CIRCUITS

Restriction to the course : DC : Direct Current Circuits

Pumping the Charges:

Capacitors are OK but very limited.

→ Charge Pump : EMF device EMF : $\mathcal{E}$

↳ "Electromotive Force" (outdated definition)

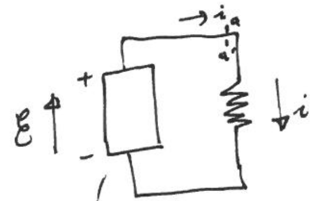
previously (up until now) Electric field produced forces that moved the charge carriers

the motion of charge carriers in terms of now: Required Energy

→ an EMF device supplies the energy for the motion via the work of it exerts.

A common EMF device : BATTERY

Most influences our lives : electric generator
solar cells
fuel cells
also living things (electric eels, human beings ("matrix"))



↳ motion of positive charge carriers opposite of the $\vec{E}$ field between the terminals (inside the EMF device)

→ Thus there must be some source of energy within the device.

Energy Source: Chemical : battery, fuel cell
Mechanical : Electric generator
Also, even temperature differences (thermopiles, sun)

in dt time, dq charge passes through a cross-section $a-a'$

dW to move dq

$\mathcal{E} = \frac{dW}{dq}$ Work per unit charge that the device does to move charges from low potential terminal to the high potential terminal

$[\mathcal{E}] = \frac{J}{C} = \text{Volt}$

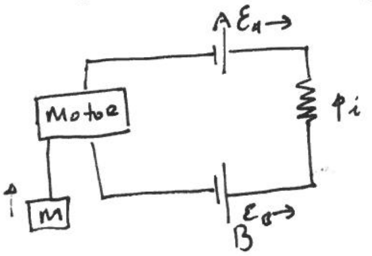
Ideal EMF: The potential difference between the terminals is equal to the emf of the device.

(e.g., a 12V battery always supplies a potential difference of 12V, always)

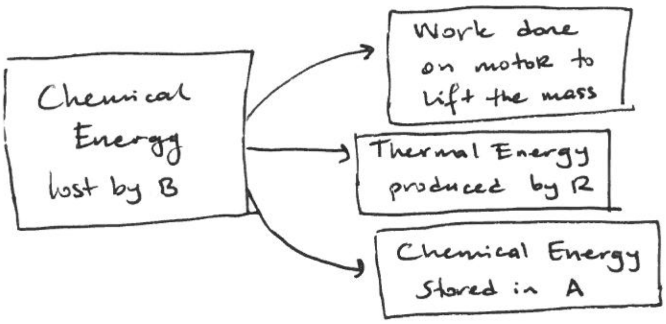
Real Life: The EMF device has an internal Resistance

("but" When it's not connected $V_{terminals} = \mathcal{E}$)

When the EMF device is connected, the device transfers energy to the charge carriers passing through it.



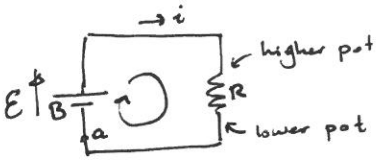
: $\mathcal{E}_B > \mathcal{E}_A$ → B is charging A



Calculating the current in a single-loop circuit

i.) via Energy Conservation:

$P = i^2 R$ → in a time interval dt, an amount of energy $i^2 R\,dt$ will appear in the resistor as thermal energy.



$dq = i\,dt$ will have moved through battery B

$dW = \mathcal{E}\,dq = \mathcal{E}\,i\,dt$

$\rightarrow \mathcal{E}\,i\,dt = i^2 R\,dt$

$\mathcal{E} = iR \longrightarrow \boxed{i = \frac{\mathcal{E}}{R}}$

ii.) via Potential:

The sum of the changes in potential encountered in a complete traversal of any loop of a current must be zero :: **Kirchhoff's Loop Rule**

the potential at a : V_a (1)

when we pass the battery, the change in the potential $= \mathcal{E}$ (2)

R: $V = iR$: it must decrease since we are moving from higher to lower potential

$\Rightarrow \Delta V = -iR$ (3)

