A NEW LOOK AT THE ELECTRICAL POTENTIAL

Induced electric fields are produced not by static charges but by a changing magnetic flux.

Although electric fields produced in either way exert forces on charged particles, there is an important difference between them: the field lines of induced electric fields form closed loops. Field lines produced by static charges never do so but must start on positive charges and end on negative charges.

⇒ Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

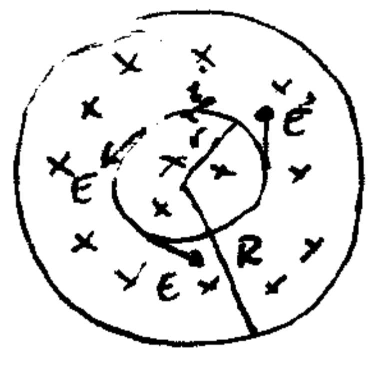
$$\hookrightarrow \oint \vec{E} \cdot d\vec{s} = 0$$

However, if there is a changing Φ_B, this integral is <u>not</u> zero but $-\frac{d\Phi_B}{dt}$

⇒ There is a contradiction

⇒ We must conclude that electric potential has no meaning for electric fields associated with induction.

<u>Example</u>:



$R = 8.5\,cm$

$\frac{dB}{dt} = 0.13\,T/s$

i) $E(r \leq R)$

$$\oint \vec{E}\,d\vec{s} = E \oint ds = E(2\pi r)$$

$$\Phi_B = BA = B(\pi r^2)$$

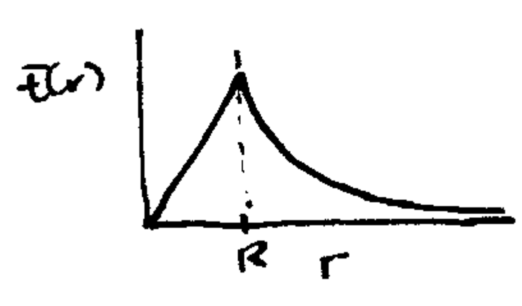
$$E(2\pi r) = \pi r^2 \frac{dB}{dt}$$

$$E = \frac{r}{2}\frac{dB}{dt}$$

ii) $E(r \geq R)$

$$\Phi_B = BA = B(\pi R^2)$$

$$E = \frac{R^2}{2r}\frac{dB}{dt}$$



INDUCTORS AND INDUCTANCE

A capacitor can be used to produce a desired electric field. Likewise, an inductor (—ℓℓℓℓℓℓ—) can be used to produce a desired magnetic field.

We will consider a long solenoid as our basic type of inductor.
(Like parallel-plate capacitors for $\vec{E}$)

if there is a current i passing through a solenoid, then the <u>inductance</u> of the inductor is defined as:

$$L = \frac{N\Phi_B}{i}$$

$N\Phi_B$ → magnetic flux linkage

L → The measure of the flux linkage produced by the inductor per unit current.

$$[L] = \frac{Tm^2}{A} = \text{Henry} = H$$

Inductance of a Solenoid

$$N\Phi_B = (n\ell)(BA)$$

$$B = \mu_0 i n$$

$$L = \frac{N\Phi_B}{i} = \frac{(n\ell)(BA)}{i} = \frac{(n\ell)(\mu_0 i n)(A)}{i} = \mu_0 n^2 \ell A$$

→ inductance per length : $\frac{L}{\ell} = \mu_0 n^2 A$ (solenoid)

↳ inductance depends only on the geometry of the device!

$$\Rightarrow [\mu_0] = \frac{Tm}{A} = \frac{H}{m} \quad \left(\mu_0 = 4\pi \times 10^{-7} \begin{array}{c} Tm/A \\ H/m \end{array}\right)$$

SELF-INDUCTION

If two coils are near each other, a current in one coil produces a magnetic field through the second coil. If i is changed $\longrightarrow$ flux is changed and an induced emf appears in the second coil, as well as in the first coil.

$\Rightarrow$ An induced emf $\mathcal{E}_L$ appears in any coil in which the current is changing

$\Rightarrow$ Self induction.

