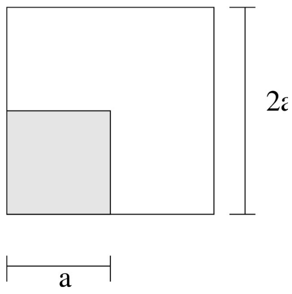
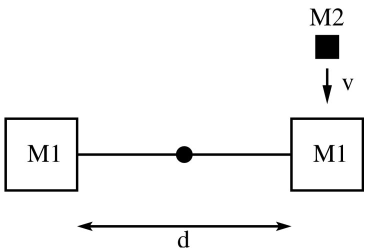
Automotive Engineering, FIZ137 2nd Midterm Exam | Instructor: Emre S. Taşcı | 18 / 12 / 2015

Choose 5 questions among the 7



1) Calculate the position of the center of mass of the system when a quarter (¼th) of a square of side 2a is removed from one of its edges. (Assume uniform mass distribution)



2) Two boxes, each having a mass of M_1 are attached to a massless rod of length d. The rod is able to rotate around a pivot attached at its center.

At t=0, as the rod is situated parallel to the ground, a particle of mass M_2 hits the box on the right edge with a speed of v and sticks to it.

a) What is the angular velocity ω (along with its direction) of the system just after the collision? (in terms of M_1, M_2, v and d) (15 points)

b) How many radians does the system rotate before it stops momentarily? (in terms of M_1, M_2, v, d and g) (5 points)

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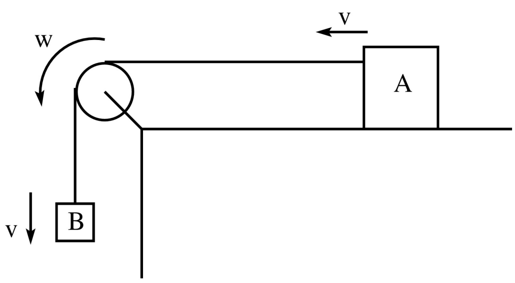
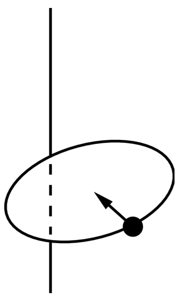
3) A pulley with radius R and moment of inertia I spins on a frictionless axle. A rope on the pulley holds two blocks of m_A and m_B. The coefficient of kinetic friction between block A and the table is μ_k. When the system is released from rest, block B descends (goes downwards).

Calculate the speed of the block B in terms of the distance d that it has traversed and g, m_A ,m_B, I, R, μ_k.





4) A disk is of radius R and mass M_d is being rotated with an angular velocity of ω_0 around its edge, with a bug of mass m_b standing on its outer edge.

Find the new angular speed of the system and the change in kinetic energy when the bug walks

a) to the center (10 points)

b) to the connection point (10 points)

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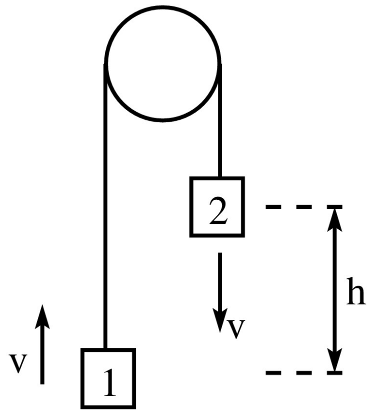
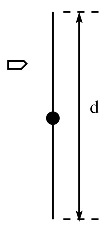
5) A pulley-mass system consists of two blocks of masses m_1 and m_2 ($m_2>m_1$); a pulley of radius R_0, mass M_P; and a string of negligable mass. The system is released from rest, and the friction is negligable. The pulley rotates around a pivot driven through its center of mass.

Find the expression for the speed v of either block after it has moved a distance h (in terms of m_1, m_2, g and h).

(For practicality, you can assume that at the beginning, m_2 is positioned h distance above m_1)





6) A uniform rod of length d and mass M_{rod} is attached from its center. A bullet of mass m_b hits and exists the rod at a point $\frac{d}{4}$ from the center.The bullet's initial and final speeds are v_i and v_f ($v_i>v_f$).

