SOLUTIONS

Automotive Engineering, FIZ138 2nd Midterm Exam | Instructor: Emre S. Taşcı | 6 / 5 / 2016

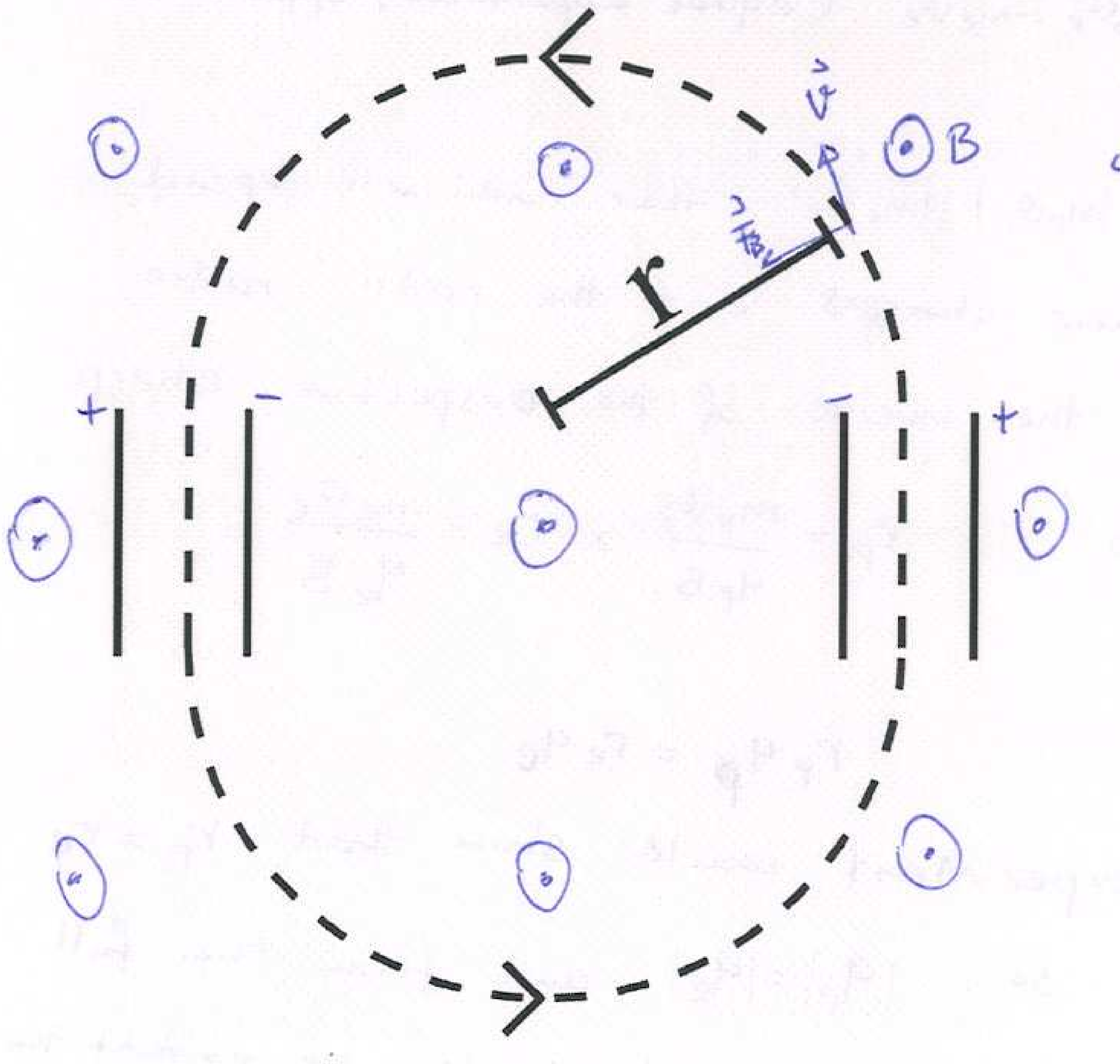
Name: Number: Signature:

Mark the 5 questions you want to be evaluated from (each question is worth 20 points):

Mark:	Q1	Q2	Q3	Q4	Q5	Q6	Q7
Grade:							

Q1) An electron travels a path as shown in the figure (semi-circular outside the plates, linear inside the plates; the plates are uniformly charged). There is also a uniform magnetic field **B** present.

In order to maintain such a path:
a) Indicate the direction of the magnetic field. (5)
b) Which side of each capacitor should be at a higher potential with respect to its counterpart? (5)
c) Calculate the electric field between the plates (in terms of m_e, e, B and r). (8)
d) To calculate the potential difference between the plates, which additional quantity do we require? (2)



c) $F = \frac{m v^2}{r} = e v B \Rightarrow v = \frac{eBr}{m}$

$eE = evB = \frac{e^2 B^2 r}{m} \rightarrow E = \frac{eB^2 r}{m}$

d) The distance between the plates:

$\Delta V = \int_0^d \vec{E}\, d\vec{x}$

Q2) A beta decay (β-decay) is defined as the transformation of a neutron into a proton and an electron (or vice versa).