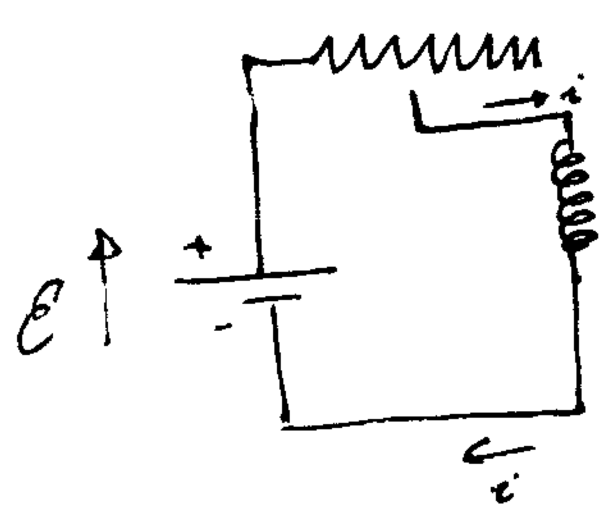
$$N\Phi_B = L i$$

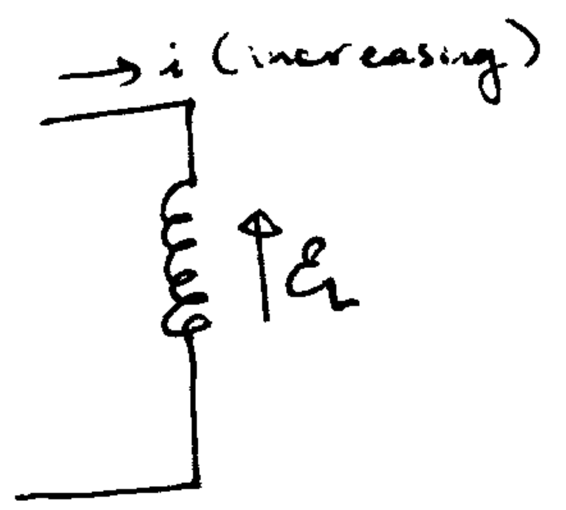
$$\mathcal{E}_L = -\frac{d(N\Phi_B)}{dt}$$

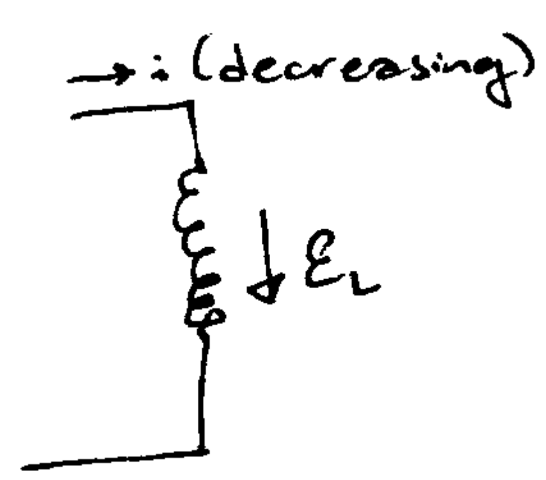
$$\Rightarrow \mathcal{E}_L = -L\frac{di}{dt} \quad \text{(Self-induced emf)}$$

Thus, in any inductor (coil, solenoid or toroid), a self-induced emf appears whenever the current changes with time. The magnitude of the current has no influence on the magnetitude of the induced emf; only the rate of change of the current counts.

According to Lenz's law, the induced emf will oppose the charge.



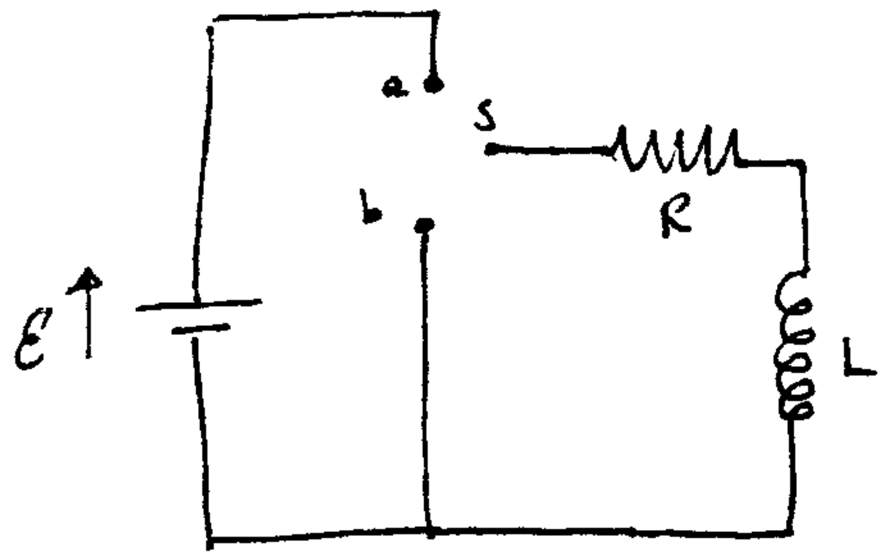




RL - Circuits

In RC: Charging → $q = C\mathcal{E}(1 - e^{-t/\tau_c})$; $\tau_c = RC$ ← capacitive time constant

Discharging → $q = q_0 e^{-t/\tau_c}$ — the capacitor does not immediately charge or discharge



S closed on a: The current in the resistor starts to rise. If there was no inductor, the current would rise rapidly to a steady value $\mathcal{E}/R$.

