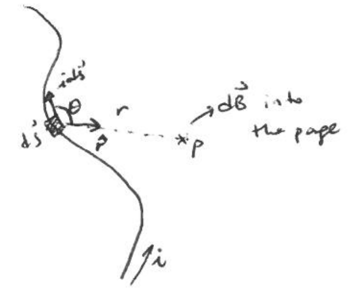
MAGNETIC FIELDS DUE TO CURRENTS

A moving charged particle produces a magnetic field around itself

$\longrightarrow$ a current of moving charged particles produces a magnetic field around the current

$\Rightarrow$ electromagnetism

Magnetic Trains : $\underbrace{\text{Maglev}}$ trains
Magnetic Levitation



$\vec{B}$ is summable but it's a little bit more difficult wrt $\vec{E}$ because dq is a scalar whereas $i\vec{ds}$ is a vector.

$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds \sin\theta}{r^2}$$

μ_0 → permeability constant

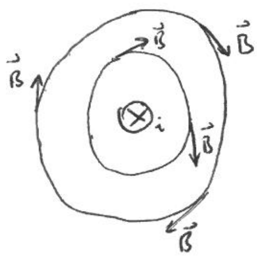
$$\mu_0 = 4\pi \times 10^{-7}\ \text{Tm/A} \simeq 1.26 \times 10^{-6}\ \text{Tm/A}$$

direction of $\vec{dB}$: $\vec{ds} \times \hat{r}$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{i\,\vec{ds} \times \hat{r}}{r^2} \quad \text{(Biot-Savart Law)}$$

Magnetic Field due to a Current in a Long Straight Wire

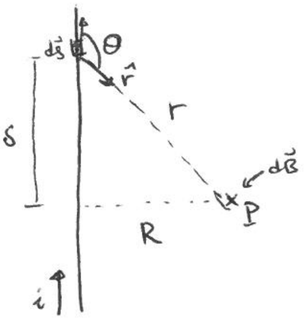
$$B = \frac{\mu_0 i}{2\pi r}$$ (Direct Result; will be derived on the next page)



(i is into the page)

Right hand Rule : Grasp the cable such that your thumb points along the current. Then your curled fingers denote the direction of $\vec{B}$.

$\vec{B}$ at any point is tangent to a magnetic field line



$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds \sin\theta}{r^2}$$

$$B = \int_{s=-\infty}^{s=\infty} dB = 2\int_{s=0}^{\infty} dB = \frac{\mu_0 i}{2\pi} \int_0^{\infty} \frac{\sin\theta\, ds}{r^2}$$

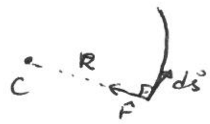
$$r = \sqrt{s^2+R^2}$$

$$\sin\theta = \sin(\pi-\theta) = \frac{R}{\sqrt{s^2+R^2}}$$

$$\rightarrow B = \frac{\mu_0 i}{2\pi} \int_0^{\infty} \frac{R\,ds}{(s^2+R^2)^{3/2}} = \frac{\mu_0 i}{2\pi R} \left[\frac{s}{(s^2+R^2)^{1/2}} \right]_0^{\infty} = \frac{\mu_0 i}{2\pi R}$$] B due to an infinite wire

$$\frac{\mu_0 i}{4\pi R}$$] B due to a semi-infinite wire

Magnetic Field due to a Current in a Circular Arc of WIRE



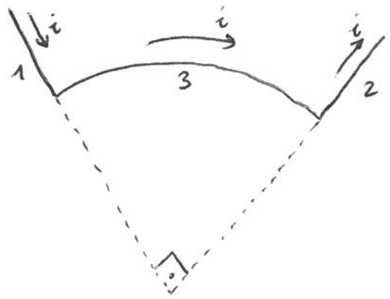
$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds \sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi} i \frac{ds}{R^2}$$

$$B = \int dB = \int_0^{\varphi} \frac{\mu_0}{4\pi} \frac{i \overbrace{R\,d\varphi}^{ds = R\,d\varphi}}{R^2} = \frac{\mu_0 i}{4\pi} \int_0^{\varphi} d\varphi$$

$$B = \frac{\mu_0 i \varphi}{4\pi R}$$] B at the center of the arc

if $\varphi = 2\pi \rightarrow B = \frac{\mu_0 i}{2R}$] B at the center of a full circle

