BBM 201
Final Exam
Time: 60 minutes

Answer the questions in the spaces provided on the question sheets.
KEEP YOUR CELLPHONE TURNED OFF UNTIL THE EXAM IS OVER.

Name: SOLUTIONS

1. For the tree below, write

   (a) (6 points) the preorder traversal,

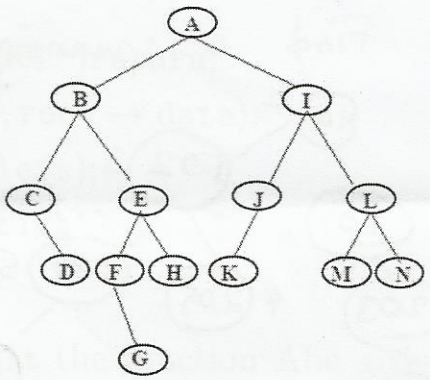
   ABCDEFGHIJKLMN

   (b) (6 points) the inorder traversal,

   CDBFGEHAKJIMLN

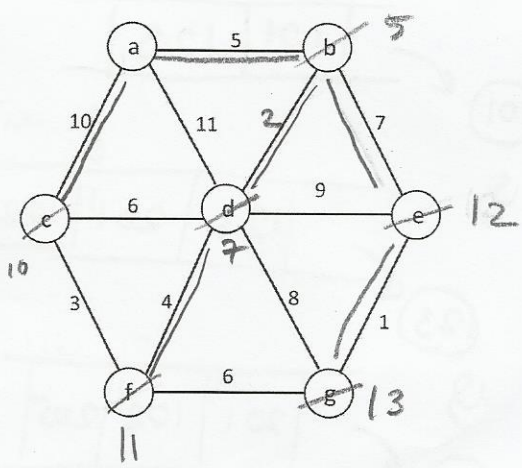
   (c) (6 points) the postorder traversal.

   DCGFHEBKJMNLIA

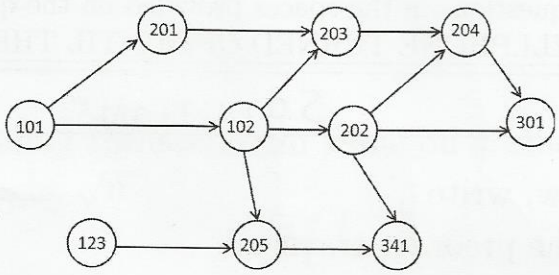


2. (12 points) Show the output of Prim's algorithm for this graph using start vertex a. Show your work by filling the table for each added vertex.

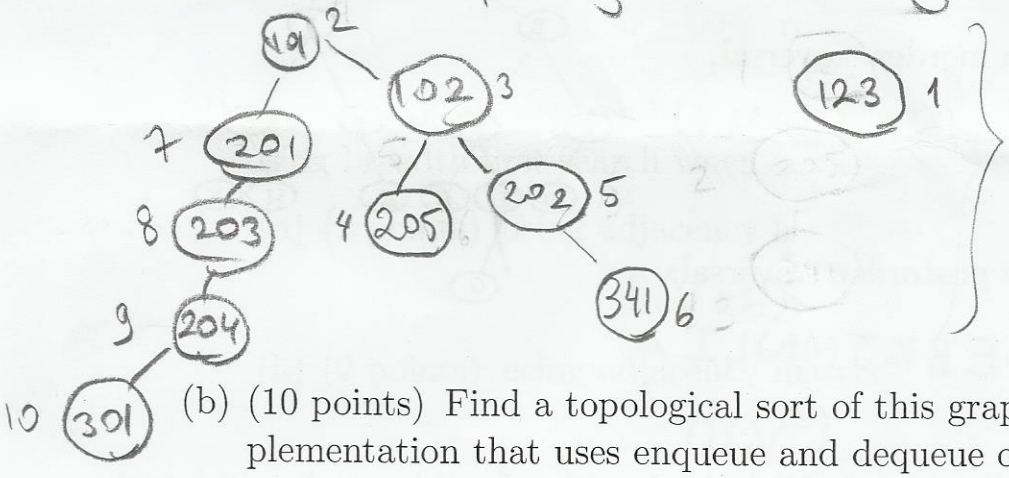
step	edge added	Weights compared
1	ab	5, 10, 11
2	bd	7, 10, 11, 12
3	ac	10, 13, 12, 16, 11, 15
4	df	12, 13, 15, 16
5	be	15, 17, 16
6	eg	15, 17



