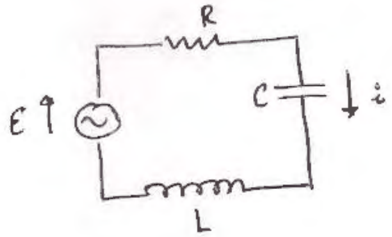
THE SERIES RLC CIRCUIT

$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$ (applied emf)

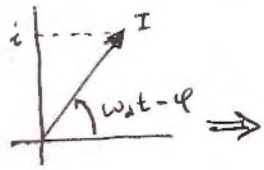


R, L, C in series

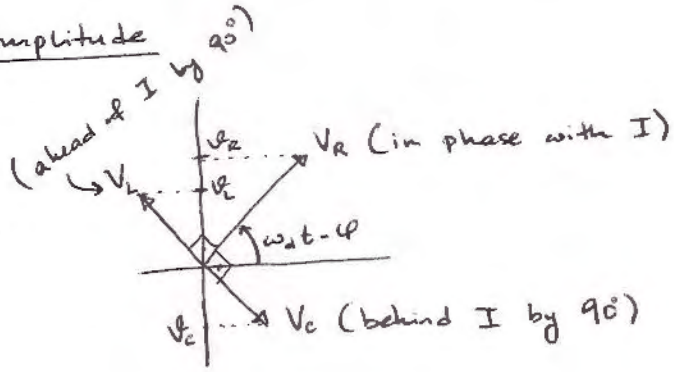
$\longrightarrow i = I \sin(\omega_d t - \varphi)$

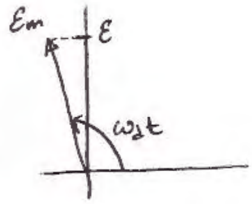
I: amplitude, φ: phase

The Current Amplitude



$\Longrightarrow$

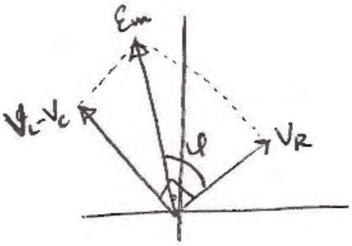




$\mathcal{E} = v_R + v_c + v_L$ (at any time)

As the phasors rotate together, this equality always holds.

$\Rightarrow$ $\mathcal{E}_m$ must be equal to the vector sum of the three voltage phasors V_R, V_c and V_L



$$\mathcal{E}_m^2 = V_R^2 + (V_L - V_c)^2$$
$$= (IR)^2 + (IX_L - IX_c)^2$$

$$\Rightarrow I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_c)^2}}$$

$\longrightarrow$ impedance Z of the circuit for the driving angular frequency ω_d:

$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

$\Rightarrow I = \frac{\mathcal{E}_m}{Z}$ $\quad X_c = \frac{1}{\omega_d C}$, $X_L = \omega_d L$

$$\Rightarrow I = \frac{\mathcal{E}_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$

I depends on the difference between $\omega_d L$ and $\frac{1}{\omega_d C}$

$\Updownarrow \qquad \Updownarrow$

X_L and X_c

$\hookrightarrow$ doesn't matter which of the quantities is greater as the difference is squared.

The current we have been describing is the "steady-state" current that occurs after the alternating emf has been applied for some time. When the emf is first applied to a circuit, a brief "transient current" occurs. Its duration (before settling down into the steady state current) is determined by the time constants $\tau_L = \frac{L}{R}$ and $\tau_C = RC$ as the inductive and capacitive elements "turn on". The transient current can, for example destroy a motor on start-up if it is not properly taken into account in the motor's circuit design.

The Phase Constant

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}$$

$$\tan\varphi = \frac{X_L - X_C}{R}$$ → depending on the relative values of the reactances X_L and X_C, it can be one of the three:

i) $X_L > X_C$: The circuit is said to be more inductive than capacitive.

