VII

POTENTIAL ENERGY AND CONSERVATION OF ENERGY

Energy is a great tool for Physicists as it allows us to work in a scalar Landscape (as opposed to Force fields). One general type of Energy is potential energy U. Technically, potential energy is energy that can be associated with the configuration (arrangement) of a system with interacting objects. (exerting Forces)

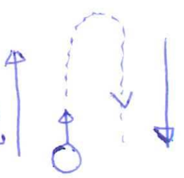
Imagine a bungee-jumper plunging from a Bridge: the system consists of the Earth and the jumper. The configuration of the system changes (the distance between the two decreases). We can account for the jumper's motion and increase in kinetic energy by defining a gravitational potential Energy U. This is the energy associated with the seperation of two objects that attract each other by gravitational force.

When the jumper steetches the cord, that system consists of the jumper and the cord. The force between the two is an elastic (spring-like) force. The configuration of the system changes (the cord stretches). We can account for the jumper's decrease in kinetic energy and the cord's increase in length by defining an elastic potential Energy U.

Physics determines how the potential energy of a system can be calculated so that energy might be stored or put to use.

WORK and POTENTIAL ENERGY

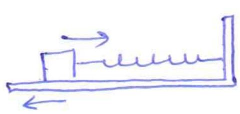
$W_g<0$



$W_g>0$: As the ball begins to fall back, the gravitational force now transfers energy from the gravitational potential energy of the Ball-Earth system to the kinetic energy of the Ball.

$\Rightarrow \Delta U = -W$

↳ Work done by the gravitational force during ascent is negative ⇒ The force transfers energy from the kinetic energy of the ball (Now we know where it stores that Energy.)



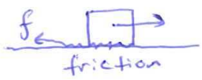
Energy from the kinetic energy of the block ⇄ Elastic potential Energy of the spring-block system

Conservative & Non-Conservative Forces

1) The system consists of two or more objects.
2) A force acts between a particle-like object in the system and the rest of the system.
3) When the system's configuration changes, the force does work (call it W_1) on the particle-like object, transferring energy between the kinetic energy K of the object and some other type of energy of the system.
4) When the configuration change is reversed, the force reverses the energy transfer, doing work W_2 in the process.

In a situation where $W_1 = -W_2$ is always true, the other type of energy is a potential energy and the force is a conservative force such as: Gravitational and elastic potential energies.

Kinetic Frictional force & drag force are non-conservative forces.



: Friction force transfers energy from the kinetic energy to a type of energy called thermal energy. (it's something to do with random motions of atoms and molecules)

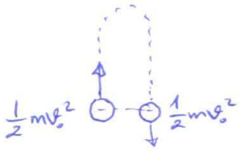
Thermal Energy can not be transferred back to Kinetic Energy of the Block by kinetic frictional force.

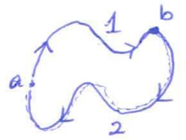
$\Rightarrow$ Thermal Energy is not a Potential Energy.

When only conservative forces act on a particle like object, we can greatly simplify otherwise difficult problems involving motion of the object.

PATH INDEPENDENCE OF CONSERVATIVE FORCES

The primary test for determining whether a force is conservative or non-conservative is this: Let the force act on a particle that moves along any <u>closed path</u>. The force is conservative only if the total energy it transfers to and from during the round trip is zero.



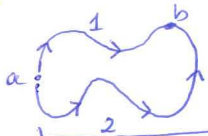


$\Rightarrow$ Any choice of path between the two points gives the same amount of work:

$W_1 = -W_2 \Rightarrow$

$W_1 + W_2 = 0$

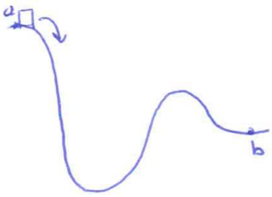
A round trip gives a total work of zero



$W_1 = W_2$

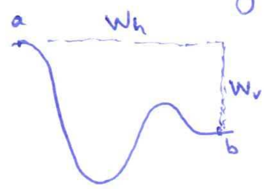
$\hookrightarrow$ The work done for each of the paths is equal to each other.