Example:



$$dB_1 = \frac{\mu_0}{4\pi} \frac{i\,ds \sin\theta}{r^2} = \frac{\mu_0 i\,ds \sin 0^\circ}{4\pi r^2} = 0$$

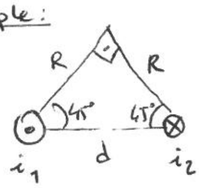
$\rightarrow B_1 = 0$

$B_2 = 0$

$$B_3 = \frac{\mu_0 i (\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}$$

(direction: inside the plane = ⊗)

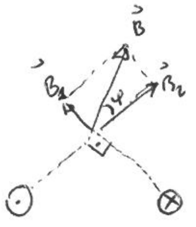
Example:



$i_1 = 15A$, $i_2 = 32A$, $d = 5.3\,cm$

$$B_1 = \frac{\mu_0 i_1}{2\pi R}, \quad B_2 = \frac{\mu_0 i_2}{2\pi R}$$

$$\cos 45^\circ = \frac{R}{d} \rightarrow R = d\cos 45^\circ$$



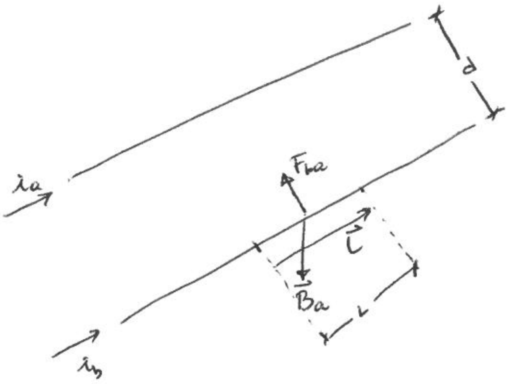
$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d \cos 45^\circ} \sqrt{i_1^2 + i_2^2}$$

$$= \dots = 1.89 \times 10^{-4}\,T \approx 190\,\mu T$$

$$\tan\varphi = \frac{B_1}{B_2}$$

$$\varphi = \tan^{-1}\frac{i_1}{i_2} = \dots = 25^\circ$$

FORCE BETWEEN TWO PARALLEL CURRENTS



$\vec{B}_a$ is the magnetic field at wire b produced by the current in wire a

$\vec{F}_{ba}$ is the resulting force acting on wire b because it carries a current in $\vec{B}_a$

$$B_a = \frac{\mu_0 i_a}{2\pi d} \quad \text{(direction: downwards at wire b)}$$

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

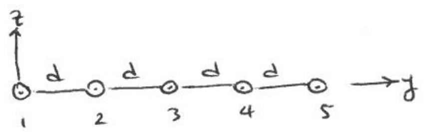
$$\vec{L} \perp \vec{B}_a \rightarrow F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d}$$

direction of $\vec{F}_{ba}$: $\vec{L} \times \vec{B}_a$

$\rightarrow \vec{F}_{ba}$ directed towards wire a

$\Rightarrow$ two wires with parallel currents attract each other (wires with antiparallel currents repel each other)

Example:



Force on the wires?

$d = 8\,cm$

$L = 10\,m$

$i = 3A \odot$

$$\vec{F}_1 = \frac{\mu_0 i^2 L}{2\pi}\left(\frac{1}{d} + \frac{1}{2d} + \frac{1}{3d} + \frac{1}{4d}\right)\hat{j}$$

$$= \ldots = 4.69 \times 10^{-4}\,N\,\hat{j}$$

$$\vec{F}_2 = \frac{\mu_0 i^2 L}{2\pi}\left(\frac{1}{2d} + \frac{1}{3d}\right)\hat{j}$$

$$= \ldots = 1.88 \times 10^{-4}\,N\,\hat{j}$$

$\vec{F}_3 = 0$ (symmetry)

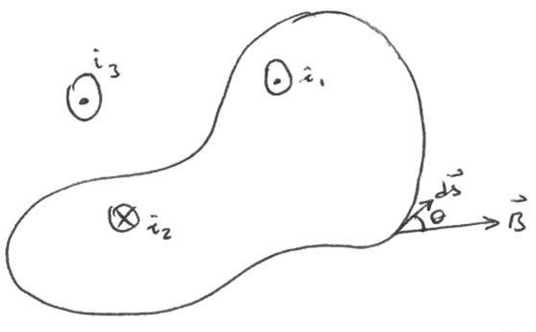
$\vec{F}_4 = -\vec{F}_2$; $\vec{F}_5 = -\vec{F}_1$

