HYPOTHESES TESTS

Hypotheses Tests for the Mean of the Normal Population

Let $X_1, X_2, ..., X_n$ be a random sample from normal distribution with the mean μ and the variance σ^2. It is shown as $X_1, X_2, ..., X_n \sim N(\mu, \sigma^2)$, then

Two-sided	One(right)-sided	One(left)-sided
$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$
$H_1 : \mu \neq \mu_0$	$H_1 : \mu > \mu_0$	$H_1 : \mu < \mu_0$
	$H_0 : \mu \leq \mu_0$	$H_0 : \mu \geq \mu_0$
	$H_1 : \mu > \mu_0$	$H_1 : \mu < \mu_0$

If population variance σ^2 is known,

Test statistic, $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \geq z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \geq z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \leq -z_\alpha$, H_0 is rejected.

If population variance σ^2 is unknown,

If the sample size n is enough large ($n \geq 30$), (Large sample size)

Test statistic, $z = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$ table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \geq z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \geq z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \leq -z_\alpha$, H_0 is rejected.

NOT: If the population distribution is different from normal distribution, when $n \geq 30$ (Central Limit Theorem) the test statistics given above are used.

If the sample size n is not enough large ($n<30$), (Small sample size)

Test statistic, $t = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$ table value, $t_{\alpha/2, n-1}$, $t_{\alpha, n-1}$ and $-t_{\alpha, n-1}$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|t| \geq t_{\alpha/2, n-1}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $t \geq t_{\alpha, n-1}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $t \leq -t_{\alpha, n-1}$, H_0 is rejected.

Here, S is the standard deviation of the sample.

Hypotheses Tests for the Population Variance σ^2

Let $X_1, X_2, ..., X_n$ be a random sample from normal distribution, shown as $X_1, X_2, ..., X_n \sim N(\mu, \sigma^2)$

Two-sided	One(right)-sided	One(left)-sided
$H_0 : \sigma^2 = \sigma_0^2$	$H_0 : \sigma^2 = \sigma_0^2$	$H_0 : \sigma^2 = \sigma_0^2$
$H_1 : \sigma^2 \neq \sigma_0^2$	$H_1 : \sigma^2 > \sigma_0^2$	$H_1 : \sigma^2 < \sigma_0^2$
	$H_0 : \sigma^2 \leq \sigma_0^2$	$H_0 : \sigma^2 \geq \sigma_0^2$
	$H_1 : \sigma^2 > \sigma_0^2$	$H_1 : \sigma^2 < \sigma_0^2$

Test statistic, $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$.

Table value, $\chi^2_{\alpha/2, n-1}$, $\chi^2_{1-\alpha/2, n-1}$, $\chi^2_{\alpha, n-1}$, $\chi^2_{1-\alpha, n-1}$

Decision: According to alternative hypothesis given above, If alternative hypothesis is two-sided: If $\chi^2 \geq \chi^2_{\alpha/2,n-1}$ or $\chi^2 \leq \chi^2_{1-\alpha/2,n-1}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $\chi^2 \geq \chi^2_{\alpha,n-1}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $\chi^2 \leq \chi^2_{1-\alpha,n-1}$, H_0 is rejected.

The Hypotheses Tests for the Comparison of Two Normal Population Variances

Let $X_{11}, X_{12}, ..., X_{1n_1}$ and $X_{21}, X_{22}, ..., X_{2n_2}$ be independent random samples from normal distributions, shown as $X_{11}, X_{12}, ..., X_{1n_1} \sim N(\mu_1, \sigma_1^2)$ and $X_{21}, X_{22}, ..., X_{2n_2} \sim N(\mu_2, \sigma_2^2)$.

$$H_0: \sigma_1^2 = \sigma_2^2$$
$$H_1: \sigma_1^2 \neq \sigma_2^2$$

Test statistic is: if $s_1^2 \geq s_2^2$, then $f = \dfrac{s_1^2}{s_2^2} \geq f_{\alpha/2,\ n_1-1, n_2-1}$

and

if $s_2^2 \geq s_1^2$, then $f = \dfrac{s_2^2}{s_1^2} \geq f_{\alpha/2,\ n_2-1, n_1-1}$, H_0 is rejected.