What is the rod's angular velocity after the collision? (In terms of M_{rod}, m_b, d)

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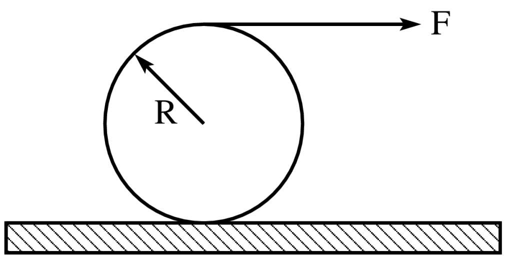
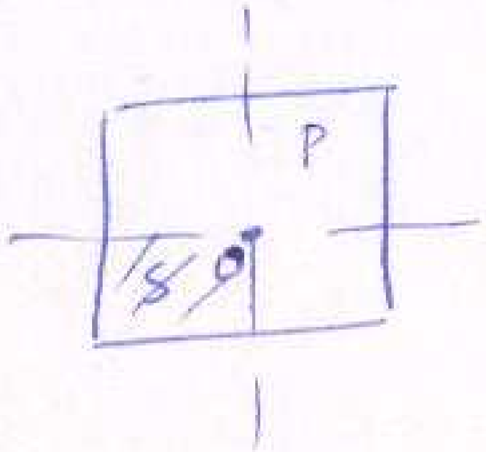
7) A rope tied to a pulley (of radius R) is being pulled by a force F, thereby rolling the pulley smoothly. The coefficient of friction between the pulley and the ground is μ, and the mass of the circular pulley is m.

a) Calculate the translational acceleration *a* of the pulley. (In terms of m and F) (15 Points)
b) Calculate the frictional force *f* between the pulley and the ground. (In terms of m and F) (5 Points)



Bonus Question: When we are considering a rolling wheel on a surface with friction present between them, when should we use which coefficient of friction (kinetic (μ_k) or static (μ_s)) and why? (10 Points)

1) $P = ?$ m_p



$S: \left(-\frac{a}{2}, -\frac{a}{2}\right)$, $d = \frac{a\sqrt{2}}{2}$ m_s

$C: (0,0)$ $m_s + m_p$

$$X_{s+p,x} = \frac{m_s x_s + m_p x_p}{m_s + m_p} = 0 \longrightarrow \frac{-\frac{a}{2} m_s + m_p x_p}{m_s + m_p} = 0$$

$\Rightarrow x_p = \frac{a}{2} \frac{m_s}{m_p} = \frac{a}{2} \cdot \frac{m}{3m} = \frac{a}{6}$

