Chapter 1 - Measurement

40 Using conversions and data in the chapter, determine the number of hydrogen atoms required to obtain 1.0 kg of hydrogen. A hydrogen atom has a mass of 1.0 u.

42. (a) In atomic mass units, the mass of one molecule is (16 + 1 + 1)u = 18 u. Using Eq. 1-9, we find

$$18\,\text{u} = (18\,\text{u})\left(\frac{1.6605402\times10^{-27}\,\text{kg}}{1\,\text{u}}\right) = 3.0\times10^{-26}\,\text{kg}.$$

(b) We divide the total mass by the mass of each molecule and obtain the (approximate) number of water molecules:

$$N \approx \frac{1.4\times10^{21}}{3.0\times10^{-26}} \approx 5\times10^{46}.$$

••26 One cubic centimeter of a typical cumulus cloud contains 50 to 500 water drops, which have a typical radius of 10 μm. For that range, give the lower value and the higher value, respectively, for the following. (a) How many cubic meters of water are in a cylindrical cumulus cloud of height 3.0 km and radius 1.0 km? (b) How many 1-liter pop bottles would that water fill? (c) Water has a density of 1000 kg/m^3. How much mass does the water in the cloud have?

26. (a) The volume of the cloud is $(3000\text{ m})\pi(1000\text{ m})^2 = 9.4\times10^9\text{ m}^3$. Since each cubic meter of the cloud contains from 50×10^6 to 500×10^6 water drops, then we conclude that the entire cloud contains from 4.7×10^{18} to 4.7×10^{19} drops. Since the volume of each drop is $\frac{4}{3}\pi(10\times10^{-6}\text{ m})^3 = 4.2\times10^{-15}\text{ m}^3$, then the total volume of water in a cloud is from 2×10^3 to 2×10^4 m^3.

(b) Using the fact that $1\text{ L} = 1\times10^3\text{cm}^3 = 1\times10^{-3}\text{m}^3$, the amount of water estimated in part (a) would fill from 2×10^6 to 2×10^7 bottles.

(c) At 1000 kg for every cubic meter, the mass of water is from 2×10^6 to 2×10^7 kg. The coincidence in numbers between the results of parts (b) and (c) of this problem is due to the fact that each liter has a mass of one kilogram when water is at its normal density (under standard conditions).

Chapter 2 – Motion Along A Straight Line

••7 Two trains, each having a speed of 30 km/h, are headed at each other on the same straight track. A bird that can fly 60 km/h flies off the front of one train when they are 60 km apart and heads directly for the other train. On reaching the other train, the bird flies directly back to the first train, and so forth. (We have no idea *why* a bird would behave in this way.) What is the total distance the bird travels before the trains collide?

7. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h, the total time that elapses before they crash is t = (60 km)/(60 km/h) = 1.0 h. During this time, the bird travels a distance of $x = vt$ = (60 km/h)(1.0 h) = 60 km.

••37 Figure 2-26 depicts the motion of a particle moving along an x axis with a constant acceleration. The figure's vertical scaling is set by x_s = 6.0 m. What are the (a) magnitude and (b) direction of the particle's acceleration?

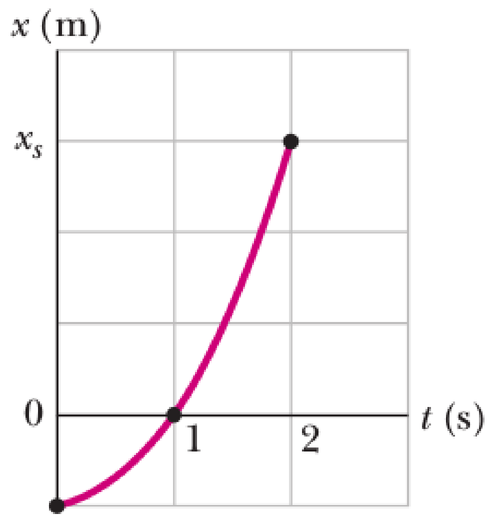


Fig. 2-26 Problem 37.

••38 (a) If the maximum acceleration that is tolerable for passengers in a subway train is 1.34 m/s^2 and subway stations are located 806 m apart, what is the maximum speed a subway train can attain between stations? (b) What is the travel time between stations? (c) If a subway train stops for 20 s at each station, what is the maximum average speed of the train, from one start-up to the next? (d) Graph x, v, and a versus t for the interval from one start-up to the next.

37. (a) From the figure, we see that $x_0 = -2.0$ m. From Table 2-1, we can apply

$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2$$

with t = 1.0 s, and then again with t = 2.0 s. This yields two equations for the two unknowns, v_0 and a:

$$0.0 - (-2.0 \text{ m}) = v_0 (1.0 \text{ s}) + \frac{1}{2} a (1.0 \text{ s})^2$$
$$6.0 \text{ m} - (-2.0 \text{ m}) = v_0 \ (2.0 \text{ s}) + \frac{1}{2} a (2.0 \text{ s})^2 .$$

Solving these simultaneous equations yields the results $v_0 = 0$ and $a = 4.0 \text{ m/s}^2$.

(b) The fact that the answer is positive tells us that the acceleration vector points in the $+x$ direction.

38. We assume the train accelerates from rest ($v_0 = 0$ and $x_0 = 0$) at $a_1 = +1.34 \text{ m/s}^2$ until the midway point and then decelerates at $a_2 = -1.34 \text{ m/s}^2$ until it comes to a stop $(v_2 = 0)$ at the next station. The velocity at the midpoint is v_1, which occurs at $x_1 = 806/2 = 403$m.

(a) Equation 2-16 leads to

$$v_1^2 = v_0^2 + 2a_1 x_1 \quad \Rightarrow \quad v_1 = \sqrt{2\left(1.34 \text{ m/s}^2\right)\left(403 \text{ m}\right)} = 32.9 \text{ m/s}.$$

(b) The time t_1 for the accelerating stage is (using Eq. 2-15)

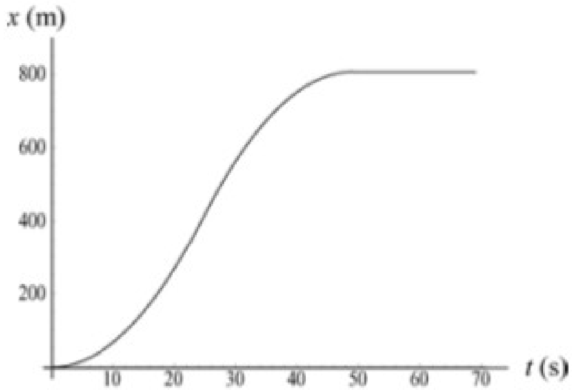
$$x_1 = v_0 t_1 + \frac{1}{2} a_1 t_1^2 \quad \Rightarrow \quad t_1 = \sqrt{\frac{2\left(403 \text{ m}\right)}{1.34 \text{ m/s}^2}} = 24.53 \text{ s}.$$

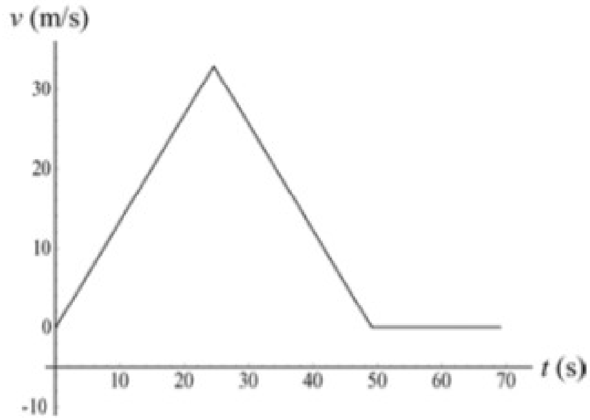
Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain t = 49.1 s for the travel time between stations.

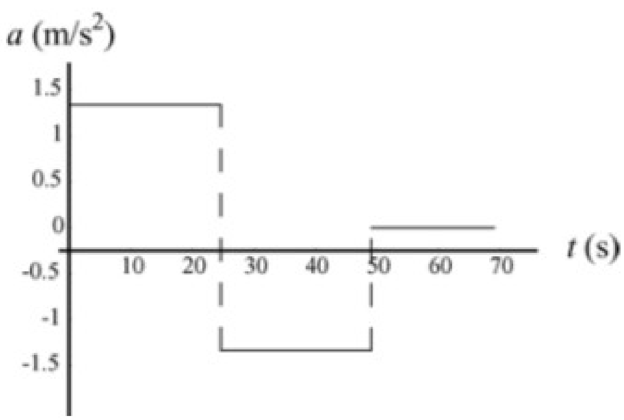
(c) With a "dead time" of 20 s, we have $T = t + 20 = 69.1$ s for the total time between start-ups. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{806 \text{ m}}{69.1 \text{ s}} = 11.7 \text{ m/s} .$$

(d) The graphs for x, v and a as a function of t are shown below. The third graph, $a(t)$, consists of three horizontal "steps" — one at 1.34 m/s^2 during $0 < t < 24.53$ s, and the next at -1.34 m/s^2 during $24.53 \text{ s} < t < 49.1$ s and the last at zero during the "dead time" $49.1 \text{ s} < t < 69.1$ s).







