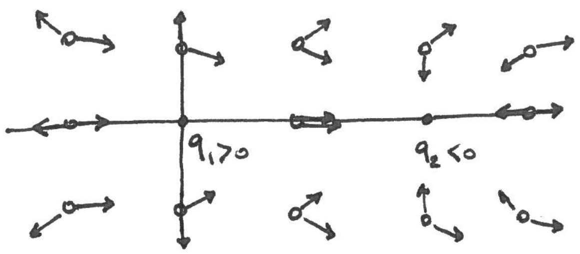
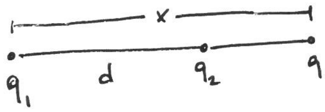
21-13

$q_3 > 0$ assumed — if $q_3 < 0$ the arrows will be reversed.



Only on the x-axis the two forces lie parallel/anti-parallel to each other $\Rightarrow y_{q_3} = 0$

Limiting our case to the x-axis:



$$\frac{k\,q_1 q_3}{x^2} + \frac{k\,q_2 q_3}{(x-d)^2} = 0 \rightarrow \frac{q_1}{x^2} = -\frac{q_2}{(x-d)^2}$$

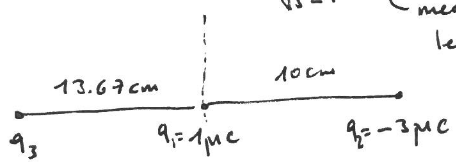
$$\Rightarrow \boxed{-\frac{q_1}{q_2} = \left(\frac{x}{x-d}\right)^2}$$

Ex : $q_1 = 1\mu C$, $q_2 = -3\mu C$, $L = 10cm$

$$\frac{1}{3} = \left(\frac{x}{x-10}\right)^2 \rightarrow \sqrt{3}\,x = x - 10$$

$$x = \frac{-10}{\sqrt{3}-1} \simeq -13.67cm$$

meaning left of the origin / q_1

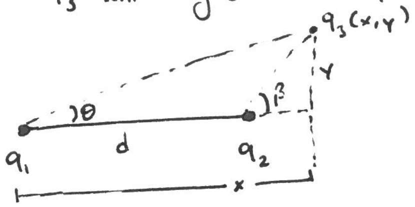


Analytical Solution

$\Sigma F_x = \Sigma F_y = 0$ so that q_3 will stay where it's put

$$|\vec{F}_{13}| = k\,\frac{q_1 q_3}{(x^2+y^2)}$$

$$|\vec{F}_{23}| = k\,\frac{q_2 q_3}{[(x-d)^2+y^2]}$$



$\Sigma F_x = 0$:

$$k\,\frac{q_1 q_3}{(x^2+y^2)}\underbrace{\frac{x}{\sqrt{x^2+y^2}}}_{\cos\theta} + k\,\frac{q_2 q_3}{[(x-d)^2+y^2]}\frac{(x-d)}{\sqrt{(x-d)^2+y^2}} = 0$$

$$\rightarrow \frac{q_1}{(x^2+y^2)^{3/2}}\,x = -\frac{q_2}{[(x-d)^2+y^2]^{3/2}}(x-d) \quad (1)$$

$\Sigma F_y = 0$:

$$k\,\frac{q_1 q_3}{(x^2+y^2)}\frac{y}{\sqrt{x^2+y^2}} + k\,\frac{q_2 q_3}{[(x-d)^2+y^2]}\frac{y}{\sqrt{(x-d)^2+y^2}} = 0$$

$$\rightarrow \frac{q_1}{(x^2+y^2)^{3/2}}\,y = -\frac{q_2}{[(x-d)^2+y^2]^{3/2}}\,y \quad (2)$$

i) if $y \neq 0$ we can eliminate y in (2):

$$\frac{q_1}{(x^2+y^2)^{3/2}} = -\frac{q_2}{[(x-d)^2+y^2]^{3/2}}$$

from (1)

$$-\frac{q_2}{[(x-d)^2+y^2]^{3/2}}\frac{(x-d)}{x} = -\frac{q_2}{[(x-d)^2+y^2]^{3/2}}$$