AMPERE'S LAW

Analogous to Gauss' Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

Closed loop



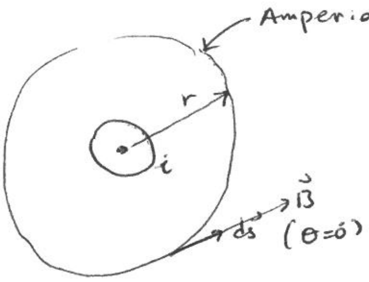
$$\vec{B} \cdot d\vec{s} = B \cos\theta \, ds$$

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = \mu_0 i_{enc}$$

$$i_{enc} = i_1 - i_2$$

$$\oint B \cos\theta \, ds = \mu_0 (i_1 - i_2)$$

Magnetic Field Outside a long Straight Wire with Current



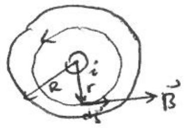
$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = B \oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i$$

$$\rightarrow B = \frac{\mu_0 i}{2\pi r} \quad \text{(outside straight wire)}$$

Magnetic Field Inside a long straight wire

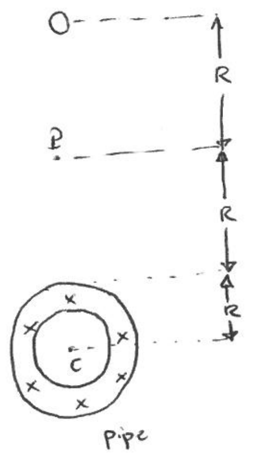
Uniformly distributed Current



$$\left.\begin{aligned} \oint \vec{B} \cdot d\vec{s} &= B \oint ds = B(2\pi r) \\ i_{enc} &= i \frac{\pi r^2}{\pi R^2} \end{aligned}\right) \quad B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$

$$\Rightarrow B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \quad \text{(inside straight wire)}$$

Example:



A long circular pipe with outside radius $R = 2.6\,cm$ carries a uniformly distributed current $i = 8\,mA$ into the page.

A wire runs parallel to the pipe at a distance $3R$ from center to center.

Find the current in the wire such that the net magnetic field at point P has the same magnitude as the net magnetic field at the center of the pipe but in the opposite direction.

Direction:

$B_c(2\pi 3R) = \mu_0 i_{wire}$

$$B_c = \frac{\mu_0 i_{wire}}{6\pi R}$$

$\underbrace{B_{P,wire}}_{\text{close}} > \underbrace{B_{c,wire}}_{\text{far from the wire}}$

$B_P = B_c$ ← only contribution from the wire

↗ both from the pipe and the wire

So, pipe's field should decrease it

→ current in the wire: ⊗

$$B_{P,pipe}(2\pi 2R) = \mu_0 i$$

$$B_{P,pipe} = \frac{\mu_0 i}{4\pi R} \qquad B_{P,wire} = -\frac{\mu_0 i_{wire}}{2\pi R}$$

$$\frac{\mu_0 i_{wire}}{6\pi R} = -\frac{\mu_0 i_{wire}}{2\pi R} + \frac{\mu_0 i}{4\pi R}$$

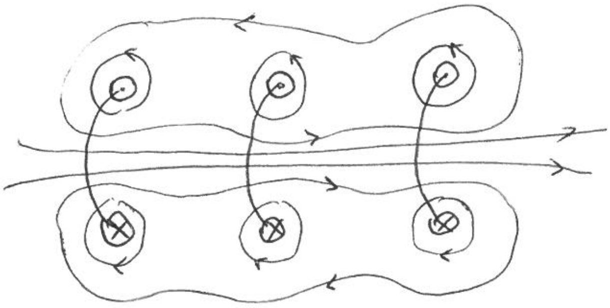
$$i_{wire}\left(\frac{1}{6} + \frac{1}{2}\right) = \frac{i}{4} \rightarrow i_{wire}\frac{4}{6} = \frac{i}{4} \Rightarrow i_{wire} = \frac{3}{8} i$$