Chapter 3 – Vectors

••40 Displacement $\vec{d}_1$ is in the yz plane 63.0° from the positive direction of the y axis, has a positive z component, and has a magnitude of 4.50 m. Displacement $\vec{d}_2$ is in the xz plane 30.0° from the positive direction of the x axis, has a positive z component, and has magnitude 1.40 m. What are (a) $\vec{d}_1 \cdot \vec{d}_2$, (b) $\vec{d}_1 \times \vec{d}_2$, and (c) the angle between $\vec{d}_1$ and $\vec{d}_2$?

40. The displacement vectors can be written as (in meters)

$$\vec{d}_1 = (4.50\text{ m})(\cos 63^\circ\,\hat{\text{j}} + \sin 63^\circ\,\hat{\text{k}}) = (2.04\text{ m})\,\hat{\text{j}} + (4.01\text{ m})\,\hat{\text{k}}$$
$$\vec{d}_2 = (1.40\text{ m})(\cos 30^\circ\,\hat{\text{i}} + \sin 30^\circ\,\hat{\text{k}}) = (1.21\text{ m})\,\hat{\text{i}} + (0.70\text{ m})\,\hat{\text{k}}\,.$$

(a) The dot product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\vec{d}_1 \cdot \vec{d}_2 = (2.04\,\hat{\text{j}} + 4.01\hat{\text{k}}) \cdot (1.21\hat{\text{i}} + 0.70\hat{\text{k}}) = (4.01\hat{\text{k}}) \cdot (0.70\hat{\text{k}}) = 2.81\,\text{m}^2.$$

(b) The cross product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\begin{aligned}\vec{d}_1 \times \vec{d}_2 &= (2.04\,\hat{\text{j}} + 4.01\hat{\text{k}}) \times (1.21\hat{\text{i}} + 0.70\hat{\text{k}}) \\ &= (2.04)(1.21)(-\hat{\text{k}}) + (2.04)(0.70)\hat{\text{i}} + (4.01)(1.21)\hat{\text{j}} \\ &= (1.43\,\hat{\text{i}} + 4.86\,\hat{\text{j}} - 2.48\,\hat{\text{k}})\,\text{m}^2.\end{aligned}$$

(c) The magnitudes of $\vec{d}_1$ and $\vec{d}_2$ are

$$d_1 = \sqrt{(2.04\text{ m})^2 + (4.01\text{ m})^2} = 4.50\text{ m}$$
$$d_2 = \sqrt{(1.21\text{ m})^2 + (0.70\text{ m})^2} = 1.40\text{ m}.$$

Thus, the angle between the two vectors is

$$\theta = \cos^{-1}\left(\frac{\vec{d}_1 \cdot \vec{d}_2}{d_1 d_2}\right) = \cos^{-1}\left(\frac{2.81\,\text{m}^2}{(4.50\text{ m})(1.40\text{ m})}\right) = 63.5^\circ.$$

63 Here are three vectors in meters:

$$\vec{d}_1 = -3.0\hat{\text{i}} + 3.0\hat{\text{j}} + 2.0\hat{\text{k}}$$
$$\vec{d}_2 = -2.0\hat{\text{i}} - 4.0\hat{\text{j}} + 2.0\hat{\text{k}}$$
$$\vec{d}_3 = 2.0\hat{\text{i}} + 3.0\hat{\text{j}} + 1.0\hat{\text{k}}.$$

What results from (a) $\vec{d}_1 \cdot (\vec{d}_2 + \vec{d}_3)$, (b) $\vec{d}_1 \cdot (\vec{d}_2 \times \vec{d}_3)$, and (c) $\vec{d}_1 \times (\vec{d}_2 + \vec{d}_3)$?

63. The three vectors are

$$\vec{d}_1 = -3.0\,\hat{i} + 3.0\,\hat{j} + 2.0\,\hat{k}$$
$$\vec{d}_2 = -2.0\,\hat{i} - 4.0\,\hat{j} + 2.0\,\hat{k}$$
$$\vec{d}_3 = 2.0\,\hat{i} + 3.0\,\hat{j} + 1.0\,\hat{k}.$$

(a) Since $\vec{d}_2 + \vec{d}_3 = 0\,\hat{i} - 1.0\,\hat{j} + 3.0\,\hat{k}$, we have

$$\vec{d}_1 \cdot (\vec{d}_2 + \vec{d}_3) = (-3.0\,\hat{i} + 3.0\,\hat{j} + 2.0\,\hat{k}) \cdot (0\,\hat{i} - 1.0\,\hat{j} + 3.0\,\hat{k})$$
$$= 0 - 3.0 + 6.0 = 3.0 \text{ m}^2.$$

(b) Using Eq. 3-30, we obtain $\vec{d}_2 \times \vec{d}_3 = -10\,\hat{i} + 6.0\,\hat{j} + 2.0\,\hat{k}$. Thus,

$$\vec{d}_1 \cdot (\vec{d}_2 \times \vec{d}_3) = (-3.0\,\hat{i} + 3.0\,\hat{j} + 2.0\,\hat{k}) \cdot (-10\,\hat{i} + 6.0\,\hat{j} + 2.0\,\hat{k})$$
$$= 30 + 18 + 4.0 = 52 \text{ m}^3.$$

(c) We found $\vec{d}_2 + \vec{d}_3$ in part (a). Use of Eq. 3-30 then leads to

$$\vec{d}_1 \times (\vec{d}_2 + \vec{d}_3) = (-3.0\,\hat{i} + 3.0\,\hat{j} + 2.0\,\hat{k}) \times (0\,\hat{i} - 1.0\,\hat{j} + 3.0\,\hat{k})$$
$$= (11\,\hat{i} + 9.0\,\hat{j} + 3.0\,\hat{k}) \text{ m}^2$$

Chapter 4 – Motion in Two and Three Dimensions

••32 GO You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-35). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

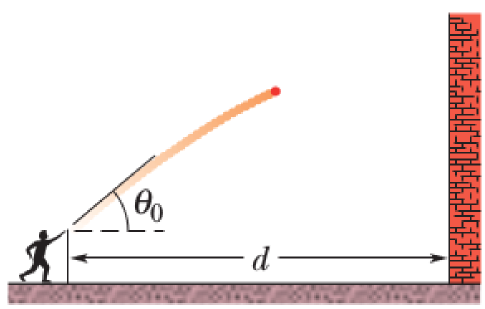


Fig. 4-35 Problem 32.

32. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the release point (the initial position for the ball as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of throw (shown in the figure). Since the horizontal component of the velocity of the ball is $v_x = v_0 \cos 40.0°$, the time it takes for the ball to hit the wall is

$$t = \frac{\Delta x}{v_x} = \frac{22.0 \text{ m}}{(25.0 \text{ m/s}) \cos 40.0°} = 1.15 \text{ s}.$$

(a) The vertical distance is

$$\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = (25.0 \text{ m/s}) \sin 40.0°(1.15 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.15 \text{ s})^2 = 12.0 \text{ m}.$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value: $v_x = v_0 \cos 40.0° = 19.2$ m/s.

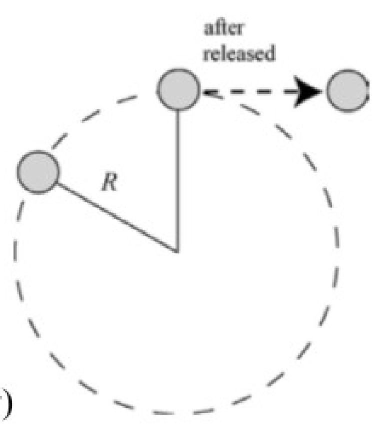
(c) The vertical component becomes (using Eq. 4-23)

$$v_y = v_0 \sin\theta_0 - gt = (25.0 \text{ m/s}) \sin 40.0° - (9.80 \text{ m/s}^2)(1.15 \text{ s}) = 4.80 \text{ m/s}.$$

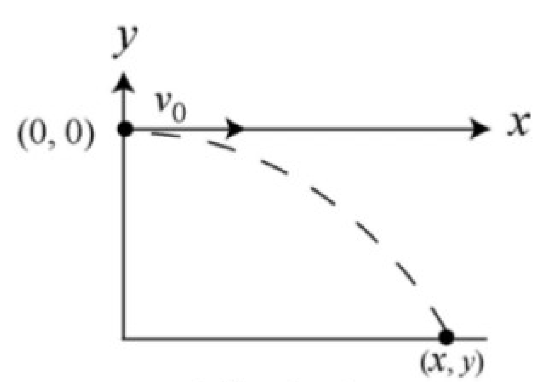
(d) Since $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

•••67 SSM WWW A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2.0 m above level ground. The string breaks, and the stone flies off horizontally and strikes the ground after traveling a horizontal distance of 10 m. What is the magnitude of the centripetal acceleration of the stone during the circular motion?

