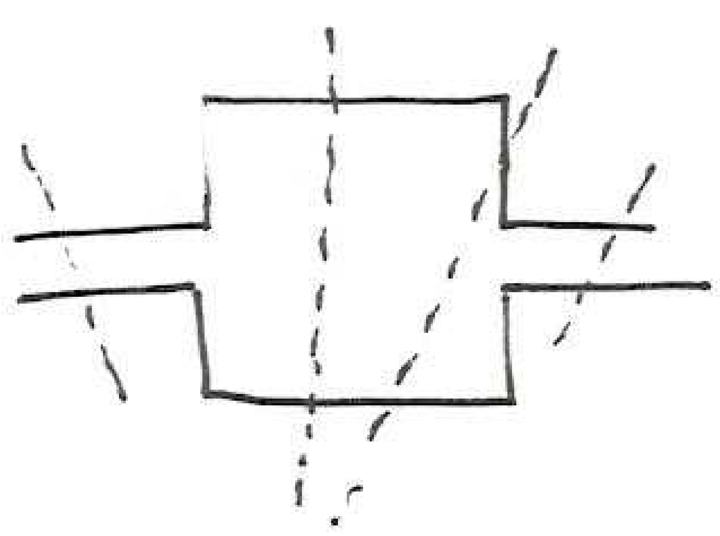
$\Rightarrow$ CURRENT: <u>Steady</u> currents of <u>conduction</u> <u>electrons</u> moving through <u>metallic conductors</u>



isolated conducting loop
regardless of having excess (net) charge,
every part is at the same potential.
$\rightarrow$ No electric field can exist within it.



$i \equiv \frac{dq}{dt} \longrightarrow q = \int dq = \int_0^t i\,dt$: Charge that passes through in a time interval.



The current has the same value (assuming the current is steady)

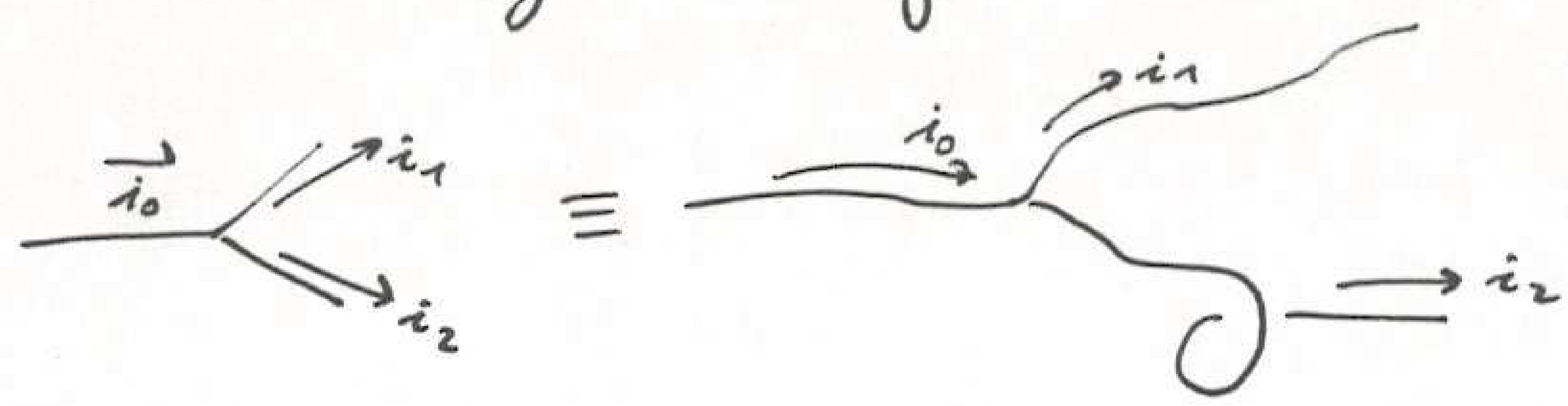
an electron enters from one side
$\longrightarrow$ another leaves from the other side

water hose & drops
$\Rightarrow$ Amount of water in the hose is a conserved quantity.

1 Ampere $= 1A = 1$ Coulomb per second $= 1\ C/s$

↳ André-Marie Ampère (1779-1836)

Current : scalar but often represented with an arrow to indicate the charge is moving.



