Computer Engineering, FIZ137 1st Midterm Exam | Inst.: R. Ellialtıoğlu & E. Taşcı | 22 / 11 / 2016 | 90 minutes

Name:

Student Number:

Section: (RE 25, ET 26)

Equations of motion for constant acceleration:

$v = v_0 + at;\quad (x - x_0) = v_0 t + \frac{1}{2}at^2;\quad v^2 = v_0^2 + 2a(x - x_0);$

$(x - x_0) = \frac{1}{2}(v_0 + v)t;\quad (x - x_0) = vt - \frac{1}{2}at^2$

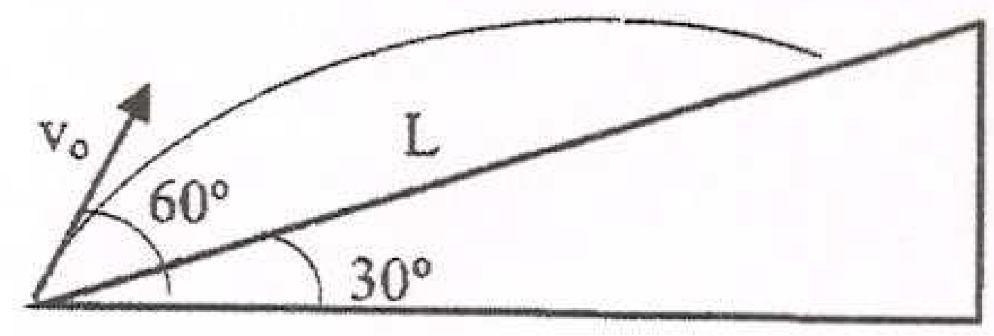
Questions

1. Some of the physical laws belonging to alternative universes/computer games are listed below where α, β, γ, δ, ϵ are constants. Analyze each of them and derive the units of the constants in the equations in order to have in principle correct equations. (4 points each)
[F: Force; K: Kinetic Energy; U: Potential Energy; ρ: density; μ_k: Kinetic friction coefficient; P: Power]

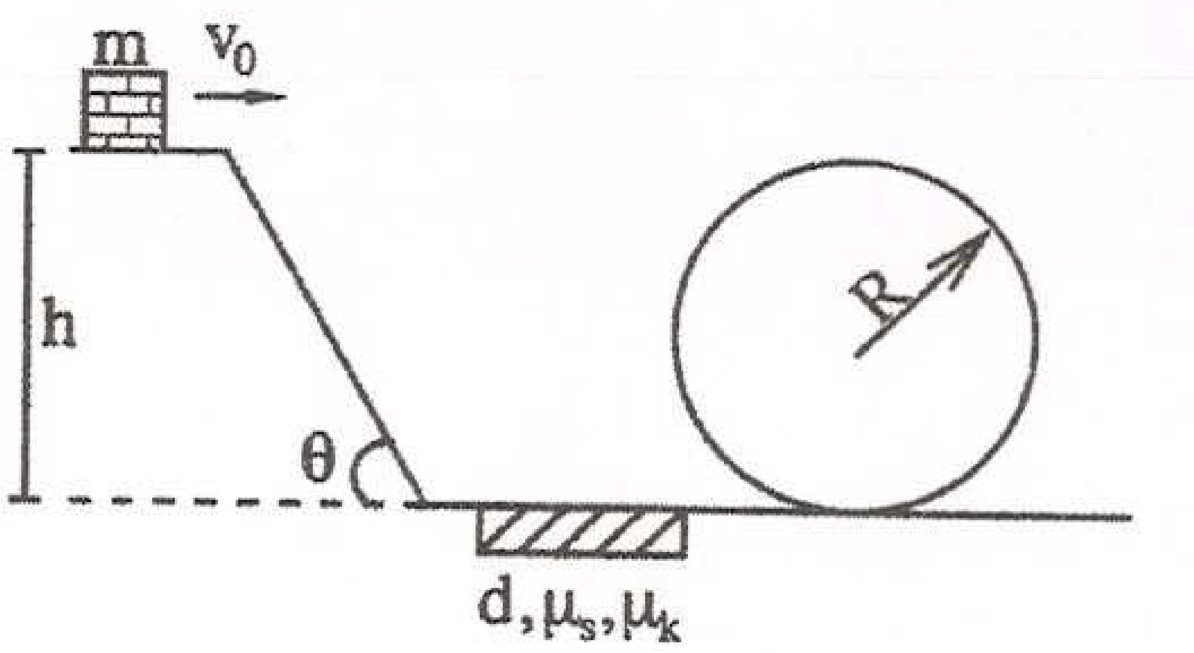
$$K = \alpha m v \qquad P = \beta \frac{ma}{t^2} \qquad F = \gamma m v^2 \qquad v = \sqrt{v_0^2 + \delta \frac{F}{m}} \qquad \mu_k = \epsilon \frac{F \rho x}{m a^2}$$