$\Rightarrow V_a + \mathcal{E} - iR = V_a$ (analogy with heights)

(1) (2) (3)

$\rightarrow \mathcal{E} - iR = 0$

$\Rightarrow \boxed{i = \frac{\mathcal{E}}{R}}$

Resistance Rule: For a move through a resistance in a direction same as the current, the change in the potential is : $-iR$

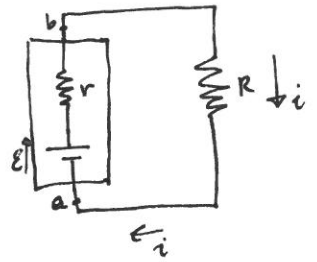
(opposite direction of the current : iR)

EMF Rule: For a move through an ideal emf device in the same direction of the emf arrow, the change in the potential is $\mathcal{E}$

(for the opposite direction : $-\mathcal{E}$)

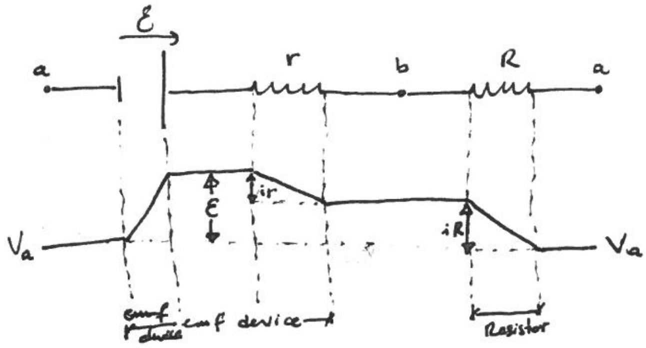
Other single-loop Circuits

Internal Resistance

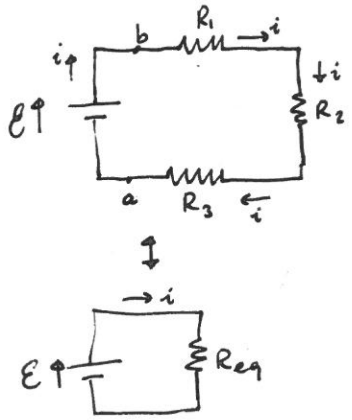


$$\mathcal{E} - ir - iR = 0$$

$$i = \frac{\mathcal{E}}{R+r} \quad \left(\rightarrow \; r \rightarrow 0 : \; i = \frac{\mathcal{E}}{R} \right)$$



Resistances in Series



The resistances in series have the identical current i

The Sum of potential differences across the resistances is equal to the applied potential difference of V

Resistances in series can be Replaced with an equivalent Resistance R_{eq} that has the same current i and the same total potential difference V as the original resistances.

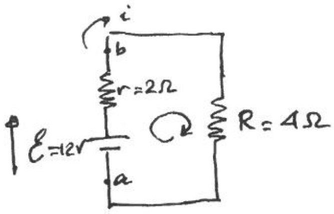
$$\rightarrow \text{i)} \quad \mathcal{E} - iR_1 - iR_2 - iR_3 = 0 \rightarrow i = \frac{\mathcal{E}}{R_1 + R_2 + R_3}$$

$$\text{ii)} \quad \mathcal{E} - iR_{eq} = 0 \rightarrow i = \frac{\mathcal{E}}{R_{eq}}$$

$$\text{i + ii} : \Rightarrow R_{eq} = R_1 + R_2 + R_3$$

(in general) $R_{eq} = \sum_{j=1}^{n} R_j$ <u>Resistances in Series</u>

Potential Difference Between two points



$V_a + \mathcal{E} - ir = V_b$

$V_b - V_a = \mathcal{E} - ir$

$i = \dfrac{\mathcal{E}}{R+r} \longrightarrow V_b - V_a = \mathcal{E} - \dfrac{\mathcal{E}}{R+r} r = \dfrac{\mathcal{E}}{R+r} R$

$V_b - V_a = \dfrac{12V}{4\Omega + 2\Omega} 4\Omega = 8V$

Suppose we move counter-clockwise:

$V_a + iR = V_b$

$V_b - V_a = iR = \dfrac{\mathcal{E}}{R+r} R$ ✓

* Potential Difference Across a Real Battery

$V = \mathcal{E} - ir$

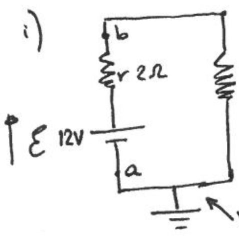
$(\mathcal{E} = 12V,\ r = 2\Omega, R = 4\Omega) \longrightarrow$ $\mathcal{E} = 12V$ but $V = 8V$

↳ note that the result depends on the value of the current through the battery.

different circuit → different current ⇒ different V

* Grounding a circuit

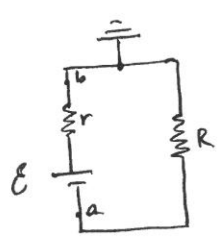
i)



V=0 at the grounding point → $V_a = 0$ → $V_b = 8V$

In both cases $V_b - V_a = 8V$

ii)



→ $V_b = 0$ → $V_a = -8V$

Power, Potential and EMF

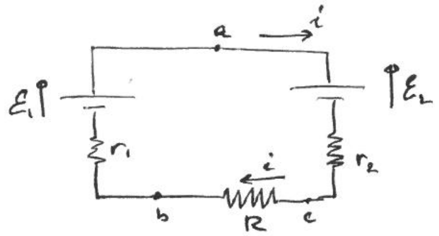
the net rate of energy transfer from the emf device to charge carriers

$P = iV$, $V = \mathcal{E} - ir \Rightarrow P = i(\mathcal{E} - ir) = i\mathcal{E} - i^2 r$

$P_r = i^2 r$ (internal dissipation rate)

$\Rightarrow P_{emf} = i\mathcal{E}$ (power of emf device)

Example



$\mathcal{E}_1 = 4.4V$ $\mathcal{E}_2 = 2.1V$

$r_1 = 2.3\Omega$ $r_2 = 1.8\Omega$ $R = 5.5\Omega$

a.) Current in the circuit?

a. $-\mathcal{E}_1 + ir_1 + iR + \mathcal{E}_2 = 0$

$\longrightarrow i = \dfrac{\mathcal{E}_1 - \mathcal{E}_2}{R + r_1 + r_2} = \ldots = 0.2396A \simeq 240mA$

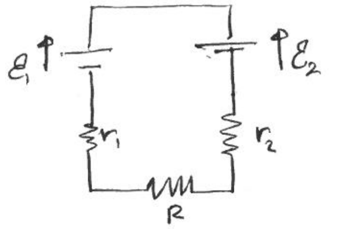
b.) Potential Diff between the terminals of Battery 1?

b. $V_b - ir_1 + \mathcal{E}_1 = V_a$

$V_a - V_b = -ir_1 + \mathcal{E}_1 = \ldots = 3.84V \simeq 3.8V > 0$

less than the emf of the battery.

Example



a.) What value must R have if the current in the circuit is 1mA?

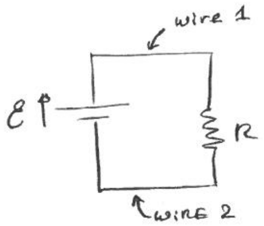
$\mathcal{E}_1 = 2V$, $\mathcal{E}_2 = 3V$, $r_1 = r_2 = 3\Omega$

$i = \dfrac{\mathcal{E}_2 - \mathcal{E}_1}{r_1 + r_2 + R} \longrightarrow R = \dfrac{\mathcal{E}_2 - \mathcal{E}_1}{i} - r_1 - r_2 = \ldots = 9.9 \times 10^2 \Omega$

b) What is the rate at which thermal energy appears in R?