$i_0 = i_1 + i_2$

Direction of Currents:

electrons vs. protons/+ charges (Charge carriers)

! [A current arrow is drawn in the direction in which positive charge carriers would move even if the actual charge carriers are negative and move in the opposite direction.

Most of the time, the assumed motion of positive charge carriers in one direction has the same effect as the actual motion of negative charge carriers in the opposite direction.

CURRENT DENSITY

Flow of charge through a cross-section of a conductor at a particular point.

$\vec{J}$ = Current Density (same direction as the velocity $\vec{v}$ of the positive charges moving
↔ opposite if negative)

Current per unit area

$\rightarrow \quad i = \int \vec{J}\, d\vec{A}$

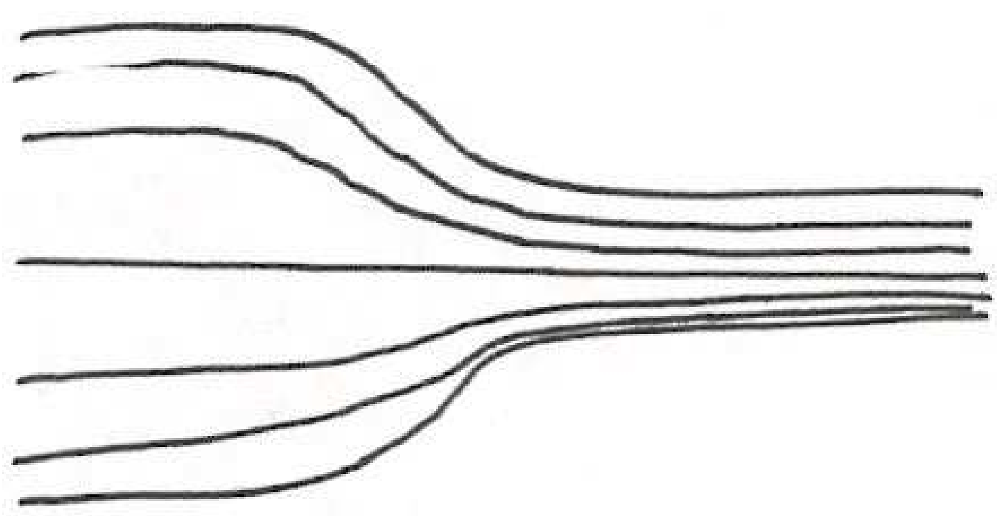
if current is uniform across the surface
and parallel to $d\vec{A}$ $\longrightarrow$ normal to the surface!

$\longrightarrow$ $\vec{J}$ is also uniform and parallel to $d\vec{A}$

$$i = \int J\, dA = J \int dA = JA$$

$$\longrightarrow J = \frac{i}{A} \quad \longrightarrow \quad [\,|J|\,] = \frac{A}{m^2}$$

Streamlines:



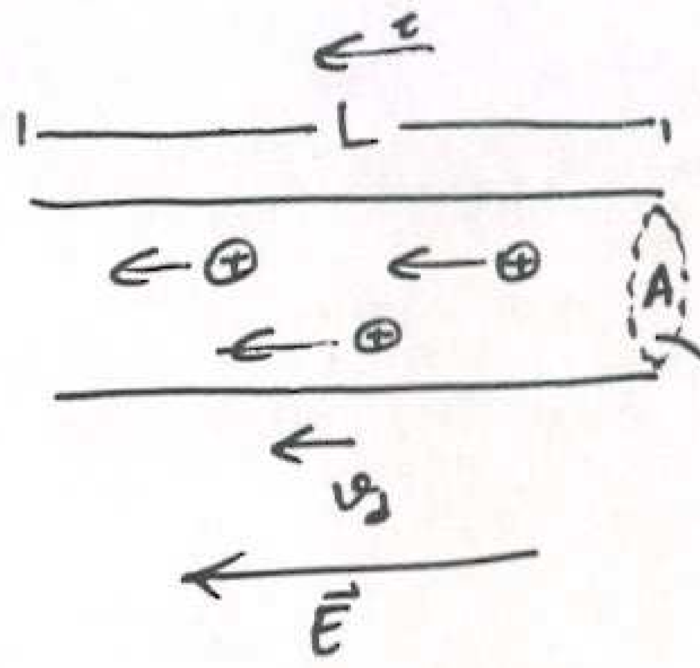
Because the Charge is conserved, the amount of charge and thus the amount of current can not change. However, the current density does change – it is greater in the narrower conductor. The spacings of the streamlines become denser.

