MAGNETIC FIELDS — Halliday & Resnick 9th Ed. 11/4/2017

28.10 A proton travels through uniform $\vec{E}$ and $\vec{B}$. $\vec{B} = -2.50\hat{i}$ mT. At one instant, the velocity of the proton is $\vec{v} = 2000\hat{j}$ m/s. At that instant and unit vector notation, what is the net force acting on the proton if $\vec{E}$ is

a) $4.00\hat{k}$ V/m, b) $-4.00\hat{k}$ V/m, c) $4.00\hat{i}$ V/m

a) $\vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B} = (1.6\times10^{-19}C)\left[4.\,V/m\,\hat{k} + 2000\,m/s\,\hat{j}\times(-2.5\times10^{-3}T)\hat{i}\right]$

$= 1.44\times10^{-18}N\,\hat{k}$

b) $\vec{F} = (1.6\times10^{-19}C)\left[-4\,V/m\,\hat{k} + 2000\,m/s\,\hat{j}\times -2.5\,mT\,\hat{i}\right]$

$= 1.6\times10^{-19}N\,\hat{k}$

c) $\vec{F} = (1.6)\times10^{-19}C)\left[4\,V/m\,\hat{i} + 2000\,m/s\,\hat{j}\times(-2.5\times10^{-3}T)\hat{i}\right]$

$= (6.41\times10^{-19}N)\hat{i} + (8.01\times10^{-19}N)\hat{k}$

28.29. An e^- follows a helical path in a uniform magnetic field of 0.3T. The pitch of the path is 6.00 μm and the magnitude of the mag. force on electron is 2×10^{-15} N. What is electron's speed?

distance traveled parallel to $\vec{B}$: $d_{\parallel} = v_{\parallel}T = v_{\parallel}\frac{2\pi m_e}{|q|B}$

$qvB = \frac{mv^2}{r}$

$\rightarrow v_{\parallel} = \frac{d_{\parallel}eB}{2\pi m_e} = 50.3$ km/s

$F_B = |q|\,v_{\perp}B_{\perp} \rightarrow v_{\perp} = 41.7$ km/s $\Rightarrow v = \sqrt{v_{\perp}^2 + v_{\parallel}^2} = 65.3$ km/s

28.38 In a certain cyclotron a proton moves in a circle of radius 0.5 m. The magnitude of the magnetic field is 1.2 T.

a) What is the oscillator freq.?

b) " " K.E. of the proton (in eV)?

$$f_{osc} = \frac{qB}{2\pi m_p}$$

1

$$T = \frac{2\pi r}{v}, \quad qvB = \frac{mv^2}{r} \;\to\; r = \frac{mv}{qB}$$

$$f = \frac{v}{2\pi r} = \frac{v \, qB}{2\pi m v} = \underline{\frac{qB}{2\pi m}} = \frac{(1.6\times10^{-19}\,c)(1.2\,T)}{2\pi\,(1.67\times10^{-27}\,kg)} = 1.83\times10^{7}\,Hz$$

b) $r = \frac{m_p v}{qB} = \frac{m_p \cdot \sqrt{2K}}{qB\sqrt{m_p}} = \frac{\sqrt{2m_p K}}{qB}$

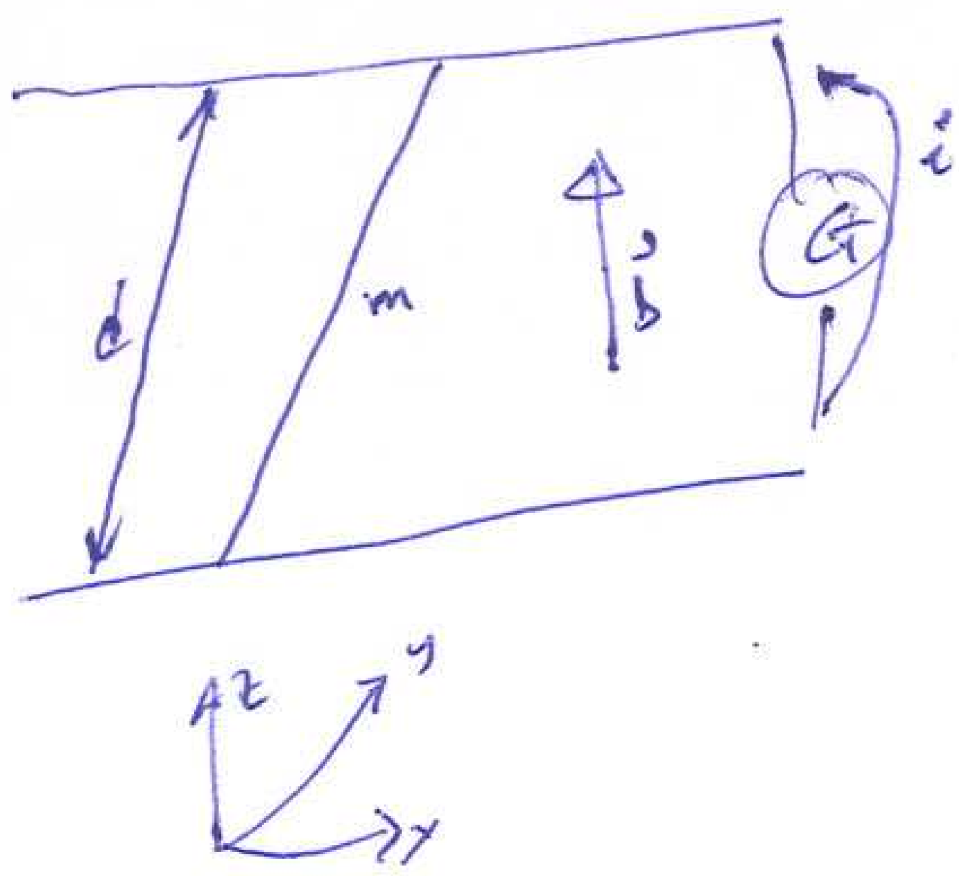
$K = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2K}{m}}$