3. (a) (10 points) Find a topological sort of the graph below using depth first search. **Explain how depth first search is used to give your answer.**

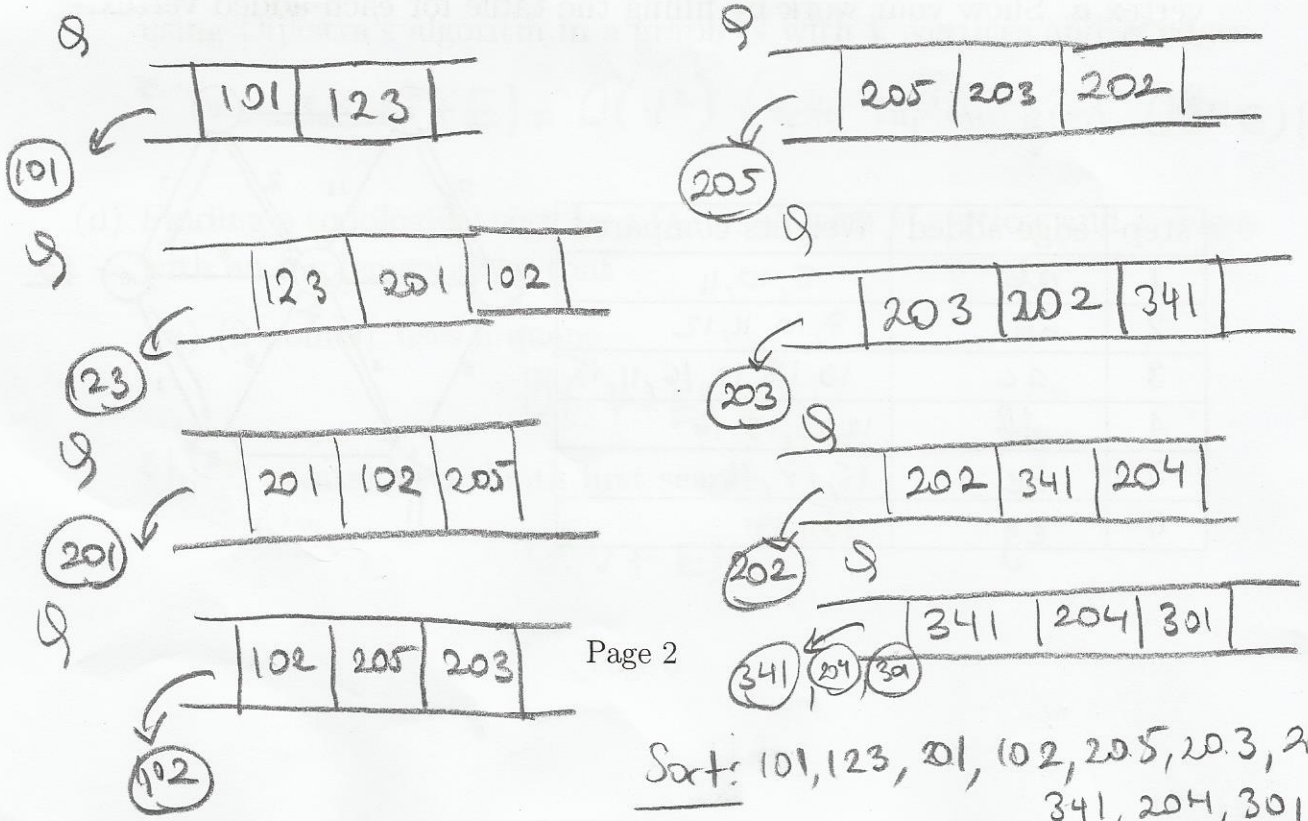


Find a spanning forest using DFS:



123, 101, 102, 205, 202, 341, 201, 203, 204, 301

(b) (10 points) Find a topological sort of this graph above using an implementation that uses enqueue and dequeue of vertices. **Show the queue at each step of this algorithm.**



Sort: 101, 123, 201, 102, 205, 203, 202, 341, 204, 301

4. The input argument of the function "Abc" is the root of a tree, whose vertices are defined using "Node" with fields "left", a pointer to the left child, and "right", a pointer to the right child.

```
Line1   struct Node{
Line2       char data;
Line3       struct Node * left;
Line4       struct Node * right;
Line5   };
Line6   void Abc(struct Node * root)
Line7   {
Line8       if(root == NULL)return;
Line9       printf("%c",root → data);
Line10      Abc(root → left);
Line11      Abc(root → right);
Line12  }
```

Which lines should be interchanged so that the function Abc gives the following traversals? (It may be also a valid answer not to interchange any lines.)

(a) (3 points) preorder traversal

No change

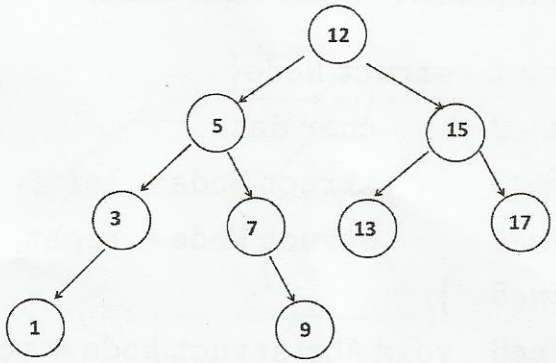
(b) (3 points) inorder traversal

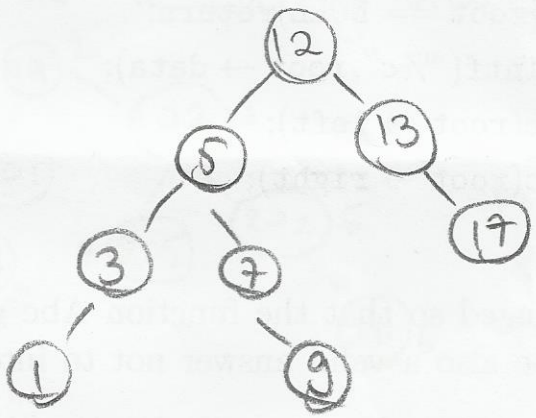
Line 9, 10, 11 → Line 10, 9, 11

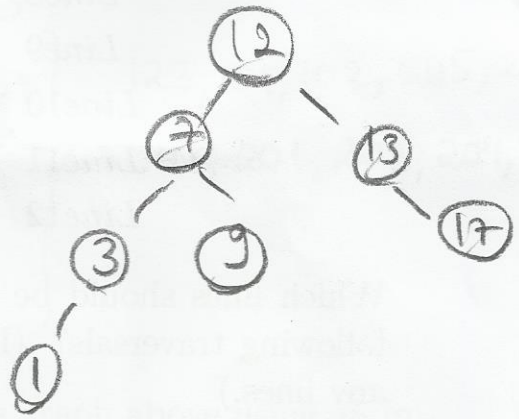
(c) (3 points) postorder traversal

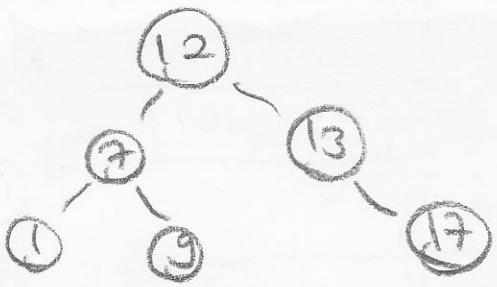
Line 9, 10, 11 → Line 10, 11, 9

5. (12 points) Remove the values 15, 5 and 3 from this tree <u>in this order</u>. **Show the remaining tree after each removal.**