2. At a certain instant, a 2 kg object is acted on by a force **F** = 4**i** – 2**j** + 5**k** [N] while having a velocity **v** = – 2**i** + 3**k** [m/s]. What is the instantaneous rate at which the force does work on the object? At a later time, the velocity consists of only a y component and the instantaneous power is – 8 W. If the force is unchanged, what is the kinetic energy of the object just then? (20 points)

3. A ball is thrown with an initial velocity of magnitude v_o = 10 m/s, which makes an angle 60° with the horizontal, from the lowest point of a 30° incline plane. Find the distance L travelled by the ball on the incline. (cos30° = sin60° = √3/2; sin30° = cos60° = 1/2; g = 10 m/s²). (20 points)

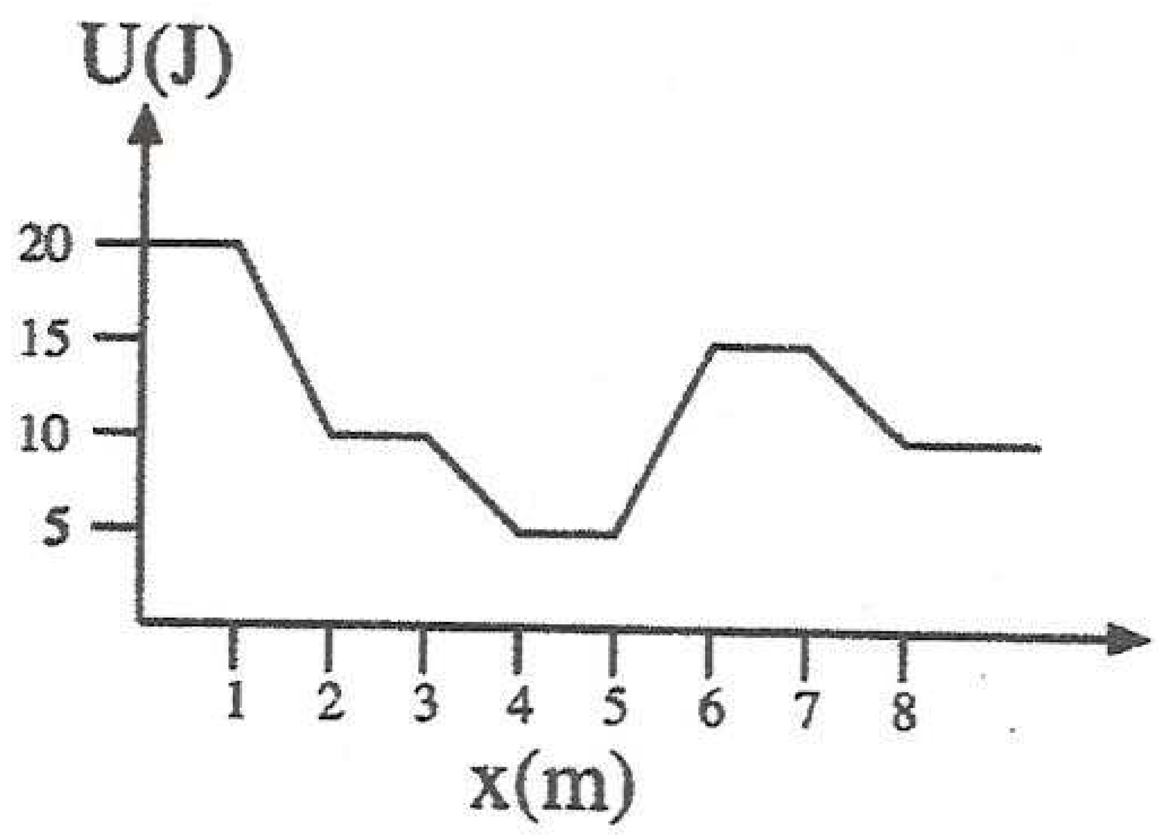


4. A block of bricks, mass m is released from a height of h with an initial horizontal speed of v_o. Shortly after it slides down an inclined plane of angle θ and enters a region of d length where there is a friction characterized by static and kinetic frictional coefficients of us μ_s and μ_k, respectively, present between the block and the floor. This region is followed by a circular loop of radius R. (Other than the shaded d region, there is no friction present)



For the particle to be able to complete the loop just without falling down, derive an equation between R and other relevant quantities. Indicate the unrelated quantities among the given ones, if there are any. (20 points)

5. Regarding the given Potential Energy vs. Position graph:
a) Plot the corresponding Force vs. Position graph (7 points)
b) Calculate the work done on the particle by the given interactions to move it from x = 0 m to x = 8 m (6 points)
c) For a particle with mechanical energy E_{mec} = 13 J, released from x = 4 m and initially moving in the positive x-direction, identify the turning and equilibrium points, if there are any. (7 points)



1) a) $K = \alpha m v$

$$J \equiv kg\frac{m^2}{s^2} = [|\alpha|]\, kg \frac{m}{s}$$

$$[|\alpha|] = \frac{m}{s} \Rightarrow [|\alpha|] = \frac{[|L|]}{[|T|]}$$

b) $P = \beta \frac{ma}{t^2}$

$$W \equiv \frac{J}{s} = \frac{kg\, m^2/s^2}{s} = [|\beta|]\, \frac{kg\, m/s^2}{s^2}$$

$$[|\beta|] = m.s \Rightarrow [|\beta|] = [|L|][|T|]$$

c) $F = \gamma m v^2$

$$N \equiv kg\, m/s^2 = [|\gamma|]\, kg \frac{m^2}{s^2}$$

$$\Rightarrow [|\gamma|] = \frac{1}{[|L|]}$$

d) $\left[\left|\delta \frac{F}{m}\right|\right] = [|v^2|]$

$$[|\delta|]\frac{kg\, m/s^2}{kg} = \frac{m^2}{s^2} \Rightarrow [|\delta|] = m \Rightarrow [|\delta|] = [|L|]$$

e) $\mu_k = \epsilon \frac{F \rho x}{m a^2}$

$$1 = [|\epsilon|]\frac{kg\, m/s^2\; kg/m^3 \cdot m}{kg\; m^2/s^4} = [|\epsilon|]\, \frac{kg\, s^2}{m^3}$$

$$\rightarrow [|\epsilon|] = \frac{m^3}{kg\, s^2} \Rightarrow [|\epsilon|] = \frac{[|L|]^3}{[|M|][|T|]^2}$$

2) $P = \frac{W}{\Delta t} = \frac{\Delta K}{\Delta t}$

$\Delta K = -\Delta U$

$F = -\frac{\Delta U}{\Delta x} \Rightarrow -\Delta U = F\Delta x$

$\Rightarrow P = \frac{F\Delta x}{\Delta t}$, $\Delta t \to 0$: $P = \vec{F}\frac{d\vec{x}}{dt} = \vec{F}\vec{v}$