DRIFT SPEED

When the conductor does not have a current through it, the conduction electrons move randomly such that the net displacement is zero. But, when there is a current, they actually still move randomly, but now they tend to drift with a drift speed v_d.
("Sadly..") it is very small compared to random motion speed.

$$v_d \cong 10^{-5} - 10^{-4}\ m/s$$

$$v_{random} \cong 10^{6}\ m/s$$



n: volumetric charge density
(number of charges per unit volume)

$\rightarrow \quad q = (nAL)e$

A: cross section area

$t = \dfrac{L}{v_d}$ (time needed to cross a length of L with a speed of v_d)

$$i = \frac{q}{t} = \frac{nAL}{L/v_d} = nAev_d$$

$$J = \frac{i}{A} \qquad v_d = \frac{i}{nAe} = \frac{J}{ne}$$

$$\longrightarrow \vec{J} = (ne)\vec{v}_d :$$

- positive carriers: $\vec{J}, \vec{v}_d$ same dir.
- negative carriers: $\vec{J}, \vec{v}_d$ opposite dir.

Ex: A steady current of 2.5A flows in a wire for 4 minutes.

a) How much charge passed by any point in the circuit?

$$\Delta Q = I\,\Delta t$$
$$= (2.5\ C/s)(240s) = 600\ C$$

b) How many electrons would this be?

$$1e = 1.6\times10^{-19}\ C \longrightarrow 600C = 3.8\times10^{21}\ \text{electrons}.$$

Ex: a) $R = 2.0mm$, $\vec{J}$ uniform $= 2\times10^5\ A/m^2$ $\quad i\Big|_{r=\frac{R}{2}}^{R} = ?$

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\frac{3R^2}{4}$$

$$R \to 2mm \longrightarrow A' = 9.424\times10^{-6}\ m^2$$

$$i = JA' = (2\times10^5\ A/m^2)(9.424\times10^{-6}\ m^2)$$
$$= 1.9\ A$$

b.) $R = 2.0mm$, $\vec{J}$ non-uniform $\Rightarrow J(r) = ar^2$

$a = 3\times10^{4}\ A/m^4$

$$i = \int \vec{J}\,d\vec{A} \qquad \vec{J}\cdot d\vec{A} = J\,dA$$

$$\rightarrow i = \int \vec{J}\,d\vec{A} = \int J\,dA = \int_{R/2}^{R} ar^2\, 2\pi r\, dr = 2\pi a\int_{R/2}^{R} r^3 dr$$

$$= 2\pi a\left[\frac{r^4}{4}\right]_{R/2}^{R} = \frac{\pi}{a}\left[R^4 - \frac{R^4}{16}\right] = \frac{15}{32}\pi a R^4 = 7.1A$$