<u>Example</u>: A 20kg block of slippery cheese that slides along a frictionless path from a to b. The cheese travels a total distance of 2.0m along the track and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?



We can not use $W_g = mgd\cos\varphi$ since φ changes along the way (even if we knew it, it still would be very difficult to calculate)

Because $\vec{F}_g$ is conservative, we can find the work using a convenient path:



$W_h = mgd\cos 90° = 0$

$W_v = mgd\cos 0° = (2.0kg)(9.8m/s^2)(0.80m)(1) = 15.7J$

$\longrightarrow$ Total Work Done: $W = W_h + W_v = 0 + 15.7J = 15.7J \simeq \underline{16J}$

DETERMINING POTENTIAL ENERGY VALUES

$\Delta U = -W$

$$W = \int_{x_i}^{x_f} F(x)\,dx \Longrightarrow \Delta U = -\int_{x_i}^{x_f} F(x)\,dx$$

Force is conservative $\longrightarrow$ the work is the same for all paths between two paths.

GRAVITATIONAL POTENTIAL ENERGY

$F_g = -mg$ ↓ Along -y direction $\rightarrow \Delta U = -\int_{y_i}^{y_f} F(y)dy = -\int_{y_i}^{y_f}(-mg)dy = mg\int_{y_i}^{y_f} dy = mgy\Big|_{y_i}^{y_f}$

$$\Rightarrow \Delta U = mg(y_f - y_i) = mg\,\Delta y \begin{cases} y_f > y_i \Rightarrow \Delta U > 0 \\ y_f < y_i \Rightarrow \Delta U < 0 \end{cases}$$

‼ Only changes (ΔU) in potential energy are meaningful!
(However, to simplify calculations, we usually choose reference points where $(U_i(y_i) = 0 \Rightarrow U(y) = mgy)$)

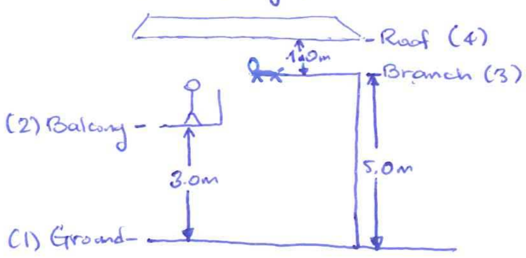
ELASTIC POTENTIAL ENERGY

$F_x = -kx \longrightarrow \Delta U = -\int_{x_i}^{x_f}(-kx)dx = k\int_{x_i}^{x_f} x\,dx = \frac{1}{2}kx^2\Big|_{x_i}^{x_f}$

$$\Rightarrow \Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

$U_i(x_i) = 0 \Rightarrow \quad U(x) = \frac{1}{2}kx^2$ (it is always ≥ 0)

<u>Example:</u> Choosing Reference levels for the gravitational Potential Energy for a cat lying on a branch.
$m = 2.0\,kg$, the cat lies about 5.0 m above the ground.



a) Calculate the potential energy of the cat for:

(1) Ground is taken to be reference point:

$U = mgy = (2.0kg)(9.8m/s^2)(5.0m) = 98J$

(2) Balcony ...

$U = mgy = mg(2.0m) = 39J$

(3) Branch ...

$U = mgy = mg(0) = 0$

(4) Roof ...

$U = mg(-1.0m) = 19.6J \simeq -20J$

b) If the cat falls down to the ground, what is the change ΔU for each Reference point?

Since the total displacement $\Delta y = -5.0m$ for each of the reference points ((1): (0)−(5.0m); (2): (−3.0m)−(2.0m); (3): (−5.0m)−(0); (4): (−6.0m)−(−1.0m))
The change in the potential energy is independent of the reference point!

$$\Delta U = mg\,\Delta y = (2.0kg)(9.8m/s^2)(-5.0m) = -98J$$

CONSERVATION OF MECHANICAL ENERGY

$E_{mec} = U + K$

Conservative Force: $\underbrace{\Delta K = W \; ; \; \Delta U = -W}$

↳ $\Delta K = -\Delta U$: One of these energies increases as much as the other decreases

$\rightarrow K_2 - K_1 = -(U_2 - U_1)$

$\Rightarrow K_2 + U_2 = K_1 + U_1$

(The sum of K and U for any state of the system) = (The sum of K and U for any other state of the system) : When the system is isolated and only conservative forces act on the system.

