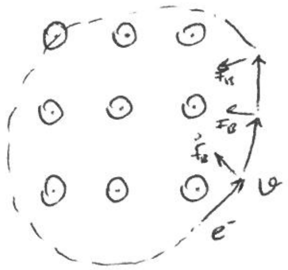
A Circulating Charged Particle

Moving in a circle at constant speed.

Net Force: Constant in magnitude and always towards the center (perpendicular to particle's velocity)



$$\vec{F}_B = q\vec{v}\times\vec{B}$$

$$\vec{F} = m\vec{a} \longrightarrow a = \frac{v^2}{r}$$

$$F = m\frac{v^2}{r}$$

$$|q|vB = \frac{mv^2}{r}$$

$$r = \frac{mv}{|q|B}$$

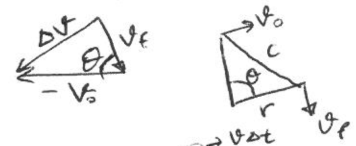
Angular velocity & accelleration revisited:

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\alpha = \frac{a_T}{r}$$

$$F = ma_r = \frac{mv^2}{r} = mr\omega^2$$

(centripetal acc.)



$$\overset{v\Delta t}{c}\ \frac{c}{r} = \frac{\Delta v}{v}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

period: $T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B}$

frequency: $f = \frac{1}{T} = \frac{|q|B}{2\pi m}$

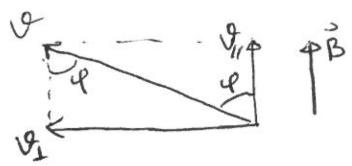
angular Frequency: $\omega = 2\pi f = \frac{|q|B}{m}$

→ do not Depend on the speed of the particle!

(alas, radius does)

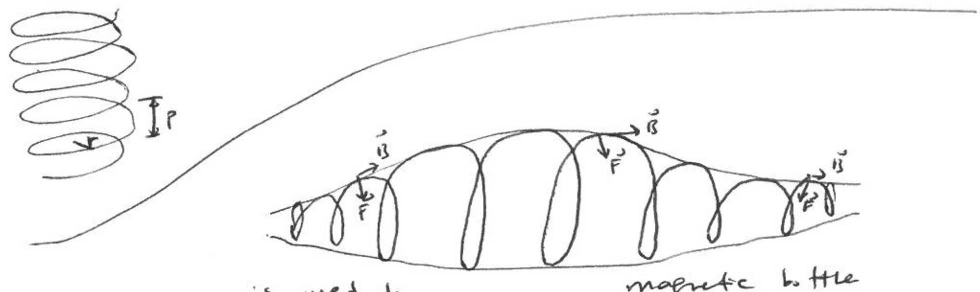
HELICAL PATHS

If the velocity of a charged particle has a component parallel to the (uniform) magnetic field, the particle will move in a helical path about the direction of the field vector.



$v_{\parallel} = v\cos\varphi$ → determines the pitch of the helix (distance Between adjacent turns)

$v_{\perp} = v\sin\varphi$ → determines the Radius of the helix



is used to confine particles

magnetic bottle (non-uniform $\vec{B}$)