$P = i^2 R = \ldots = 9.9 \times 10^{-4} W$

Example



$R = 6\,\Omega$

$\mathcal{E} = 12V$ (ideal)

$l_{wires} = 20\,cm$

$r_{wires} = 1\,m$

Copper wires ($\rho = 1.69 \times 10^{-8}\,\Omega m$)

a.) $V_R = ?$

$$R_{wire} = \rho \frac{L}{A} = \rho \frac{L}{\pi r^2} = \dots = 0.0011\,\Omega$$

$$R_{tot} = 2R_{wire} + R = 6.0022\,\Omega$$

$$i = \frac{\mathcal{E}}{R_{tot}} = \dots = 1.9993\,A$$

$$R = 6\,\Omega \longrightarrow V = iR = \dots = 11.996\,V \approx 12V$$

b.) Each of the two sections of the wire?

$$V_{wire} = iR_{wire} = \dots = 2.15\,mV$$

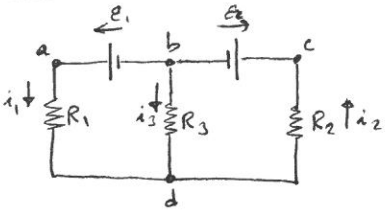
c.) Rate of Energy lost to thermal Energy in R?

$$P = i^2 R = 23.98\,W \approx 24W$$

d.) Rate of energy lost in each section of wire?

$$P = i^2 R_{wire} = 4.3\,mW$$

MULTILOOP CIRCUITS



Two junctions : b, d

Three branches : bad, bcd, bd
left right central

d : $i_1 + i_3 = i_2$

The sum of currents entering any junction must be equal to the sum of currents leaving that junction.

KIRCHHOFF'S JUNCTION RULE

→ Conservation of charge for a steady flow.

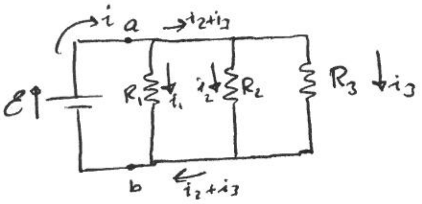
Three loops : $\underbrace{\overset{\text{left}}{\text{badb}}}$ $\underbrace{\overset{\text{right}}{\text{bcdb}}}$ $\overset{\text{big loop}}{\text{badcb}}$

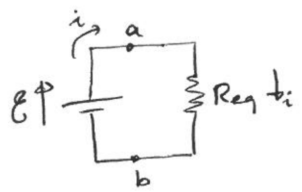
$\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0$ $\quad -i_3 R_3 - i_2 R_2 - \mathcal{E}_2 = 0$ $\quad \mathcal{E}_1 - i_1 R_1 - i_2 R_2 - \mathcal{E}_2 = 0$

3 eqns, 3 unknowns

$$\begin{pmatrix} 1 & -1 & 1 \\ R_1 & 0 & -R_3 \\ 0 & -R_2 & -R_3 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix}$$

RESISTANCES IN PARALLEL





All of the resistances have the same potential difference V

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad i_3 = \frac{V}{R_3}$$

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$i = \frac{V}{R_{eq}}$$

$$\Longrightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

2R: $R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2}$; $\dfrac{1}{R_{eq}} = \displaystyle\sum_{j=1}^{n} \frac{1}{R_j}$ (n resistances in parallel)

The Voltmeter & Amperemeter

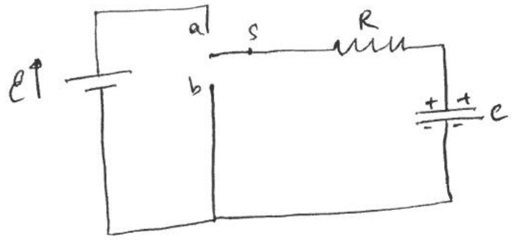
R_A of the amperemeter must be very much smaller.

R_v of the voltmeter must be very much larger.

A, V, Ω → multimeter

RC Circuits

Time Varying Current



Charging S: a

Discharging S: b

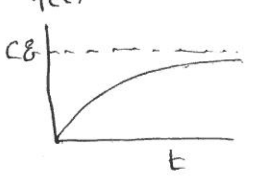
Charging: $\mathcal{E} - iR - \underbrace{\frac{q}{C}}_{} = 0$

potential difference across capacitor] negative because the capacitor's top plate is at a higher potential than the lower → there is a drop as we move down.

$i = \frac{dq}{dt}$: the charge on the capacitor increases as the current is flowing with time.