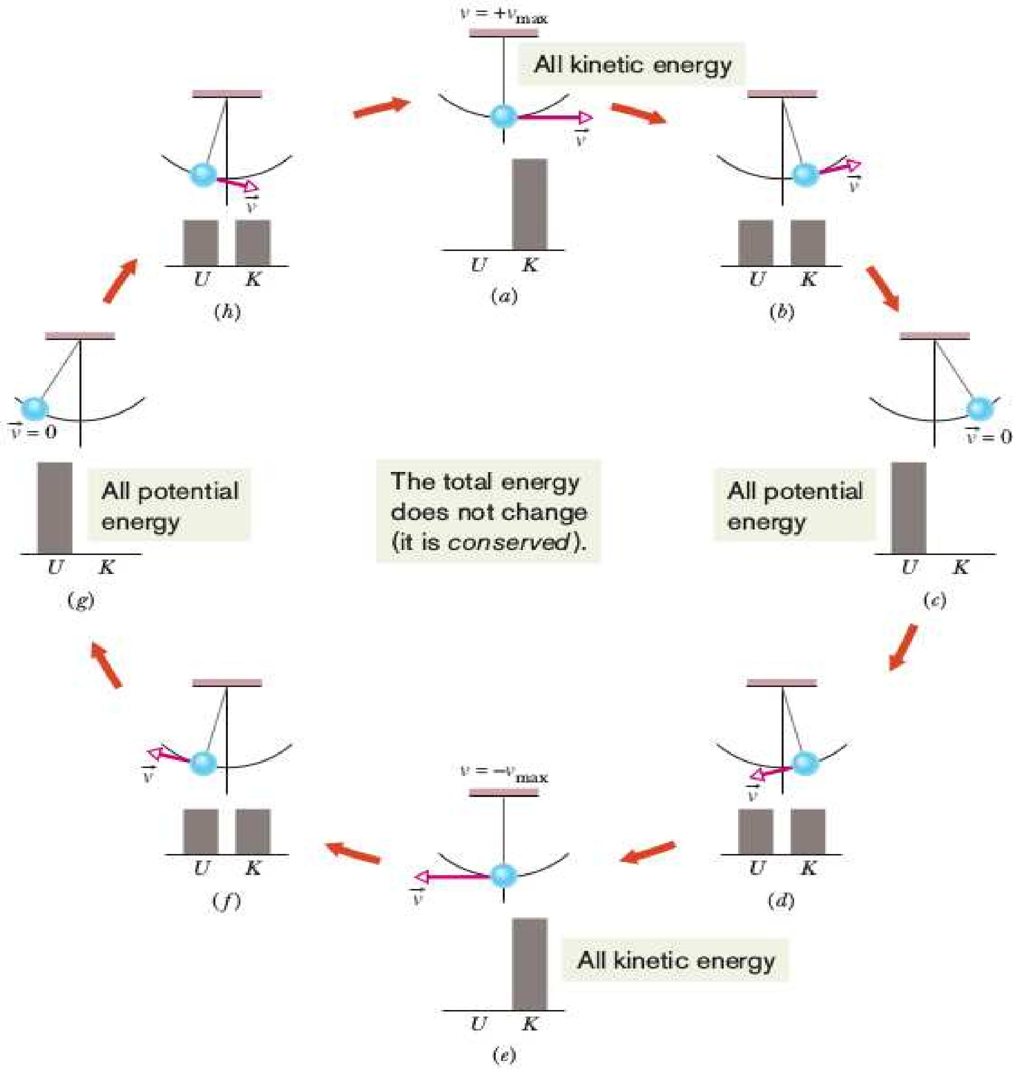
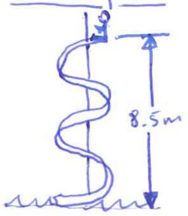