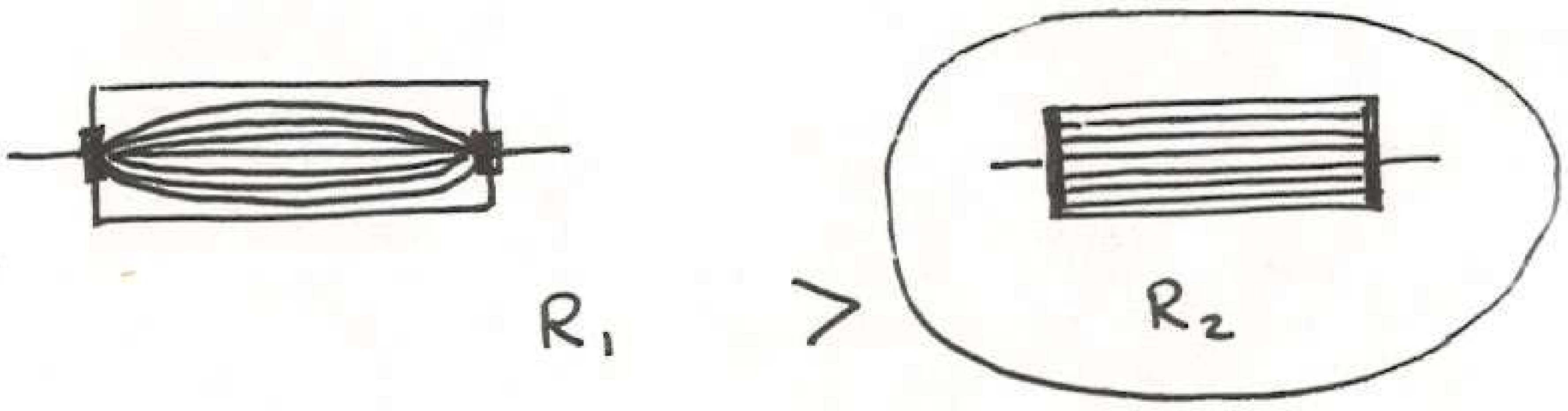
Resistance & Resistivity

$$R = \frac{V}{i} \qquad [|R|] = \text{ohm} = \Omega = \frac{V}{A}$$

$i = \frac{V}{R} \rightarrow$ for a given V,

greater the resistance,
→ smaller the current

Dependent on the manner of potential difference is applied to it.



$R_1 > R_2$

← we will always assume this kind of attachment/connection

generality: $\rho = \frac{E}{J}$ resistivity of the material

$$\frac{V/m}{A/m^2} = \frac{V}{A} m = \Omega m$$

$\vec{E} = \rho \vec{J}$

$\rho \leftrightarrow \sigma$

resistivity

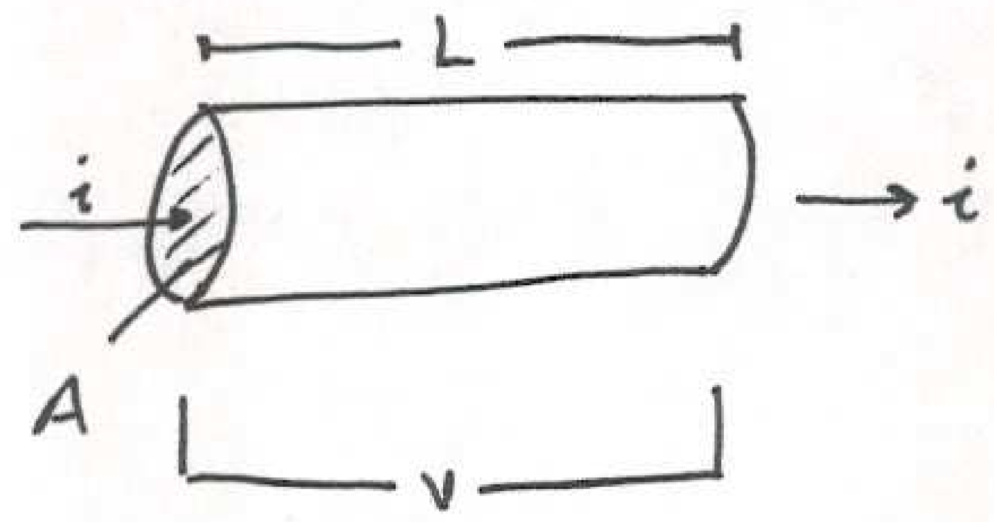
conductivity $\rightarrow \sigma = \frac{1}{\rho}$, $[|\sigma|] = \underbrace{(\Omega m)^{-1}}_{\text{mho/m}}$

mho ↓ Ω^{-1}

$\vec{J} = \sigma \vec{E}$

	$\rho\ (\Omega m)$
Silver	1.62×10^{-8}
Copper	1.69×10^{-8}
Gold	2.35×10^{-8}
Silicon, pure	2.5×10^{3}
Silicon, n-type	8.7×10^{-4}
glass	$10^{10} - 10^{14}$
Quartz	10^{16}