$$K = \frac{(rqB)^2}{2m_p} = \frac{\left[(0.5\,m)(1.6\times10^{-19}\,c)(1.2\,T)\right]^2}{2\,(1.67\times10^{-27}\,kg)\cdot(1.6\times10^{-19}\,J/eV)}$$

$$= 1.72\times10^{7}\,eV$$

28.46.



$m = 24.1\,mg$

$d = 2.56\,cm$

$\vec{B} = 56.3\,mT\,\hat{k}$

$i = 9.13\,mA$

4

at $t = 61.1\,ms$ → what are the wire's

a) speed

b) direction of motion (left or right)?

$$v = at = \frac{F_B t}{m} = \frac{idBt}{m}$$

$$= \frac{(9.13\times10^{-3}\,A)(2.56\times10^{-2}\,m)(5.63\times10^{-2}\,T)(0.0611\,s)}{2.41\times10^{-5}\,kg}$$

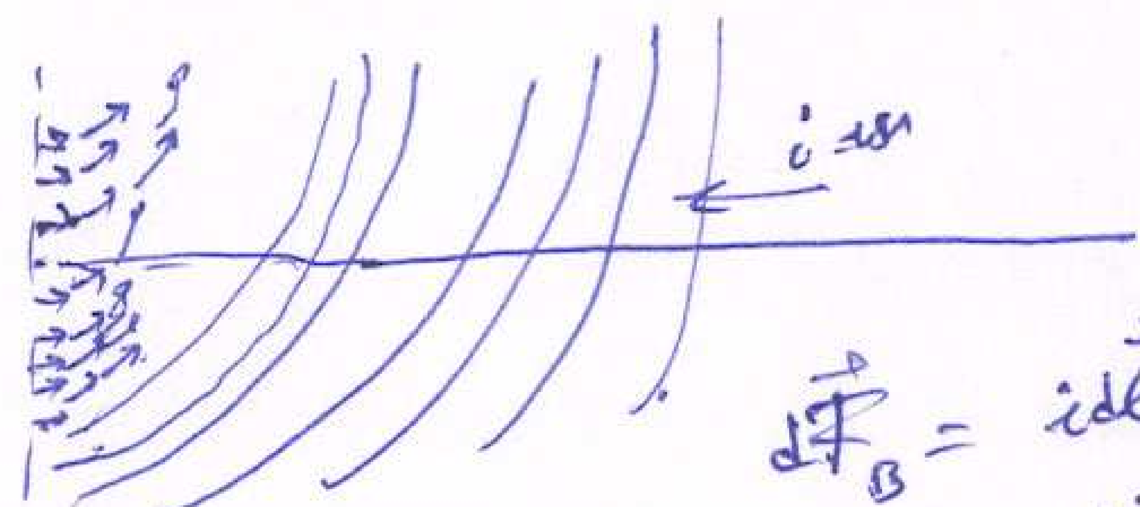
$$= 3.34\times10^{-2}\,m/s$$

(left)

28.48: A long, rigid conductor, lying along an x axis, carries a current of 5A in negative x-direction.

A magnetic field $\vec{B}$ is present, $\vec{B} = 3\hat{i} + 8x^2\hat{j}$ x: meter B: mT

Find, in unit vector notation, the force on the 2.0 m segment of the conductor that lies between x = 1 m and x = 3 m.



$d\vec{F}_B = i d\vec{l} \times \vec{B}$

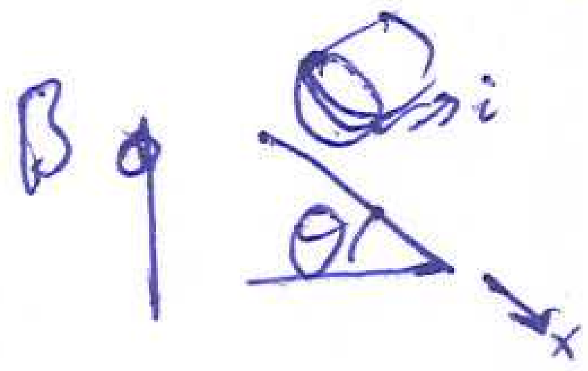
$d\vec{l} = dx\,\hat{i}$, $\vec{B} = B_x\hat{i} + B_y\hat{j}$

$\vec{F}_B = \int i\, d\vec{l} \times \vec{B} = \int_{x_i}^{x_f} i\, dx\,\hat{i} \times (B_x\hat{i} + B_y\hat{j}) = i\int_{x_i}^{x_f} B_y dx\,\hat{k}$

$= (-5.0A)\int_1^3 (8x^2 dx)(m.mT)\hat{k} = (-0.35N)\hat{k}$

28.51. A wood cylinder of mass m = 0.250kg and length L = 0.100 m with N = 10.0 turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder.