Fig. 8-7 A pendulum, with its mass concentrated in a bob at the lower end, swings back and forth. One full cycle of the motion is shown. During the cycle the values of the potential and kinetic energies of the pendulum–Earth system vary as the bob rises and falls, but the mechanical energy E_{mec} of the system remains constant. The energy E_{mec} can be described as continuously shifting between the kinetic and potential forms. In stages (*a*) and (*e*), all the energy is kinetic energy. The bob then has its greatest speed and is at its lowest point. In stages (*c*) and (*g*), all the energy is potential energy. The bob then has zero speed and is at its highest point. In stages (*b*), (*d*), (*f*), and (*h*), half the energy is kinetic energy and half is potential energy. If the swinging involved a frictional force at the point where the pendulum is attached to the ceiling, or a drag force due to the air, then E_{mec} would not be conserved, and eventually the pendulum would stop.

(frictionless)

Example: Find the child's speed at the bottom of the water-slide



! We cannot find her speed by using acceleration since we don't know the slope/angle of the slide!

Two forces acting: $\vec{F}_g$ and $\vec{F}_N$ but F_N is always proportional to displacement, so no work is done by $\vec{F}_N$.

$$\underbrace{E_{mec,b}}_{\text{bottom}} = \underbrace{E_{mec,t}}_{\text{top}}$$

$$\frac{1}{2}mv_b^2 + mgy_b = \frac{1}{2}mv_t^2 + mgy_t \rightsquigarrow v_b^2 = v_t^2 + 2g(y_t - y_b)$$

Initial speed $v_t = 0$, $y_t - y_b = 8.5m \rightarrow v_b = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(8.5m)} = 13m/s$

(But, we couldn't calculate if the time it takes to reach to the bottom was asked)