$$= \dots = 3\,mA$$

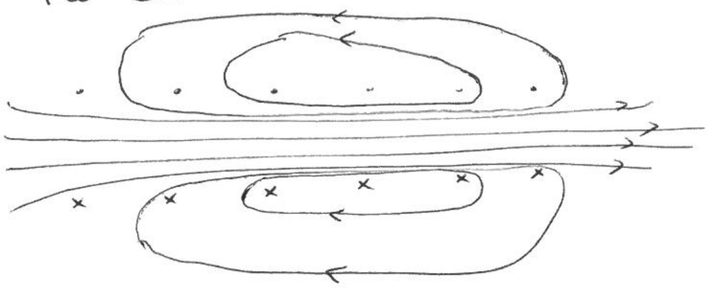
SOLENOIDS AND TOROIDS

Magnetic Field of a Solenoid

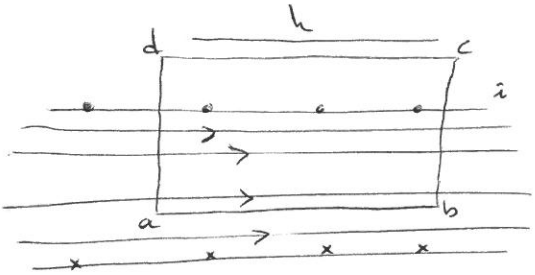
Solenoid: Long, tightly wound helical coil of wire (Assume $L \gg R$)



In an ideal solenoid (infinitely long and consists of tightly packed turns of square wire) the field inside the coil is uniform and parallel to the solenoid axis.



Ideal solenoid: The magnetic field outside the solenoid is zero



ideal

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

$$\oint \vec{B} \cdot d\vec{s} = \underbrace{\int_a^b \vec{B} \cdot d\vec{s}}_{Bh} + \underbrace{\int_b^c \vec{B} \cdot d\vec{s}}_{\substack{0\ (\theta = 90^\circ) \\ \vec{B} \perp d\vec{s}}} + \underbrace{\int_c^d \vec{B} \cdot d\vec{s}}_{\substack{0 \\ \text{No } \vec{B}}} + \underbrace{\int_d^a \vec{B} \cdot d\vec{s}}_{\substack{0 \\ \vec{B} \perp d\vec{s}}}$$

$$\Rightarrow \oint \vec{B}\, d\vec{s} = Bh$$

number of turns per unit length

$$i_{enc} = i(nh)$$

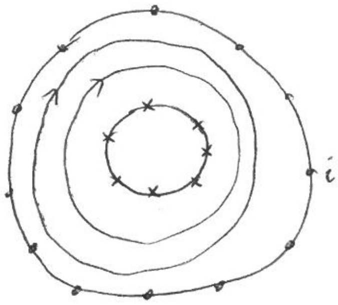
$$Bh = \mu_0 i n h$$

$$B = \mu_0 i n \quad \text{(ideal solenoid)}$$

→ the magnetic field magnitude B within a solenoid does not depend on the diameter or the length of the solenoid and B is uniform over the cross-section

Analogy: B: solenoid ⟷ E: parallel plates

Magnetic Field of a Toroid



Toroid: A closed-loop solenoid

$$B 2\pi r = \mu_0 i N \quad \leftarrow \text{total number of turns}$$

$$B = \frac{\mu_0 i N}{2\pi} \frac{1}{r}$$

← unlike the solenoid B is not uniform over the cross-section.

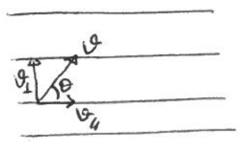
Example: An electron is shot into one end of a solenoid. As it enters the uniform magnetic field within the solenoid, its speed is $v = 800 m/s$ and its velocity vector makes an angle of $\theta = 30°$ with the central axis of the solenoid. The solenoid carries $i = 4A$ and has $N = 8000$ turns along its length.