$\rightarrow \frac{x-d}{x} = 1$ which means $d = 0$ whereas $d > 0$

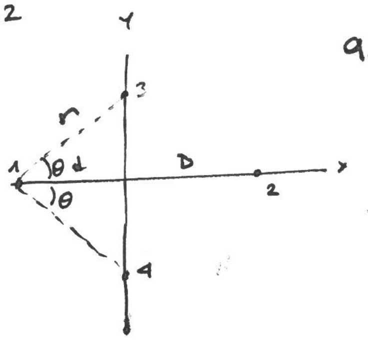
$\Rightarrow$ no solution !

ii) if $y = 0$, (1) becomes:

$$\frac{q_1\,x}{x^3} = -\frac{q_2}{(x-d)^3}(x-d)$$

$$\boxed{-\frac{q_1}{q_2} = \left(\frac{x}{x-d}\right)^2}$$

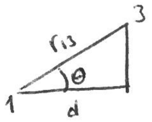
21-22



$q_2, q_3 = q_4, \theta, d$ given.

if net force on $q_1 = 0$ what is the distance D?

$$|\vec{F}_{31}| = k\frac{q_3 q_1}{r^2} = k\frac{q_3 q_1}{d^2}\cos^2\theta = |\vec{F}_{41}|$$



$$\cos\theta = \frac{d}{r_{13}}$$

$$\rightarrow r_{13} = \frac{d}{\cos\theta}$$

$$\vec{F}_{31} = k\frac{q_3 q_1}{d^2}\cos^2\theta\left[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$$

$\vec{F}_{31}$ & $\vec{F}_{41}$: y-components are in opposite direction
→ cancel each other

x-components are in the same direction
→ add up

$$\sum \vec{F}_x = 0 : \quad 2k\frac{q_3 q_1}{d^2}\cos^2\theta\cos\theta + k\frac{q_2 q_1}{(d+D)^2} = 0$$

$|\vec{F}_{31_x}| = \vec{F}_{41_x}$

$$\frac{2 q_3 \cos^3\theta}{d^2} = -\frac{q_2}{(d+D)^2}$$

$$(d+D)^2 = \left[-\frac{q_2}{2q_3\cos^3\theta}\right]d^2$$

$$d + D = \sqrt{-\frac{q_2}{2q_3\cos^3\theta}}\,d$$

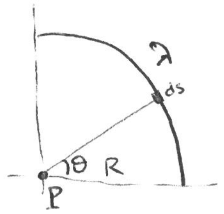
$$D = \left(\sqrt{-\frac{q_2}{2q_3\cos^3\theta}} - 1\right)d$$

Ex: $d = 2\,cm$, $q_2 = 8\times10^{-19}\,C$

$q_3 = q_4 = -1.6\times10^{-19}$

$\theta = 30^\circ \rightarrow D = 1.9238\,cm$

~ 21-26



$\vec{E}_P = ?$

$ds = R\,d\theta$

$dq = \lambda\, ds = R\lambda\, d\theta$

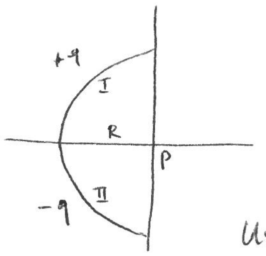
$$d\vec{E} = k\frac{dq}{R^2}(\hat{r}) = -k\frac{dq}{R^2}\overbrace{\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]}^{\hat{r}} = -\frac{\overbrace{kR\lambda d\theta}^{dq}}{R^2}\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]$$

$\hat{r} = -\hat{i}\cos\theta - \hat{j}\sin\theta$: Should be from the source to the point, hence the (-) signs!

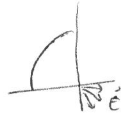
$$\vec{E} = -\frac{k}{R}\lambda\int_0^{\pi/2}\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]d\theta = -\frac{k\lambda}{R}\left[\hat{i}\left(\sin\tfrac{\pi}{2} - \sin 0\right) - \hat{j}\left(\cos\tfrac{\pi}{2} - \cos 0\right)\right]$$

$$= -\frac{k\lambda}{R}\left[\hat{i} + \hat{j}\right] \quad (*)$$

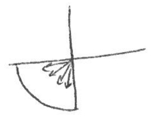
21-26



E_I : $\hat{r} = \hat{i}\cos\theta - \hat{j}\sin\theta$, $\lambda > 0$



E_{II} : $-\hat{r} = -\hat{i}\cos\theta - \hat{j}\sin\theta$, $\lambda < 0$



Using (*) :