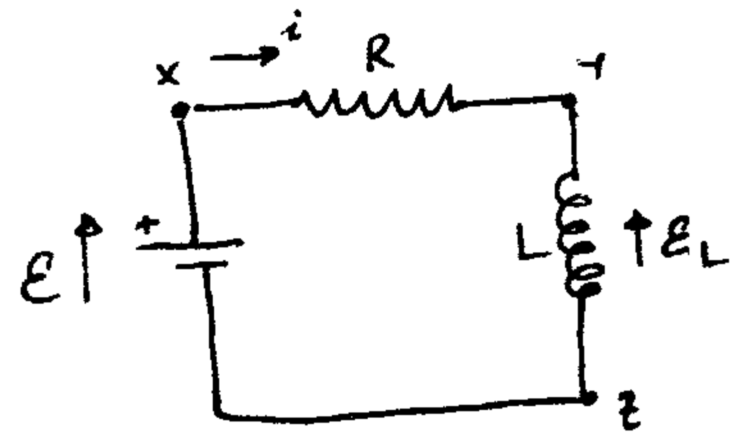
Because of the inductor, however, a self induced emf $\mathcal{E}_L$ appears on the circuit:

$$\mathcal{E}_L = -L\frac{di}{dt}$$

As long as $\mathcal{E}_L$ is present, the current will be less than $\mathcal{E}/R$

As time goes on, the rate at which the current increases becomes less rapid and the magnitude of the self-induced emf which is proportional to di/dt becomes smaller. Thus, the current in the circuit approaches $\mathcal{E}/R$ asymptotically.

→ Initially, an inductor acts to oppose changes in the current through it. A long time later, it acts like an ordinary connecting wire.



$$-iR - L\frac{di}{dt} + \mathcal{E} = 0$$

$$L\frac{di}{dt} + Ri = \mathcal{E} \quad \text{(RL circuit)}$$

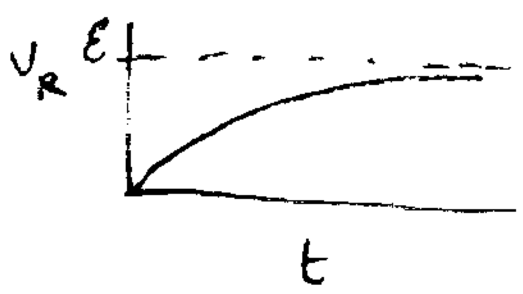
$$\rightarrow i = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

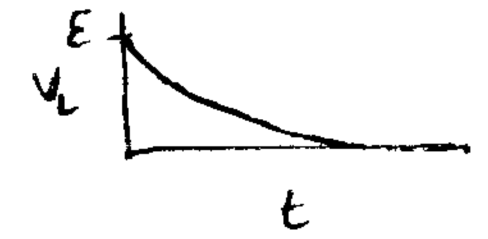
$t = 0 \rightarrow i = 0$
$t \to \infty \rightarrow i \to \frac{\mathcal{E}}{R}$

$$\boxed{i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)} \quad \text{(Rise of the current)}$$

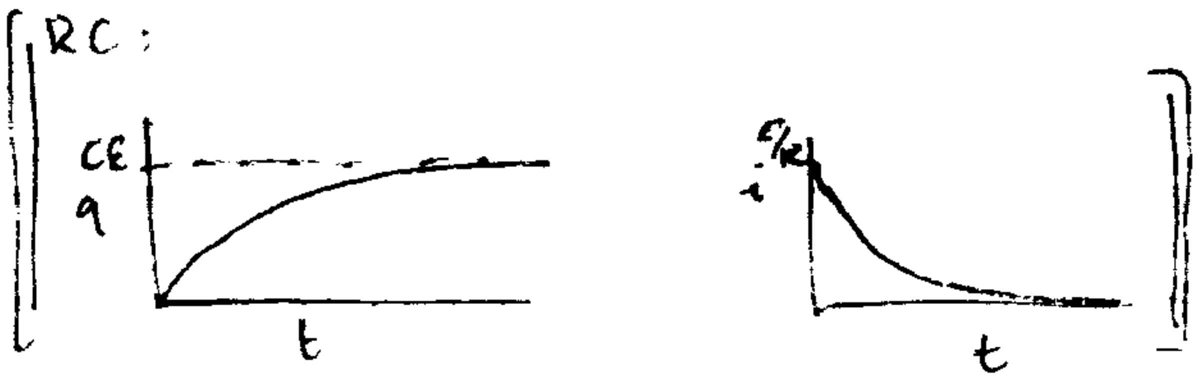
$V_R = iR$, $V_L = L\frac{di}{dt}$

$\tau_L = \frac{L}{R}$: inductive time constant





RC:



$$\tau_L = \frac{L}{R} \rightsquigarrow [|\tau_L|] = \frac{H}{\Omega} = \frac{H}{\Omega}\underbrace{\left(\frac{V\cdot s}{H.A}\right)}_{\equiv 1}\underbrace{\left(\frac{\Omega\, A}{V}\right)}_{\equiv 1} = \underline{s}$$