How many revolutions does the electron make along its helical path within the solenoid by the time it emerges from the solenoid's other end?

$$|q| \vec{v}_\perp \times \vec{B} = \frac{mv^2}{r}$$

$$r = \frac{mv}{|q|B}$$

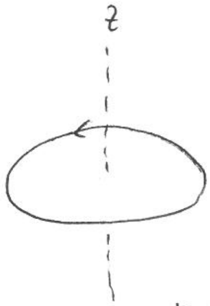
$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi m}{eB}$$



$$t = \frac{L}{v_\parallel} \qquad B = \mu_0 i n = \mu_0 i \frac{N}{L}$$

$$\frac{t}{T} = \underbrace{\left(\frac{L}{v\cos\theta}\right)}_{t} \frac{e\overbrace{\left(\mu_0 i \frac{N}{L}\right)}^{B}}{2\pi m} = \ldots = 1.6 \times 10^6$$

A Current-Carrying Coil as a Magnetic Dipole



$\vec{\tau} = \vec{\mu} \times \vec{B}$

magnetic dipole moment: $\mu = N i A$

$$B(z) = \frac{\mu_0 i R^2}{2(R^2+z^2)^{3/2}}$$

$$z \gg R: \quad B(z) \approx \frac{\mu_0 i R^2}{2 z^3} \Rightarrow \frac{\mu_0}{2\pi} \frac{N i \overbrace{(\pi R^2)}^{A}}{z^3} \qquad \vec{B} \parallel \vec{\mu}$$

$$\Rightarrow \vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3} \quad \text{(current carrying coil)}$$

A current carrying coil as a magnetic dipole:

1) it experiences a torque when placed in an external magnetic field.

2) it generates its own intrinsic magnetic field.

INDUCTION & INDUCTANCE

A current produces a magnetic field

$\updownarrow$

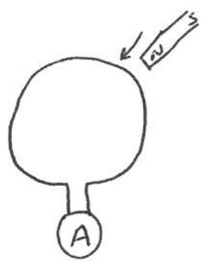
A magnetic field can produce an electric field that can derive a current

$\Rightarrow$ FARADAY'S LAW OF INDUCTION

(example: electroguitars)

TWO EXPERIMENTS

1st

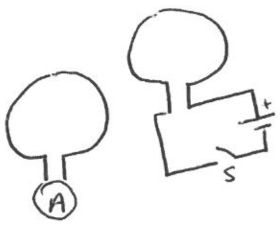


1) A current appears only if there is a relative motion between the loop and the magnet.
2) Faster motion produces a greater current.
3) N ↙ current direction X N →

   N ↙ current direction X S ↙

- Current: induced current
- Work done per unit charge to produce the current: induced emf
   - process: induction

2nd



At the switch's opening and closing only, not when the current is constant, a current is measured on the sensitive ampermeter.

The induced emf and induced current in these experiments are apparently caused when something changes — but what is that "something"?

FARADAY'S LAW OF INDUCTION

Changing the amount of magnetic field passing through the loop

→ an emf and a current can be induced.

This is proportional to the change of the number of magnetic field lines passing thragh the loop.

$\left(\Phi_E = \int \vec{E} \cdot d\vec{A} \right)$ $\quad \Phi_B = \int \vec{B} \cdot d\vec{A}$ magnetic flux through area A

Suppose that loop lies in a plane & $\vec{B}$ is $\perp$

$$B\,dA \cos 0^\circ = B\,dA$$

$$\underbrace{\Phi_B}_{\text{unit}} = BA$$

$\hookrightarrow 1\ T\,m^2 = 1\,Wb = 1$ Weber

Faraday's Law: The magnitude of the emf $\mathcal{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

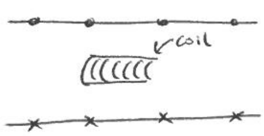
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

We can change the flux by:

1) Changing the magnitude of the B
2) Changing the total area of the coil or the area that lies within B
3) Changing the angle between the magnetic field direction and the plane of the coil

Example: Induced emf in a coil due to a solenoid



$n = 220$ turns/cm

$i = 1.5A$

diameter $D = 3.2cm$

The current in the solenoid is reduced to zero at a steady rate in 25 ms

Coil's diameter $d = 2.1cm$

$N = 130$ turns

What is the magnitude of emf that is induced while the current in the solenoid is changing?