The cylinder is released on a plane inclined at an angle θ to the horizontal. If there is a uniform vertical $\vec{B}$ of 0.5 T, what is the least current i through the coil that keeps the cylinder rolling down the plane?

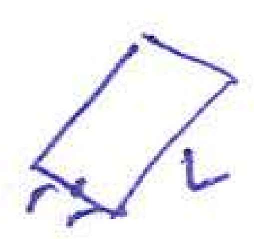


Forces

$mg\sin\theta - f = ma$

Torque

$f.r - \mu B\sin\theta = I\alpha$



Steady: $a = 0$, $\alpha = 0$ $\rightarrow$ $mgr = \mu B$

Area loop = 2r.L

$\mu = NiA = Ni\,2rL$

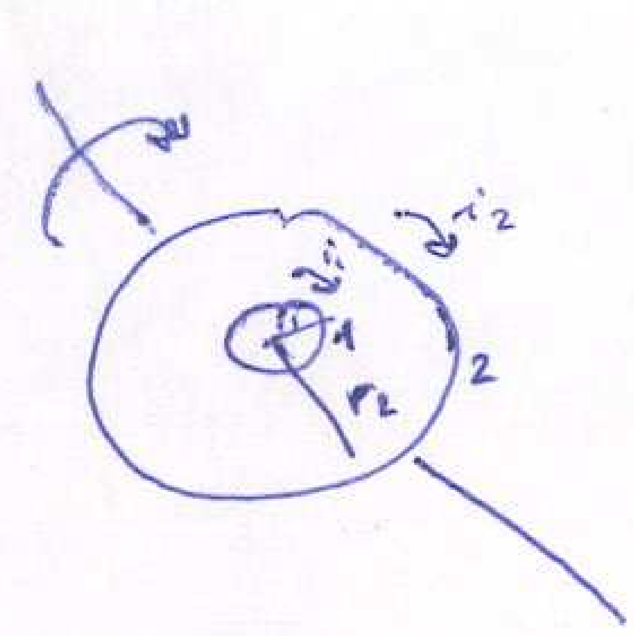
$\rightarrow mgr = Ni2rLB$

$i = \frac{mg}{2NLB} = \frac{0.25kg\ 9.8m/s^2}{2(10)(0.1m)(0.5T)} = 2.45A$

3

18/4/2017

29.16 Two concentric circular loops carrying current in the same direction (in the same plane)



$r_1 = 1.50\,cm$, $r_2 = 2.50\,cm$

$i_1 = 4.00\,mA$, $i_2 = 6\,mA$

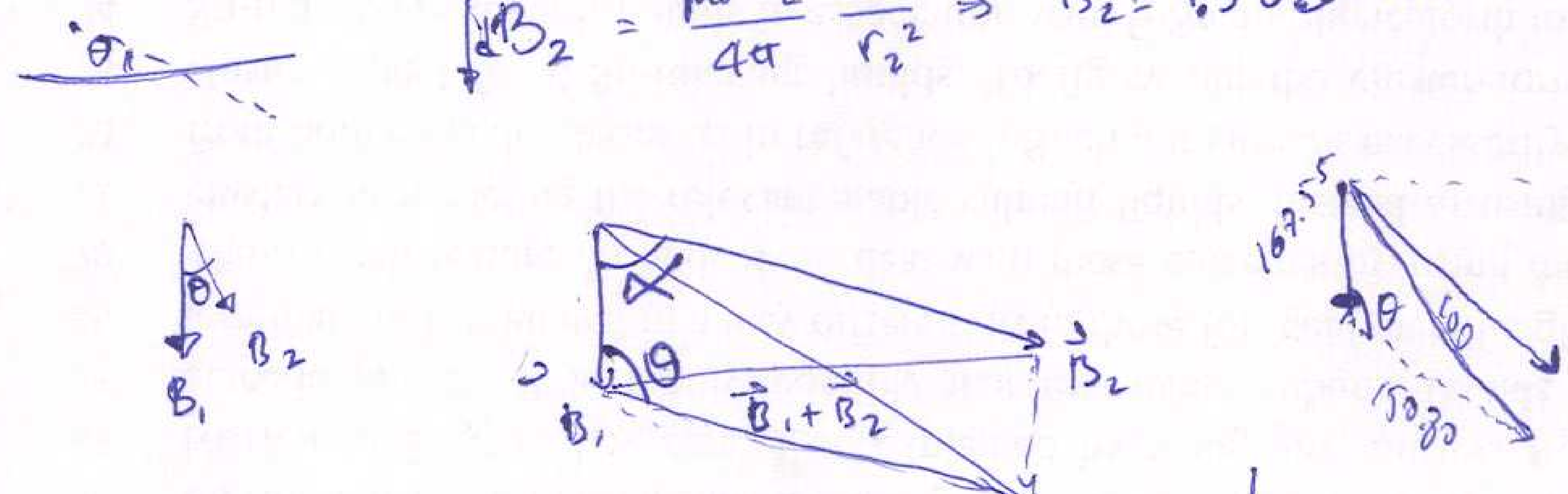
Loop 2 is to be rotated about a diameter while the net magnetic field $\vec{B}$ → set up by the two loops at their common center is measured. Through what angle must loop 2 be rotated so that the magnitude of the net field is $100\,nT$?