$\underbrace{R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E}}$ (Charging eqn.)

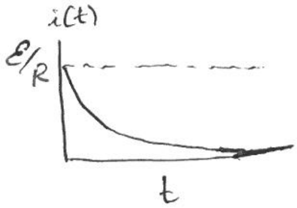
Solution to the charging eqn.



$t=0: \; q=0$

$q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$ Charging a capacitor

$t \to \infty: \; q = C\mathcal{E}$



$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$

$i_0 = \frac{\mathcal{E}}{R}, \quad i_{t\to\infty} = 0$

$q = CV_C \rightarrow V_C = \frac{q}{C} = \mathcal{E}\left(1 - e^{-t/RC}\right)$ → $t=0: \; V_C = 0$; → $t\to\infty \;\; V_C = \mathcal{E}$

The time constant

RC has dimensions of time

$\underbrace{RC}$ → Capacitive time constant

$$\tau \equiv RC$$

$$t = \tau = RC: \quad q = C\mathcal{E}(1 - e^{-1}) = 0.63\,C\mathcal{E}$$

greater τ, greater charging time

Discharging a capacitor

Switch S: b

$$-iR + \frac{q}{C} = 0$$

$i = -\frac{dq}{dt}$ → the charge on the capacitor is decreasing with time as the current flows in the circuit

$$\rightarrow R\frac{dq}{dt} + \frac{q}{C} = 0 \rightarrow q = \underbrace{q_0}_{CV_0} e^{-t/RC}$$

at τ: $q = q_0 e^{-1} = 37\%$ of the initial charge

$$i = \frac{dq}{dt} = \underbrace{-}_{\text{charge is decreasing}}\left(\frac{q_0}{RC}\right) e^{-t/RC} \quad \text{(discharging)}$$

Capacitance Derivation of $q = C\mathcal{E}(1 - e^{-t/RC})$

$$R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E} \qquad \frac{dq}{dt} + \frac{q}{RC} = \frac{\mathcal{E}}{R}$$

$$\underbrace{q_p}_{\text{particular solution}}$$

$$q = q_p + Ke^{-at} \qquad a = \frac{1}{RC}$$

$t \to \infty$ $\frac{dq}{dt} = 0$ → no further current

$q = q_p$

$q_p = C\mathcal{E}$

$\Rightarrow q = C\mathcal{E} + Ke^{-at}$

$t = 0$: $q = 0$

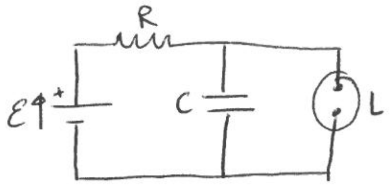
$0 = C\mathcal{E} + K$

$K = -C\mathcal{E}$

$\Rightarrow q = C\mathcal{E} - C\mathcal{E}e^{-t/RC}$

$q = C\mathcal{E}(1 - e^{-t/RC})$

Example (Quiz)



$V_L = 72V$ Current through the lamp only when V reaches V_L

$V = 95V$

$C = 0.15\mu F$

In order to have the Lamp flashes two times per second, what should the value of R be?

$$V_L = \mathcal{E}(1 - e^{-t/RC})$$

$$R = \frac{\overset{\swarrow 0.5s}{t}}{C \ln[\mathcal{E}/(\mathcal{E}-V_L)]} = \ldots = 2.35 \times 10^6 \Omega$$

$$\frac{V_L}{\mathcal{E}} = 1 - e^{-t/RC}$$

$$1 - \frac{V_L}{\mathcal{E}} = e^{-t/RC}$$

$$-t/RC = \ln\left(\frac{\mathcal{E}-V_L}{\mathcal{E}}\right)$$

$$t/RC = \ln\left(\frac{\mathcal{E}}{\mathcal{E}-V_L}\right)$$

$$R = \frac{t}{C\ln\left(\frac{\mathcal{E}}{\mathcal{E}-V_L}\right)}$$

MAGNETIC FIELDS

Magnets → Fridge Magnets to CD & DVD, harddisks, speakers, security alarms

What Produces a magnetic field?

electric charge → $\vec{E}$

Magnetic charge $\xrightarrow{?}$ $\vec{B}$

↳ magnetic monopoles?