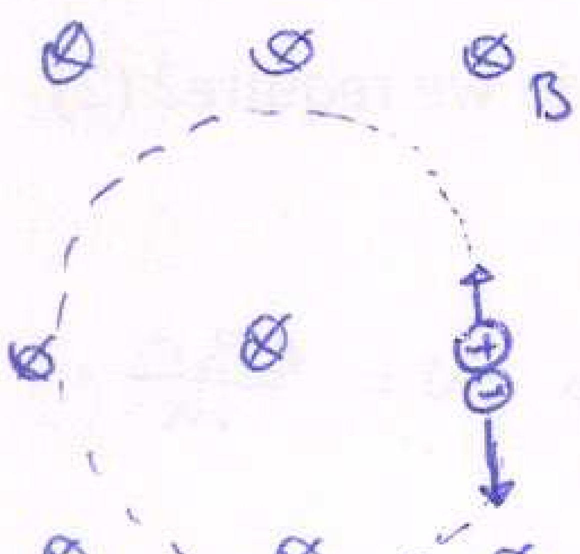
Assume that the only particles involved are the neutron, proton and electron; the neutron was at rest before decay; and also that the momentum is conserved. Proton's mass and charge are initially known. Ignore the Coulombic interactions.

a) Devise an experimental setup that you could use this process to "discover" the charge of the electron. (10P)

b) Draw the expected paths of the particles in your experiment, along with directions of motion. If the particles collide with each other, calculate the time it takes until the collision occurs; if not, then calculate the time it takes for each to have moved a distance of (2mv)/(qB) from their starting point. (10P)

[Bonus: Try to draw the patterns of the emerging proton and the electron in the case when the neutron decays while it is traveling]

a.)



$m_p v_p + m_e v_e = 0 \rightarrow m_p v_p = -m_e v_e$ (equal magnitudes, opposite directions)

$r = \frac{mv}{qB}$: Since $|m_p v_p| = |m_e v_e|$, their radii will depend on their charges and the radii' ratio gives the inverse of the respective charge ratio!

$r_p = \frac{m_p v_p}{q_p B}$, $r_e = \frac{m_e v_e}{q_e B}$

$r_p q_p = r_e q_e$

The experiment would reveal that $r_p = r_e$

so: $|q_p| = |q_e|$ and from the full circle it would be evident that

$q_e = -q_p = -e$

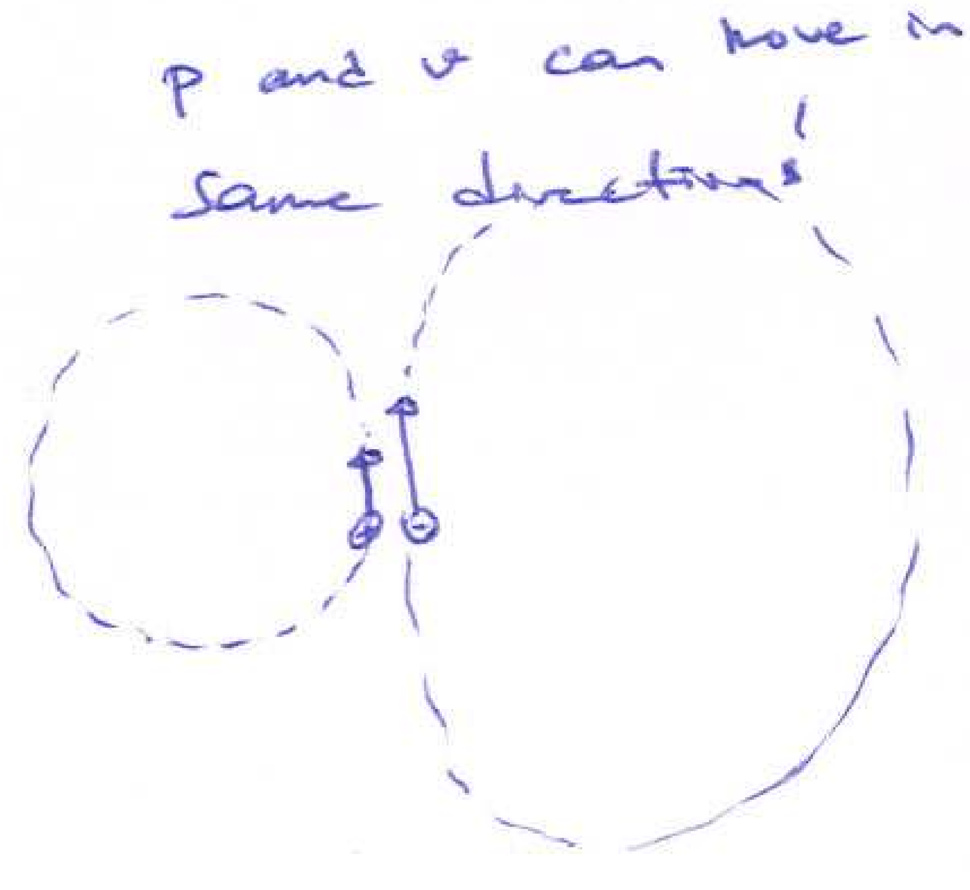
b) $m_e = m_n - m_p$

$2\pi r = v_p \tau + v_e \tau$

$2\pi r = \frac{eBr}{m_p}\tau + \frac{eBr}{m_e}\tau$

$\Rightarrow \tau = \frac{2\pi}{eB}\left(\frac{m_p m_e}{m_p + m_e}\right)$

Bonus: Since $m_p v_p + m_e v_e = m_n v_n > 0$, p and e can move in the same direction!