$4\pi \times 10^{-7}\,Tm$

$1.2566371 \times 10^{-6}\,Tm$

$B_1 = \frac{\mu_0 i_1}{4\pi} \frac{dl}{r_1^2} \Rightarrow B_1 = 1.6755 \times 10^{-7}\,T = 167.55\,nT$

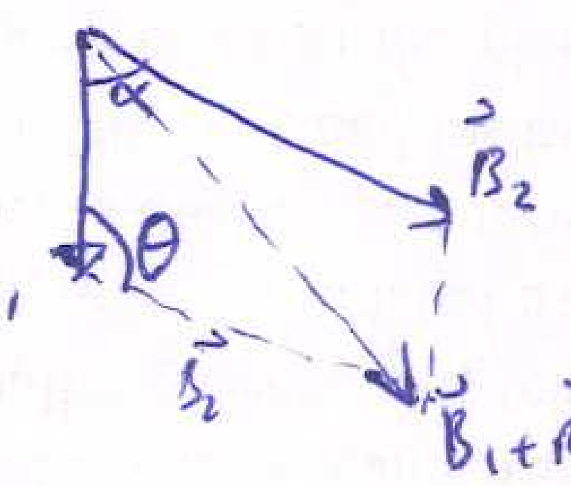
$B_2 = \frac{\mu_0 i_2}{4\pi} \frac{dl}{r_2^2} \Rightarrow B_2 = 1.5080 \times 10^{-7}\,T = 150.80\,nT$



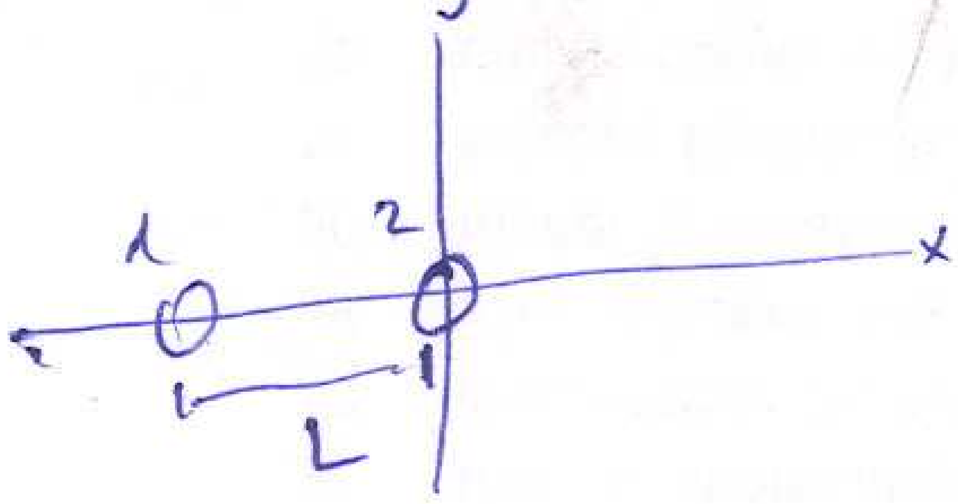
$100^2 = B_1^2 + B_2^2 - 2B_1B_2\cos\theta$

$\Rightarrow \theta = 36^\circ$

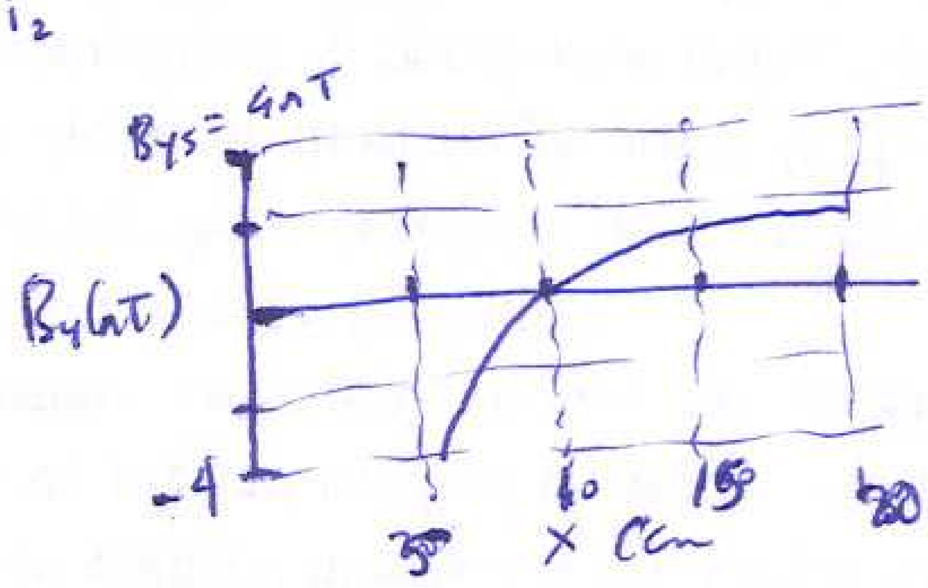
$\alpha = 180^\circ - \theta = 144^\circ$



29.22 Figure shows two long parallel wires carrying current and sep. by distance L.



$\frac{i_1}{i_2} = 4$, directions unknown.



a.) at what value of $x > 0$ is B_y maximum?

b) if $i_2 = 3\,mA$ → what is the value of that maximum

c) what are the directions of i_1 and i_2?