Ex: Helical Motion of a charged particle in $\vec{B}$

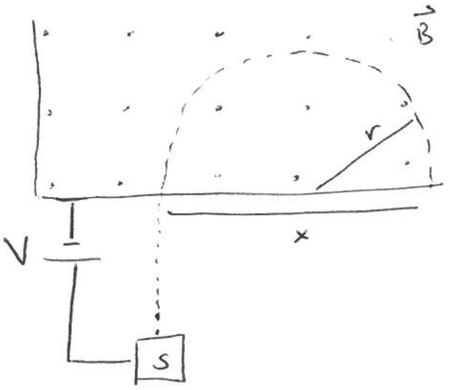
$K = 22.5\,eV$

$|\vec{B}| = 4.55\times10^{-4}\,T \qquad \hat{vB} = 65.5^\circ$

$p = ?$

$$p = v_{\parallel} T = (v\cos\varphi)\frac{2\pi m}{|q|B} = \dots = 9.16\,cm$$

Ex: Uniform Circular Motion of a charged particle in $\vec{B}$



$B = 80\,mT$

$V = 1000\,V$

$q = 1.60222\times10^{-19}$

$x = 1.6254\,m$

$m = ? \quad (u = 1.6605\times10^{-27}\,kg)$

$$\frac{1}{2}mv^2 - qV = 0 \rightarrow v = \sqrt{\frac{2qV}{m}}$$

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}$$

$$\Rightarrow m = \frac{B^2 q x^2}{8V} = \dots = 3.3863\times10^{-25}\,kg$$
$$= 203.93\,u$$

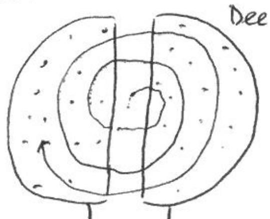
Cyclotrons and Synchotrons

For e^- (low mass, r is ok)

but for p → the required distance is too long.

Solution: Speed up moderately in some short distance.
Then use $\vec{B}$ to cause the particle circle back
and move through the potential difference V again.

Cyclotron:



Dee: hollow D-shaped objects made of sheet copper.

Oscillator alternates the electric potential difference across the gap between dees, so the particle is accelarated first towards one dee and the next so forth...

→

A proton will move towards the negatively charged Dee and enter it. Once inside, it is shielded from the electric field by the copper walls. No $\vec{E}$ but $\vec{B}$ so it will move in a circular path.

$$r = \frac{mv}{|q|B}$$

The moment the particle emerges, the potential difference is reversed (by the oscillator) so it will move to the other dee.

The frequency (→ independent of v!) of the photon circulating in the magnetic field must be equal to the fixed frequency of the oscillator.

$f = f_{osc}$: resonance condition

$$\frac{|q|B}{2\pi m} = f_{osc} \rightarrow \boxed{|q|B = 2\pi m f_{osc}}$$

PROTON SYNCHOTRON

proton energies > 50 MeV : the cyclotron fails (relativistic effects)

Real Life : the frequency of revolutions decreases steadily at high speeds.

another problem : for a 500 GeV proton in 1.5 T $\vec{B}$

$r = 1.1$ km.

$\Rightarrow$ B and f_{osc} are made to vary with time during the accelleration circle

Fermilab Synchotron : 6.3 km, 1 TeV (10^{12} eV)

Ex: Accelerating charged particle in a cyclotron

$f_{osc} = 12$ MHz

dee R = 53 cm

a.) B needed to accelerate deutron? $m_d = 3.34 \times 10^{-27}$ (2×p)

$|q|B = 2\pi m f_{osc}$

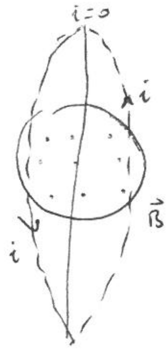
$B = \frac{2\pi m f_{osc}}{|q|} = \ldots = 1.57$ T

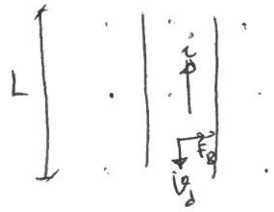
b.) Resulting K.E. of the deuterons?