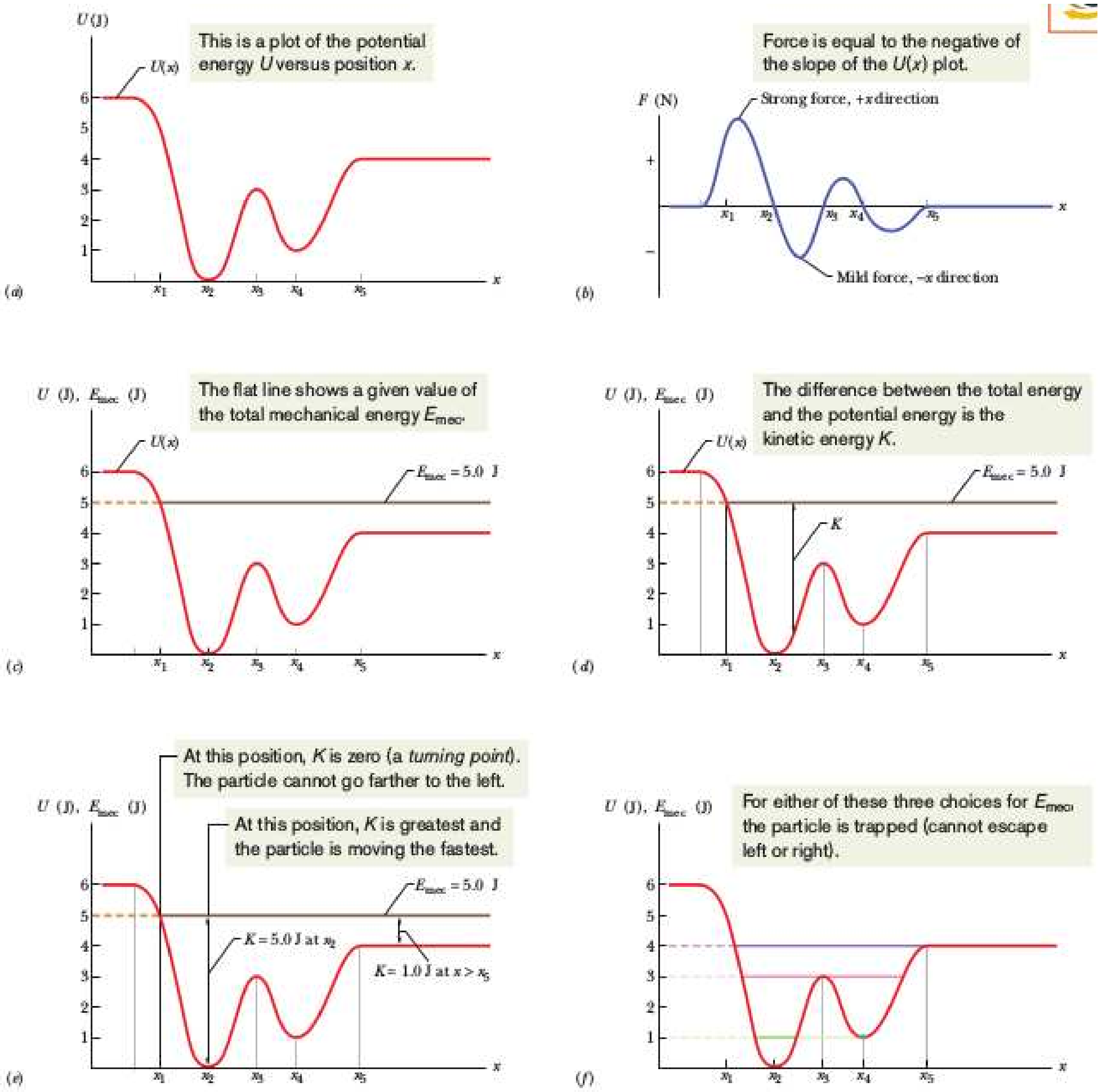


Fig. 8-9 (*a*) A plot of $U(x)$, the potential energy function of a system containing a particle confined to move along an x axis. There is no friction, so mechanical energy is conserved. (*b*) A plot of the force $F(x)$ acting on the particle, derived from the potential energy plot by taking its slope at various points. (*c*)–(*e*) How to determine the kinetic energy. (*f*) The $U(x)$ plot of (*a*) with three possible values of E_{mec} shown.

READING A POTENTIAL ENERGY CURVE

Assume conservative forces act on 1-D motion.

Suppose we know $U(x)$ and want to find force:

$$\Delta U(x) = -W = -F(x)\Delta x$$

$\Delta x \to 0$: $F(x) = -\dfrac{dU(x)}{dx}$ (Refer to (a) & (b) of Fig. 8-9 in the previous page for $U(x) \leftrightarrow F(x)$ relation)

Check: $U(x) = \frac{1}{2}kx^2 \longrightarrow F(x) = -kx$ ✓ (Spring-Block system)

$U(x) = mgx \longrightarrow F(x) = -mg$ ✓ (Particle-Earth system)

* Turning Points: In the absence of a non-conservative force, the mechanical energy E of a system is constant and equals to:

$$E_{mec} = U(x) + K(x)$$

$$\longrightarrow K(x) = E_{mec} - U(x)$$

Referring to (c) & (d) of fig. 8-9: Suppose $E_{mec} = 5.0J$ (c)

$\longrightarrow$ (d): $K(x) = E_{mec} - U(x) \Rightarrow$ (e): $K(x > x_5) = 5.0J - 4.0J = 1.0J$

K greatest when $x = x_2$ $(5.0J - 0 = 5.0J)$

K lowest when $x = x_1$ $(5.0J - 5.0J = 0J)$

$K = \frac{1}{2}mv^2 \geq 0$: always positive or zero, can never be negative

$\longrightarrow$ the particle can never move to the left of x_1 where $E_{mec} - U$ is negative.

$\Rightarrow$ As the particle moves from x_2 to x_1, K decreases (particle slows) until $K = 0$ at x_1 (the particle stops).