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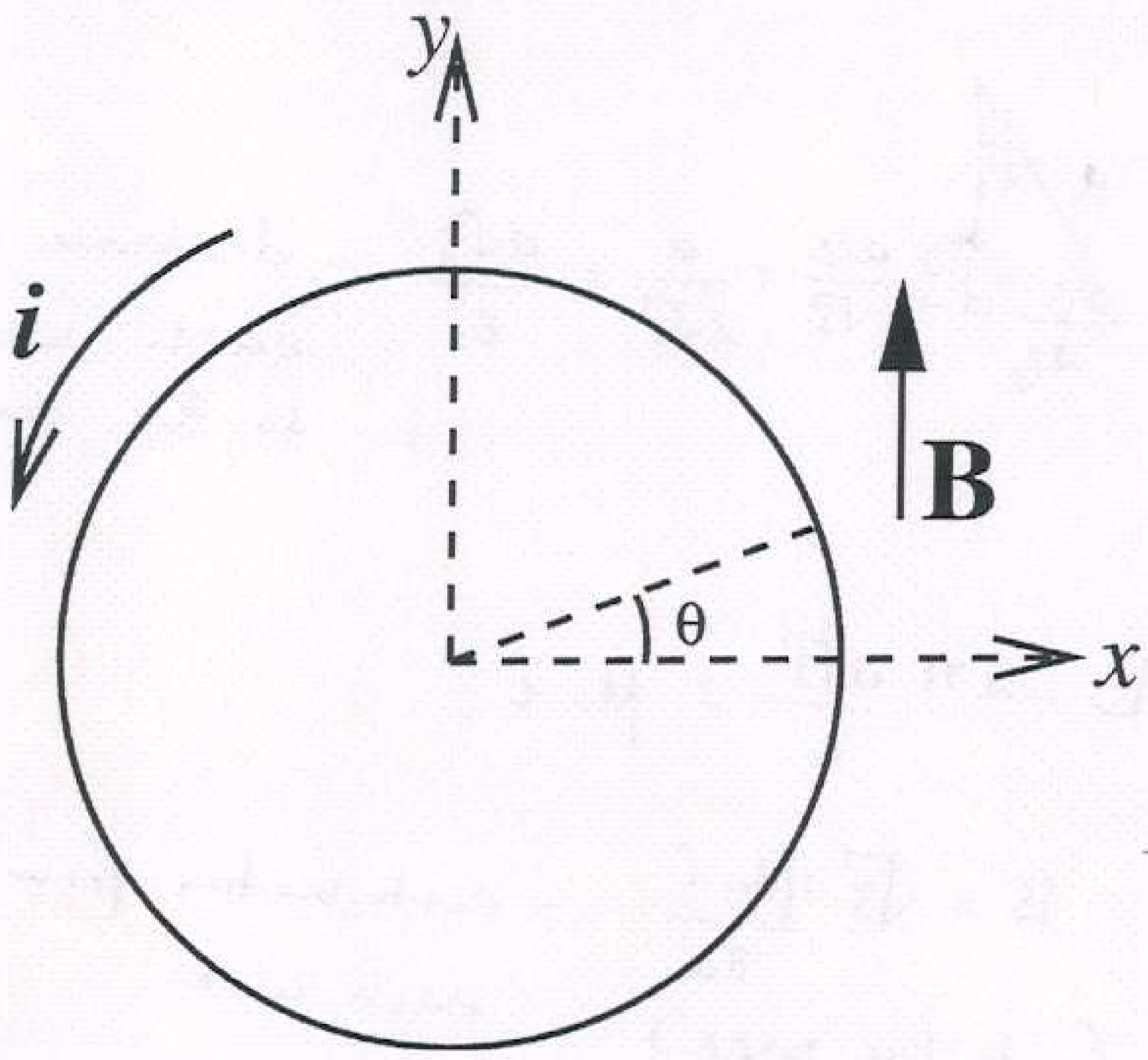
Q3) A current i is passing through a circular wire positioned in the xy-plane and suspended at a space station with no gravity. At time t=0, a uniform magnetic field in the positive y direction is switched on as shown in the figure.