How:

i) Moving charges like a current in a wire → electromagnet

ii) By means of elementary particles such as electrons → intrinsic magnetic field around them adds up to make a permanent magnet.

Definition of $\vec{B}$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$\vec{F}_E$ → Measured Force

q → test charge

Since we have not yet found a magnetic Monopole, we can't do it like that.

→ $\vec{F}_B$ exerted on a moving electronically charged test particle.

$$|\vec{F}_B| \propto v \sin\varphi$$

$$\vec{F}_B \perp \vec{v} \Rightarrow B = \frac{F_B}{|q| v}$$

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$|\vec{F}_B| = |q| v B \sin\varphi$$

↳ φ: angle Between $\vec{v}$ and $\vec{B}$

Finding the magnetic force on a particle

$F_B \propto q B v$

$q=0$ or $v=0 \longrightarrow F=0$

$F=0$ also if $\vec{v} \parallel \vec{B}$ ($\varphi = 0°$ or $\varphi = 180°$)

and $F = F_{max}$ if $\varphi = 90°$

$\Rightarrow$ Right hand Rule: Tip of the fingers point in the $\vec{v}$ direction, palm is turned in the direction of $\vec{B}$

$\rightarrow$ Thumb points the force due to $\vec{v} \times \vec{B}$

(if $q<0 \rightarrow$ opposite direction)

$\vec{F}_B$ is always perpendicular to $\vec{v}$ and $\vec{B}$

$\rightarrow \vec{F}_B$ never has a component parallel to $\vec{v}$

$\rightarrow \vec{F}_B$ can not change a particle's speed v (kinetic energy)

but only the direction of $\vec{v}$.

$$[|B|] = \frac{N}{C\, m/s} = 1 \text{ Tesla} = 1T$$

$$1T = 1 \frac{\text{Newton}}{C/s\ m} = 1 \frac{N}{A\, m}$$

$$1T = \underbrace{10^4}_{} \text{ Gauss} = 10^4 G$$

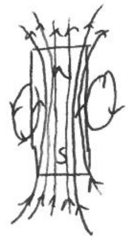
$\hookrightarrow$ Earth's surface

Surface of a neutron star	10^8 T
Near big electro-magnet	1.5 T
Near small electro-magnet	10^{-2} T
Earth's Surface	10^{-4} T
Space	10^{-10} T
Smallest value in a magnetically shielded Room	10^{-14} T

Magnetic Field Lines

1) The direction of the tangent to a magnetic field line at any point gives the direction of $\vec{B}$ at that point.

2) Spacing : denser lines $\longrightarrow$ stronger $\vec{B}$.

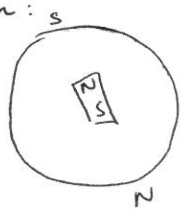


The lines leave from one end (north pole : N)
and enter from the other (south pole : S)

$\rightarrow$ magnetic $\underbrace{\text{dipole}}_{\text{two poles}}$

Opposite Poles attract each other
Like poles repel each other

Earth :



But we already call S as North Pole
so "geomagnetic north pole"

Example: $\vec{B}$ ($=1.2\,mT$) upward

proton K.E. $= K = 5.3\,MeV$ south to north

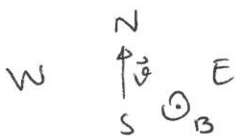
$m_p = 1.67\times10^{-27}\,kg$, $F_B = ?$

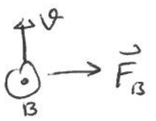
$$K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}} = 3.2\times10^{9}\,m/s$$

$$F_B = |q|\,vB\sin\varphi \overset{90^\circ}{=}\dots = 6.1\times10^{-15}\,N$$

$$a = \frac{F_B}{m} = 3.7\times10^{12}\,m/s^2$$

Direction : ?