Checking the F-x graph, we see that at x_1, the force on the particle is positive (because the slope $\frac{dU}{dx}$ is negative), so the particle does not remain at x_1 but instead begins to move to the right, opposite its earlier motion ($K = 0 \to v = 0$, $F > 0 \to$ in the + x-direction).

Hence, x_1 is a <u>turning point</u>. There is <u>no</u> turning point (where $K = 0$) <u>on the right side of the graph</u> $\longrightarrow$ When the particle heads to the right, it will continue indefinitely.

* Equilibrium Points: (Refer to (f) of Fig. 8-9)

If $E_{mec} = 4.0\,J$ $\longrightarrow$ The turning point shifts from x_1 to somewhere between x_1 and x_2.

Also, at any point to the right of x_5, $E_{mec} = U$

$\longrightarrow K = 0, \; -\frac{dU}{dx} = 0 \Rightarrow F = 0$

$\Rightarrow v = 0$, No force acts on it so it must be stationary.

A particle at such a position is said to be in <u>neutral equilibrium</u>. (e.g. A marble placed on a horizontal tabletop).

If $E_{mec} = 3.0\,J$ $\longrightarrow$ There are two turning points: One is between x_1 and x_2, and the other is between x_4 and x_5.

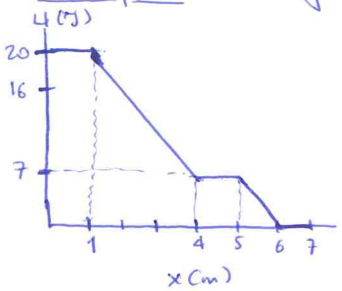
In Addition, x_3 is a point at which $K=0$ and $F=0$ $\longrightarrow$ the particle remains stationary. However, if it is displaced even slightly in either direction, a non-zero force pushes it further along the same direction and the particle continues to move.

A particle at such a position is said to be in <u>unstable equilibrium</u>. (e.g., A marble balanced on top of a bowling ball)

If $E_{mec} = 1.0\,J$ $\longrightarrow$ If we place it at x_4, it is stuck there. It can't move to the left or right by itself because to do so would require a negative kinetic energy. If we push it left or right, a restoring force appears that moves it back towards x_4. Such a particle is said to be in stable equilibrium. (e.g., a marble placed at the bottom of a hemispheric bowl)

If we place it in the cup-like potential well centered at x_2, it is between two turning points. It can still move somewhat but only partway to x_1 or x_3.

Example: Reading a Potential Energy Graph



A 2.0 kg particle moves along on x-axis in one dimensional motion while a conservative force along x-axis acts on it.

At $x = 6.5\,m$, the particle has velocity:

$$\vec{v}_0 = (-4.00\,m/s)\,\hat{i}$$

a) Find its speed at $x_1 = 4.5\,m$

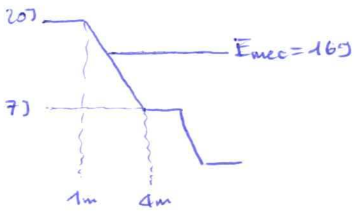
$x = 6.5\,m \rightarrow U = 0 \Rightarrow E_{mec} = K = \frac{1}{2}(2.0\,kg)(4.0\,m/s)^2 = 16\,J$

$X_1 = 4.5\,m \rightarrow U_1 = 7\,J \Rightarrow K(x_1) = E_{mec} - U(x_1) = 16\,J - 7\,J = 9\,J$

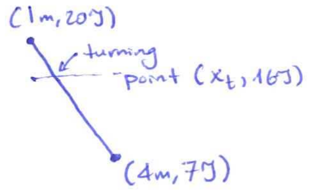
$K_1 = \frac{1}{2} m v_1^2 \Rightarrow 9\,J = \frac{1}{2}(2.0\,kg) v_1^2 \Rightarrow v_1 = 3.0\,m/s$

b) Where is the particle's turning point located?