Calculating Resistance From Resistivity



Streamlines uniform

$$E = \frac{V}{L} \qquad J = \frac{i}{A}$$

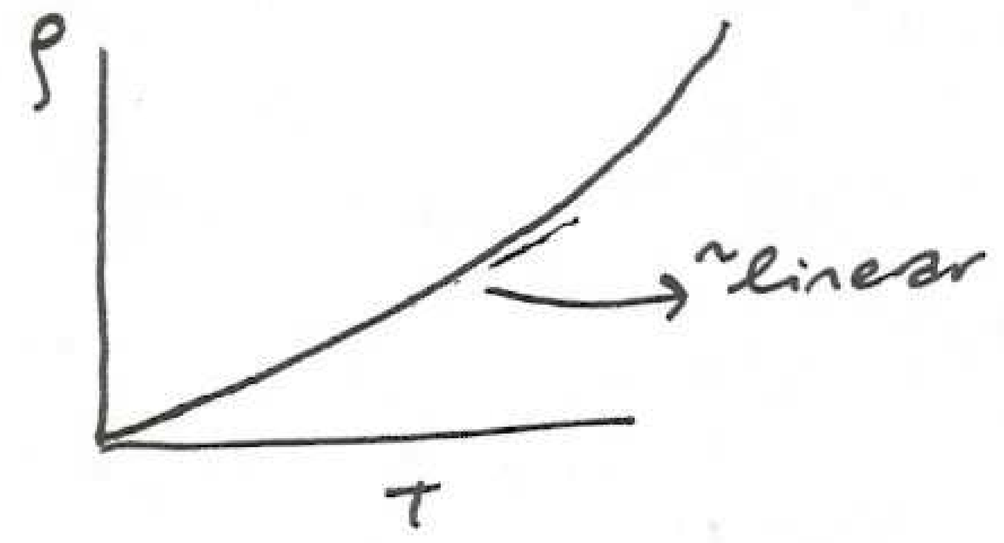
$$\rho = \frac{E}{J} = \frac{V/L}{i/A} \Rightarrow R = \rho \frac{L}{A}$$

R

V, i, R : macroscopic quantities

E, J, ρ : microscopic quantities

Variation with Temperature



$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

usually $T_0 = 293°K$ (room temp, $20°C$)

temperature coefficient of resistivity

Ex 7

1.2 cm × 1.2 cm × 15 cm

rectangular block, iron ($\rho = 9.68 \times 10^{-8}\ \Omega m$)

1) R if the square ends are connected? $\rightarrow A = (1.2 cm)^2 = 1.44 \times 10^{-4} m^2$

$L = 0.15 m$

2.) R if the rectangular ends are connected?