$$\Phi_B = BA$$

$$B = \mu_0 i n$$

$$\mathcal{E} = -\overset{130}{N} \frac{d\Phi_B}{dt}$$

$$A = \frac{1}{4}\pi d^2 \left(= 3.464 \times 10^{-4} m^2 \right)$$

$$n = 220 \text{ turns/cm} = 22000 \text{ turns/m}$$

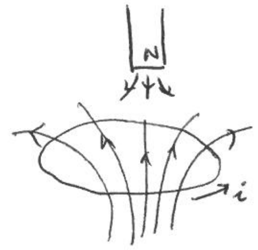
$$\Phi_{B,i} = BA = (\mu_0 i n) A = \ldots = 1.44 \times 10^{-5} Wb$$

$$\frac{d\Phi_B}{dt} = \frac{\Delta \Phi_B}{\Delta t} = \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} = \frac{(0 - 1.44 \times 10^{-5}) Wb}{25 \times 10^{-3} s} = \ldots = -5.76 \times 10^{-4} V$$

$$\mathcal{E} = N \frac{d\Phi_B}{dt} = 130 \times 5.76 \times 10^{-4} V = 7.5 \times 10^{-2} V = 75 mV$$

Lenz's Law

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.



* Opposition to Pole movement
* opposition To the flux change

(Also refer to the figure on the next page)

Example:

$r = 0.2\,m$ $\quad B = (4t^2+2t+3)\,T$

$\mathcal{E}_{bat} = 2V$

$R = 2.0\,\Omega$

a) $t = 10 \rightarrow \mathcal{E}_{ind} = ?$

$$\mathcal{E}_{ind} = \frac{d\Phi_B}{dt} = \frac{dBA}{dt} = A\frac{dB}{dt}$$

$$A = \frac{1}{2}\pi r^2$$

$$\rightarrow \mathcal{E}_{ind} = A\frac{dB}{dt} = \frac{\pi r^2}{2}\frac{d}{dt}(4t^2+2t+3) = \frac{\pi r^2}{2}(8t+2)$$

$$t = 10s \rightarrow \mathcal{E}_{ind} = 5.152V \simeq 5.2V$$

i_{ind}, $\mathcal{E}_{ind}$

b.) $i\ (t=10)$?

$$i = \frac{\mathcal{E}_{net}}{R} = \frac{\mathcal{E}_{ind} - \mathcal{E}_{bat}}{R} = \frac{5.152V - 2V}{2\Omega} = 1.58A \simeq 1.6A$$

i

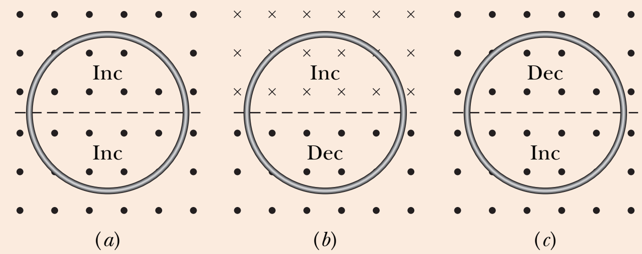
nears the loop with its magnetic field $\vec{B}$ directed *downward*, the flux through the loop increases. To oppose this increase in flux, the induced current i must set up its own field $\vec{B}_{ind}$ directed *upward* inside the loop, as shown in Fig. 30-5*a*; then the upward flux of field $\vec{B}_{ind}$ opposes the increasing downward flux of field $\vec{B}$. The curled–straight right-hand rule of Fig. 29-21 then tells us that i must be counterclockwise in Fig. 30-5*a*.

Note carefully that the flux of $\vec{B}_{ind}$ always opposes the *change* in the flux of $\vec{B}$, but that does not always mean that $\vec{B}_{ind}$ points opposite $\vec{B}$. For example, if we next pull the magnet away from the loop in Fig. 30-4, the flux Φ_B from the magnet is still directed downward through the loop, but it is now decreasing. The flux of $\vec{B}_{ind}$ must now be downward inside the loop, to oppose the *decrease* in Φ_B, as shown in Fig. 30-5*b*. Thus, $\vec{B}_{ind}$ and $\vec{B}$ are now in the same direction.

In Figs. 30-5*c* and *d*, the south pole of the magnet approaches and retreats from the loop, respectively.

CHECKPOINT 2

The figure shows three situations in which identical circular conducting loops are in uniform magnetic fields that are either increasing (Inc) or decreasing (Dec) in magnitude at identical rates. In each, the dashed line coincides with a diameter. Rank the situations according to the magnitude of the current induced in the loops, greatest first.