$r = \frac{mv}{|q|B} \rightarrow v = \frac{R|q|B}{m} = \ldots = 3.99 \times 10^7$ m/s

$K = \frac{1}{2}mv^2 = \ldots = 2.7 \times 10^{-12}$ J $\simeq 17$ MeV

Magnetic Force on a Current Carrying Line





$$q = it = i\frac{L}{v_d}$$

$$F_B = q v_d B \sin\varphi = \frac{iL}{v_d} v_d B \sin 90^\circ$$

$$F_B = iLB$$

$$\boxed{\text{general: } \vec{F}_B = i\vec{L} \times \vec{B}}$$

$\vec{L}$: Line segment in the direction along the current

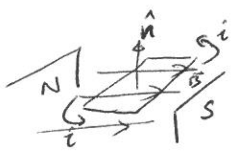
$$F_B = iLB\sin\varphi$$

(in practice we define B from F, L, i)

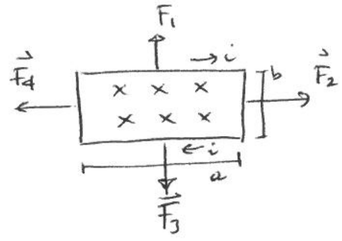
if the wire is not straight or the field is not uniform:

$$d\vec{F}_B = i\,d\vec{L} \times \vec{B}$$

Torque on a Current Loop



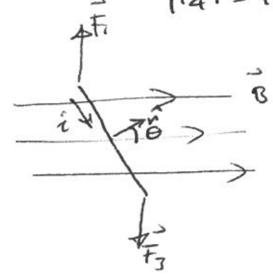
$\hat{n}$: normal to the plane of the loop



Sides 1 & 3 : perpendicular to $\vec{B}$

$$F_2 = ibB\sin(90-\theta) = ibB\cos\theta$$

$|F_4| = |F_2|$, opp. dir. → they cancel each other



F_2 ⊙

F_4 ⊗

$$|\vec{F}_1| = |\vec{F}_3| = iaB$$

$$\tau' = \left(iaB\frac{b}{2}\sin\theta\right) + \left(iaB\frac{b}{2}\sin\theta\right) = iabB\sin\theta$$

N loops: $\tau = N\tau' = NiabB\sin\theta = \underbrace{(NiA)}_{\text{properties of the coil}}B\sin\theta$

Circular Loop

$$\tau = (N i \pi r^2) B \sin\theta$$

The magnetic field tends to rotate the loop such that $\hat{n}$ is parallel to the field.

application: in motors, i is reversed back and forth.

Magnetic Dipole Moment

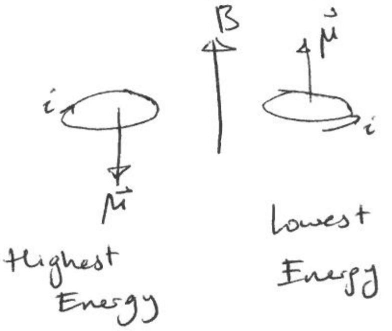
$\mu = N i A$ (magnetic moment)

$$\hat{\mu} = \hat{n}$$

$$[|\mu|] = A\, m^2$$

$\rightarrow \tau = \mu B \sin\theta$

$\rightarrow \vec{\tau} = \vec{\mu} \times \vec{B}$ (electric dipole: $\vec{\tau} = \vec{p} \times \vec{E}$)



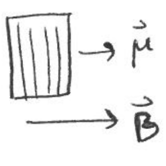
$U(\theta) = -\vec{p} \cdot \vec{E}$

$\updownarrow$

$U(\theta) = -\vec{\mu} \cdot \vec{B} \rightarrow [|\mu|] = \frac{J}{T}$ $(= A\, m^2)$

$W_a = U_f - U_i$

Ex: Rotating a Magnetic Dipole in B



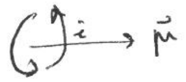
$N = 250$ turns

$A = 2.52 \times 10^{-4}\, m^2$

$i = 100\, mA$

$B = 0.85\, T$

direction of i?



How much work should the torque applied by an external agent have to do on the coil to rotate it $90°$ from its initial position so that $\vec{\mu} \perp \vec{B}$?

$W_a = U_f - U_i$

$= U(90°) - U(0°) = -\mu B \cos 90° - (-\mu B \cos 0°) = 0 + \mu B = \mu B$

$W_a = (\underbrace{N i A}_{\mu}) B = \ldots = 5.355 \times 10^{-6}\, J$