67. The stone moves in a circular path (top view shown below left) initially, but undergoes projectile motion after the string breaks (side view shown below right).



(top view)



(side view)

Since $a = v^2 / R$, to calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed. Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0 t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). It hits the ground at $x = 10$ m and $y = -2.0$ m.

Formally solving the y-component equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\ \text{m})\sqrt{-\frac{9.8\ \text{m/s}^2}{2(-2.0\ \text{m})}} = 15.7\ \text{m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v_0^2}{R} = \frac{(15.7\ \text{m/s})^2}{1.5\ \text{m}} = 160\ \text{m/s}^2.$$

Note: The above equations can be combined to give $a = \dfrac{gx^2}{-2yR}$. The equation implies that the greater the centripetal acceleration, the greater the initial speed of the projectile, and the greater the distance traveled by the stone. This is precisely what we expect.

••80 GO A 200-m-wide river flows due east at a uniform speed of 2.0 m/s. A boat with a speed of 8.0 m/s relative to the water leaves the south bank pointed in a direction 30° west of north. What are the (a) magnitude and (b) direction of the boat's velocity relative to the ground? (c) How long does the boat take to cross the river?

80. This is a classic problem involving two-dimensional relative motion. We align our coordinates so that *east* corresponds to $+x$ and *north* corresponds to $+y$. We write the vector addition equation as $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$. We have $\vec{v}_{WG} = (2.0\angle 0°)$ in the magnitude-angle notation (with the unit m/s understood), or $\vec{v}_{WG} = 2.0\hat{\text{i}}$ in unit-vector notation. We also have $\vec{v}_{BW} = (8.0\angle 120°)$ where we have been careful to phrase the angle in the 'standard' way (measured counterclockwise from the $+x$ axis), or $\vec{v}_{BW} = (-4.0\hat{\text{i}} + 6.9\hat{\text{j}})$ m/s.

(a) We can solve the vector addition equation for $\vec{v}_{BG}$:

$$\vec{v}_{BG} = v_{BW} + \vec{v}_{WG} = (2.0 \text{ m/s})\hat{\text{i}} + (-4.0\hat{\text{i}} + 6.9\hat{\text{j}}) \text{ m/s} = (-2.0 \text{ m/s})\hat{\text{i}} + (6.9 \text{ m/s})\hat{\text{j}}.$$

Thus, we find $|\vec{v}_{BG}| = 7.2$ m/s.

(b) The direction of $\vec{v}_{BG}$ is $\theta = \tan^{-1}[(6.9 \text{ m/s})/(-2.0 \text{ m/s})] = 106°$ (measured counterclockwise from the +x axis), or 16° west of north.

(c) The velocity is constant, and we apply $y - y_0 = v_y t$ in a reference frame. Thus, in the *ground* reference frame, we have $(200 \text{ m}) = (7.2 \text{ m/s})\sin(106°)t \rightarrow t = 29$ s. Note: If a student obtains "28 s," then the student has probably neglected to take the y component properly (a common mistake).

Chapter 5 – Force and Motion I

••11 A 2.0 kg particle moves along an x axis, being propelled by a variable force directed along that axis. Its position is given by $x = 3.0 \text{ m} + (4.0 \text{ m/s})t + ct^2 - (2.0 \text{ m/s}^3)t^3$, with x in meters and t in seconds. The factor c is a constant. At $t = 3.0$ s, the force on the particle has a magnitude of 36 N and is in the negative direction of the axis. What is c?

•••12 GO Two horizontal forces $\vec{F}_1$ and $\vec{F}_2$ act on a 4.0 kg disk that slides over frictionless ice, on which an xy coordinate system is laid out. Force $\vec{F}_1$ is in the positive direction of the x axis and has a magnitude of 7.0 N. Force $\vec{F}_2$ has a magnitude of 9.0 N. Figure 5-32 gives the x component v_x of the velocity of the disk as a function of time t during the sliding. What is the angle between the constant directions of forces $\vec{F}_1$ and $\vec{F}_2$?

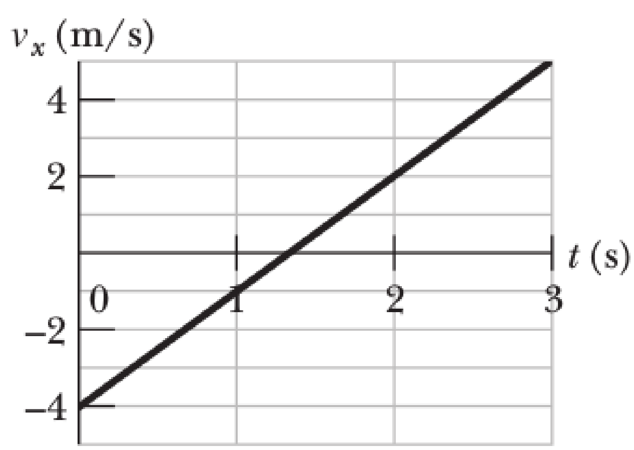


Fig. 5-32 Problem 12.

11. The velocity is the derivative (with respect to time) of given function x, and the acceleration is the derivative of the velocity. Thus, $a = 2c - 3(2.0)(2.0)t$, which we use in Newton's second law: $F = (2.0 \text{ kg})a = 4.0c - 24t$ (with SI units understood). At $t = 3.0$ s, we are told that $F = -36$ N. Thus, $-36 = 4.0c - 24(3.0)$ can be used to solve for c. The result is $c = +9.0 \text{ m/s}^2$.

12. From the slope of the graph we find $a_x = 3.0 \text{ m/s}^2$. Applying Newton's second law to the x axis (and taking θ to be the angle between F_1 and F_2), we have

$$F_1 + F_2 \cos\theta = m\,a_x \quad \Rightarrow \quad \theta = 56°.$$

••34 GO In Fig. 5-40, a crate of mass $m = 100$ kg is pushed at constant speed up a frictionless ramp ($\theta = 30.0°$) by a horizontal force $\vec{F}$. What are the magnitudes of (a) $\vec{F}$ and (b) the force on the crate from the ramp?

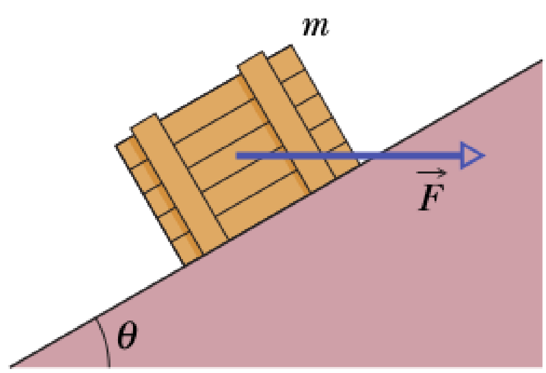
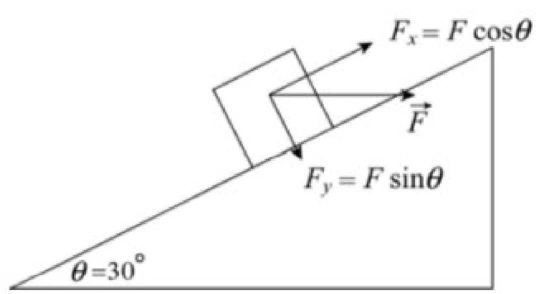


Fig. 5-40 Problem 34.

••35 The velocity of a 3.00 kg particle is given by $\vec{v} = (8.00t\hat{\mathrm{i}} + 3.00t^2\hat{\mathrm{j}})$ m/s, with time t in seconds. At the instant the net force on the particle has a magnitude of 35.0 N, what are the direction (relative to the positive direction of the x axis) of (a) the net force and (b) the particle's direction of travel?

34. We resolve this horizontal force into appropriate components.



(a) Newton's second law applied to the x-axis produces

$$F\cos\theta - mg\sin\theta = ma.$$

For $a = 0$, this yields $F = 566$ N.

(b) Applying Newton's second law to the y axis (where there is no acceleration), we have

$$F_N - F\sin\theta - mg\cos\theta = 0$$

which yields the normal force $F_N = 1.13 \times 10^3$ N.