$\rightarrow$ where $v = 0 \rightarrow K = 0 \leftrightarrow$ where $E_{mec} = U$



$\rightarrow$



slope of the line:

$$\frac{16\,J - 20\,J}{x - 1\,m} = \frac{20\,J - 7\,J}{1\,m - 4\,m}$$

$$\Rightarrow x = \frac{(-4.0\,J)(-3\,m)}{13\,J} + 1\,m = 0.9\,m + 1.0\,m = \underline{\underline{1.9\,m}}$$

c) Evaluate the force acting on the particle, when it's in the region:

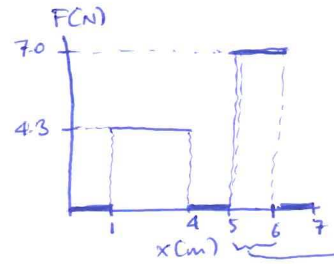
$1.9\,m < x < 4.0\,m$

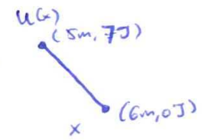
$F(x) = -\frac{dU(x)}{dx}$: negative slope of the graph

$\rightarrow F = -\frac{20\,J - 7.0\,J}{1.0\,m - 4.0\,m} = -\frac{13\,J}{-3.0\,m} = \underline{4.3\,N}$: in the positive x direction

$\Rightarrow$ thus the particle initially moving to the left is stopped by the force and then sent to $\underline{\text{Rightwards}}$.

d) Plot F-x graph:





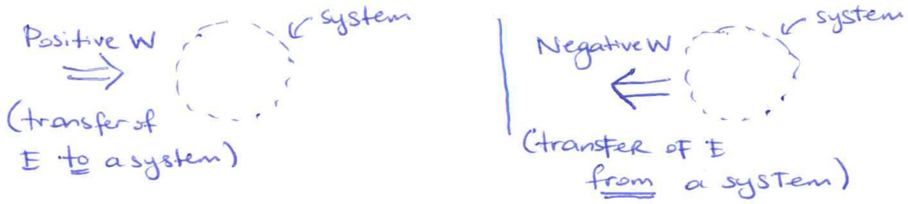
x (m)	U (J)
5.0	7.0
6.0	0

$\rightarrow -\text{slope} = -\frac{(7.0 - 0)\,J}{(5.0 - 6.0)\,m} = \underline{\underline{7.0\,J}}$

WORK DONE ON A SYSTEM BY AN EXTERNAL FORCE

We had defined Work as the energy transferred to-or-from an object by means of a force acting on the object. We can now extend that definition to an external Force acting on a system of objects:

> WORK is energy transferred To or From a system By means of an external Force acting on that system.



When more than one force act on a system, their net work is the energy transferred to or from the system.

For a single particle, the work done on the system can only change the kinetic energy of the system $\longrightarrow \Delta K = W$

However, if a system is more complicated, an external force can change other forms of energy as well.

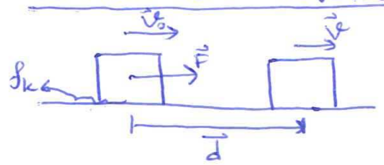
A) NO FRICTION INVOLVED

To hurl a Ball, you first squat and cup your hands under the Ball on the floor. Then you rapidly straighten up while also pulling your hands up sharply launching the ball upward, at about face level. During your upward motion, your applied force on the Ball obviously does Work; that is, it's an external force that transfers entrgy, but to what system?

We check to see which energies change. There's a change ΔK in the ball's kinetic Energy (since it gains an initial speed), and because the Ball and earth become more separated, there is also ΔU in the gravitational potential energy of the ball-Earth system. To include both changes, we consider the ball-earth system:

$$W = \Delta K + \Delta U \Rightarrow W = \Delta E_{mec}$$: Work done on a system, no friction involved.

B) FRICTION INVOLVED



A constant horizontal force $\vec{F}$ pulls a block, increasing its velocity from $\vec{v}_0$ to $\vec{v}$ through a displacement $\vec{d}$ while a constant kinetic frictional force f_k from the floor acts on the Block.