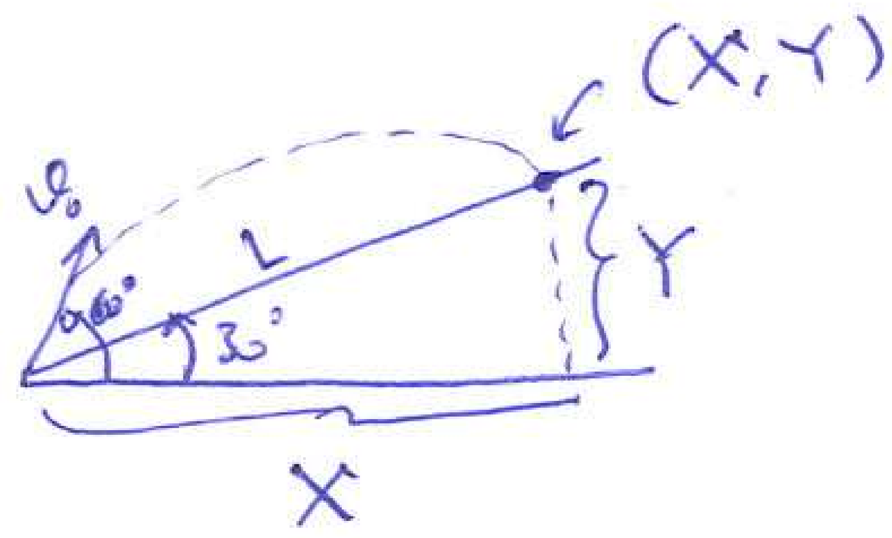
a) $\left.\begin{array}{l}\vec{F} = (4\hat{i} - 2\hat{j} + 5\hat{k})\,N \\ \vec{v} = (-2\hat{i} + 3\hat{k})\,m/s\end{array}\right\} \rightarrow P = \vec{F}\cdot\vec{v} = (-8+15)W = \underline{7W}$

b) $\vec{v} = v_y\hat{j} \Rightarrow -8\,W = (4\hat{i} - 2\hat{j} + 5\hat{k})N\,(v_y\hat{j})$

$$-8W = -2v_y N \Rightarrow v_y = \frac{-8W}{-2N} = 4\,m/s$$

$$\Rightarrow K = \frac{1}{2}mv^2 = \frac{1}{2}(2kg)(4\,m/s)^2 = \underline{\underline{16J}}$$

3)



$X = L\cos 30°$

$Y = L\sin 30°$

$v_x = v_0 \cos 60°$

$v_y = v_0 \sin 60°$

$X = v_x t = v_0\cos 60° t \rightarrow L\cos 30° = v_0 \cos 60° t$

$\Rightarrow t = \frac{L\cos 30°}{v_0\cos 60°}$ (1)

$Y = v_y t - \frac{1}{2}gt^2 = L\sin 30°$

(1) $\Rightarrow \cancel{v_0}\sin 60°\left(\frac{\cancel{L}\cos 30°}{\cancel{v_0}\cos 60°}\right) - \frac{1}{2}g\left(\frac{L^{\cancel{2}}\cos^2 30°}{v_0^2\cos^2 60°}\right) = \cancel{L}\sin 30°$