(W $\rightarrow$ E)

if it was an electron :

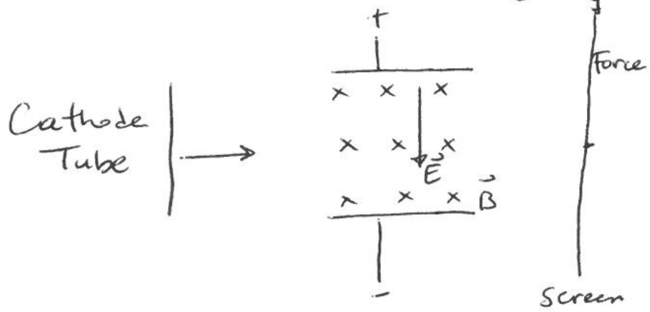
(E $\rightarrow$ W)
opposite direction.

⊙ coming towards you (out of the page)	⊗ moving away from you (into the page)

CROSSED FIELDS: DISCOVERY OF ELECTRON

if $\vec{E} \neq 0$ & $\vec{B} \neq 0$ & $\vec{E} \perp \vec{B}$ → CROSSED Fields

1897 J.J. Thomson @ Cambridge University



Force on e^-: up due to $\vec{E}$

down due to $\vec{B}$

$$y = \frac{qEL^2}{2mv^2} \quad (1)$$

L: length of the plates

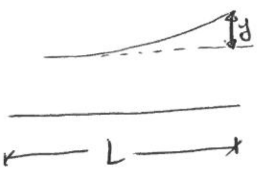
When B and E is adjusted such that they cancel each other:

$$|q|E = |q| v B \sin(90^\circ) = |q| v B$$

$$\Rightarrow v = \frac{E}{B} \quad (2)$$

$$(1) + (2): \quad \boxed{\frac{m}{|q|} = \frac{B^2 L^2}{2yE}}$$

Measurable



$L = vt \rightarrow t = \frac{L}{v}$

$F = qE$

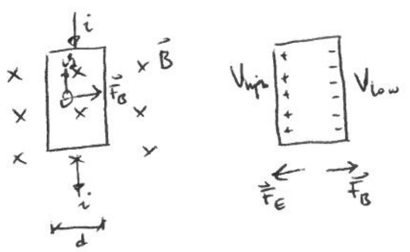
$a = \frac{F}{m} = \frac{qE}{m}$

$y = \frac{1}{2}at^2 = \frac{1}{2} \frac{qE}{m} t^2 = \frac{1}{2} \frac{qE}{m} \frac{L^2}{v^2}$

Derivation of (1):

CROSSED FIELDS: THE HALL EFFECT

1879 Edwin Hall (24 years old)



$V = Ed$

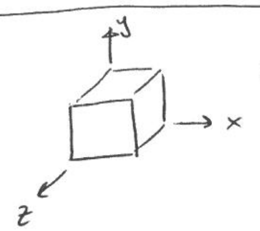
$eE = e v_d B$

$v_d = \frac{J}{ne} = \frac{i}{neA}$

$$\rightarrow n = \frac{Bi}{Vle}$$

$l = \frac{A}{d}$: thickness

Potential Difference Setup Across a moving Conductor



metal

$d = 1.5\,cm$

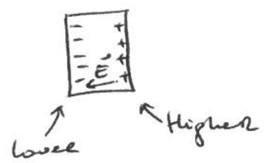
moving in positive y direction,

constant $v = 4.0\,m/s$

$$\vec{B} = 0.05\,\hat{k}\ T$$

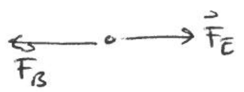
a.) Which cube face is at a lower effective potential? and which is at a higher potential?

$q < 0 \quad \vec{F}_B : -\hat{i}$ direction



b.) What is the potential Difference?

$\vec{F}_E = q\vec{E} \quad (q<0 \rightarrow \vec{F}_E$: rightwards$)$



Eventually: $F_B = F_E$

$$V = Ed$$

$$F_E = |q|E$$

$$F_B = |q|vB\sin\varphi \leftarrow 90^\circ$$

$$F_E = F_B$$

$$|q|E = |q|vB$$

$$E = vB$$

$$V = Ed = vBd \quad (= 3\,mV)$$