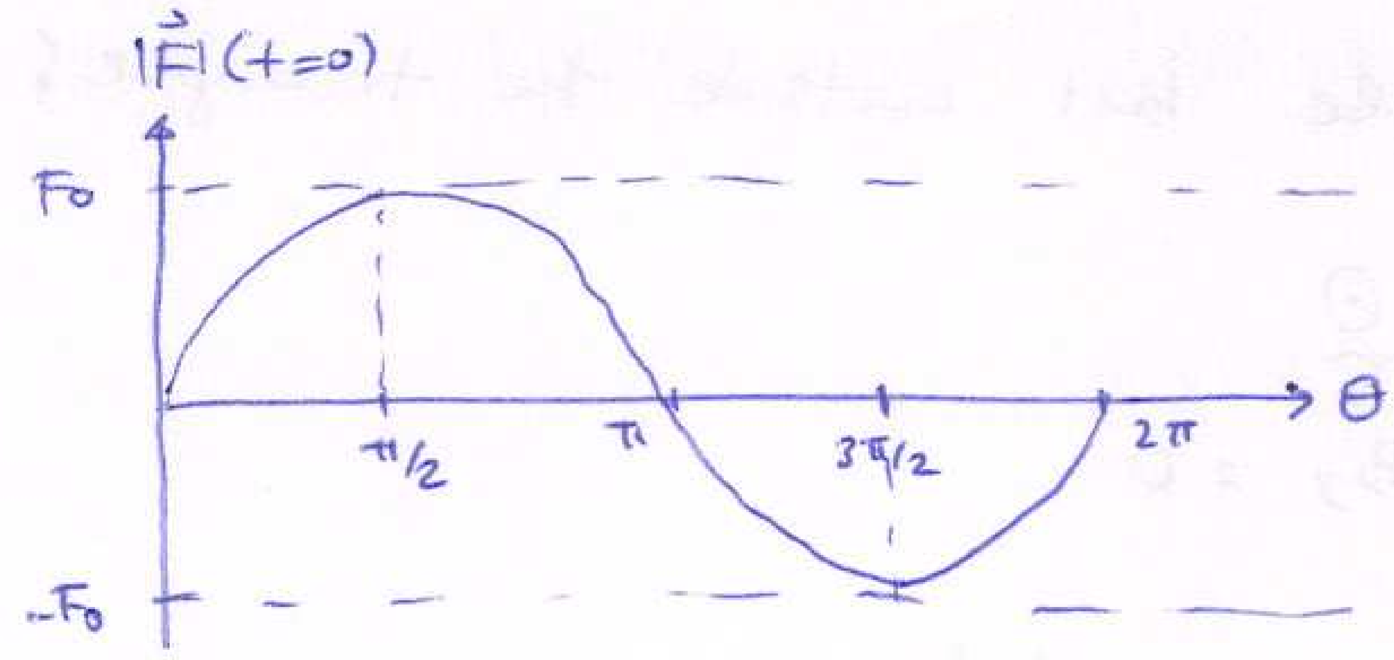
a) Plot the F vs. θ graph at $t = 0$ where F is the force exerted on the wire by the magnetic field and θ is the angular position (from 0 to 2π). (6P)

b) Plot the F vs. t graphs for $\theta = 0$ and $\pi/2$ positions of the wire, indicate the directions of each. (8P)

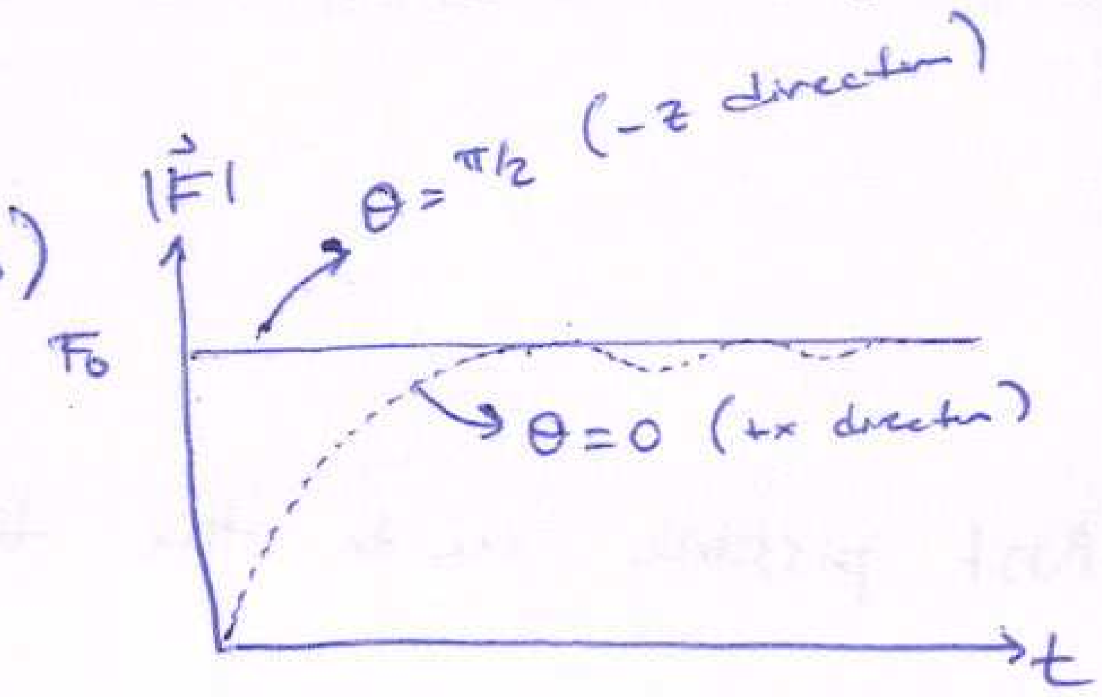
c) Plot the graph of the angle between the magnetic dipole moment of the circuit μ and the magnetic field with respect to the time. (6P)

When drawing the graphs that involve time, be careful about the behavior of the system as $t \to \infty$ – is it periodical or does it eventually reach an equilibrium?