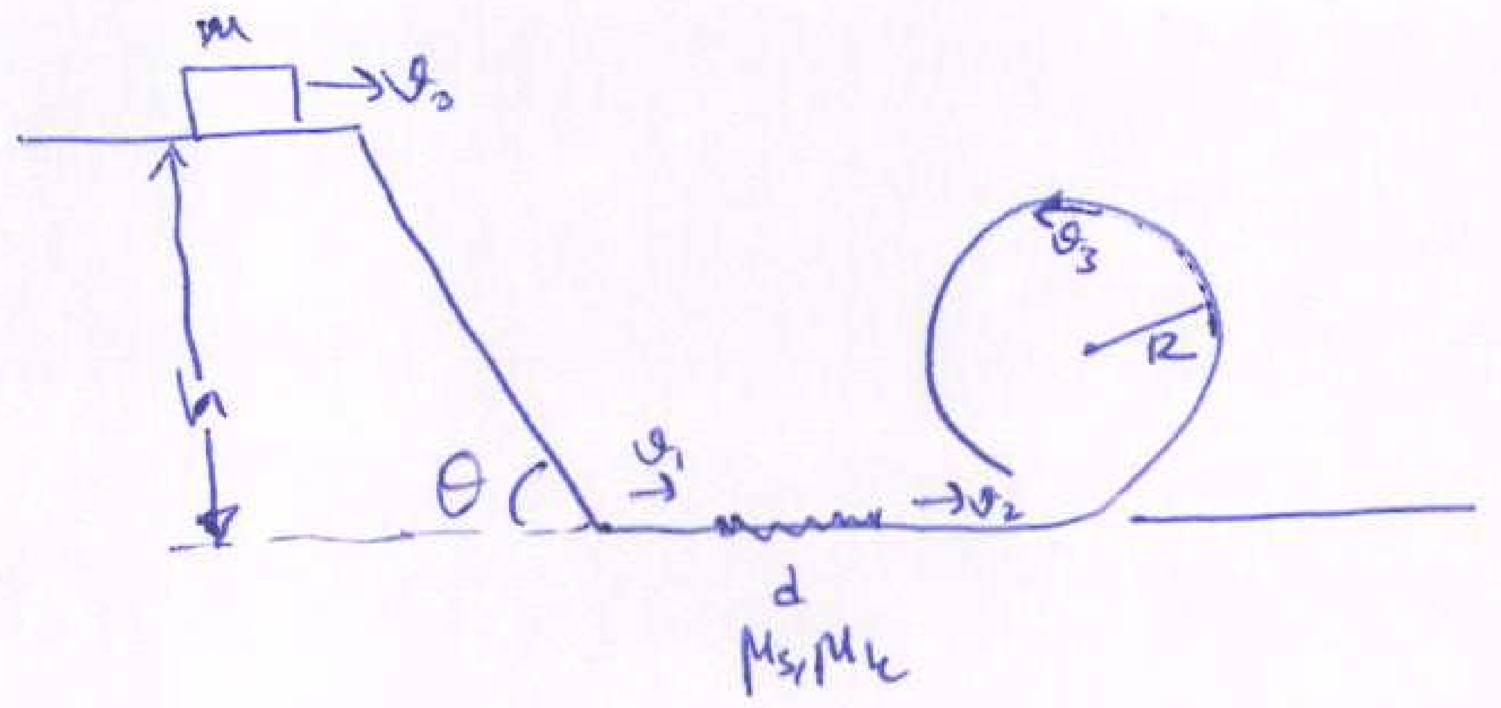
$$\frac{\sin 60°\cos 30°}{\cos 60°} - \frac{1}{2}g\frac{L\cos^2 30}{v_0^2\cos^2 60°} = \sin 30°$$

$$\frac{\sqrt{3}/2\;\sqrt{3}/2}{1/2} - \frac{1}{2}(10\,m/s^2)\frac{L\,(\sqrt{3}/2)^2}{(10\,m/s)^2\,(1/2)^2} = \frac{1}{2} \rightarrow \frac{3}{2} - \frac{L\,3/4}{(10m)\,1/2} = \frac{1}{2}$$

$$\frac{3}{20m}L = \frac{3}{2} - \frac{1}{2} = 1$$

$$\Rightarrow L = \frac{20m}{3} = 6.\bar{6}\,m$$

4)



$$\cancel{m}gh + \frac{1}{2}\cancel{m}\vartheta_0^2 = \frac{1}{2}\cancel{m}\vartheta_1^2 \quad (\vartheta_1 > \vartheta_0)$$

$$2gh + \vartheta_0^2 = \vartheta_1^2 \quad (1)$$

$\Delta K = W_f$, $W_f < 0$ (Energy is transferred <u>from</u> the block by the friction Force)