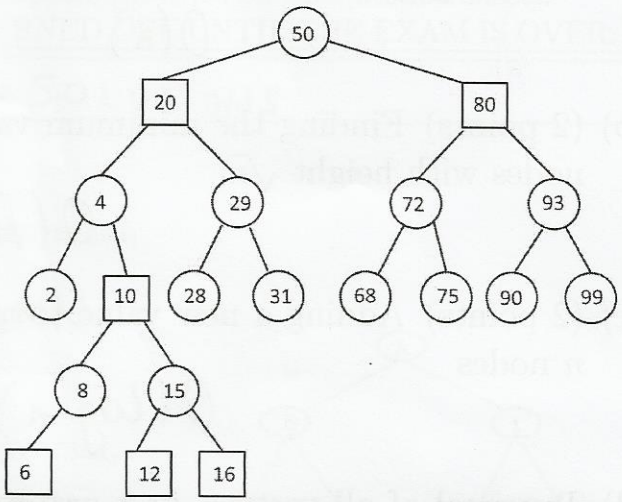


6. Consider the balanced red and black tree, where nodes in □ are red and nodes in ○ are black. Apply each operation <u>to the tree below</u> and show each step by showing the changes on the position and the color of the nodes.

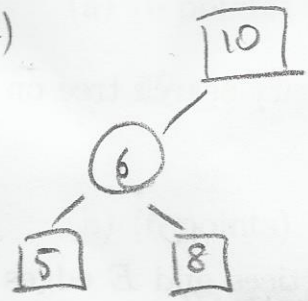
   (a) (3 points) Add 5

   (b) (4 points) Add 7

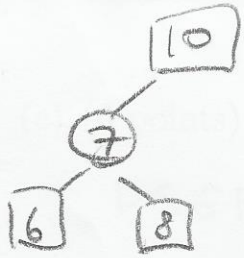
   (c) (6 points) Delete 2



a)

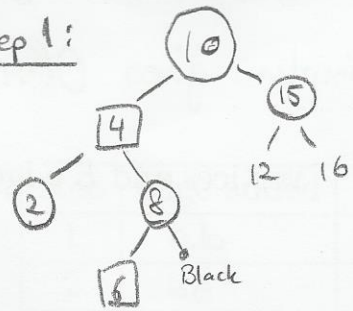


b)



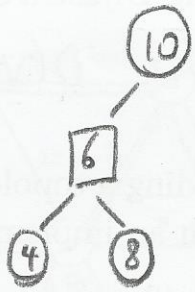
c) - Sibling of 2 is not black

Step 1:

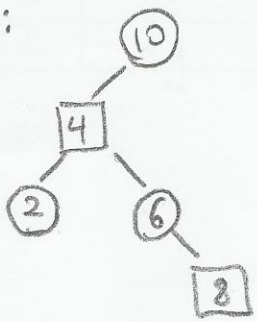


- Further away (from 2) child of 8 is black

- Rotation toward 2:



Step 2:



7. Write the **worst-case** running time of the following implementations using big Oh notation.

   (a) (2 points) Searching a given value on a binary search tree on n nodes with height $n/2$

   $O(n)$

   (b) (2 points) Finding the minimum value on a binary search tree on n nodes with height $\sqrt{n}$

   $O(\sqrt{n})$

   (c) (2 points) Adding a new value to a balanced binary search tree on n nodes

   $O(\log n)$

   (d) Traversal of all vertices in a graph G with V vertices and E edges using breadth first search when

      (a) (2 points) using adjacency list

      $O(V+E)$

      (b) (2 points) using adjacency matrix

      $O(V^2)$

   (c) (2 points) Finding minimum-cost paths from a single source vertex using Dijkstra's algoritm in a graph G with V vertices and E edges.

   $O(V^2+E) = O(V^2)$ (better implem. gives $O((V+E)\log V)$)

   (d) Finding a topological sort for a DAG, G, with V vertices and E edges with an implementation that

      (a) (2 points) uses a queue

      $O(V+E)$

      (b) (2 points) uses depth first search

      $O(V+E)$