$B_y = 0$ at $x = 10\,cm$

→ currents are in opposite directions; $i_1 = 4 i_2$ $(i_1 > i_2)$

2-1

$B_y = \frac{\mu_0 i_1}{2\pi(L+x)} - \frac{\mu_0 i_2}{2\pi x} = \frac{\mu_0 i_2}{2\pi}\left(\frac{4}{L+x} - \frac{1}{x}\right)$ → where is this max? $\Rightarrow \frac{dB_y}{dx} = 0$

$\frac{dB_y}{dx} = \frac{\mu_0 i_2}{2\pi}\left(\frac{-4}{(L+x)^2} + \frac{1}{x^2}\right) = \frac{\mu_0 i_2}{2\pi}\left(\frac{-4x^2 + L^2 + x^2 + 2Lx}{x^2(L+x)^2}\right) = \frac{\mu_0 i_2}{2\pi}\left(\frac{-3x^2 + 2Lx + L^2}{x^2(L+x)^2}\right)$

$\Rightarrow 3x^2 - 2Lx - L^2 = 0$ $\quad x = \frac{2L \pm \sqrt{4L^2 + 12L^2}}{6} = \frac{2L \pm 4L}{6}$ $\quad x_1 = L$ ✓ $\quad x_2 = -\frac{L}{3}$ ✗

29.22 Cont'd

18/4/2017

$X=L \rightarrow B_y = \frac{\mu_0 i_2}{2\pi}\left(\frac{4}{2L} - \frac{1}{L}\right) = \frac{\mu_0 i_2}{2\pi L}$

$x = 10\,cm \quad B_y = 0 \rightarrow \frac{\mu_0 i_2}{2\pi}\left(\frac{4}{L+10} - \frac{1}{10}\right) = 0 \rightarrow \frac{4}{L+10} = \frac{1}{10} \rightarrow 40 = L+10$

$\Rightarrow L = 30\,cm.$

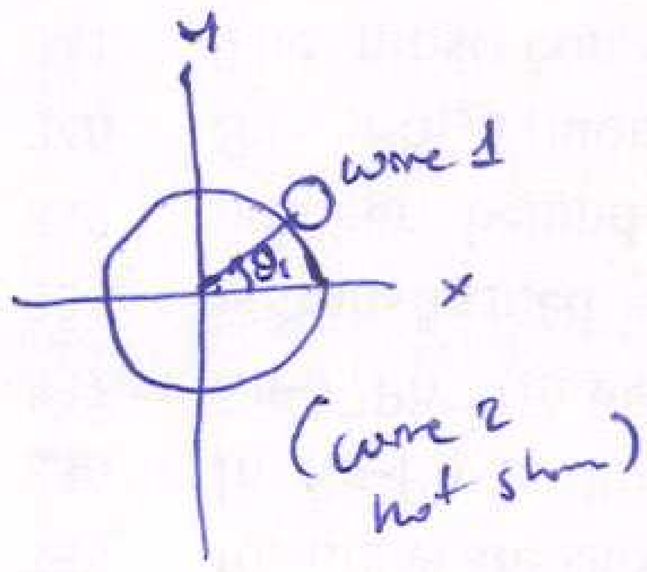
a.) $x = L = 30\,cm$

b) $i_2 = 0.003A \rightarrow \frac{\mu_0 i_2}{2\pi L} = 2\,nT$

c) $x \rightarrow 0 \quad B_y < 0 \rightarrow i_2 \otimes$

$i_1 \odot$

29.30 Two long straight thin wires with current lie against an equally long plastic cylinder, at radius R=20cm.



With wire 2 fixed, wire 1 is moved around the cylinder

$\Theta_1 = 0 \ldots 180°$

Net magnetic field at the center is measured.