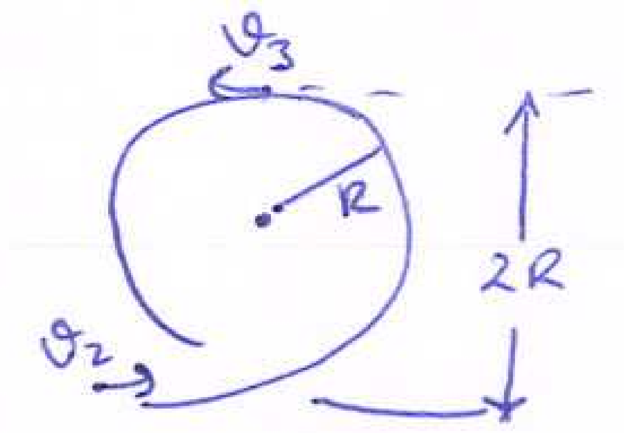
$(\vartheta_2 < \vartheta_1)$

$$\underbrace{\frac{1}{2}\cancel{m}\vartheta_2^2 - \frac{1}{2}\cancel{m}\vartheta_1^2}_{\Delta K} = \underbrace{-f_k d}_{W_f}$$

$$= -\cancel{m}g\mu_k d$$

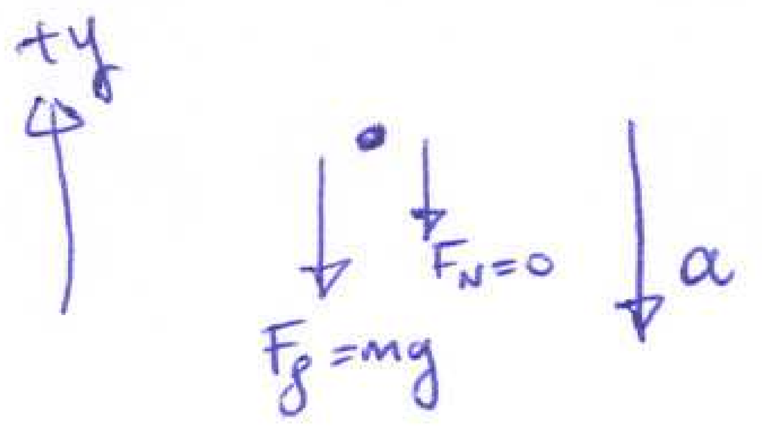
$$\vartheta_2^2 = -2g\mu_k d + \vartheta_1^2$$

$$(1) \Rightarrow \quad = -2g\mu_k d + 2gh + \vartheta_0^2 \quad (2)$$



$$\frac{1}{2}m\vartheta_2^2 = mg(2R) + \frac{1}{2}m\vartheta_3^2$$

$$\vartheta_3^2 = \vartheta_2^2 - 4gR \quad (3)$$



$$0 - \cancel{m}g = \cancel{m}(-a) = \cancel{m}\left(-\frac{\vartheta_3^2}{R}\right)$$

$$g = \frac{\vartheta_3^2}{R} \Rightarrow \vartheta_3^2 = gR$$

$$(3) \Rightarrow \quad \vartheta_2^2 - 4gR = gR$$

$$\vartheta_2^2 = 5gR \quad (4)$$

$$(2) + (4): \quad 5gR = \vartheta_0^2 + 2gh - 2g\mu_k d$$

$$\boxed{R = \frac{\vartheta_0^2}{5g} + \frac{2}{5}(h - \mu_k d)}$$

$\hookrightarrow$ R is independent of m, θ and μ_s.

5) a) $F = -\frac{dU}{dx}$, U linear $\Rightarrow$ $F = -\frac{\Delta U}{\Delta x}$