$$R = \frac{\rho L}{A} = 100 \mu\Omega$$

$A = (1.2)(15) \times 10^{-4} m^2$

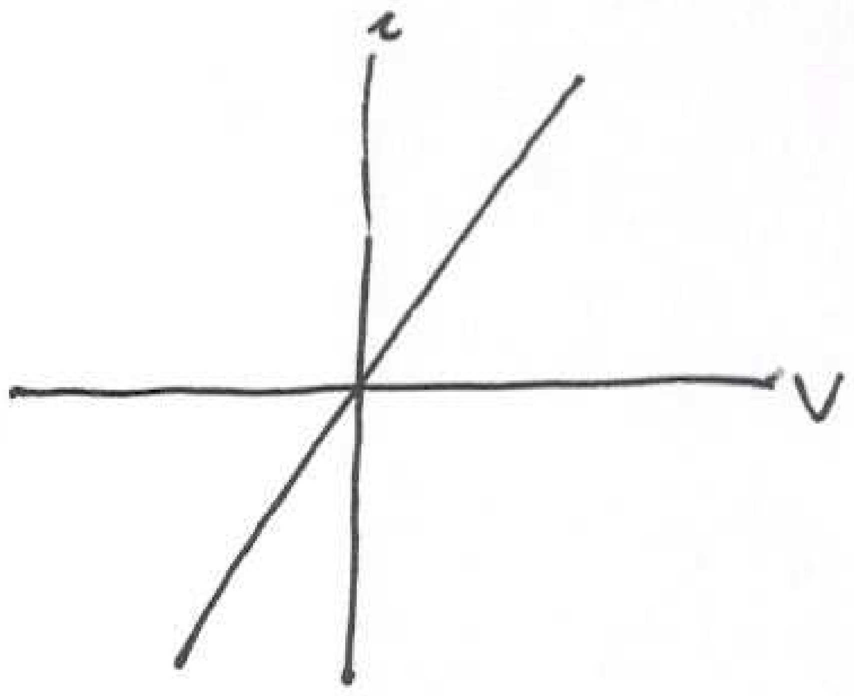
$L = 0.012 m$

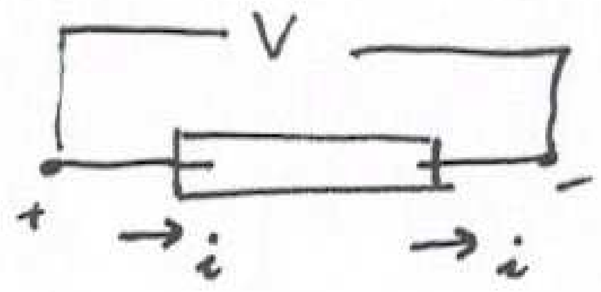
$$R = \frac{\rho L}{A} = 0.65 \mu\Omega$$

OHM'S LAW

A resistor is a conductor with a specified Resistance.

Same Resistance value no matter what the magnitude and direction of the applied voltage.

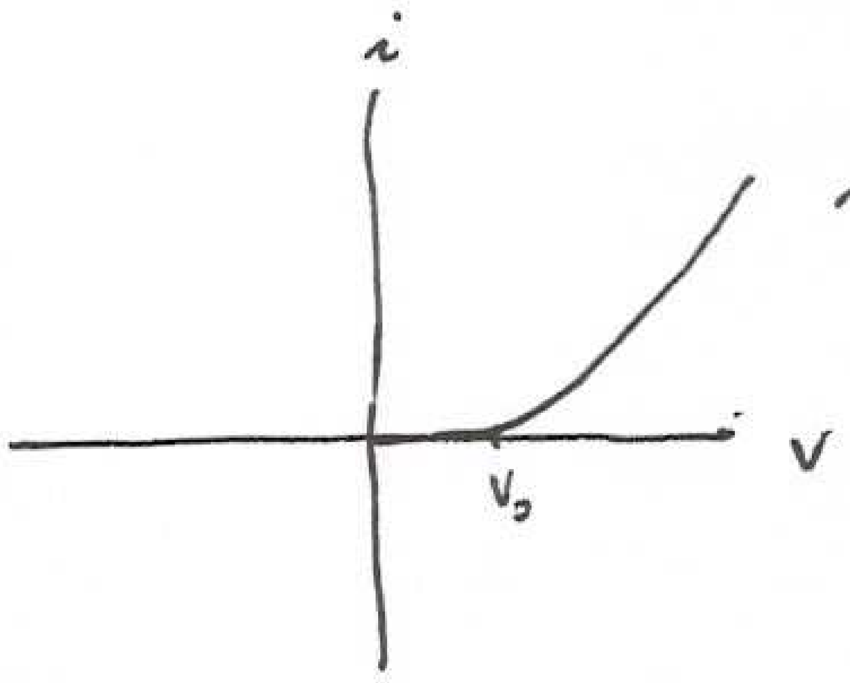




i/V = const.

$$R = \frac{V}{i}$$

counter-example: <u>diode</u>



A current can exist only when V is positive and applied Voltage $> V_0$.

→ Ohm's Law is an assertion that the current through a device is always directly proportional to the potential diff applied to the device.

A conducting device obeys Ohm's Law when the resistance of the device is independent of the magnitude and polarity of the applied voltage difference.

$V = iR$ → Resistance eqn. applies to all conducting devices.

"Resistance at 'that' value of V"

OHM'S Law: i vs. V is linear,
R is independent of V

OHM'S LAW : Devices ⟶ material

$$\vec{E} = \rho \vec{J}$$

A conducting material obeys OHM'S Law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

All Homogenous materials, whether they are conductors or semiconductors obey OHM's Law within some range of values of the electric field. If the field is too strong, there will be departures from OHM's Law.

A MICROSCOPIC VIEW OF OHM'S LAW

Free electrons move among atoms, interacting only with atoms (assuming, not with each other).

→ They should have a Maxwellian speed distribution (Like molecules in a gas).

→ the average speed depends on temperature.

However, electron motion is governed not by the laws of classical physics but those of quantum physics.