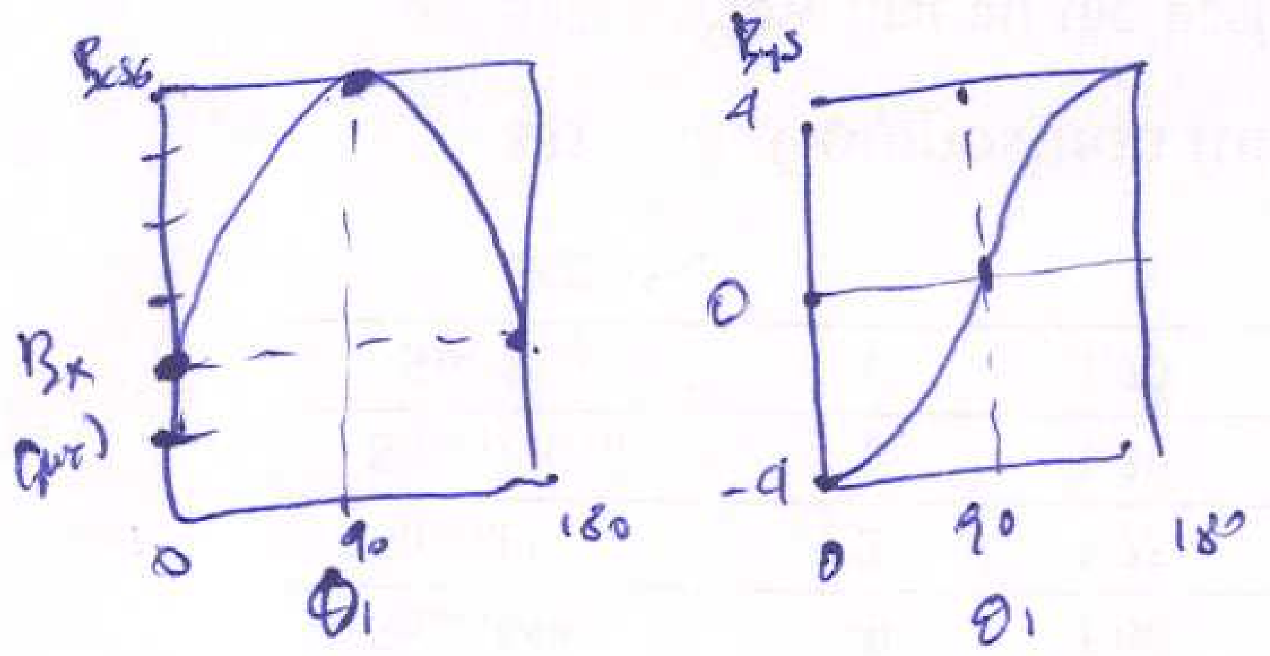
$B_{xs} = 6\,\mu T$

$B_{ys} = 4\,\mu T$

a.) at what angle θ_2 is wire 2 located?

b) Size and direction of current 1

c) " " " 2



a) when $\theta_1 = 90° \rightarrow$ no $B_y \rightarrow$ wire $2\theta_2$ $\frac{\pi}{2}$ / $-\frac{\pi}{2}$ ← otherwise wire 1 couldn't move over it

b). $\theta_1 = 90° \quad B_{net\,x} = 6\,\mu T$

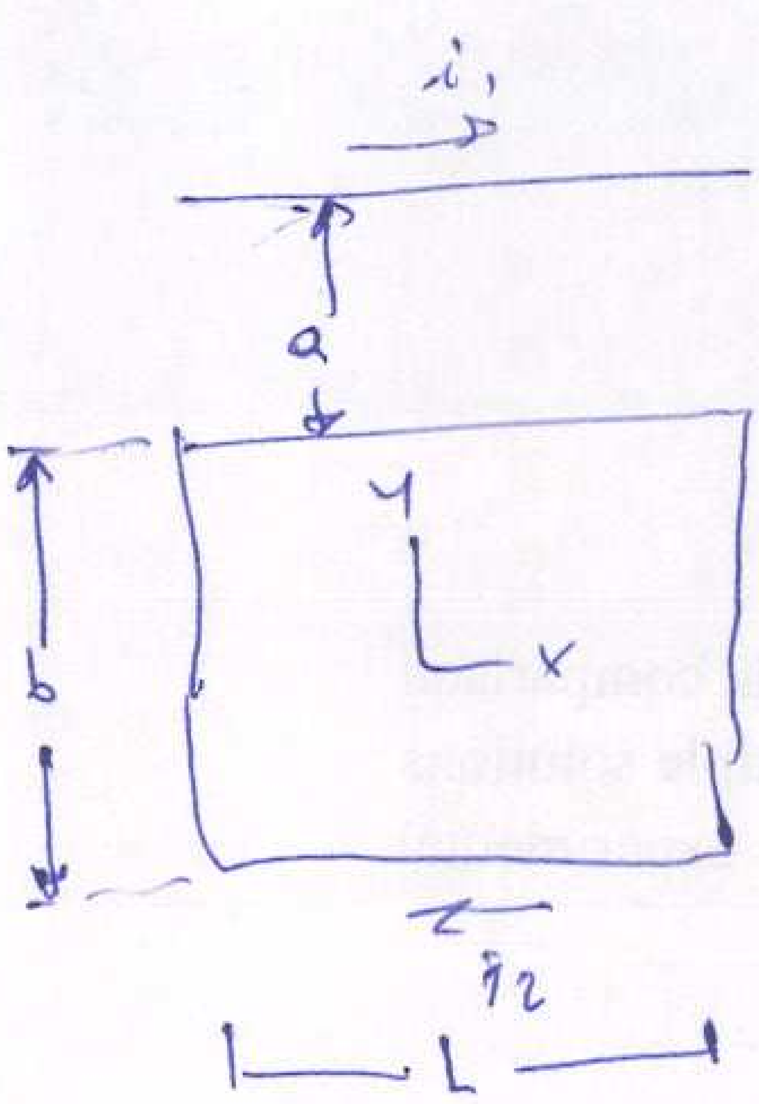
$\theta_1 = 0° \quad B_{net\,x} = 2\,\mu T = B_{2x}$ ← This is due to $i_2 \Rightarrow B_{1x} = 6\,\mu T - 2\,\mu T = 4\,\mu T$