y_p = similar/symmetric to x_p

$= \frac{a}{6}$

$\Rightarrow \boxed{(x_p, y_p) = \left(\frac{a}{6}, \frac{a}{6}\right)}$

$\rightarrow$ distance $= \frac{a\sqrt{2}}{6}$ $(\theta = 45°)$

2) a) $L_i = L_f$

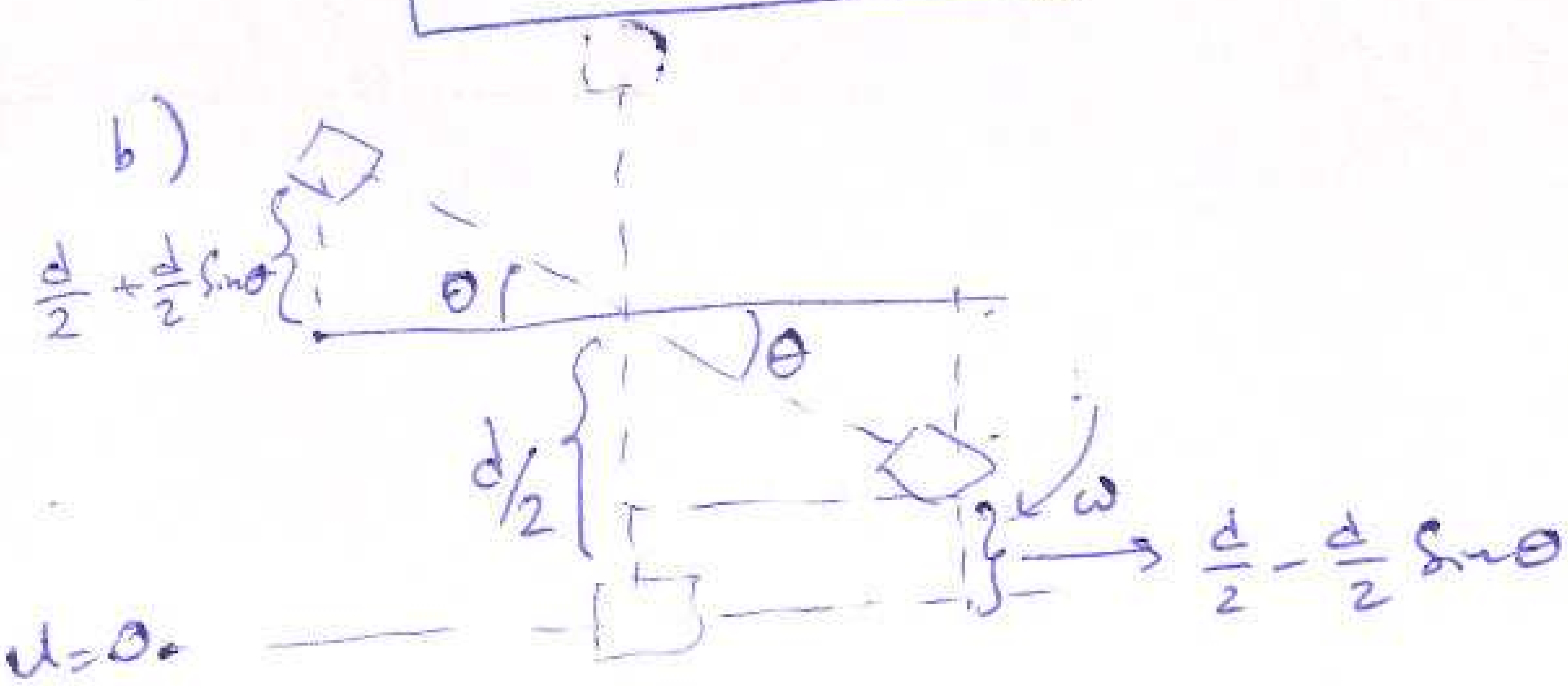
$M_2 v \frac{d}{2} = I_{system} \cdot \omega$

$I_{system} = \sum_{i=1}^{3} m_i r_i^2 = \frac{M_1 d^2}{4} + \frac{M_1 d^2}{4} + M_2 \frac{d^2}{4} = \frac{d^2}{4}(2M_1 + M_2)$

$\rightarrow M_2 v \frac{d}{2} = \frac{d^2}{4_2}(2M_1 + M_2)\omega$

$\rightarrow \boxed{\omega = \frac{2M_2 v}{(2M_1 + M_2) d}}$

b)



$U_i + K_i = U_f + K_f$

$\left(2M_1 \cdot g\frac{d}{2} + M_2 g \frac{d}{2}\right) + \frac{1}{2} I_{sys}\omega^2 = M_1 g\left(\frac{d}{2} - \frac{d}{2}\sin\theta\right)$
$+ M_2 g\left(\frac{d}{2} - \frac{d}{2}\sin\theta\right)$
$+ M_1 g\left(\frac{d}{2} + \frac{d}{2}\sin\theta\right)$

$$\theta = \sin^{-1}\left[-\frac{M_2 v^2}{gd(2M_1 + M_2)}\right]$$

3) $U_1 = m_B g d$, work done by friction: $W = \mu_k m_A g d$

$K_1 = 0$

$K_2 = \frac{1}{2} m_A v^2 + \frac{1}{2} m_B v^2 + \frac{1}{2} I\omega^2$, $\omega = \frac{v}{R}$

$= \frac{1}{2}\left(m_A + m_B + \frac{I}{R^2}\right)v^2$,

$U_2 = 0$

$U_1 + K_1 \overset{+W}{=} U_2 + K_2 \Rightarrow m_B g d - \mu_k m_A g d = \frac{1}{2}\left(m_A + m_B + \frac{I}{R^2}\right)v^2 \Rightarrow v = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$