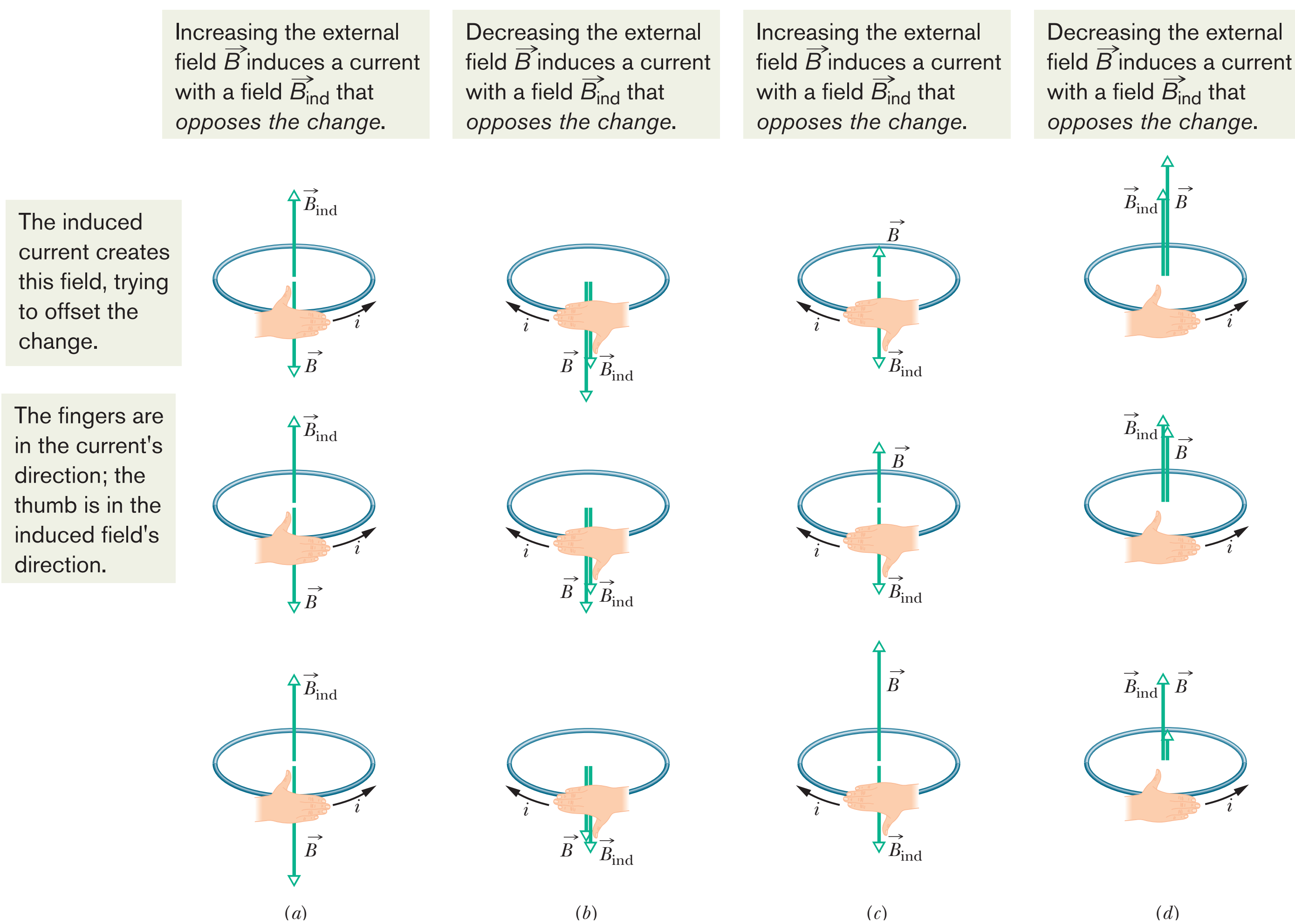
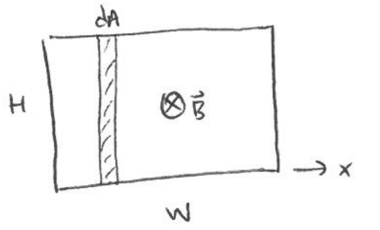


Fig. 30-5 The direction of the current i induced in a loop is such that the current's magnetic field $\vec{B}_{ind}$ opposes the *change* in the magnetic field $\vec{B}$ inducing i. The field $\vec{B}_{ind}$ is always directed opposite an increasing field $\vec{B}$ (a, c) and in the same direction as a decreasing field $\vec{B}$ (b, d). The curled–straight right-hand rule gives the direction of the induced current based on the direction of the induced field.