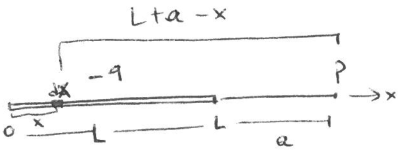
$$\left.\begin{aligned}\vec{E}_I &= \frac{k\lambda}{R}\left[\hat{i} - \hat{j}\right] \\ \vec{E}_{II} &= -\frac{k\lambda}{R}\left[\hat{i} + \hat{j}\right]\end{aligned}\right\} \quad \vec{E}_{TOT} = -\frac{k\lambda}{R}2\hat{j}$$

$$\lambda \cdot \frac{\pi R}{2} = q \rightarrow \lambda = \frac{2q}{\pi R}$$

$$\vec{E}_{TOT} = -\frac{4kq}{\pi R^2}\hat{j}$$

Ex: $q = 4.5 \times 10^{-12}\,C$, $R = 5 \times 10^{-2}\,m$ $\Big\} \rightarrow \vec{E}_{TOT} = 20.604\ N/C$

22.31



q, L, a given; λ : uniform

$$\lambda = -\frac{q}{L} \quad (<0)$$

$dq = \lambda dx$

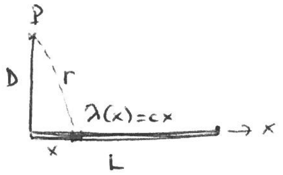
$$dE_x = k\frac{dq}{(L+a-x)^2} = k\frac{\lambda dx}{(L+a-x)^2} = -\frac{kq\,dx}{L(L+a-x)^2}$$

$$E_x = \int_0^L dE = -\frac{kq}{L}\int_0^L \frac{dx}{(L+a-x)^2} = -\frac{kq}{L}\left[\frac{1}{L+a-x}\right]_{x=0}^{L}$$

$$E_x = -\frac{kq}{La}\left(\frac{L}{L+a}\right) = -\frac{kq}{a(L+a)}$$

Ex: $q = 4.23\times10^{-15}$ C, $L = 0.0815$ m, $a = 0.12$ m $\}$ $E_x = -1.57\times10^{-3}$ N/C

24-40



$V_\infty = 0$, D, L given, $\lambda(x) = cx$

a.) $V_P = ?$ $\qquad dq = \lambda dx$, $r = \sqrt{x^2+D^2}$

$$dV_P = \frac{k\,dq}{r} = \frac{k\lambda\,dx}{\sqrt{x^2+D^2}}$$

$\lambda(x) = cx$

$$V_P = \int_0^L \frac{k\lambda\,dx}{\sqrt{x^2+D^2}} = kc\int_0^L \frac{x\,dx}{\sqrt{x^2+D^2}} = kc\left[\sqrt{D^2+x^2}\right]_{x=0}^{L}$$

$$V_P = kc\left(\sqrt{L^2+D^2} - D\right)$$

Ex: $c = 49.9\times10^{-12}$ C/m², $D = 3.56$ cm, $L = 10$ cm $\}$ $V_P = 3.1648\times10^{-2}$ V

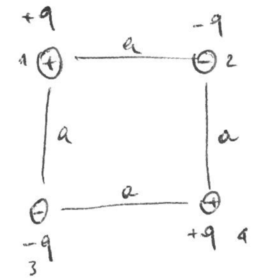
b.) E_y at P

$$D \to y : \quad V(y) = kc\left(\sqrt{L^2+y^2} - y\right)$$

$$E_y = -\frac{\partial V}{\partial y} = kc\left(1 - \frac{y}{\sqrt{L^2+y^2}}\right)$$

Ex: $c, D, L \cdots \rightarrow E_y(y=D) = 0.29815$ N/C

24-43



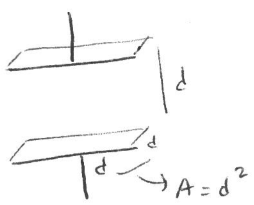
How much work is required to set up this?

$$V_\infty = 0 \rightarrow U_i = 0$$

$$U_f = kq^2\left(\underset{1\text{-}2}{-\frac{1}{a}} \underset{1\text{-}3}{-\frac{1}{a}} \underset{1\text{-}4}{+\frac{1}{a\sqrt{2}}} \underset{2\text{-}4}{-\frac{1}{a}} \underset{3\text{-}4}{-\frac{1}{a}} \underset{2\text{-}3}{+\frac{1}{a\sqrt{2}}}\right) = \frac{2kq^2}{a}\left(\frac{1}{\sqrt{2}} - 2\right)$$