i) x: 0 to 1m : $U = \text{const.} \rightarrow F_{01} = 0$

ii) x: 1m to 2m : $F_{12} = -\frac{(20-10)\,J}{(1-2)\,m} = 10N$

iii) x: 2m to 3m : $U = \text{const} \longrightarrow F_{23} = 0$

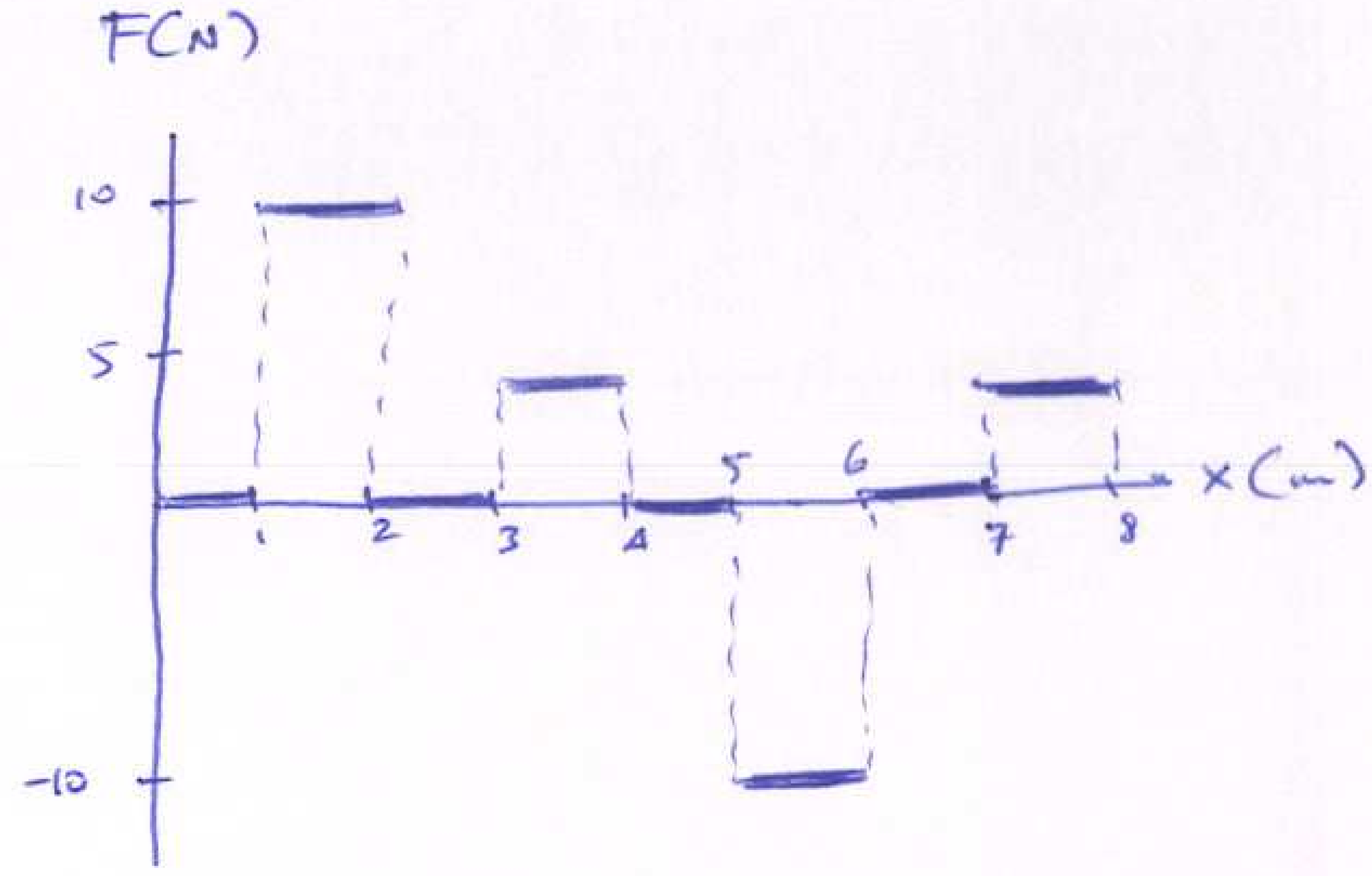
iv) x: 3m to 4m : $F_{34} = -\frac{(10-5)\,J}{(3-4)\,m} = 5N$