35. The acceleration vector as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(8.00t\,\hat{\text{i}} + 3.00t^2\,\hat{\text{j}}\right)\text{ m/s} = (8.00\,\hat{\text{i}} + 6.00t\,\hat{\text{j}})\text{ m/s}^2.$$

(a) The magnitude of the force acting on the particle is

$$F = ma = m\,|\,\vec{a}\,| = (3.00)\sqrt{(8.00)^2 + (6.00t)^2} = (3.00)\sqrt{64.0 + 36.0\,t^2}\text{ N}.$$

Thus, $F = 35.0$ N corresponds to $t = 1.415$ s, and the acceleration vector at this instant is

$$\vec{a} = [8.00\,\hat{\text{i}} + 6.00(1.415)\,\hat{\text{j}}]\text{ m/s}^2 = (8.00\text{ m/s}^2)\,\hat{\text{i}} + (8.49\text{ m/s}^2)\hat{\text{j}}.$$

The angle $\vec{a}$ makes with $+x$ is

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{8.49\text{ m/s}^2}{8.00\text{ m/s}^2}\right) = 46.7°.$$

(b) The velocity vector at $t = 1.415$ s is

$$\vec{v} = \left[8.00(1.415)\,\hat{\text{i}} + 3.00(1.415)^2\,\hat{\text{j}}\right]\text{m/s} = (11.3\text{ m/s})\,\hat{\text{i}} + (6.01\text{ m/s})\hat{\text{j}}.$$

Therefore, the angle $\vec{v}$ makes with $+x$ is

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{6.01\text{ m/s}}{11.3\text{ m/s}}\right) = 28.0°.$$

••53 In Fig. 5-48, three connected blocks are pulled to the right on a horizontal frictionless table by a force of magnitude $T_3 = 65.0$ N. If $m_1 = 12.0$ kg, $m_2 = 24.0$ kg, and $m_3 = 31.0$ kg, calculate (a) the magnitude of the system's acceleration, (b) the tension T_1, and (c) the tension T_2.

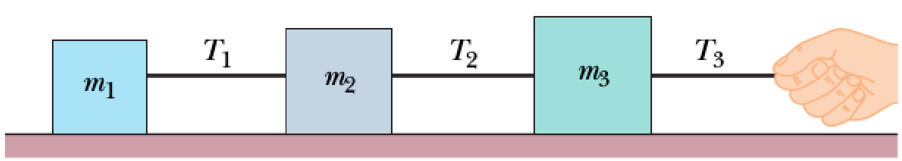


Fig. 5-48 Problem 53.

53. We apply Newton's second law first to the three blocks as a single system and then to the individual blocks. The $+x$ direction is to the right in Fig. 5-48.

(a) With $m_{sys} = m_1 + m_2 + m_3 = 67.0$ kg, we apply Eq. 5-2 to the x motion of the system, in which case, there is only one force $\vec{T}_3 = +\vec{T}_3\ \hat{\text{i}}$. Therefore,

$$T_3 = m_{sys} a \quad \Rightarrow \quad 65.0\,\text{N} = (67.0\,\text{kg})a$$

which yields $a = 0.970$ m/s^2 for the system (and for each of the blocks individually).

(b) Applying Eq. 5-2 to block 1, we find

$$T_1 = m_1 a = (12.0\,\text{kg})\left(0.970\,\text{m/s}^2\right) = 11.6\,\text{N}.$$

(c) In order to find T_2, we can either analyze the forces on block 3 or we can treat blocks 1 and 2 as a system and examine its forces. We choose the latter.

$$T_2 = (m_1 + m_2)a = (12.0\text{ kg} + 24.0\text{ kg})\left(0.970\text{ m/s}^2\right) = 34.9\text{ N}.$$

71 **SSM** Figure 5-60 shows a box of dirty money (mass $m_1 = 3.0$ kg) on a frictionless plane inclined at angle $\theta_1 = 30°$. The box is connected via a cord of negligible mass to a box of laundered money (mass $m_2 = 2.0$ kg) on a frictionless plane inclined at angle $\theta_2 = 60°$. The pulley is frictionless and has negligible mass. What is the tension in the cord?

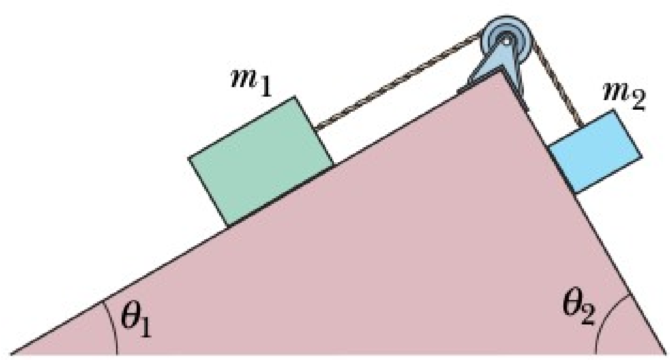
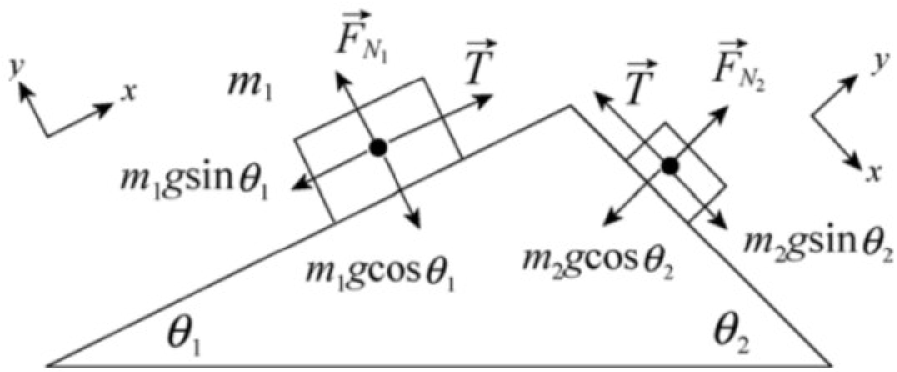


Fig. 5-60 Problem 71.

71. The $+x$ axis is "uphill" for $m_1 = 3.0$ kg and "downhill" for $m_2 = 2.0$ kg (so they both accelerate with the same sign). The x components of the two masses along the x axis are given by $m_1 g \sin\theta_1$ and $m_2 g \sin\theta_2$, respectively. The free-body diagram is shown below. Applying Newton's second law, we obtain

$$T - m_1 g \sin\theta_1 = m_1 a$$
$$m_2 g \sin\theta_2 - T = m_2 a.$$



Adding the two equations allows us to solve for the acceleration:

$$a = \left(\frac{m_2 \sin\theta_2 - m_1 \sin\theta_1}{m_2 + m_1}\right) g$$

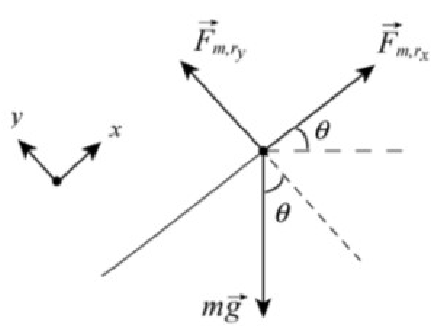
With $\theta_1 = 30°$ and $\theta_2 = 60°$, we have a = 0.45 m/s^2. This value is plugged back into either of the two equations to yield the tension

$$T = \frac{m_1 m_2 g}{m_2 + m_1}(\sin\theta_2 + \sin\theta_1) = 16 \text{ N}.$$

Note: In this problem we find $m_2 \sin\theta_2 > m_1 \sin\theta_1$, so that $a > 0$, indicating that m_2 slides down and m_1 slides up. The situation would reverse if $m_2 \sin\theta_2 < m_1 \sin\theta_1$. When $m_2 \sin\theta_2 = m_1 \sin\theta_1$, $a = 0$, and the two masses hang in balance. Notice also the symmetry between the two masses in the expression for T.

91 SSM A motorcycle and 60.0 kg rider accelerate at 3.0 m/s^2 up a ramp inclined 10° above the horizontal. What are the magnitudes of (a) the net force on the rider and (b) the force on the rider from the motorcycle?

91. The free-body diagram is shown below. Note that F_{m,r_y} and F_{m,r_x}, respectively, are thought of as the y and x components of the force $\vec{F}_{m,r}$ exerted by the motorcycle on the rider.



(a) Since the net force equals ma, then the magnitude of the net force on the rider is (60.0 kg) (3.0 m/s^2) = 1.8×10^2 N.