Example:



$B = 4t^2x^2$

$W = 3m$

$H = 2m$

$\mathcal{E}$ $(t = 0.10s)$?

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\,dA = \int BH\,dx = \int 4t^2x^2H\,dx$$

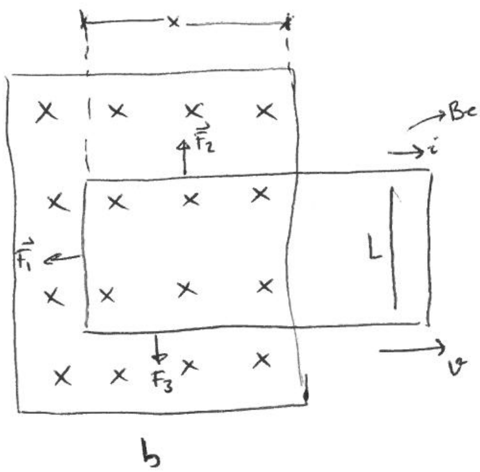
$$\Phi_B = 4t^2H \int_0^3 x^2\,dx = 4t^2H \left[\frac{x^3}{3}\right]_0^3 = 72t^2$$

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(72t^2)}{dt} = 144t$$

$t = 0.10s \rightarrow \mathcal{E} \approx 14V$ $\quad i: \circlearrowleft$

Induction and Energy Transfers

Energy is always transferred to thermal energy during the process because of the electrical resistance of the loop (unless the loop is superconducting)



→Because the flux is decreasing

$P = Fv$ Rate at which you do work.

$\Phi_B = BA = BLx$

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt}BLx = BL\frac{dx}{dt} = BLv$$

$$i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$$

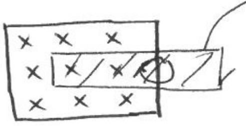
$\vec{F}_1 = i\vec{L} \times \vec{B}$ $\qquad \vec{F}_2$ cancels $\vec{F}_3$

$|\vec{F}| = |\vec{F}_1| = iLB\sin 90° = iLB$

$$F = \frac{B^2L^2v}{R}$$

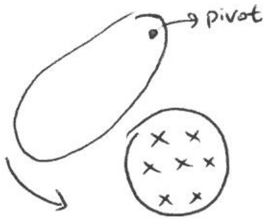
$$P = Fv = \frac{B^2L^2v^2}{R} \quad \leftrightarrow \quad P = i^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$$

Eddy Currents



→ solid conducting plate out of a magnetic field.

e^-'s all the way  whirlpool ("eddy")

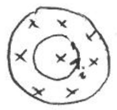


Each time it passes, a portion of its mechanical energy is transferred to its thermal energy.

After several swings, no mechanical energy remains and the warmed-up plate just hangs from its pivot.

Induced Electric Fields

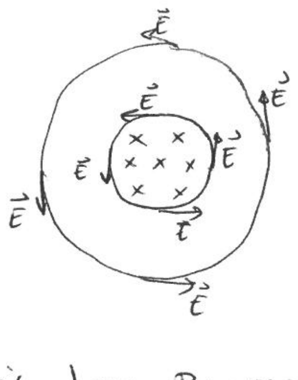
Copper ring of radius r



 i → So there must be an $\vec{E}$

↑ induced electric field

⇒ A changing magnetic field produces an electric field.



Faraday's Law Re-visited:

$$W = \mathcal{E} q_0 \,,\quad W = \int \vec{F}\, d\vec{s} = (q_0 E)(2\pi r)$$

$$\Rightarrow \mathcal{E} = 2\pi r E$$

$$W = \oint \vec{F}\, d\vec{s} = q_0 \oint \vec{E}\, d\vec{s} \rightarrow \mathcal{E} = \oint \vec{E}\, d\vec{s}$$

$$\left.\begin{array}{l} \mathcal{E} = \oint \vec{E}\, d\vec{s} \\ \mathcal{E} = -\dfrac{d\Phi_B}{dt} \end{array}\right\} \rightarrow \boxed{\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$

FARADAY'S LAW