Block: $F_{net,x} = ma$

$\rightarrow F - f_k = ma$: forces constant $\longleftrightarrow \vec{a}$ is constant

$$\Rightarrow v^2 = v_0^2 + 2ad$$

$$\rightarrow a = \frac{v^2 - v_0^2}{2d}$$

$$F - f_k = m\left(\frac{v^2 - v_0^2}{2d}\right) = \frac{mv^2}{2d} - \frac{mv_0^2}{2d}$$

$$\rightarrow Fd = \underbrace{\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2}_{\Delta K} + f_k d$$

$$\Rightarrow Fd = \Delta K + f_k d$$

In a more general situation (like pulling the block up a ramp), there can also be a change in potential energy as well. To include such a change, we generalize the equation as:

$$Fd = \Delta E_{mec} + \underbrace{f_k d}$$

$\hookrightarrow$ manifests itself as heat $\Rightarrow$ thermal Energy

$\rightarrow \Delta E_{th} = f_k d$ (Experimentally derived)

$$\Rightarrow \underbrace{Fd} = \Delta E_{mec} + \Delta E_{th}$$

Work done by the external force $\vec{F}$; but on which system is the work done?

$\Rightarrow$ Block's mechanical energy & thermal Energy of Block-Floor

$\Rightarrow$ Block-Floor system

$\Rightarrow \boxed{W = \Delta E_{mec} + \Delta E_{th}}$: Work done on a system, friction involved.

CONSERVATION OF ENERGY

Energy transfer $\longleftrightarrow$ money transfer between accounts

$\Rightarrow$ energy can not magically appear / disappear.

$\Rightarrow$ **LAW OF CONSERVATION OF ENERGY:**
The total energy E of a system can change only by amounts of energy that are transfered to or from the system.

$$W = \Delta E = \underbrace{\Delta E_{mec}}_{\Delta K + \Delta U} + \underbrace{\Delta E_{th}}_{\text{Thermal Energy}} + \underbrace{\Delta E_{int}}_{}$$

$\hookrightarrow$ types of internal Energy other than thermal Energy

! This is actually an assumption — we have not derived it from basic physics principle. Rather, it is a law based on countless experiments without having found not even one exception.

ISOLATED SYSTEM

If a system is isolated from its environment
$\longrightarrow$ no energy is transferred to or from.

$\Rightarrow$ The total Energy E of an isolated system can't change.

Kinetic $\rightleftarrows$ Potential
Kinetic $\rightarrow$ Thermal

The total of all types of energy in the system can not change.

e.g., Rock-climber Descending:

Potential $\longrightarrow$ Kinetic $\longrightarrow$ Thermal Energy (Ropes & Rings)

Thus, for an isolated system:

$$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0$$

$$\Delta E_{mec} = E_{mec,2} - E_{mec,1} \longrightarrow \underline{E_{mec,2}} = E_{mec,1} - \Delta E_{th} - \Delta E_{int},$$

$\hookrightarrow$ We can relate the total Energy of an isolated system at one instant to another instant <u>without</u> considering the energies at intermediate times.

EXTERNAL FORCES AND INTERNAL ENERGY TRANSFERS

An external force can change the K or U of an object without doing work on the object — i.e. without transferring energy to the object. Instead, the force is responsible for transfers of energy from one type to another inside the object.

ice skater pushes away a railing

Her kinetic energy increases because of an external force $\vec{F}$ on her from the rail. However that force does not transfer energy from the rail to her. Thus the force does no work on her.

the rail does not gain speed or elevated.

Rather, her kinetic energy increases as a result of internal transfers from the biochemical energy in her muscles.

Assume constant a (no friction) $v_0 \rightarrow v$: $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d$

$$K - K_0 = F\cos\varphi\, d$$

$$\Delta K = Fd\cos\varphi$$

$$\Delta U + \Delta K = Fd\cos\varphi$$

the force on the right side does no work on the object but still responsible for the energy.

POWER (Extended Definition) : $P_{avg} = \dfrac{\Delta E}{\Delta t}$

$\Delta t \rightarrow 0$: $P = \dfrac{dE}{dt}$

Rate at which work is done by a force (old definition)

Rate at which energy is transferred by a force (Extended Definition)