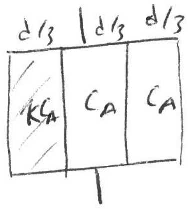
$$W = -\Delta U = -U_f = \frac{2kq^2}{a}\left(2 - \frac{1}{\sqrt{2}}\right)$$

$$\text{Ex: } \left.\begin{array}{l} q = 2.3\times10^{-12}\,C \\ a = 64\text{ cm}\end{array}\right\} \quad W = 1.9214\times10^{-13}\,J$$



$$C_0 = \epsilon_0 \frac{A}{d} = \epsilon_0 d$$

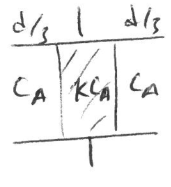
i)



$$C_A = \epsilon_0 \frac{d/3 \cdot d}{d} = \epsilon_0 \frac{d}{3} = \frac{C_0}{3}$$

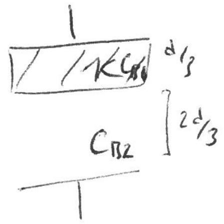
$$C_{TOT} = \frac{2}{3}C_0 + k\frac{C_0}{3} = \frac{k+2}{3}C_0$$

ii)



$$C_{TOT} = \frac{k+2}{3}C_0$$

iii)

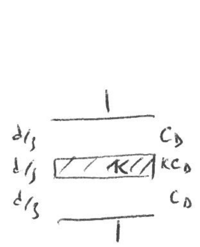


$$C_{B1} = \epsilon_0 \frac{\overset{A=d^2}{d^2}}{d/3} = 3\epsilon_0 d = 3C_0$$

$$C_{B2} = \epsilon_0 \frac{d^2}{2d/3} = \frac{3}{2}\epsilon_0 d = \frac{3}{2}C_0$$

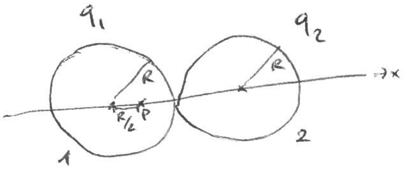
$$C_{TOT} = \left[\frac{1}{3kC_0} + \frac{2}{3C_0}\right]^{-1} = \left(\frac{1+2k}{3kC_0}\right)^{-1} = \frac{3k}{1+2k}C_0$$

iv)



$$C_D = \epsilon_0 \frac{d^2}{d/3} = 3C_0 \qquad C_{TOT} = \left[\frac{1}{3C_0} + \frac{1}{3kC_0} + \frac{1}{3C_0}\right]^{-1} = \frac{3k}{1+2k}C_0$$

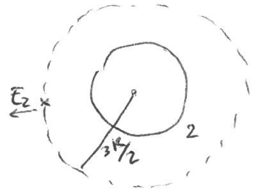
23-54



Uniformly distributed charges throughout volume

if electric field at P = 0, then $\frac{q_2}{q_1} = ?$

i)

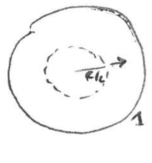


$\vec{E}_2$: $-\hat{i}$ direction.

$$\underbrace{E_2 . 4\pi\left(\frac{3R}{2}\right)^2}_{\text{gauss shell surface area}} = \frac{q_2}{\epsilon_0}$$

$$\rightarrow E_2 = \frac{q_2}{4\pi\epsilon_0}\left(\frac{2}{3R}\right)^2$$

ii)



E_1 : $\hat{i}$ direction

$$q_{enc} = \rho \cdot \frac{4}{3}\pi\left(\frac{R}{2}\right)^3$$

$\downarrow$

$$\text{volume charge density} = \frac{q_1}{\frac{4}{3}\pi R^3}$$

$$\underbrace{E_1 . 4\pi\left(\frac{R}{2}\right)^2}_{\text{gauss shell surface area}} = \frac{q_{enc}}{\epsilon_0}$$

$$E_1 . 4\pi\left(\frac{R}{2}\right)^2 = \underbrace{\frac{q_1}{\frac{4}{3}\pi R^3}}_{\rho}\frac{1}{\epsilon_0}\underbrace{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}_{\text{volume of the enclosed sphere}}$$