(b) We apply Newton's second law to the x axis: $F_{m,r_x} - mg\sin\theta = ma$, where m = 60.0 kg, a = 3.0 m/s^2, and θ = 10°. Thus, $F_{m,r_x} = 282$ N Applying it to the y axis (where there is no acceleration), we have

$$F_{m,r_y} - mg\cos\theta = 0$$

which produces $F_{m,r_y} = 579$ N. Using the Pythagorean theorem, we find

$$\sqrt{F_{m,r_x}^2 + F_{m,r_y}^2} = 644 \text{ N.}$$

Now, the magnitude of the force exerted on the rider by the motorcycle is the same magnitude of force exerted by the rider on the motorcycle, so the answer is 6.4×10^2 N, to two significant figures.

Chapter 6 – Force and Motion II

••16 A loaded penguin sled weighing 80 N rests on a plane inclined at angle θ = 20° to the horizontal (Fig. 6-23). Between the sled and the plane, the coefficient of static friction is 0.25, and the coefficient of kinetic friction is 0.15. (a) What is the least magnitude of the force $\vec{F}$, parallel to the plane, that will prevent the sled from slipping down the plane? (b) What is the minimum magnitude F that will start the sled moving up the plane? (c) What value of F is required to move the sled up the plane at constant velocity?

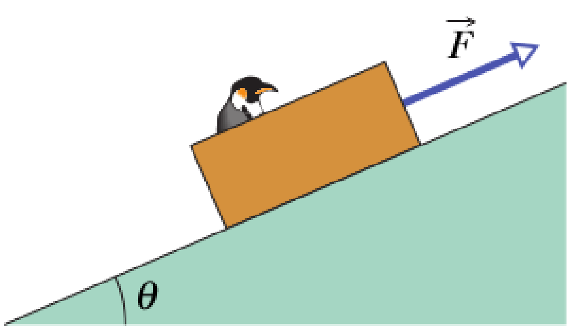


Fig. 6-23 Problems 16 and 22.

••22 GO In Fig. 6-23, a sled is held on an inclined plane by a cord pulling directly up the plane. The sled is to be on the verge of moving up the plane. In Fig. 6-28, the magnitude F required of the cord's force on the sled is plotted versus a range of values for the coefficient of static friction μ_s between sled and plane: $F_1 = 2.0$ N, $F_2 = 5.0$ N, and $\mu_2 = 0.50$. At what angle θ is the plane inclined?

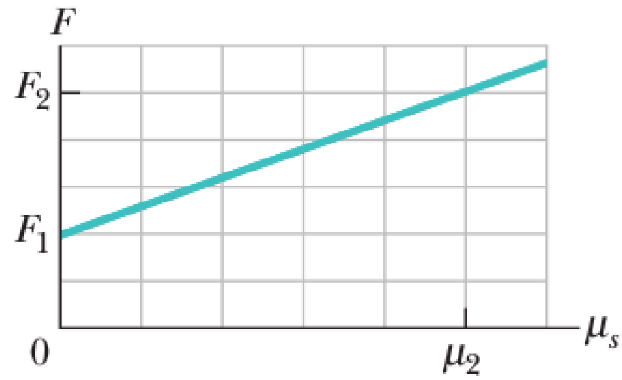


Fig. 6-28 Problem 22.

16. (a) In this situation, we take $\vec{f}_s$ to point uphill and to be equal to its maximum value, in which case $f_{s,\,\max} = \mu_s F_N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$F_{\min 1} - mg\sin\theta + f_{s,\,\max} = ma = 0$$
$$F_N - mg\cos\theta = 0$$

which (with $\theta = 20°$) leads to

$$F_{\min 1} - mg\left(\sin\theta + \mu_s\cos\theta\right) = 8.6 \text{ N}.$$

(b) Now we take $\vec{f}_s$ to point downhill and to be equal to its maximum value, in which case $f_{s,\,\max} = \mu_s F_N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$F_{\min 2} = mg\sin\theta - f_{s,\,\max} = ma = 0$$
$$F_N - mg\cos\theta = 0$$

which (with $\theta = 20°$) leads to

$$F_{\min 2} = mg\left(\sin\theta + \mu_s\cos\theta\right) = 46 \text{ N}.$$

A value slightly larger than the "exact" result of this calculation is required to make it accelerate uphill, but since we quote our results here to two significant figures, 46 N is a "good enough" answer.

(c) Finally, we are dealing with kinetic friction (pointing downhill), so that

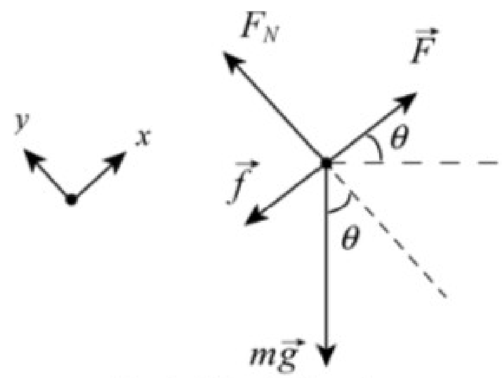
$$0 = F - mg\sin\theta - f_k = ma$$
$$0 = F_N - mg\cos\theta$$

along with $f_k = \mu_k F_N$ (where $\mu_k = 0.15$) brings us to

$$F = mg\left(\sin\theta + \mu_k \cos\theta\right) = 39 \text{ N} .$$

22. The free-body diagram for the sled is shown below, with $\vec{F}$ being the force applied to the sled, $\vec{F}_N$ the normal force of the inclined plane on the sled, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the $+x$ direction to be along the inclined plane and the $+y$ direction to be in its normal direction. The equations for the x and the y components of the force according to Newton's second law are:

$$F_x = F - f - mg\sin\theta = ma = 0$$
$$F_y = F_N - mg\cos\theta = 0.$$



Now $f = \mu F_N$, and the second equation gives $F_N = mg\cos\theta$, which yields $f = \mu mg\cos\theta$. This expression is substituted for f in the first equation to obtain

$$F = mg(\sin\theta + \mu\cos\theta)$$

From the figure, we see that $F = 2.0$ N when $\mu = 0$. This implies $mg\sin\theta = 2.0$ N. Similarly, we also find $F = 5.0$ N when $\mu = 0.5$:

$$5.0 \text{ N} = mg(\sin\theta + 0.50\cos\theta) = 2.0 \text{ N} + 0.50 mg\cos\theta$$

which yields $mg\cos\theta = 6.0$ N. Combining the two results, we get

$$\tan\theta = \frac{2}{6} = \frac{1}{3} \;\Rightarrow\; \theta = 18°.$$

75 A locomotive accelerates a 25-car train along a level track. Every car has a mass of 5.0×10^4 kg and is subject to a frictional force $f = 250v$, where the speed v is in meters per second and the force f is in newtons. At the instant when the speed of the train is 30 km/h, the magnitude of its acceleration is 0.20 m/s^2. (a) What is the tension in the coupling between the first car and the locomotive? (b) If this tension is equal to the maximum force the locomotive can exert on the train, what is the steepest grade up which the locomotive can pull the train at 30 km/h?

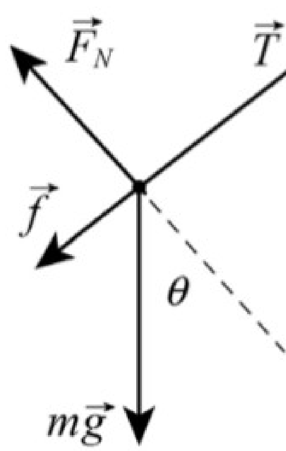
75. We may treat all 25 cars as a single object of mass $m = 25 \times 5.0 \times 10^4$ kg and (when the speed is 30 km/h = 8.3 m/s) subject to a friction force equal to

$$f = 25 \times 250 \times 8.3 = 5.2 \times 10^4 \text{ N}.$$

(a) Along the level track, this object experiences a "forward" force T exerted by the locomotive, so that Newton's second law leads to

$$T - f = ma \quad \Rightarrow \quad T = 5.2\times10^4 + (1.25\times10^6)(0.20) = 3.0\times10^5 \text{ N}.$$

(b) The free-body diagram is shown below, with θ as the angle of the incline.



The $+x$ direction (which is the only direction to which we will be applying Newton's second law) is uphill (to the upper right in our sketch).

Thus, we obtain

$$T - f - mg\sin\theta = ma$$

where we set $a = 0$ (implied by the problem statement) and solve for the angle. We obtain $\theta = 1.2°$.

96 A child places a picnic basket on the outer rim of a merry-go-round that has a radius of 4.6 m and revolves once every 30 s. (a) What is the speed of a point on that rim? (b) What is the lowest value of the coefficient of static friction between basket and merry-go-round that allows the basket to stay on the ride?

96. (a) The distance traveled in one revolution is $2\pi R = 2\pi(4.6 \text{ m}) = 29$ m. The (constant) speed is consequently $v = (29 \text{ m})/(30 \text{ s}) = 0.96$ m/s.