$\varphi > 0$ → phasor I rotates <u>behind</u> phasor $\mathcal{E}_m$

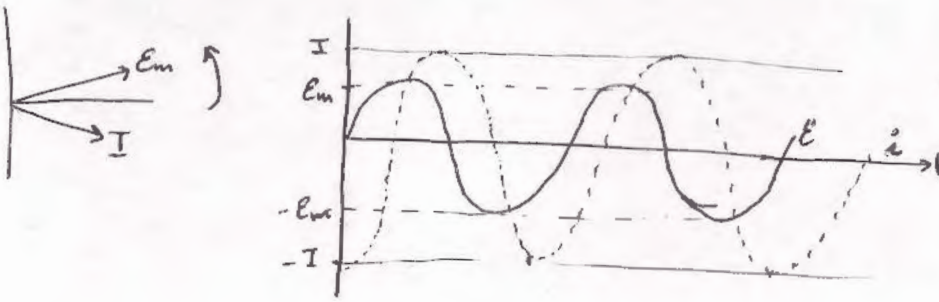
ii) $X_C > X_L$: the circuit is said to be more capacitive than inductive.

$\varphi < 0$ → I rotates <u>ahead</u> of $\mathcal{E}_m$

iii) $X_C = X_L$: The circuit is said to be in Resonance

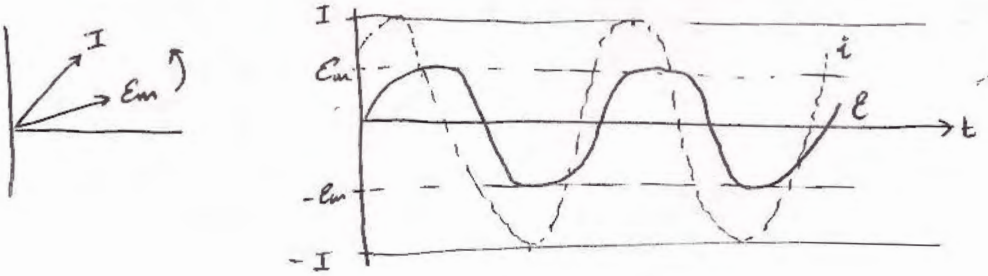
$\varphi = 0$, $\mathcal{E}_m$ & I rotate <u>together</u> in phase.

i) $\varphi > 0$



Current lags the emf, curve peaks later.

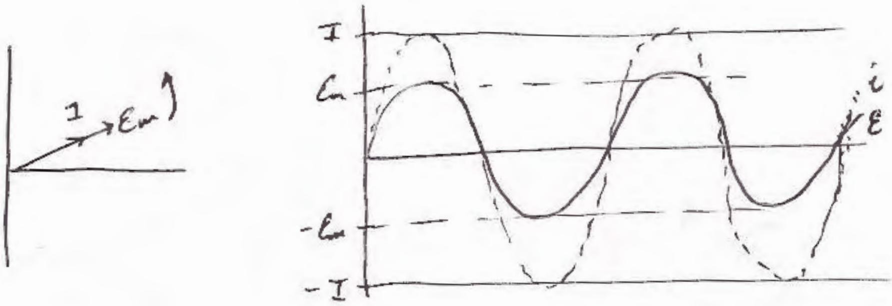
ii) $\varphi < 0$



Current leads the emf

Curve peaks earlie

ii) $\varphi = 0$



Current and the emf are in phase

They peak together.

<u>RESONANCE</u>

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$ for a given R, the amplitude is maximum when $X_L = X_C$

$$\longrightarrow \omega_d L = \frac{1}{\omega_d C} \quad \text{or} \quad \omega_d = \frac{1}{\sqrt{LC}} \quad (\text{maximum } I)$$

Since the natural angular frequency ω of the RLC is also equal to $\frac{1}{\sqrt{LC}}$, maximum value of I occurs when the driving angular frequency matches the natural angular frequency,

$\longrightarrow$ that is at <u>resonance</u>. $\omega_d = \omega = \frac{1}{\sqrt{LC}}$ (resonance)