v) x: 4m to 5m : $U = \text{const} \longrightarrow F_{45} = 0$

vi) x: 5m to 6m : $F_{56} = -\frac{(5-15)\,J}{(5-6)\,m} = -10N$

vii) x: 6m to 7m : $U = \text{const} \longrightarrow F_{67} = 0$

viii) x = 7m to 8m : $F_{78} = -\frac{(15-10)\,J}{(7-8)\,J} = 5N$

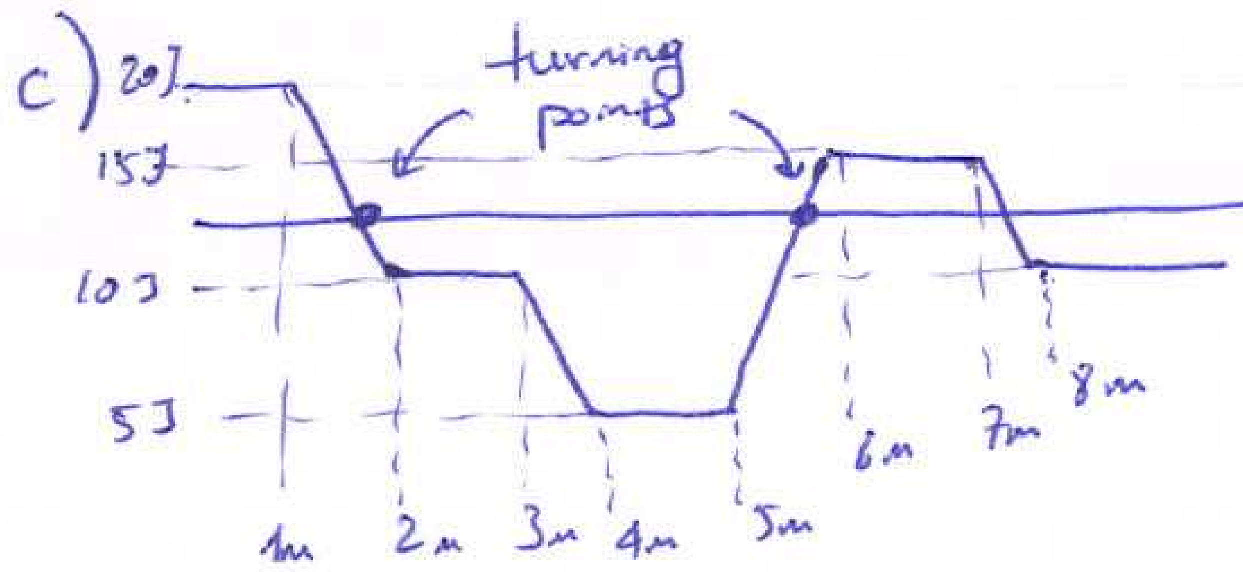


b) $W = \sum F(x)\Delta x = (10.1 + 5.1 - 10.1 + 5.1)\,J = 10\,J$

– or –

$W = -\Delta U = -(U(x=8m) - U(x=0m)) = -(10J - 20J) = 10J$

c)



Two turning points:

i) x_{t1-2} : between $x = 1m$ and $x = 2m$

ii) x_{t5-6} : between $x = 5$ and $x = 6m$

No equilibrium point in between

$K(x) + U(x) = E_{mec}$

at turning point, $K = 0$ $(v=0)$, $F \neq 0$

x_{t1-2} : $\frac{\overset{13J}{y_{t1-2}} - 20J}{x_{t1-2} - 1m} = \frac{(20-10)\,J}{(1-2)\,m} \rightarrow \frac{-7J}{x_{t1-2} - 1m} = \frac{10J}{-1m}$

$\Rightarrow x_{t1-2} - 1m = \frac{7m}{10} \rightarrow x_{t1-2} = \frac{17m}{10} = \underline{1.7m}$

x_{t5-6} : $\frac{\overset{13J}{y_{t5-6}} - 15J}{x_{t5-6} - 6m} = \frac{(5-15)\,J}{(5-6)\,m} \rightarrow \frac{-2J}{x_{t5-6} - 6m} = \frac{-10J}{-1m}$

$\Rightarrow x_{t5-6} - 6m = -\frac{2m}{10}$

$x_{t5-6} = \frac{58m}{10} = \underline{\underline{5.8m}}$