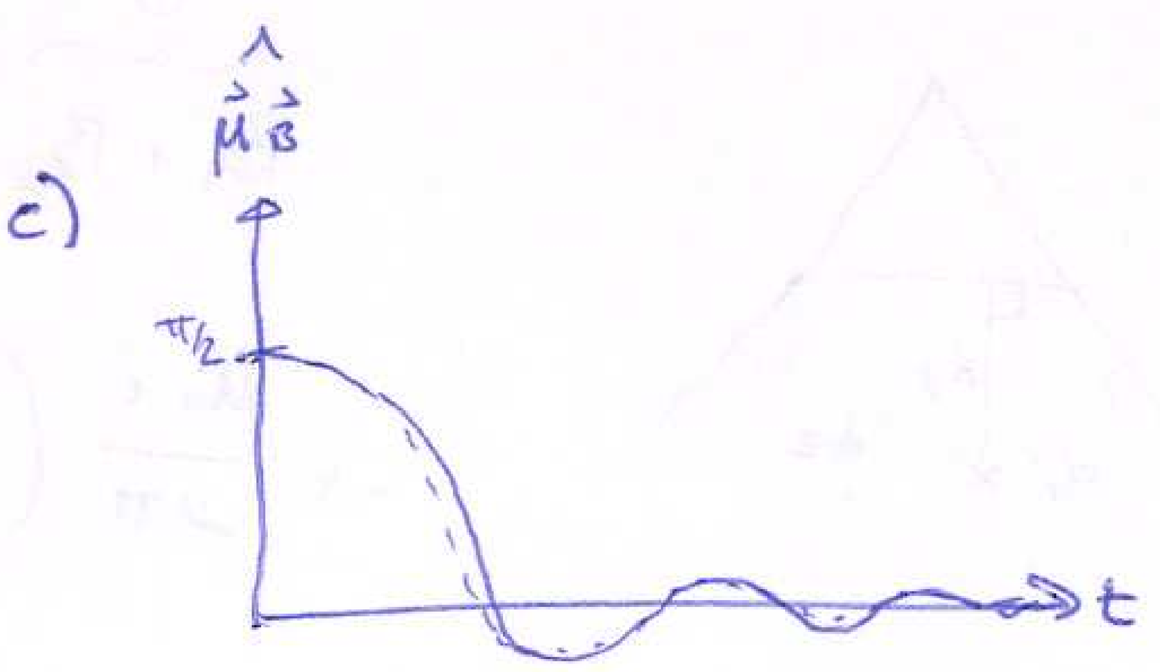
a.) $d\vec{F} = i\,d\vec{\ell} \times \vec{B} \rightarrow |d\vec{F}| = \underbrace{i\,d\ell\,B}_{dF_0} \sin\theta$



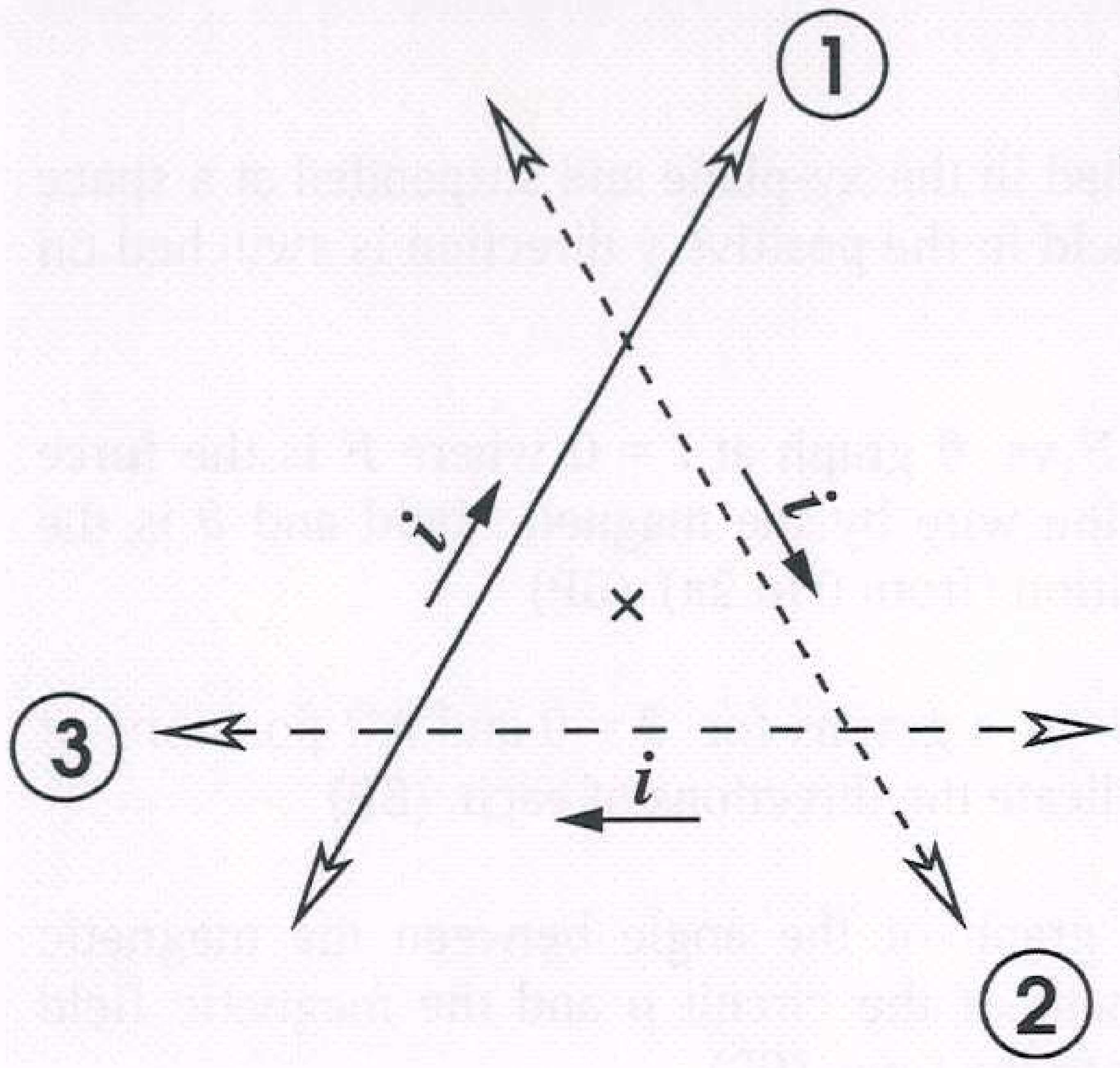
b)