→ turns out, they move in a metal with a single effective speed $v_{eff} = 1.6\times10^{6}\,m/s$

Drift speed $v_{drift} \cong 5\times10^{-7}\,m/s$: much less than the effective speed.

$$a = \frac{F}{m} = \frac{eE}{m}$$

after a typical collision, each electron will lose the information of its previous drift velocity

$$\rightarrow \quad v_d = a\tau = \frac{eE}{m}\tau$$

$$\vec{J} = ne\vec{v}_d \Rightarrow v_d = \frac{J}{ne} = \frac{eE\tau}{m}$$

$$E = \left(\frac{m}{e^2 n\tau}\right) J$$

$$\vec{E} = \rho\vec{J} : \rho = \frac{m}{e^2 n\tau} \Rightarrow \text{Resistivity is independent of } \vec{E}$$

Example: Mean free time & Mean Free path (between collisions) for the conduction electrons in copper

$$n_{cu} = 8.49\times10^{28}\,m^{-3}, \quad \rho_{cu} = 1.69\times10^{-8}\,\Omega m$$

$\tau \sim$ constant and independent of the Electric Field

$$\rho = \frac{m}{e^2 n\tau} \rightarrow \tau = \frac{m}{\underbrace{ne^2\rho}}$$

$$[ne^2\rho] = m^{-3}\,C^2\,\Omega m = \frac{C^2\Omega}{m^2} = \frac{C^2 V}{m^2 A} = \frac{C^2 J/C}{m^2\,C/s} = \frac{kg\,m^2/s^2}{m^2/s} = \frac{kg}{s}$$

$$\dots\rightarrow \tau = \frac{9.1\times10^{-31}\,kg}{3.67\times10^{-17}\,kg/s} = 2.5\times10^{-14}\,s$$

⟶ Cont'd.

mean free path λ

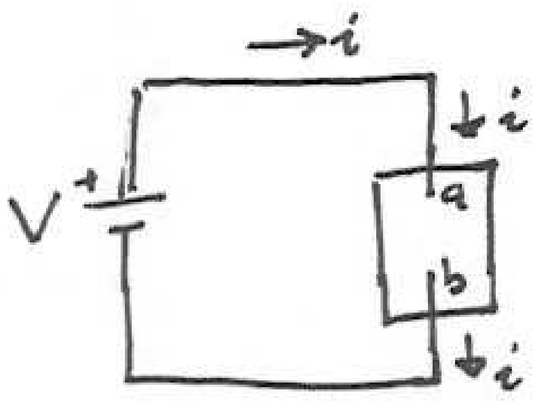
$(v_{eff} = 1.6\times10^{6}\,m/s)$

$$\lambda = v_{eff}\cdot\tau = (1.6\times10^{6}\,m/s)(2.5\times10^{-14}\,s)$$

$$= 4\times10^{-8}\,m = 40\,nm$$

~ 150 times the distance between nearest neighbor atoms in a copper lattice

Power in electric circuits



$a \to b$ $dq = i\,dt$

dq moves through a decrease in potential of magnitude V and thus its electric potential energy decreases in magnitude by the amount

$$dU = dq\cdot V = i\,dt\,V$$

Power $P = \frac{dU}{dt} = iV$ (rate of electric energy transfer)

from battery to the device

if device is a motor connected to a mechanical load,

the energy transferred as work done on the load.

If the device is a rechargable battery,

the energy is transferred to stored chemical energy.

If the device is a resistor,

the energy is transferred to internal thermal energy.

$$[|P|] = 1\ V.A = 1\left(\frac{J}{C}\right)\left(\frac{C}{s}\right) = 1\frac{J}{s} = 1\,\underline{\underline{W}}$$

As an e^- moves through a resistor at a constant drift speed, its average kinetic energy remains constant and its last potential energy appears as thermal energy.

