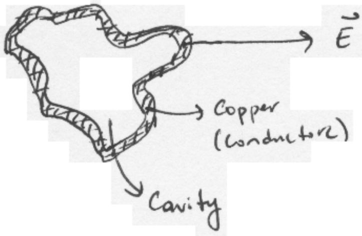
A Charged Isolated Conductor

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

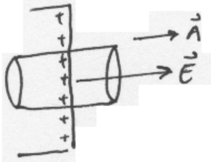


$\vec{E}$ inside is zero (otherwise, the electrons would move and there would always be a current within)

$\vec{E}$ is zero anywhere inside

$\Phi = 0 \longrightarrow q_{net} = 0$

$\Rightarrow q$ must be on the surface



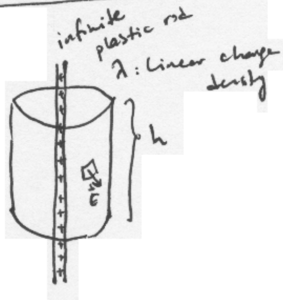
$\Phi = EA$

surface charge density

$q_{enc} = \sigma A$

$\Rightarrow \epsilon_0 EA = \sigma A \Longrightarrow E = \dfrac{\sigma}{\epsilon_0}$ (Conducting Surface)

Cylindirical Symmetry



$\Phi = EA\cos\overset{0}{\theta} = E(2\pi r h)$

$Q = \lambda h \qquad \epsilon_0 \Phi = q_{enc}$

$\epsilon_0 E(2\pi r h) = \lambda h$

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$

Compare with previously derived Result: finite-long rod (4-1)

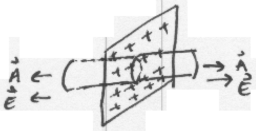
$(k = 1/4\pi\epsilon_0\ ,\ d \to \ast)$

$E = \dfrac{1}{4\pi\epsilon_0} \dfrac{\lambda}{r} \dfrac{2a}{\sqrt{a^2 + r^2}}$

$\underset{\text{infinite length}}{\underbrace{a \gg r}}$: $E = \dfrac{1}{4\pi\epsilon_0} \dfrac{\lambda}{r} \dfrac{2a}{a\sqrt{1 + \underbrace{\frac{r^2}{a^2}}_{\to 0}}} \rightarrow E_{a\to\infty} = \dfrac{\lambda}{2\pi\epsilon_0 r}$

PLANAR SYMMETRY

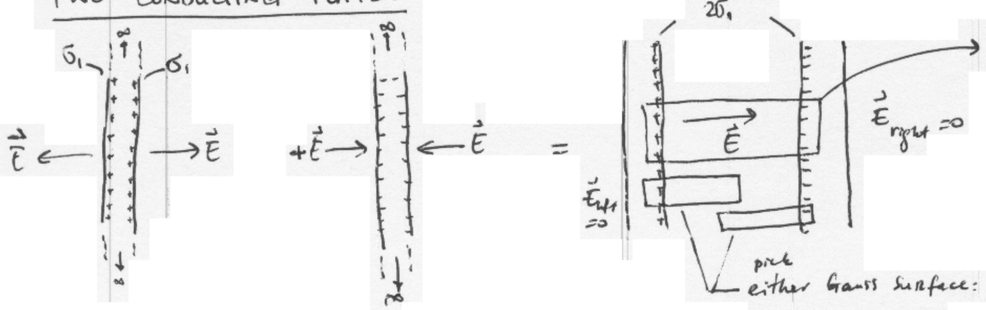
Non-conducting Sheet of charge



$$2EA = \sigma A$$

$$E = \frac{\sigma}{2\epsilon_0} \quad \text{(independent of distance!)}$$

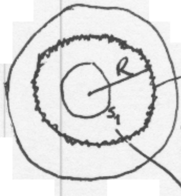
TWO CONDUCTING PLATES



If you pick the Gaussian surface like this, you are not calculating the $\vec{E}$ between the plates but that of the system!

$$E = \frac{2\sigma_1}{\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

SPHERICAL SYMMETRY



q (thin, uniformly charged sphericall shell with total charge q)

S_2, Gaussian surface, encloses the shell

S_1, Gaussian Surface, is inside the shell

* A shell of uniform charge attracts/repels a charged particle as if all the shell's charge were concentrated at the center of the shell.

* If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad r \geq R$$

$$E = 0 \quad r < R$$

Suppose a radial charge distribution in a sphere: (ρ: volume charge density)
q': enclosed charge
q: total charge (q'_{tot})

$$\left.\begin{array}{ll} r > R & E = \dfrac{1}{4\pi\epsilon_0} \dfrac{q'_{total} \to q}{R^2} \\[2ex] r \leq R & E = \dfrac{1}{4\pi\epsilon_0} \dfrac{q'}{r^2} \end{array}\right\} \quad \frac{\left(\begin{array}{c}\text{Charge enclosed by} \\ \text{sphere of radius } r\end{array}\right)}{\left(\begin{array}{c}\text{volume enclosed by} \\ \text{sphere of radius } r\end{array}\right)} = \frac{\text{full charge}}{\text{full volume}}$$

$$\rightarrow \quad \frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3} \rightarrow q' = q\,\frac{r^3}{R^3}$$

$$\Rightarrow \quad E = \left(\frac{q}{4\pi\epsilon_0 R^3}\right) r \quad (\text{uniform charge, } r \leq R)$$