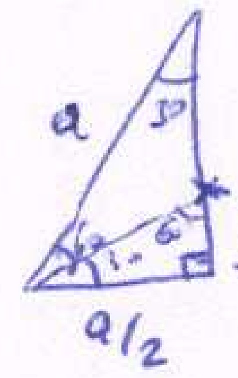
c)



Q4) Three very long wires, each carrying a current of i are arranged to yield an equilateral triangle shape of side a. Calculate the magnetic field produced at its center. (20P)



[Bonus: Try to find special points (if there are any) where the net magnetic field is zero]



$$\frac{a/2}{\sqrt{3}} = \frac{a}{2\sqrt{3}} = \frac{a\sqrt{3}}{6}$$: distance of each wire to the center.

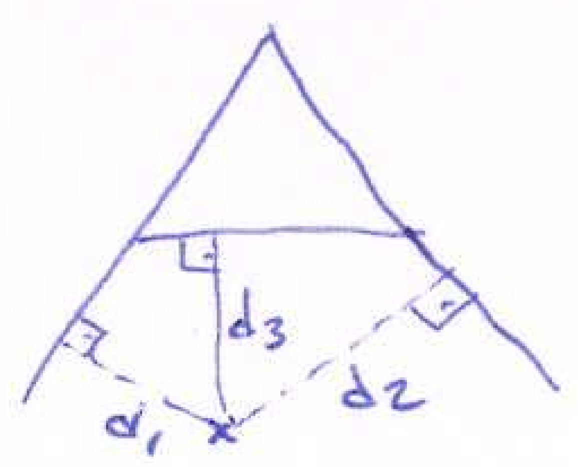
$$B \cdot 2\pi \frac{a\sqrt{3}}{6} = \mu_0 i$$

$$B = \sqrt{3}\,\frac{\mu_0 i}{\pi a}$$: contribution from each wire

(into the page)

$$\Rightarrow B_x = 3 \cdot B = 3\sqrt{3}\,\frac{\mu_0 i}{\pi a}$$

Bonus: Not possible inside the triangle but outside the triangle;



$$\underbrace{B_1 + B_2}_{\otimes} - \underbrace{B_3}_{\odot} = 0$$

$$\rightarrow \frac{\mu_0 i}{2\pi}\left(\frac{1}{d_1} + \frac{1}{d_2} - \frac{1}{d_3}\right) = 0$$

$$\rightarrow d_2 d_3 + d_1 d_3 - d_1 d_2 = 0$$ (and similar for other outside regions)

(e.g. $d_3 = 5m$, $d_1 = 1m$, $d_2 = 5/4$)

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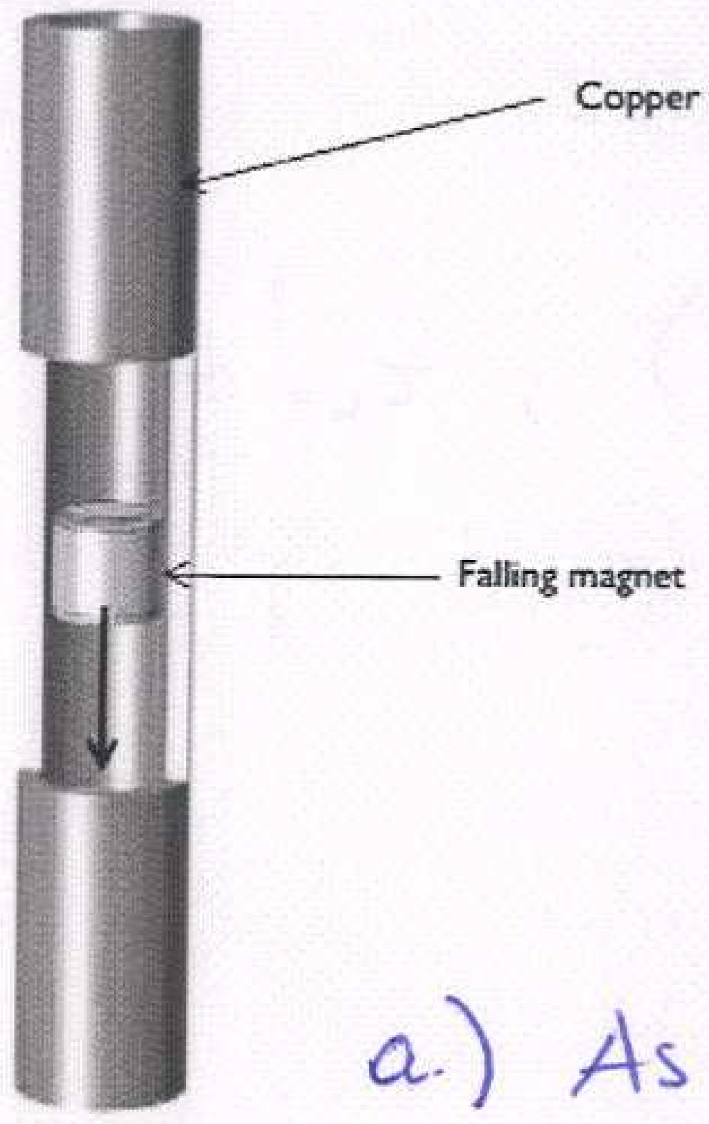
Q5) A cylindrical magnet (North pole up) is dropped into a hollow cylindrical tube made of copper (conductor).