$\rho \times V_{R/2} = q_{enc}$

$$E_1 = \frac{1}{4\pi\epsilon_0}\left(\frac{2}{R}\right)^2\left(\frac{1}{2}\right)^3 q_1$$

if net electric field at P = 0

$\rightarrow$ E_1 & E_2 : magnitudes equal, directions opposite.

$$\vec{E}_1 + \vec{E}_2 = 0 \iff E_1 = E_2$$

$$\frac{1}{4\pi\epsilon_0}\frac{1}{2R^2}q_1 = \frac{1}{4\pi\epsilon_0}\frac{4}{9R^2}q_2$$

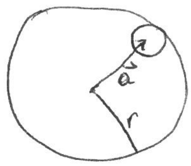
$$\Rightarrow \frac{q_2}{q_1} = \frac{9}{8} = \underline{1.125}$$

23-73 A non conducting solid sphere has a uniform volume charge density ρ. Let $\vec{r}$ be the vector from the center of the sphere to a general point P within the sphere.

a) Show that the electric field at P is given by

$$\vec{E} = \rho\vec{r}/3\epsilon_0$$

b) A spherical cavity is hollowed out of the sphere as shown in the figure. Using superposition concepts, show that the electric field at all points within the cavity is uniform and equal to



$\vec{E} = \rho\vec{a}/3\epsilon_0$ where $\vec{a}$ is the position vector from the center of the sphere to the center of cavity.

Solution

a)

$$\frac{1}{r^2}\hat{r} = \frac{1}{r^3}\vec{r} \rightarrow \vec{E} = \frac{q_{enc}}{\underbrace{4\pi\epsilon_0 r^2}_{\text{gauss surface area}}}\hat{r} = \frac{1}{4\pi\epsilon_0 r^3} q_{enc} \cdot \underbrace{\vec{r}}_{\frac{\hat{r}}{r^2}}$$

$$q_{enc} = \rho \cdot V_{gauss.\,surface} = \rho \cdot \frac{4}{3}\pi r^3$$

$$\rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0 \cancel{r^3}} \cdot \rho\frac{4}{3}\pi\cancel{r^3}\vec{r}$$

$$\Rightarrow \vec{E} = \frac{\rho\vec{r}}{3\epsilon_0}$$

b) We can think the cavity in the sphere as a superposition of the sphere + negatively charged sphere same size of the cavity

$$\Rightarrow \vec{E}(\vec{r}) = \frac{\rho\vec{r}}{3\epsilon_0} + \frac{(-\rho)(\vec{r}-\vec{a})}{3\epsilon_0} = \frac{\rho a}{3\epsilon_0}$$

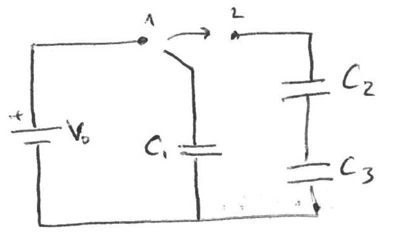
↳ independent of $\vec{r}$ hence uniform.

25-28 The Figure displays a 12.0 V battery and 3 uncharged capacitors:

$$C_1 = 4\,\mu F \; ; \; C_2 = 6\,\mu F \; ; \; C_3 = 3\,\mu F.$$

The switch is thrown to the left side until capacitor 1 is fully charged. Then the switch is thrown to the right.

What is the final charge on each capacitor?



When the swith is at 1, the charge on the C_1 is:

$$Q_i = C_1 V_0 \quad (1)$$

Then, after the switch is at 2, this charge will be distributed among the 3 capacitors

$$Q_i = Q_1 + Q_2 + Q_3$$

Equivalent capacitor for C_2 & C_3:

$$C_{eq_{23}} = \left[\frac{1}{C_2} + \frac{1}{C_3}\right]^{-1}$$

$$C_{eq_{23}} = \frac{C_2 C_3}{C_2 + C_3}$$

Since C_2 and C_3 are connected in series, they will accumulate the same amount of charge:

$$Q_2 = Q_3$$

$$\rightarrow Q_i = Q_1 + 2Q_2 \quad (2)$$

$$(1) + (2) \rightarrow \quad C_1 V_0 = Q_1 + 2Q_2 \quad (3)$$

The voltage on both sides must be the same (parallel) so:

$$V = \underbrace{\frac{Q_1}{C_1}} = \overset{V_2}{\frac{Q_2}{C_2}} + \overset{V_3}{\frac{Q_2}{C_3}} = \frac{C_2 + C_3}{C_2 C_3} Q_2 = \underbrace{\frac{Q_2}{C_{eq_{23}}}},$$

$$\rightarrow Q_2 = \frac{C_{eq_{23}}}{C_1} Q_1 \quad (4)$$

$$(3) + (4) \rightarrow \quad C_1 V_0 = Q_1 + \frac{C_{eq_{23}}}{C_1} Q_1 = \left[1 + \frac{C_{eq_{23}}}{C_1}\right] Q_1 \Rightarrow Q_1 = \frac{C_1 V_0}{1 + \frac{C_{eq_{23}}}{C_1}}, \quad Q_2 = \frac{C_{eq_{23}}}{C_1} Q_1$$

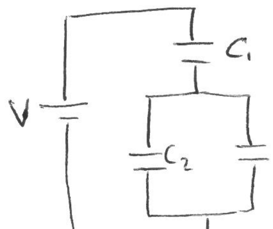
$$V_0 = 12\,V, \; \left.\begin{matrix} C_1 = 4\,\mu F \\ C_2 = 6\,\mu F \\ C_3 = 3\,\mu F \end{matrix}\right\} \rightarrow Q_1 = 32\,\mu C, \; Q_2 = 16\,\mu C$$

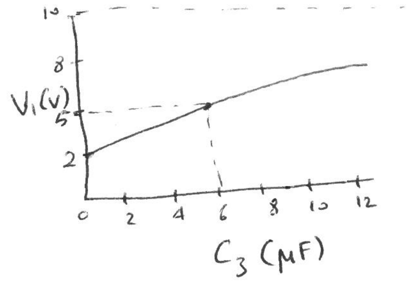
25-26 In the figure, C_3 is a variable capacitor and a voltmeter connected to C_1 yields the $V_1 - C_3$ graph.

As $C_3 \to \infty$ $V_1 \to 10V$.

What is the electrical potential V across the battery?

$C_1 = ?$ and $C_2 = ?$





$$\left.\begin{aligned} V_1(C_3=0) &= 2V \\ V_1(C_3 = 6\mu F) &= 5V \\ V_1(C_3 \to \infty) &= 10V \end{aligned}\right\}$$

$$C_{eq} = \left[\frac{1}{C_1} + \frac{1}{C_2+C_3}\right]^{-1} = \frac{C_1(C_2+C_3)}{C_1+C_2+C_3}$$

$$Q = C_{eq}.V = C_1 V_1 \Rightarrow V = \frac{C_1}{C_{eq}} V_1 = \frac{C_1+C_2+C_3}{C_2+C_3} V_1$$

in series, each of the condensator (in series) has the same charge with the same as C_{eq} charge.

special case: $C_3 \to \infty$:

$$V = \frac{C_1/C_3 + C_2/C_3 + 1}{C_2/C_3 + 1} V_1$$

$$V(C_3 \to \infty) = \frac{1}{1} V_1(C_3 \to \infty) = \underline{10 \text{ Volt}}$$

* $C_3 = 0 \to V_1 = 2$ Volt, C_1 & C_2 are in series now

$V = V_1 + V_2$ (1) $\to 10 = 2 + V_2 \to \underline{V_2 = 8 \text{ Volt}}$

$C_1 V_1 = C_2 V_2$ (2) $\to C_1 . 2 = C_2 . 8 \to \underline{C_1 = 4C_2}$ (3)

* $C_3 = 6\mu F \to V_1 = 5$ Volt

$$V = \frac{C_1+C_2+C_3}{C_2+C_3} V_1 \to \frac{V}{V_1} = \frac{C_1+C_2+C_3}{C_2+C_3}$$

10 Volt

5 Volt

6 μF

$4C_2$

$$\Rightarrow 2 = \frac{C_1 + C_2 + 6}{C_2 + 6} \to 2C_2 + 12 = 5C_2 + 6$$

$3C_2 = 6\mu F$

$\underline{C_2 = 2\mu F}$

$\to \underline{C_1 = 8\mu F}$