(b) Newton's second law (using Eq. 6-17 for the magnitude of the acceleration) leads to

$$f_s = m\left(\frac{v^2}{R}\right) = m(0.20)$$

in SI units. Noting that $F_N = mg$ in this situation, the maximum possible static friction is $f_{s,\max} = \mu_s mg$ using Eq. 6-1. Equating this with $f_s = m(0.20)$ we find the mass m cancels and we obtain $\mu_s = 0.20/9.8 = 0.021$.

Chapter 7 – Kinetic Energy and Work

••32 Figure 7-36 gives spring force F_x versus position x for the spring–block arrangement of Fig. 7-9. The scale is set by $F_s = 160.0$ N. We release the block at $x = 12$ cm. How much work does the spring do on the block when the block moves from $x_i = +8.0$ cm to (a) $x = +5.0$ cm, (b) $x = -5.0$ cm, (c) $x = -8.0$ cm, and (d) $x = -10.0$ cm?

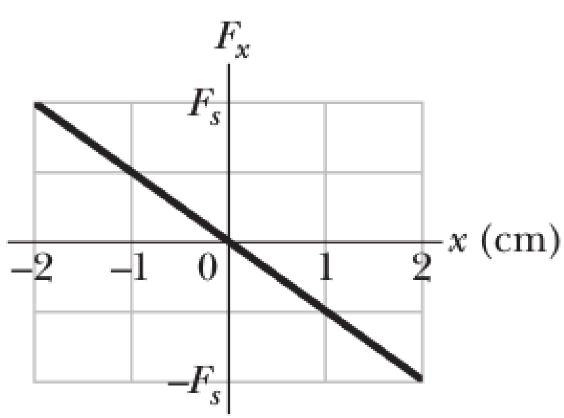


Fig. 7-36 Problem 32.

32. The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. Since $F_x = -kx$, the slope in Fig. 7-36 corresponds to the spring constant k. Its value is given by $k = 80\ \text{N/cm} = 8.0\times10^3\ \text{N/m}$.

(a) When the block moves from $x_i = +8.0\ \text{cm}$ to $x = +5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (0.050\ \text{m})^2] = 15.6\ \text{J} \approx 16\ \text{J}.$$

(b) Moving from $x_i = +8.0\ \text{cm}$ to $x = -5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.050\ \text{m})^2] = 15.6\ \text{J} \approx 16\ \text{J}.$$

(c) Moving from $x_i = +8.0\ \text{cm}$ to $x = -8.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.080\ \text{m})^2] = 0\ \text{J}.$$

(d) Moving from $x_i = +8.0\ \text{cm}$ to $x = -10.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.10\ \text{m})^2] = -14.4\ \text{J} \approx -14\ \text{J}.$$

54 The only force acting on a 2.0 kg body as the body moves along an x axis varies as shown in Fig. 7-42. The scale of the figure's vertical axis is set by $F_s = 4.0$ N. The velocity of the body at $x = 0$ is 4.0 m/s. (a) What is the kinetic energy of the body at $x = 3.0$ m? (b) At what value of x will the body have a kinetic energy of 8.0 J? (c) What is the maximum kinetic energy of the body between $x = 0$ and $x = 5.0$ m?

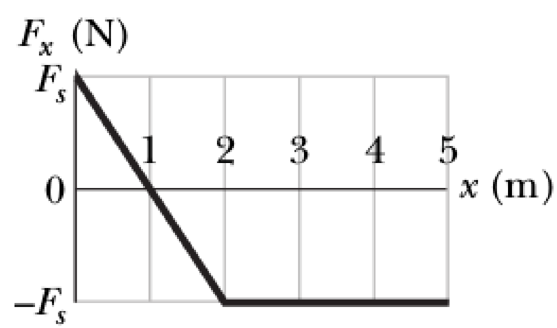


Fig. 7-42 Problem 54.

54. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. We find the area in terms of rectangular [length × width] and triangular [$\frac{1}{2}$ base × height] areas and use the work-kinetic energy theorem appropriately. The initial point is taken to be $x = 0$, where $v_0 = 4.0$ m/s.

(a) With $K_i = \frac{1}{2}mv_0^2 = 16$ J, we have

$$K_3 - K_0 = W_{0<x<1} + W_{1<x<2} + W_{2<x<3} = -4.0 \text{ J}$$

so that K_3 (the kinetic energy when $x = 3.0$ m) is found to equal 12 J.

(b) With SI units understood, we write $W_{3<x<x_f}$ as $F_x \Delta x = (-4.0 \text{ N})(x_f - 3.0 \text{ m})$ and apply the work-kinetic energy theorem:

$$K_{x_f} - K_3 = W_{3<x<x_f}$$
$$K_{x_f} - 12 = (-4)(x_f - 3.0)$$

so that the requirement $K_{xf} = 8.0$ J leads to $x_f = 4.0$ m.

(c) As long as the work is positive, the kinetic energy grows. The graph shows this situation to hold until $x = 1.0$ m. At that location, the kinetic energy is

$$K_1 = K_0 + W_{0<x<1} = 16 \text{ J} + 2.0 \text{ J} = 18 \text{ J}.$$

79 **SSM** A 2.0 kg lunchbox is sent sliding over a frictionless surface, in the positive direction of an x axis along the surface. Beginning at time $t = 0$, a steady wind pushes on the lunchbox in the negative direction of the x axis. Figure 7-49 shows the position x of the lunchbox as a function of time t as the wind pushes on the lunchbox. From the graph, estimate the kinetic energy of the lunchbox at (a) $t = 1.0$ s and (b) $t = 5.0$ s. (c) How much work does the force from the wind do on the lunchbox from $t = 1.0$ s to $t = 5.0$ s?

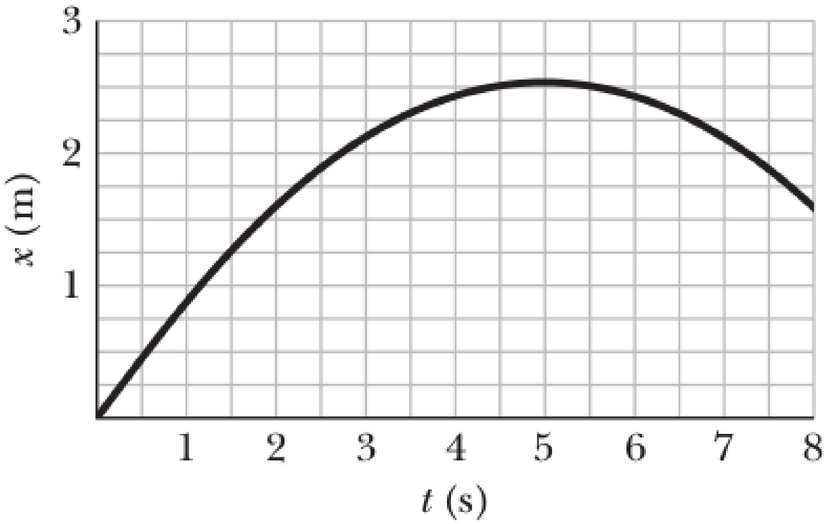


Fig. 7-49 Problem 79.

79. Figure 7-49 represents $x(t)$, the position of the lunchbox as a function of time. It is convenient to fit the curve to a concave-downward parabola:

$$x(t) = \frac{1}{10}t(10-t) = t - \frac{1}{10}t^2 .$$

By taking one and two derivatives, we find the velocity and acceleration to be

$$v(t) = \frac{dx}{dt} = 1 - \frac{t}{5} \;\text{ (in m/s)}, \quad a = \frac{d^2x}{dt^2} = -\frac{1}{5} = -0.2 \;\text{ (in m/s}^2\text{)}.$$

The equations imply that the initial speed of the box is $v_i = v(0) = 1.0\ \text{m/s}$, and the constant force by the wind is

$$F = ma = (2.0\ \text{kg})(-0.2\ \text{m/s}^2) = -0.40\ \text{N} .$$

The corresponding work is given by (SI units understood)

$$W(t) = F \cdot x(t) = -0.04t(10-t) .$$

The initial kinetic energy of the lunch box is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\ \text{kg})(1.0\ \text{m/s})^2 = 1.0\ \text{J}.$$

With $\Delta K = K_f - K_i = W$, the kinetic energy at a later time is given by (in SI units)

$$K(t) = K_i + W = 1 - 0.04t(10 - t)$$

(a) When t = 1.0 s, the above expression gives

$$K(1\ \text{s}) = 1 - 0.04(1)(10 - 1) = 1 - 0.36 = 0.64 \approx 0.6\ \text{J}$$

where the second significant figure is not to be taken too seriously.

(b) At t = 5.0 s, the above method gives $K(5.0\ \text{s}) = 1 - 0.04(5)(10 - 5) = 1 - 1 = 0$.