a) How would it be different from the free fall of a non-magnetic cylindrical shaped object, having the same mass as the magnet? (10P)

b) If the experiment is repeated with the South pole of the magnet pointing up, what would change? (4P)

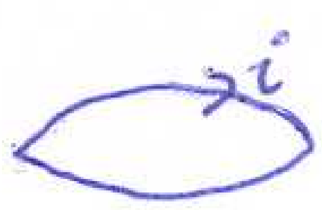
c) Indicate the induced quantities, if there are any. (6P)

[Bonus: Try to explain the principles behind the metal detectors used by the security at the airports, shopping malls, etc..]

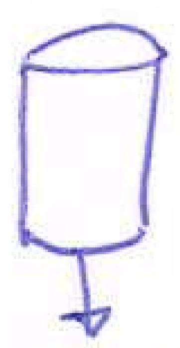
a.) As it is falling Down, the magnetic Flux around it changes (decreasing Behind, increasing ahead) and this induces a movement of the electrons in the copper, opposing the change so they would be generating a countering magnetic Field, attracting the magnet at the top and repulsing it at the bottom thus slowing it down.

b) The directions of the induced currents would be reversed but due to Lenz' Law, the countering effect would be the same.

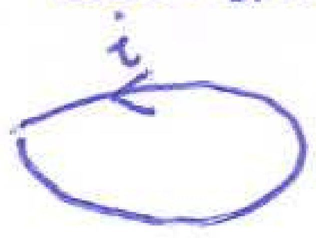
c.)



Flux decreasing → induced current supports.

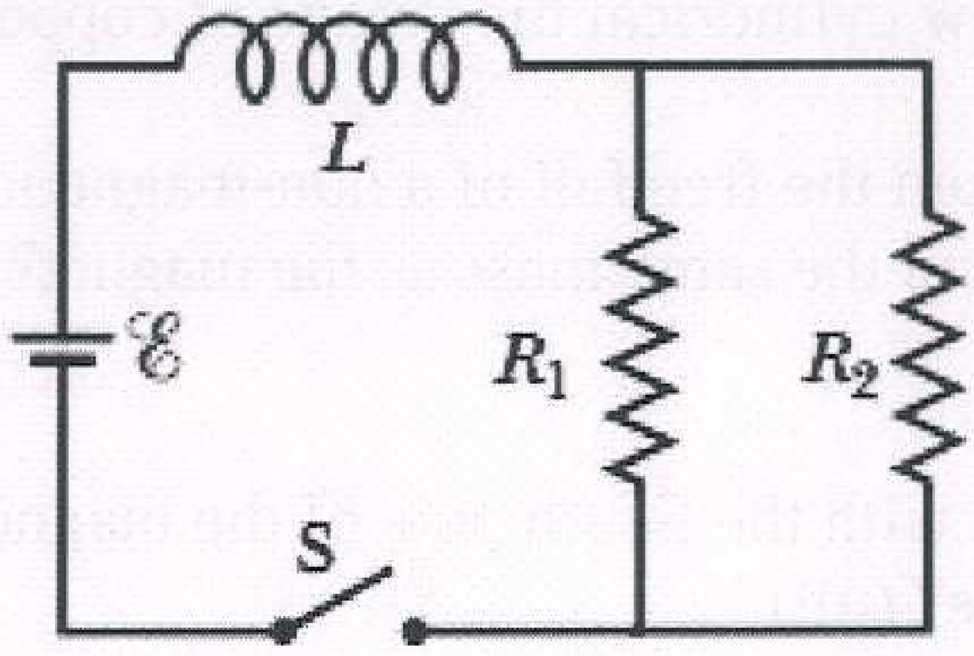


magnetic flux increasing in this direction → induced current opposes this:



Bonus: In the detectors, there is an applied static field but since the metals are being moved through, a current is induced due to the changing Flux (It's equivalent to holding the magnet and moving the tube!)

Q6) In the circuit drawn below, what must be the R_1/R_2 ratio so that the current reaches half of its maximum value in t' seconds after the switch is closed? (20P)



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$i = \underbrace{\frac{\mathcal{E}}{R_{eq}}}_{I}\left(1 - e^{-t/\tau_L}\right) \quad ; \quad \tau_L = \frac{L}{R_{eq}}$$

$$\frac{I}{2} = I\left(1 - e^{-t' R_{eq}/L}\right)$$

$$e^{-t' R_{eq}/L} = \frac{1}{2}$$

$$-\frac{t' R_{eq}}{L} = \ln\frac{1}{2} \rightarrow R_{eq} = \frac{L}{t'}\ln 2$$

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{L}{t'}\ln 2$$

dependent on R_1 !