$B_{1x} = \frac{\mu_0}{2\pi}\frac{i_1}{R} \rightarrow i_1 = 4A$, as it increases $\theta: 0 \rightarrow 90$: $\odot$

c) $\theta_1 = 0 \xrightarrow{B_{1x}=0} B_{2x} = 2\,\mu T \quad B_{2x} = \frac{\mu_0 i_2}{2\pi R} \rightarrow i_2 = 2A \quad \otimes$

2-2

29.41

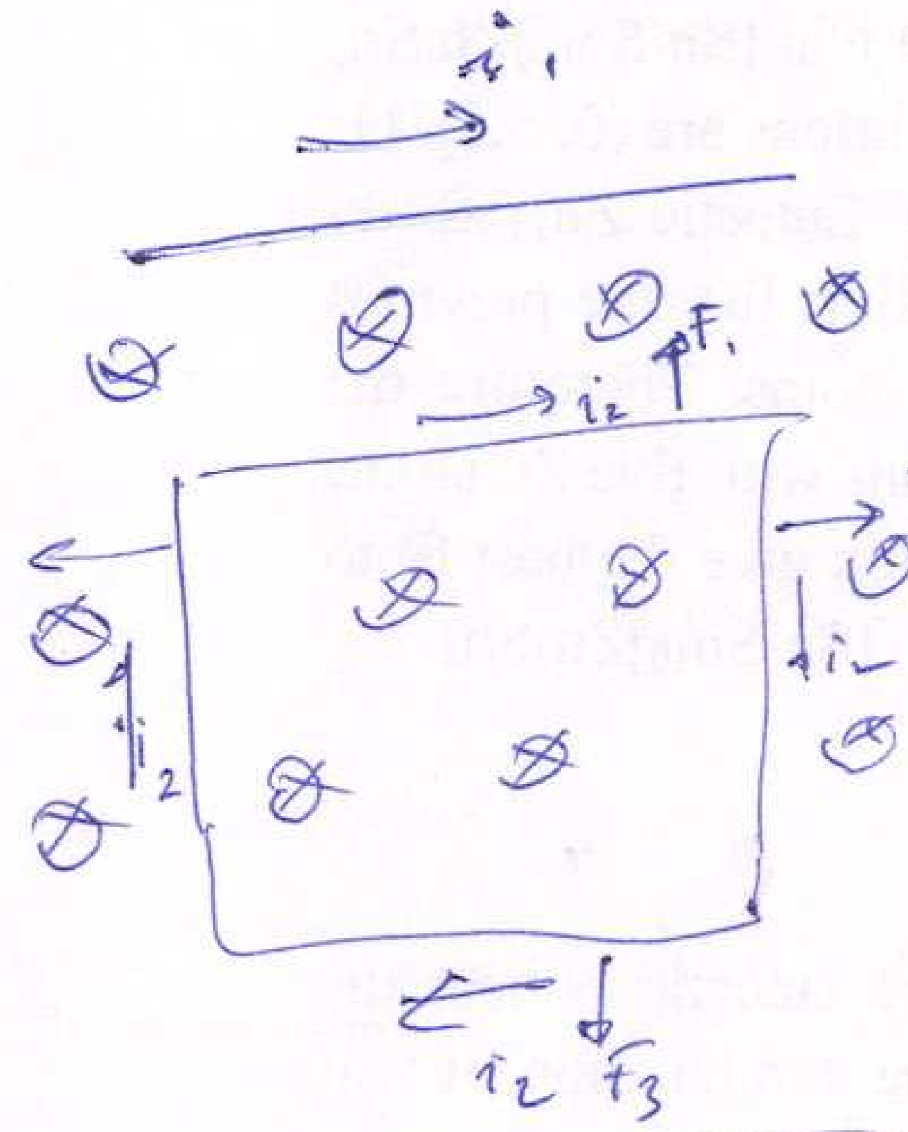


$i_1 = 30A$

$i_2 = 20A$

$a = 1cm$
$b = 8cm$
$L = 30cm$

In unit vector notation, what is the net force ~~What~~ on loop due to i_1?



$$F = i \vec{l} \times \vec{B}$$

$$\vec{B} = -\frac{\mu_0 i_1}{2\pi r}\hat{k}$$

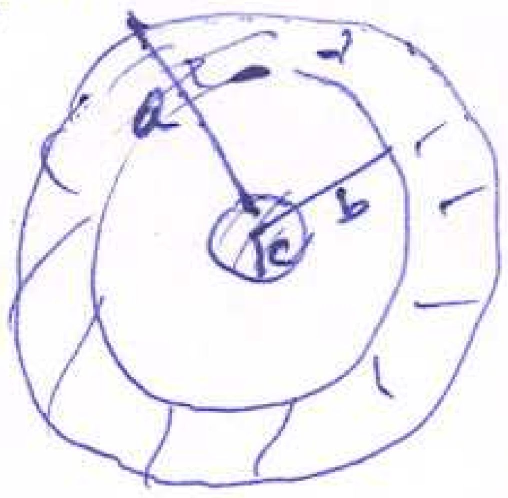
$$\vec{F}_1 + \vec{F}_3 = \frac{\mu_0 i_2 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+L}\right)\hat{j}$$

$$= 3.2 \times 10^{-3} N \hat{j}$$

$$\left(\vec{F}_2 = -\vec{F}_4 \; . \; |\vec{F}_2| = \int_a^{a+b} \frac{i_2 \mu_0 i_1}{2\pi y}\, dy \right)$$

they cancel each other out.