(c) The work done by the force from the wind from t = 1.0 s to t = 5.0 s is

$$W = K(5.0) - K(1.0\ \text{s}) = 0 - 0.6 \approx -0.6\ \text{J}.$$

Chapter 8 – Potential Energy and Conservation of Energy

••28 Figure 8-39*a* applies to the spring in a cork gun (Fig. 8-39*b*); it shows the spring force as a function of the stretch or compression of the spring. The spring is compressed by 5.5 cm and used to propel a 3.8 g cork from the gun. (a) What is the speed of the cork if it is released as the spring passes through its relaxed position? (b) Suppose, instead, that the cork sticks to the spring and stretches it 1.5 cm before separation occurs. What now is the speed of the cork at the time of release?

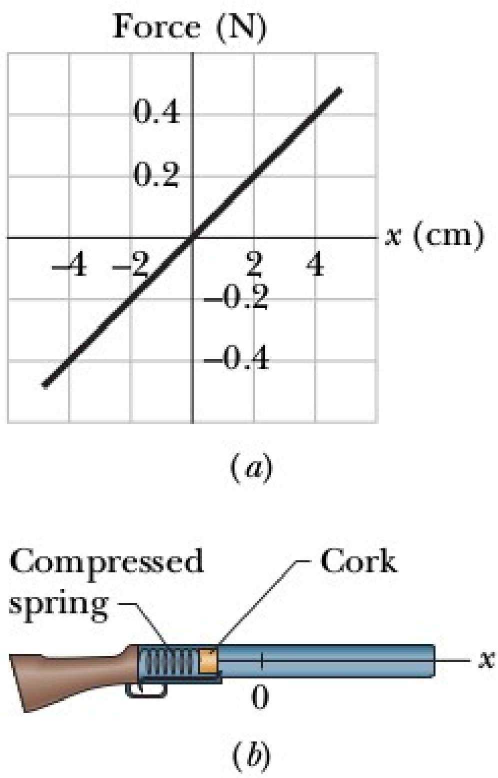


Fig. 8-39 Problem 28.

28. From the slope of the graph, we find the spring constant

$$k = \frac{\Delta F}{\Delta x} = 0.10\,\text{N/cm} = 10\,\text{N/m}.$$

(a) Equating the potential energy of the compressed spring to the kinetic energy of the cork at the moment of release, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow v = x\sqrt{\frac{k}{m}}$$

which yields $v = 2.8$ m/s for $m = 0.0038$ kg and $x = 0.055$ m.

(b) The new scenario involves some potential energy at the moment of release. With d = 0.015 m, energy conservation becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}kd^2 \Rightarrow v = \sqrt{\frac{k}{m}\left(x^2 - d^2\right)}$$

which yields $v = 2.7$ m/s.

••38 Figure 8-47 shows a plot of potential energy U versus position x of a 0.200 kg particle that can travel only along an x axis under the influence of a conservative force. The graph has these values: $U_A = 9.00$ J, $U_C = 20.00$ J, and $U_D = 24.00$ J. The particle is released at the point where U forms a "potential hill" of "height" $U_B = 12.00$ J, with kinetic energy 4.00 J. What is the speed of the particle at (a) $x = 3.5$ m and (b) $x = 6.5$ m? What is the position of the turning point on (c) the right side and (d) the left side?

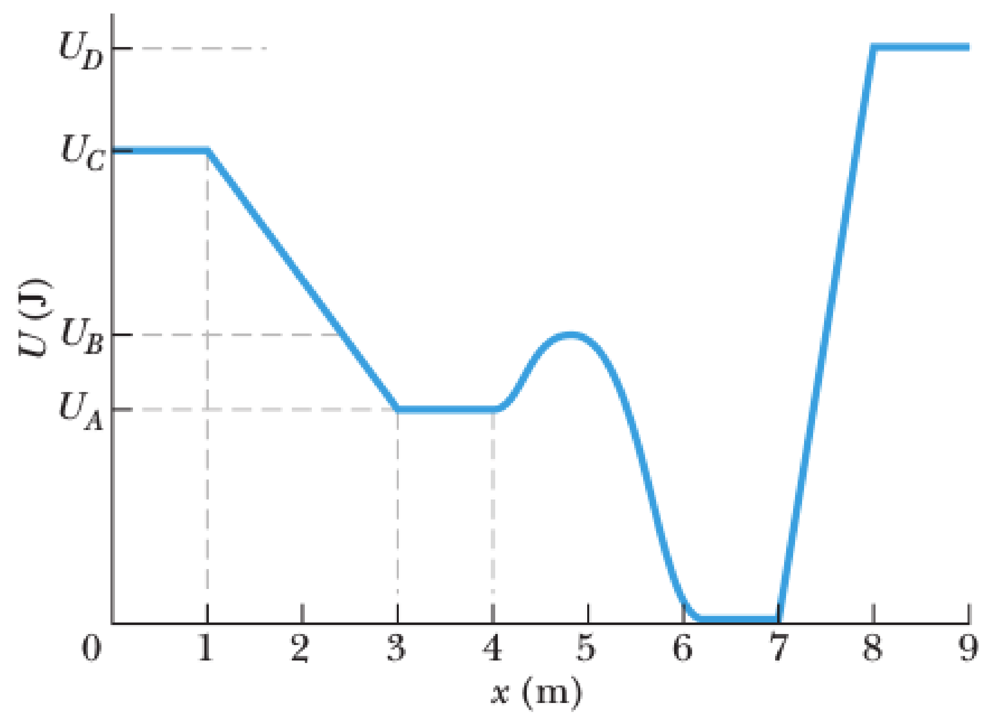


Fig. 8-47 Problem 38.

38. In this problem, the mechanical energy (the sum of K and U) remains constant as the particle moves.

(a) Since mechanical energy is conserved, $U_B + K_B = U_A + K_A$, the kinetic energy of the particle in region A ($3.00 \text{ m} \le x \le 4.00 \text{ m}$) is

$$K_A = U_B - U_A + K_B = 12.0 \text{ J} - 9.00 \text{ J} + 4.00 \text{ J} = 7.00 \text{ J}.$$

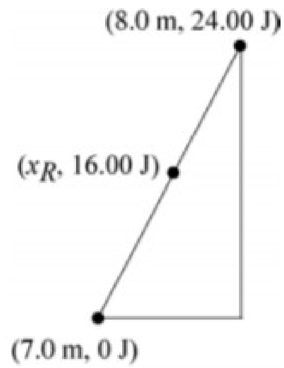
With $K_A = mv_A^2/2$, the speed of the particle at $x = 3.5$ m (within region *A*) is

$$v_A = \sqrt{\frac{2K_A}{m}} = \sqrt{\frac{2(7.00\ \text{J})}{0.200\ \text{kg}}} = 8.37\ \text{m/s}.$$

(b) At $x = 6.5$ m, $U = 0$ and $K = U_B + K_B = 12.0\ \text{J} + 4.00\ \text{J} = 16.0\ \text{J}$ by mechanical energy conservation. Therefore, the speed at this point is

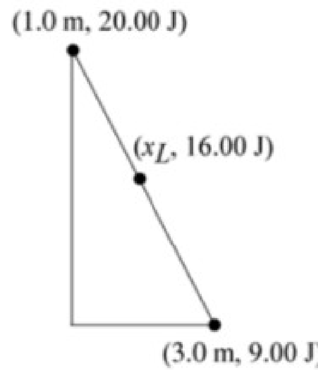
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(16.0\ \text{J})}{0.200\ \text{kg}}} = 12.6\ \text{m/s}.$$

(c) At the turning point, the speed of the particle is zero. Let the position of the right turning point be x_R. From the figure shown on the right, we find x_R to be



$$\frac{16.00\ \text{J} - 0}{x_R - 7.00\ \text{m}} = \frac{24.00\ \text{J} - 16.00\ \text{J}}{8.00\ \text{m} - x_R} \quad \Rightarrow x_R = 7.67\ \text{m}.$$

(d) Let the position of the left turning point be x_L. From the figure shown, we find x_L to be



$$\frac{16.00\ \text{J} - 20.00\ \text{J}}{x_L - 1.00\ \text{m}} = \frac{9.00\ \text{J} - 16.00\ \text{J}}{3.00\ \text{m} - x_L} \quad \Rightarrow x_L = 1.73\ \text{m}.$$

••54 A child whose weight is 267 N slides down a 6.1 m playground slide that makes an angle of 20° with the horizontal. The coefficient of kinetic friction between slide and child is 0.10. (a) How much energy is transferred to thermal energy? (b) If she starts at the top with a speed of 0.457 m/s, what is her speed at the bottom?