The mechanical energy transferred to thermal energy is dissipated (lost) because the transfer cannot be reversed.

if the device is a resistor: $R = \frac{V}{i} \to P = i^2R \leftrightarrow P = \frac{V^2}{R}$

SEMICONDUCTORS

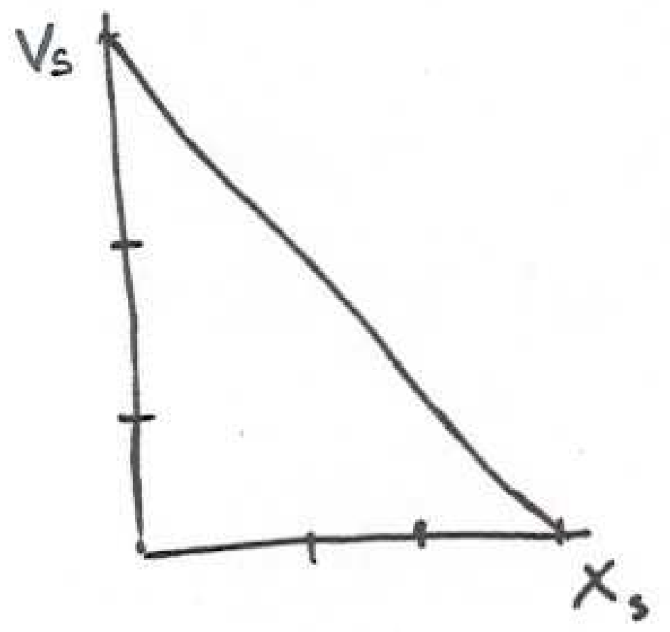
	Copper	Silicon
Type	Metal	Semiconductor
Charge carrier density (m^{-3})	8.49×10^{28}	1×10^{16}
Resistivity ρ (Ωm)	1.69×10^{-8}	2.5×10^{3}
Temperature coefficient of resistivity (K^{-1})	4.3×10^{-3}	-70×10^{-3}

↑ resistivity of silicon decreases with temp

Pure silicon is an efficient insulator. But its resistivity can be reduced in a controlled way by adding specific "impurity" atoms, i.e. by "doping".

	n-type silicon	p-type silicon
ρ (Ωm)	8.7×10^{-4}	2.8×10^{-3}

Example



Copper wire ($\rho = 1.69 \times 10^{-8} \Omega m$)

$X_s = 3m$

$V_s = 12 \mu V$

$r_{wire} = 2 \times 10^{-3} m$

$i = ?$

$A = \pi r^2$

$V = iR = i \rho \frac{L}{A} \rightarrow \ldots \rightarrow i = 0.00297 A \simeq 3 mA$

Example Nichrome wave

$A = 2.6 \times 10^{-6} m^2$

$\rho = 5 \times 10^{-7} \Omega m$

$V = 75 V$

a.) if $P = 5000 W \rightarrow L = ?$

$$P = \frac{V^2}{R} = \frac{AV^2}{\rho L} \rightarrow L = \frac{AV^2}{\rho P} = \ldots = 5.85 m$$

b.) if $V = 100V$ with the same dissipation rate, what is the L then?

$$L \propto V^2 \qquad L' = L\left(\frac{V'}{V}\right)^2 = 5.85 \left(\frac{100}{75}\right)^2 = 10.4 m$$

Example $A = 2 \times 10^{-6} m^2$

$L = 4m$

Copper wire ($\rho = 1.69 \times 10^{-8} \Omega m$)

$i = 2A$, uniform.

a.) $P = ?$ b.) $E = ?$
energy transferred in 30 mins?

$R = \rho \frac{L}{A} = 0.0338 \Omega$

a.) $P = i^2 R = \ldots = 0.135 W \rightarrow J/s$ → b.) $E_{30min} = 0.135 \; J/s \times 30 \times 60 s = 2.43 \times 10^2 J$

* Electron energies & "loose" electrons in the inner shells

Semiconducting devices such as transistors and Junction diodes are fabricated by the selective doping of different regions of the silicon with impurity atoms of different kinds.

$\rho = \frac{m}{e^2 n \tau}$ $T \uparrow$ Conductor : $n \gg$ but $\sim$ constant with respect to T

increase in ρ in metals is due to the increase in the collisions rate which lowers τ

Semi-conductor : $n <$ but increases rapidly

$\rightarrow \rho \downarrow$

Superconductors

Mercury : $R = 0$ below 4K

electron pairs

1911 : Onnes

1972 Nobel : Bardeen, Cooper, Schrieffer

2003 Nobel : Abrikosov, Ginzburg, Legget