29.87



Equal but opposite currents i are uniformly distributed.

$B(r)$ i) $r < c$
ii) $c < r < b$
iii) $b < r < a$
iv) $r > a$

Assume: i smaller + / i larger −

a) $B = \frac{\mu_0 i r}{2\pi c^2}$ $r \le c$

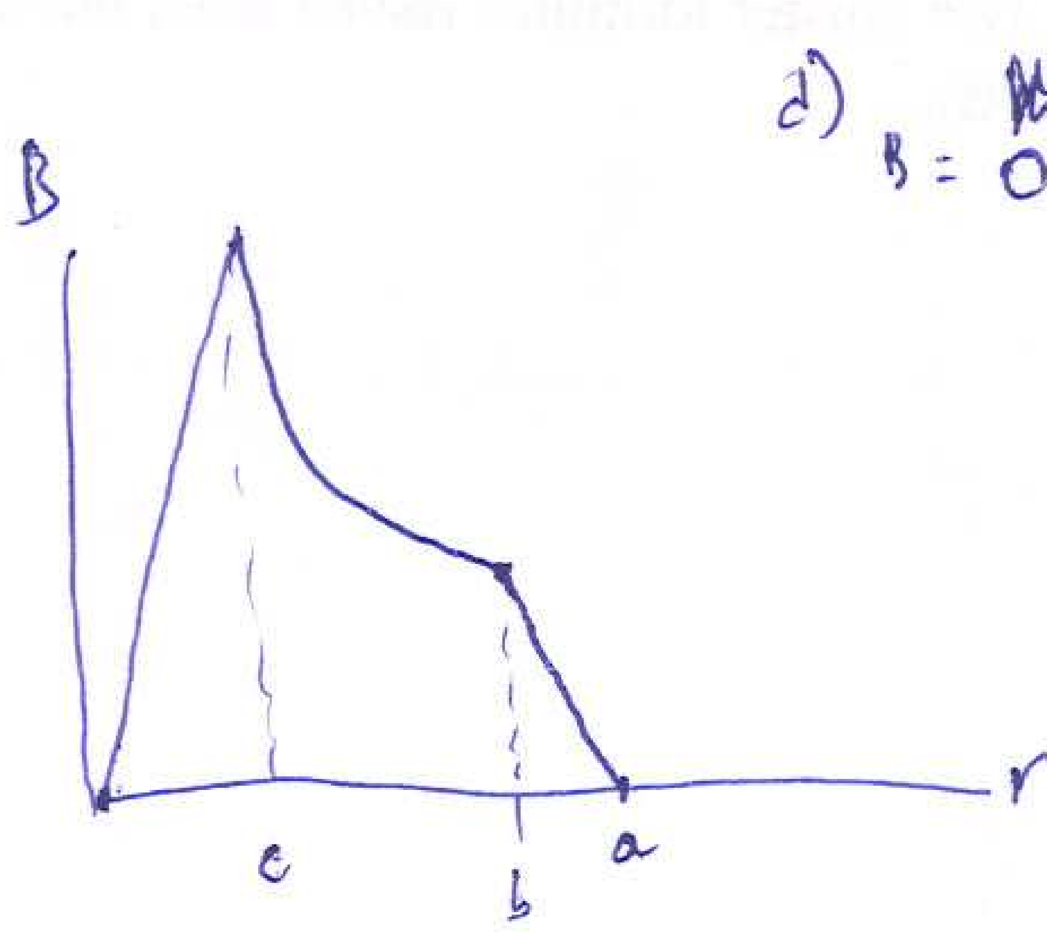
$\frac{\pi c^2}{\pi r^2} \quad \frac{i}{i'}$ $\quad i' = \frac{i r^2}{c^2}$

b) $B = \frac{\mu_0 i}{2\pi r}$ $c < r \le b$

c) $B = \frac{\mu_0 i}{2\pi r} - \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - b^2}{a^2 - b^2}\right)$
$= \frac{\mu_0 i}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right)$ $\quad b < r \le a$

$\frac{\pi(a^2 - b^2)}{\pi(r^2 - b^2)} \quad \frac{i}{i'}$ $\quad i' = \frac{r^2 - b^2}{a^2 - b^2} i$

d) ~~$\mu_0 i$~~ $B = 0$



$a \to \infty$, $b \to \infty$, $a > b$ $\Big\}$ $B = \frac{\mu_0 i}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right)$

$\frac{\mu_0 i}{2\pi r}\left(\frac{1 - r^2/a^2}{1 - 0}\right) = \frac{\mu_0 i}{2\pi r}$