$$H_0: \sigma_1^2 = \sigma_2^2 \qquad H_0: \sigma_1^2 \leq \sigma_2^2$$
$$H_1: \sigma_1^2 > \sigma_2^2 \qquad H_1: \sigma_1^2 > \sigma_2^2$$

Test statistic is: if $f = \dfrac{s_1^2}{s_2^2} \geq f_{\alpha,\ n_1-1, n_2-1}$, H_0 is rejected.

$$H_0: \sigma_1^2 = \sigma_2^2 \qquad H_0: \sigma_1^2 \geq \sigma_2^2$$
$$H_1: \sigma_1^2 < \sigma_2^2 \qquad H_1: \sigma_1^2 < \sigma_2^2$$

Test statistic is: if $f = \dfrac{s_2^2}{s_1^2} \geq f_{\alpha,\ n_2-1, n_1-1}$, H_0 is rejected.

The Hypotheses Tests to Compare the Means of Two Normal Populations

Let $X_{11}, X_{12}, ..., X_{1n_1}$ and $X_{21}, X_{22}, ..., X_{2n_2}$ be independent random samples from normal distribution, shown as $X_{11}, X_{12}, ..., X_{1n_1} \sim N(\mu_1, \sigma_1^2)$ and $X_{21}, X_{22}, ..., X_{2n_2} \sim N(\mu_2, \sigma_2^2)$

Two-sided	One(right)-sided	One(left)-sided
$H_0: \mu_1 - \mu_2 = \delta$	$H_0: \mu_1 - \mu_2 = \delta$	$H_0: \mu_1 - \mu_2 = \delta$
$H_1: \mu_1 - \mu_2 \neq \delta$	$H_1: \mu_1 - \mu_2 > \delta$	$H_1: \mu_1 - \mu_2 < \delta$
	$H_0: \mu_1 - \mu_2 \leq \delta$	$H_0: \mu_1 - \mu_2 \geq \delta$
	$H_1: \mu_1 - \mu_2 > \delta$	$H_1: \mu_1 - \mu_2 < \delta$

If σ_1^2 and σ_2^2 are <u>known,</u>

Test statistic, $z = \dfrac{\bar{x}_1 - \bar{x}_2 - \delta}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \geq z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \geq z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \leq -z_\alpha$, H_0 is rejected.

If σ_1^2 and σ_2^2 are <u>unknown,</u>

If the sample sizes are <u>n_1 and $n_2 \geq 30$, (Large sample sizes)</u>

Test statistics, $z = \dfrac{\bar{x}_1 - \bar{x}_2 - \delta}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \geq z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \geq z_{\alpha}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \leq -z_{\alpha}$, H_0 is rejected.

NOT: If the populations' distributions are different from normal distribution, when n_1 and $n_2 \geq 30$ (Central Limit Theorem) the test statistics given above are used.

<u>If the sample sizes are n_1 and $n_2 < 30$, (Small sample sizes)</u>

Firstly, whether $\sigma_1^2 = \sigma_2^2 = \sigma^2$ or not must be tested.

If $\sigma_1^2 = \sigma_2^2 = \sigma^2$,

Test statistics, $.t = \dfrac{\bar{x}_1 - \bar{x}_2 - \delta}{s_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

table value, $t_{\alpha/2, n_1+n_2-2}$, t_{α, n_1+n_2-2} and $-t_{\alpha, n_1+n_2-2}$

Pooled variance $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|t| \geq t_{\alpha/2, n_1+n_2-2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $t \geq t_{\alpha, n_1+n_2-2}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $t \leq -t_{\alpha, n_1+n_2-2}$, H_0 is rejected.

If $\sigma_1^2 \neq \sigma_2^2$,

Test statistic, $.t = \dfrac{\bar{x}_1 - \bar{x}_2 - \delta}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ table value, $t_{\alpha/2, \nu}$, $t_{\alpha, \nu}$ and $-t_{\alpha, \nu}$

Degrees of freedom $\nu = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\left(\dfrac{s_1^2}{n_1}\right)^2 \left(\dfrac{1}{n_1-1}\right) + \left(\dfrac{s_2^2}{n_2}\right)^2 \left(\dfrac{1}{n_2-1}\right)}$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|t| \geq t_{\alpha/2, \nu}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $t \geq t_{\alpha, \nu}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $t \leq -t_{\alpha, \nu}$, H_0 is rejected.