4) $I_{disk} = I_{cm} + M_d R^2 = \frac{1}{2} M_d R^2 + M_d R^2 = \frac{3}{2} M_d R^2$

$I_{bug}(d_{bug}) = M_b \cdot d_b^2$; d_b: distance to the axis of rotation $\begin{cases} \text{Initial} : d_b = 2R \\ \text{Center} : d_b = R \\ \text{final} : d_b = 0 \end{cases}$

Initial: $I_{system_{2R}} = \frac{3}{2} M_d R^2 + M_b (2R)^2 = R^2 \left(\frac{3}{2} M_d + 4 M_b \right)$

Center:

$$I_{system_R} = \frac{3}{2} M_d R^2 + M_b R^2 = R^2 \left(\frac{3}{2} M_d + M_b \right)$$

$$I_{system_{2R}} \cdot \omega_0 = I_{system_R} \cdot \omega_{d=R} \Rightarrow \omega_{d=R} = \frac{R^2 \left(\frac{3}{2} M_d + 4 M_b \right)}{R^2 \left(\frac{3}{2} M_d + M_b \right)}$$

$$\boxed{\Delta \omega = \omega_{d=R} - \omega_0}$$

$$K = \frac{1}{2} I \omega^2 \rightarrow \boxed{\Delta K = \frac{1}{2} \left(I_{S_R} \omega_R^2 - I_{S_{2R}} \omega_0^2 \right)}$$

Final:

$$I_{system_0} = \frac{3}{2} M_d R^2$$

$$\omega_{d=0} = \frac{R^2 \left(\frac{3}{2} M_d + 4 M_b \right)}{R^2 \cdot \frac{3}{2} M_d}$$

$$\boxed{\begin{array}{l} \Delta \omega = \omega_{d=0} - \omega_0 \\ \Delta K = \frac{1}{2} \left(I_{S_0} \omega_{d=0}^2 - I_{S_{2R}} \omega_0^2 \right) \end{array}}$$

5)

$E_i = U = m_2 g h$

$E_f = U + K_{trans} + K_{rot} = m_1 g h + \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2} I \omega$

$v = R_0 \omega$, $\omega = \frac{v}{R_0}$, $I = \frac{1}{2} M_p R_0^2$

$\rightarrow E_f = m_1 g h + \frac{1}{2}(m_1 + m_2 + \frac{1}{2} M_p)$

$E_i = E_f$

$m_2 g h = m_1 g h + \frac{1}{2}(m_1 + m_2 + \frac{1}{2} M_p) v^2$

$$\boxed{v = \sqrt{\frac{2(m_2 - m_1) g h}{m_1 + m_2 + \frac{1}{2} M_p}}}$$

$L_i = L_f$

6) $m_b v_i \cdot \frac{d}{4} = m_b \cdot v_f \cdot \frac{d}{4} + I_{com} \cdot \omega$

$\omega = \frac{m d}{4}(v_i - v_f) / I_{com} = \frac{m_b \cancel{d}}{\cancel{4}} \cdot \frac{\cancel{12}^3}{M_{rd} d^{\cancel{2}}}(v_i - v_f) = \boxed{\frac{3 m_b}{M_{rd} d}(v_i - v_f)}$

$I_{com} = \frac{1}{12} M_{rod} d^2$

7) a.) $ma = F - f_s$

$f_s = F - ma$

$I\alpha = \frac{Ia}{R} = \frac{mR^2}{2}\frac{a}{R} = R(F + f_s) = R(2F - ma)$

$\frac{mR}{2} a = 2FR - mRa$

$\frac{3m\cancel{R}a}{2} = 2F\cancel{R} \longrightarrow \boxed{a = \frac{4F}{3m}}$

b.) $ma = F - f_s$

$m \cdot \frac{4F}{3m} = F - f_s \longrightarrow \boxed{f_s = -\frac{F}{3}}$

Bonus: $\mu_k \rightarrow$ slips (translate)
$\mu_s \rightarrow$ rolls (rotation)