54. (a) Using the force analysis shown in Chapter 6, we find the normal force $F_N = mg\cos\theta$ (where mg = 267 N) which means

$$f_k = \mu_k F_N = \mu_k mg \cos\ \theta.$$

Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta = (0.10)(267)(6.1)\cos 20° = 1.5\times 10^2 \text{ J}.$$

(b) The potential energy change is

$$\Delta U = mg(-d \sin\ \theta) = (267 \text{ N})(-\ 6.1 \text{ m}) \sin 20° = -5.6 \times 10^2 \text{ J}.$$

The initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{267\,\text{N}}{9.8\,\text{m/s}^2}\right)(0.457\,\text{m/s}^2) = 2.8\ \text{J}.$$

Therefore, using Eq. 8-33 (with W = 0), the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{\text{th}} = 2.8 - (-5.6\times 10^2) - 1.5\times 10^2 = 4.1\times 10^2 \text{J}.$$

Consequently, the final speed is $v_f = \sqrt{2K_f/m} = 5.5\ \text{m/s}$.

•••64 In Fig. 8-55, a block is released from rest at height d = 40 cm and slides down a frictionless ramp and onto a first plateau, which has length d and where the coefficient of kinetic friction is 0.50. If the block is still moving, it then slides down a second frictionless ramp through height $d/2$ and onto a lower plateau, which has length $d/2$ and where the coefficient of kinetic friction is again 0.50. If the block is still moving, it then slides up a frictionless ramp until it (momentarily) stops. Where does the block stop? If its final stop is on a plateau, state which one and give the distance L from the left edge of that plateau. If the block reaches the ramp, give the height H above the lower plateau where it momentarily stops.

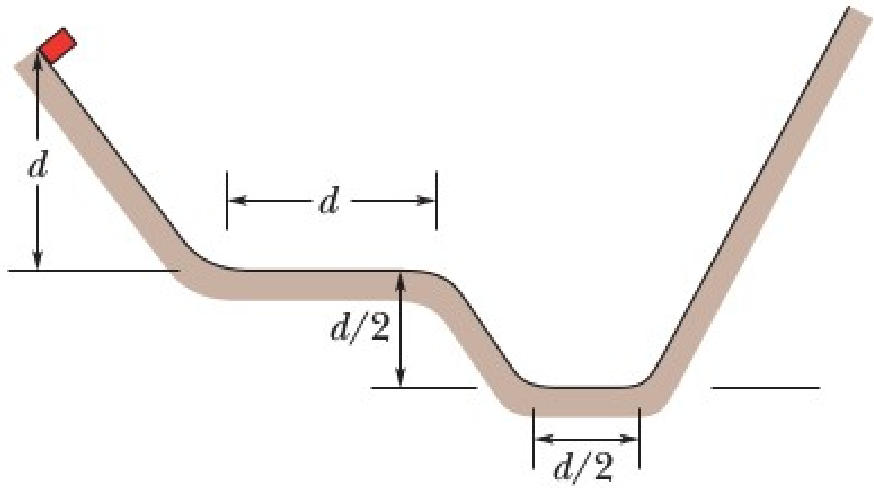


Fig. 8-55 Problem 64.

64. In the absence of friction, we have a simple conversion (as it moves along the inclined ramps) of energy between the kinetic form (Eq. 7-1) and the potential form (Eq. 8-9). Along the horizontal plateaus, however, there is friction that causes some of the kinetic energy to dissipate in accordance with Eq. 8-31 (along with Eq. 6-2 where μ_k = 0.50 and $F_N = mg$ in this situation). Thus, after it slides down a (vertical) distance d it has gained $K = \frac{1}{2}mv^2 = mgd$, some of which ($\Delta E_{\text{th}} = \mu_k mgd$) is dissipated, so that the value of kinetic energy at the end of the first plateau (just before it starts descending towards the lowest plateau) is

$$K = mgd - \mu_k mgd = \frac{1}{2}mgd\ .$$

In its descent to the lowest plateau, it gains $mgd/2$ more kinetic energy, but as it slides across it "loses" $\mu_k mgd/2$ of it. Therefore, as it starts its climb up the right ramp, it has kinetic energy equal to

$$K = \frac{1}{2}mgd + \frac{1}{2}mgd - \frac{1}{2}\mu_k mgd = \frac{3}{4}mgd\ .$$

Setting this equal to Eq. 8-9 (to find the height to which it climbs) we get $H = \frac{3}{4}d$. Thus, the block (momentarily) stops on the inclined ramp at the right, at a height of

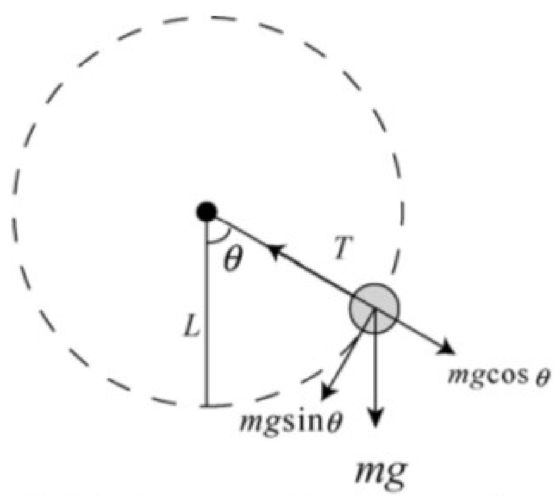
$$H = 0.75d = 0.75\,(\,40\text{ cm}) = 30\text{ cm}$$

measured from the lowest plateau.

75 **SSM** To form a pendulum, a 0.092 kg ball is attached to one end of a rod of length 0.62 m and negligible mass, and the other end of the rod is mounted on a pivot. The rod is rotated until it is straight up, and then it is released from rest so that it swings down around the pivot. When the ball reaches its lowest point, what are (a) its speed and (b) the tension in the rod? Next, the rod is rotated until it is horizontal, and then it is again released from rest. (c) At what angle from the vertical does the tension in the rod equal the weight of the ball? (d) If the mass of the ball is increased, does the answer to (c) increase, decrease, or remain the same?

75. This problem deals with pendulum motion. The kinetic and potential energies of the ball attached to the rod change with position, but the mechanical energy remains conserved throughout the process.

Let L be the length of the pendulum. The connection between angle θ (measured from vertical) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy mgh) is given by $h = L(1 - \cos\theta)$. The free-body diagram is shown below. The initial height is at $h_1 = 2L$, and at the lowest point, we have $h_2 = 0$. The total mechanical energy is conserved throughout.



(a) Initially the ball is at a height $h_1 = 2L$ with $K_1 = 0$ and $U_1 = mgh_1 = mg(2L)$. At the lowest point $h_2 = 0$, we have $K_2 = \frac{1}{2}mv_2^2$ and $U_2 = 0$. Using energy conservation in the form of Eq. 8-17 leads to

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad 0 + 2mgL = \frac{1}{2}mv_2^2 + 0 .$$

This leads to $v_2 = 2\sqrt{gL}$. With L = 0.62 m, we have

$$v_2 = 2\sqrt{(9.8 \text{ m/s}^2)(0.62 \text{ m})} = 4.9 \text{ m/s}.$$

(b) At the lowest point, the ball is in circular motion with the center of the circle above it, so $\vec{a} = v^2/r$ upward, where $r = L$. Newton's second law leads to

$$T - mg = m\frac{v^2}{r} \Rightarrow T = m\left(g + \frac{4gL}{L}\right) = 5\ mg.$$

With m = 0.092 kg, the tension is T = 4.5 N.

(c) The pendulum is now started (with zero speed) at $\theta_i = 90°$ (that is, $h_i = L$), and we look for an angle θ such that $T = mg$. When the ball is moving through a point at angle θ, as can be seen from the free-body diagram shown above, Newton's second law applied to the axis along the rod yields

$$\frac{mv^2}{r} = T - mg\cos\theta = mg(1-\cos\theta)$$

which (since $r = L$) implies $v^2 = gL(1 - \cos\theta)$ at the position we are looking for. Energy conservation leads to

$$\begin{aligned} K_i + U_i &= K + U \\ 0 + mgL &= \frac{1}{2}mv^2 + mgL\ (1-\cos\theta) \\ gL &= \frac{1}{2}(gL(1-\cos\theta)) + gL\ (1-\cos\theta) \end{aligned}$$

where we have divided by mass in the last step. Simplifying, we obtain

$$\theta = \cos^{-1}(1/3) = 71°.$$

(d) Since the angle found in (c) is independent of the mass, the result remains the same if the mass of the ball is changed.

Note: At a given angle θ with respect to the vertical, the tension in the rod is

$$T = m\left(\frac{v^2}{r} + g\cos\theta\right).$$

The tangential acceleration, $a_t = g\sin\theta$, is what causes the speed, and therefore, the kinetic energy, to change with time. Nonetheless, mechanical energy is conserved.