$\mathcal{E}_L = -L\frac{di}{dt}$; $V = iR$

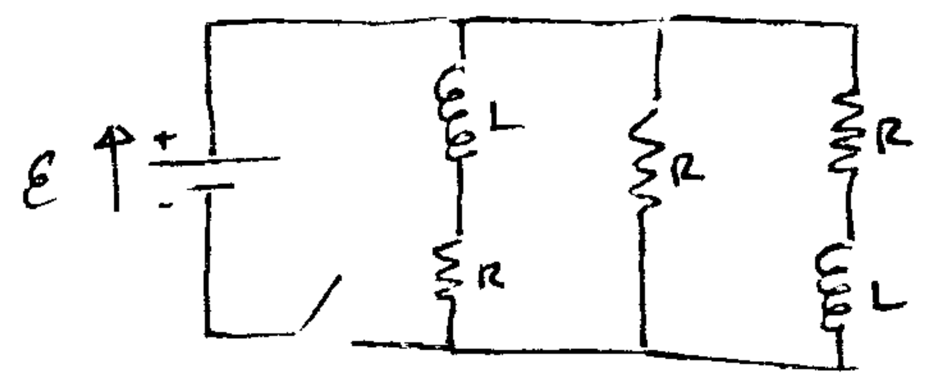
$$t = \tau_L = \frac{L}{R} \rightarrow i = \frac{\mathcal{E}}{R}\left(1-e^{-1}\right) = 0.63\,\frac{\mathcal{E}}{R}$$; 63% of maximum current

* After a long time, S-b is closed (battery removed)

$$\mathcal{E} = 0 \Rightarrow \quad L\frac{di}{dt} + iR = 0$$

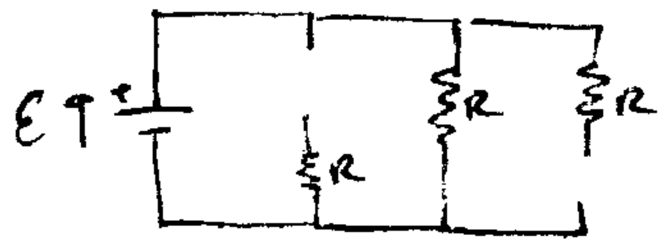
$$i = \frac{\mathcal{E}}{R}\, e^{-t/\tau_L} = i_0\, e^{-t/\tau_L}$$ (decay of the current)

Example:



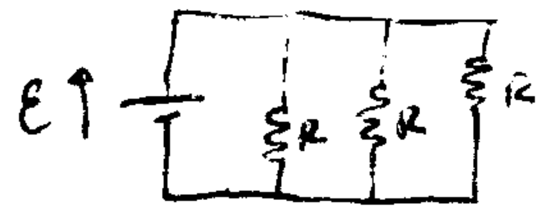
$R = 9\,\Omega$
$L = 2\,mH$
$\mathcal{E} = 18\,V$

i) Current i through the battery just after the switch is closed?
Inductors will act as broken wires (just after the switch is closed)



$$\mathcal{E} - iR = 0 \rightarrow i = \frac{\mathcal{E}}{R} = \frac{18}{9} = 2A$$

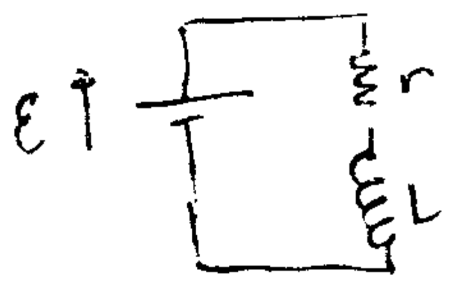
ii) Current long after the switch is closed?
Inductors will behave like ordinary wires



$$R_{eq} = \frac{R}{3} \Rightarrow i = \frac{\mathcal{E}}{R_{eq}} = \frac{18}{3} = 6A$$

Example: A solenoid : $L = 53\,mH$
$r = 0.37\,\Omega$

If it's connected to a battery, how long will it take to reach to half of its equilibrium value?



$$i = \frac{\mathcal{E}}{R}\left(1-e^{-t/\tau_L}\right) \qquad \frac{1}{2}\,\frac{\mathcal{E}}{R} = \frac{\cancel{\mathcal{E}}}{\cancel{R}}\left(1-e^{-t_0/\tau_L}\right)$$

$$e^{-t_0/\tau_L} = \frac{1}{2} \rightarrow t_0 = \tau_L \ln 2 = \frac{L}{R}\ln 2 = \dots = \underline{0.10\,s}$$