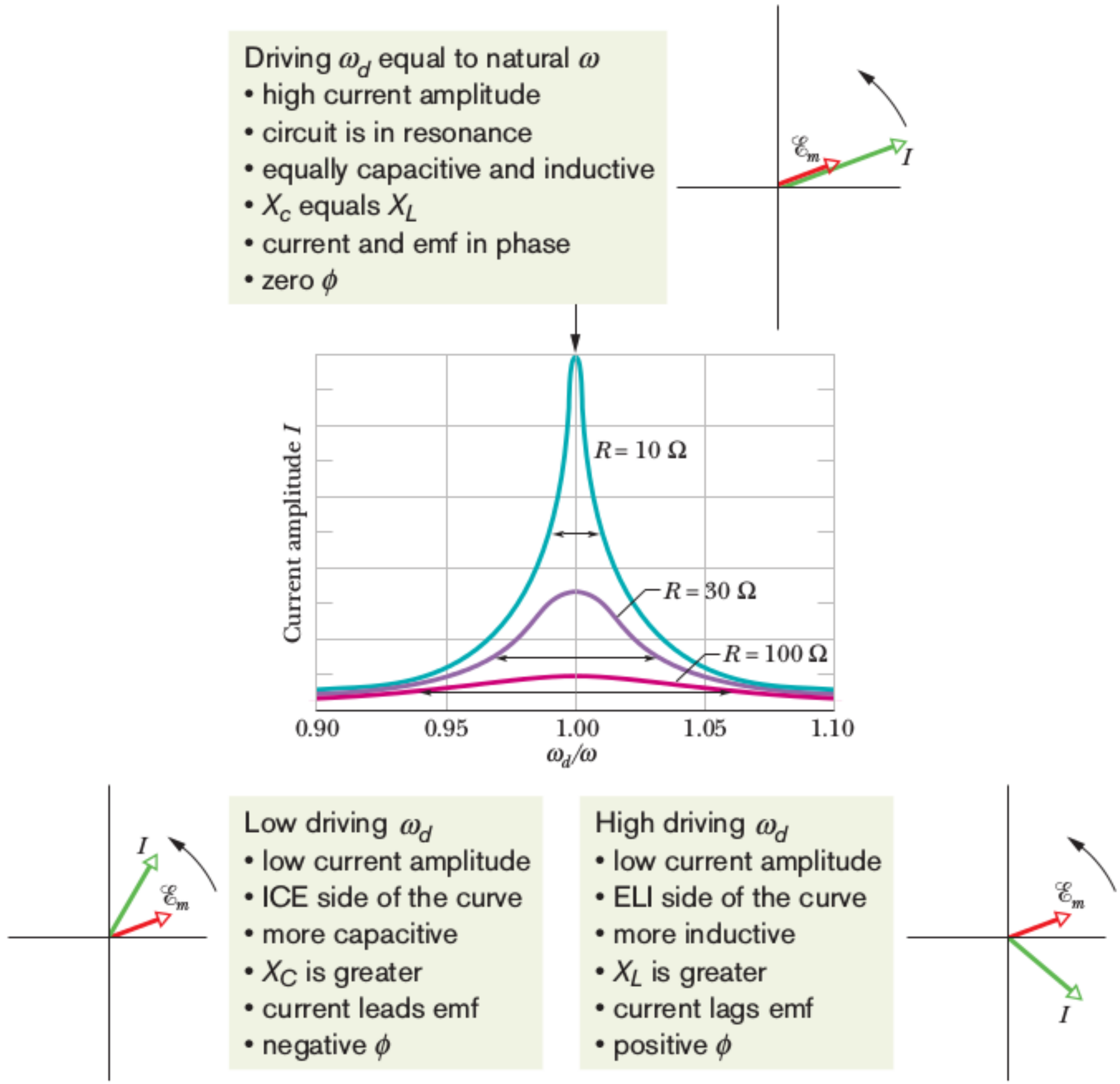


Fig. 31-16 *Resonance curves* for the driven RLC circuit of Fig. 31-7 with $L = 100\ \mu H$, $C = 100$ pF, and three values of R. The current amplitude I of the alternating current depends on how close the driving angular frequency ω_d is to the natural angular frequency ω. The horizontal arrow on each curve measures the curve's *half-width,* which is the width at the half-maximum level and is a measure of the sharpness of the resonance. To the left of $\omega_d/\omega = 1.00$, the circuit is mainly capacitive, with $X_C > X_L$; to the right, it is mainly inductive, with $X_L > X_C$.