NOT: If the population distributions are different from normal distribution, when n_1 and $n_2 \geq 30$, Central Limit Theorem is used.

The Hypotheses Tests for Paired Samples

$D_i = X_{1i} - X_{2i} \sim N(\mu_1 - \mu_2, \sigma_D^2)$ i=1,2,…,n

If σ_D^2 is known,

Two-sided	One(right)-sided	One(left)-sided
$H_0: \mu_1 - \mu_2 = d_0$	$H_0: \mu_1 - \mu_2 = d_0$	$H_0: \mu_1 - \mu_2 = d_0$
$H_1: \mu_1 - \mu_2 \neq d_0$	$H_1: \mu_1 - \mu_2 > d_0$	$H_1: \mu_1 - \mu_2 < d_0$
	$H_0: \mu_1 - \mu_2 \leq d_0$	$H_0: \mu_1 - \mu_2 \geq d_0$
	$H_1: \mu_1 - \mu_2 > d_0$	$H_1: \mu_1 - \mu_2 < d_0$

Test statistic, $z = \dfrac{\bar{d} - d_0}{\sigma_D / \sqrt{n}}$. Table value, $z_{\alpha/2}$, z_{α} and $-z_{\alpha}$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \geq z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \geq z_{\alpha}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \le -z_\alpha$, H_0 is rejected.

If σ_D^2 is <u>unknown</u>,

Two-sided	One(right)-sided	One(left)-sided
$H_0: \mu_1 - \mu_2 = d_0$	$H_0: \mu_1 - \mu_2 = d_0$	$H_0: \mu_1 - \mu_2 = d_0$
$H_1: \mu_1 - \mu_2 \ne d_0$	$H_1: \mu_1 - \mu_2 > d_0$	$H_1: \mu_1 - \mu_2 < d_0$
	$H_0: \mu_1 - \mu_2 \le d_0$	$H_0: \mu_1 - \mu_2 \ge d_0$
	$H_1: \mu_1 - \mu_2 > d_0$	$H_1: \mu_1 - \mu_2 < d_0$

Test statistic, $t = \dfrac{\bar{d} - d_0}{s_D / \sqrt{n}}$. Table value $t_{\alpha/2,n-1}$, $t_{\alpha,n-1}$ and $-t_{\alpha,n-1}$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|t| \ge t_{\alpha/2,n-1}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $t \ge t_{\alpha,n-1}$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $t \le -t_{\alpha,n-1}$, H_0 is rejected.

The Hypotheses Tests for the Population Proportion

$X \sim Binom(n, p)$

Two-sided	One(right)-sided	One(left)-sided
$H_0: p = p_0$	$H_0: p = p_0$	$H_0: p = p_0$
$H_1: p \ne p_0$	$H_1: p > p_0$	$H_1: p < p_0$
	$H_0: p \le p_0$	$H_0: p \ge p_0$
	$H_1: p > p_0$	$H_1: p < p_0$

Test statistic, $z = \dfrac{\frac{x}{n} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ Table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \ge z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \ge z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \le -z_\alpha$, H_0 is rejected.

The Hypotheses Tests to Compare Two Population Proportions

X_1 and X_2 are two random variables from a binomial distribution, shown as $X_1 \sim Binom(n_1, p_1)$ and $X_2 \sim Binom(n_2, p_2)$.

Two-sided	One(right)-sided	One(left)-sided
$H_0: p_1 - p_2 = 0$	$H_0: p_1 - p_2 = 0$	$H_0: p_1 - p_2 = 0$
$H_1: p_1 - p_2 \ne 0$	$H_1: p_1 - p_2 > 0$	$H_2: p_1 - p_2 < 0$
	$H_0: p_1 - p_2 \le 0$	$H_0: p_1 - p_2 \ge 0$
	$H_1: p_1 - p_2 > 0$	$H_2: p_1 - p_2 < 0$

Test statistic, $z = \dfrac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ Table value, $z_{\alpha/2}$, z_α and $-z_\alpha$

Decision: According to alternative hypotheses given above, respectively;

If alternative hypothesis is two-sided: If $|z| \ge z_{\alpha/2}$, H_0 is rejected.

If alternative hypothesis is one(right)-sided: If $z \ge z_\alpha$, H_0 is rejected.

If alternative hypothesis is one(left)-sided: If $z \le -z_\alpha$, H_0 is rejected.

Here, $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$.