$$\frac{R_1}{R_2} = \frac{R_1}{\frac{L}{t'}\ln 2} - 1$$

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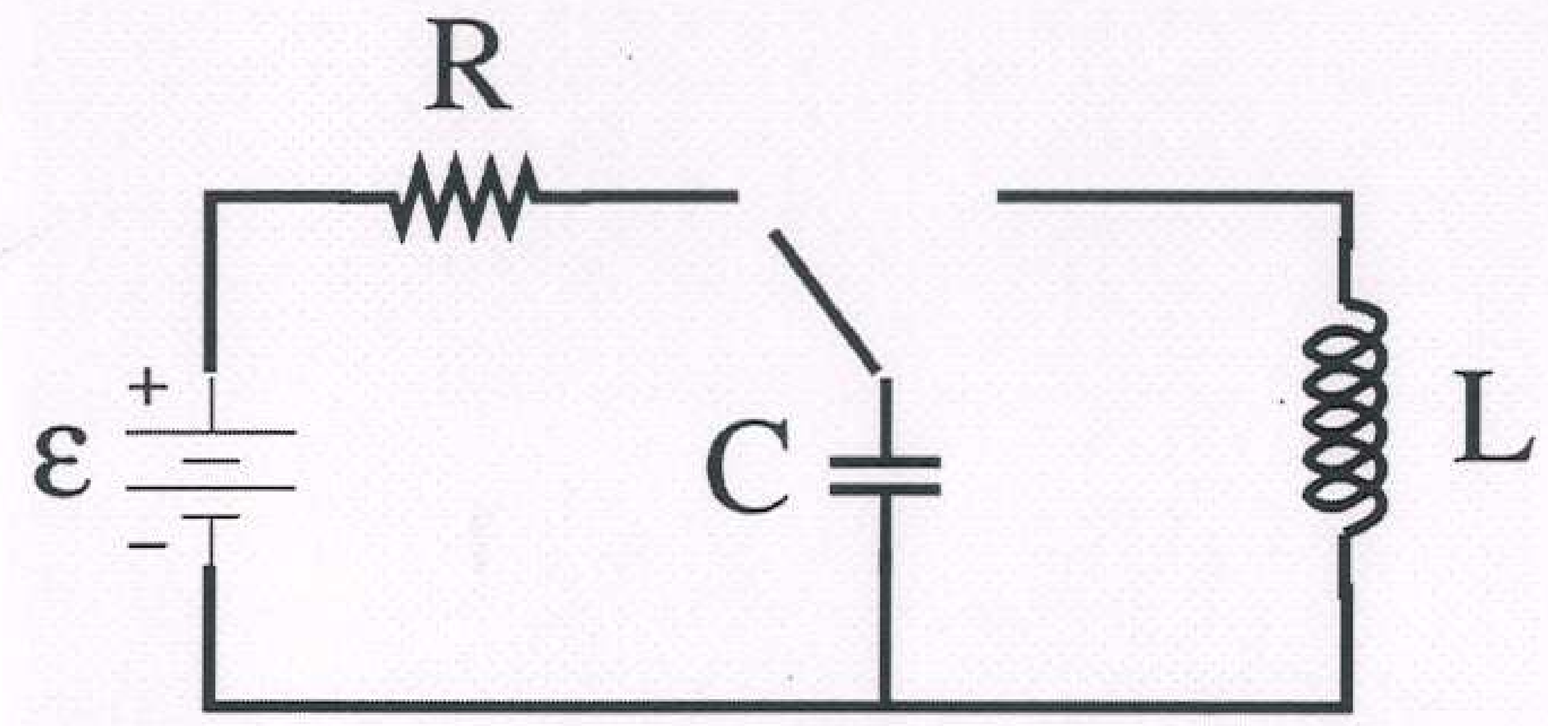

Q7) At the beginning, C is connected to R for a long time. Then at t=0, the switch is moved so that C is now connected to L.

Calculate:

a) the frequency (f), angular frequency (ω) and the period (T) of the oscillation of the circuit (4P)

b) the maximum charge that appears on the capacitor (3P)

c) the maximum current in the inductor (3P)

d) the total energy of the circuit (4P)

e) the energy stored at the inductor at t=0, T/4, T/2, 3T/4, T, 2T (3P)

f) the energy stored at the capacitor at t=0, T/4, T/2, 3T/4, T, 2T (3P)

a) $\omega = \frac{1}{\sqrt{LC}}$, $f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$, $T = \frac{1}{f} = 2\pi\sqrt{LC}$

b) $Q = CV = C\mathcal{E}$

c) $i(t) = \underline{\omega Q} \sin(\omega t)$

$\hookrightarrow I = \frac{C\mathcal{E}}{\sqrt{LC}} = \mathcal{E}\sqrt{\frac{C}{L}}$

d) $U_0 = \frac{Q^2}{2C} = \frac{C^2\mathcal{E}^2}{2C} = \frac{C\mathcal{E}^2}{2}$

e) $U_B = \frac{Li^2}{2} = \frac{L}{2}\left(\mathcal{E}\sqrt{\frac{C}{L}} \sin(\omega t)\right)^2 = \overbrace{\frac{C\mathcal{E}^2}{2}}^{U_0} \sin^2\left(\frac{2\pi}{T}t\right)$

f) $U_E = U_0 - U_B$

t	0	T/4	T/2	3T/4	T	2T
U_B	0	U_0	0	U_0	0	0
U_E	U_0	0	U_0	0	U_0	U_0