SOLUTIONS

Automotive Engineering, FIZ138 ~~2nd Midterm~~ FINAL Exam | Instructor: Emre S. Taşcı | 27 / 5 / 2016

Name: Number: Signature:

$$e=1.6\times10^{-19}\,C;\ \epsilon=9\times10^{-12}\frac{C^2}{N\,m^2};\ \mu_0=4\pi\times10^{-7}\frac{Wb}{A\,m};\ m_e=9.1\times10^{-31}\,kg$$

Mark the 5 questions you want to be evaluated from (each question is worth 20 points):

Mark:	Q1	Q2	Q3	Q4	Q5	Q6	Q7
Grade:							

Q1) a) What is the unit of E/B ratio in terms of basic SI units m, kg, s, A? (5 points)

b) Show that the following are all equivalent to Henry: (5 points)

i) J/A2 ii) Ωs iii) Wb /A

c) If a current of 8 mA runs through a wire for 10 seconds, how many electrons have passed through the circuit? (5 points)

d) Which of the following action(s) will induce a current in a loop of wire in a **uniform** magnetic field? (5 points)

i) Rotating the loop about an axis parallel to the field
ii) Moving the loop within the field
iii) Decreasing the strength of the field

a.) $F = qE$, $F = qvB$ $\longrightarrow \left[\frac{E}{B}\right] = [v] = \frac{m}{s}$

b.) i) $U = \frac{1}{2}Li^2 \rightarrow [L] = \frac{J}{A^2}$

ii) $\mathcal{E} = -L\frac{di}{dt} \rightarrow (V = A.\Omega) = [L]\frac{A}{s} \Rightarrow [L] = \Omega s$

$V = I.R$

iii) $\mathcal{E} = -L\frac{di}{dt} = -\frac{d\Phi_B}{dt} \rightarrow \frac{Wb}{s} = [L]\frac{A}{s} \Rightarrow [L] = \frac{Wb}{A}$

c.) $q = it = 8\times10^{-3}\times10\ C = 8\times10^{-2}\ C$

$1.6\times10^{-19}\ C \quad 1e^-$

$8\times10^{-2}\ C \quad x$

$\rightarrow \#_{e^-} = \frac{8\times10^{-2}}{1.6\times10^{-19}} = 5\times10^{17}$ electrons.

d) Only "decreasing the strength of the field" induces a current.

Q2) A balloon, coated with a conducting material is charged 5C and air is being pumped into. Suppose that it is always of spherical shape and its radius with respect to time is given by:

$r(t)=\alpha t$ where $\alpha=5\times10^{-2} m/s$.

a) Calculate the electric field at d=1m from its center at t=1s and t=3s (15 points)

b) Faraday's Law can also be written in terms of electric field:

$\oint \vec{E}\,d\vec{s}=-\frac{d\Phi_B}{dt}$ which says that a changing magnetic field induces an electric field.

There is a similar law (Maxwell's Law of Induction) that relates the induced magnetic field to the change in the electric field:

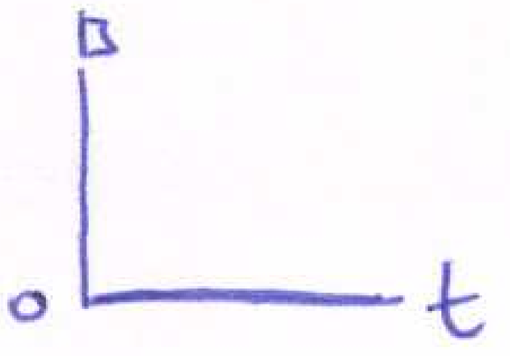
$$\oint \vec{B}\,d\vec{s}=\mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

For this example, roughly plot the induced magnetic field at a distance of d=1m from the center, with respect to time from t=0 to t=5s. (5 points)

a.) At both t=1s and t=3s, the charge inside the Gaussian surface is same (i.e. 5C)

$$E(t=1s)=E(t=3s)=\frac{5C}{9\times10^{-12}\,C^2/Nm^2\; 4\pi(1m)^2}=0.044\times10^{12}\,N/C=4.4\times10^{10}\,N/C$$

b) $\Phi_E=\oint E\cdot dA=\frac{Q_{in}}{\epsilon_0}$: $Q_{in}=\text{constant}\rightarrow\Phi_E\text{ const.}\rightarrow\frac{d\Phi_E}{dt}=0\Rightarrow B=0$



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Q3) Two capacitors with capacitances C_1 = 4μF and C_2=6μF are connected in parallel to a 120V emf device and after some long time, they are disconnected from the source and each other. If they are now connected positive plate to negative plate and negative plate to positive plate, calculate the resulting charge on each capacitor (20 Points)

Initially: $q_1 = C_1 V = 4\mu F\ \cancel{1}20V = 480\mu C$

$q_2 = 6\mu \times 120 = 720\mu C$

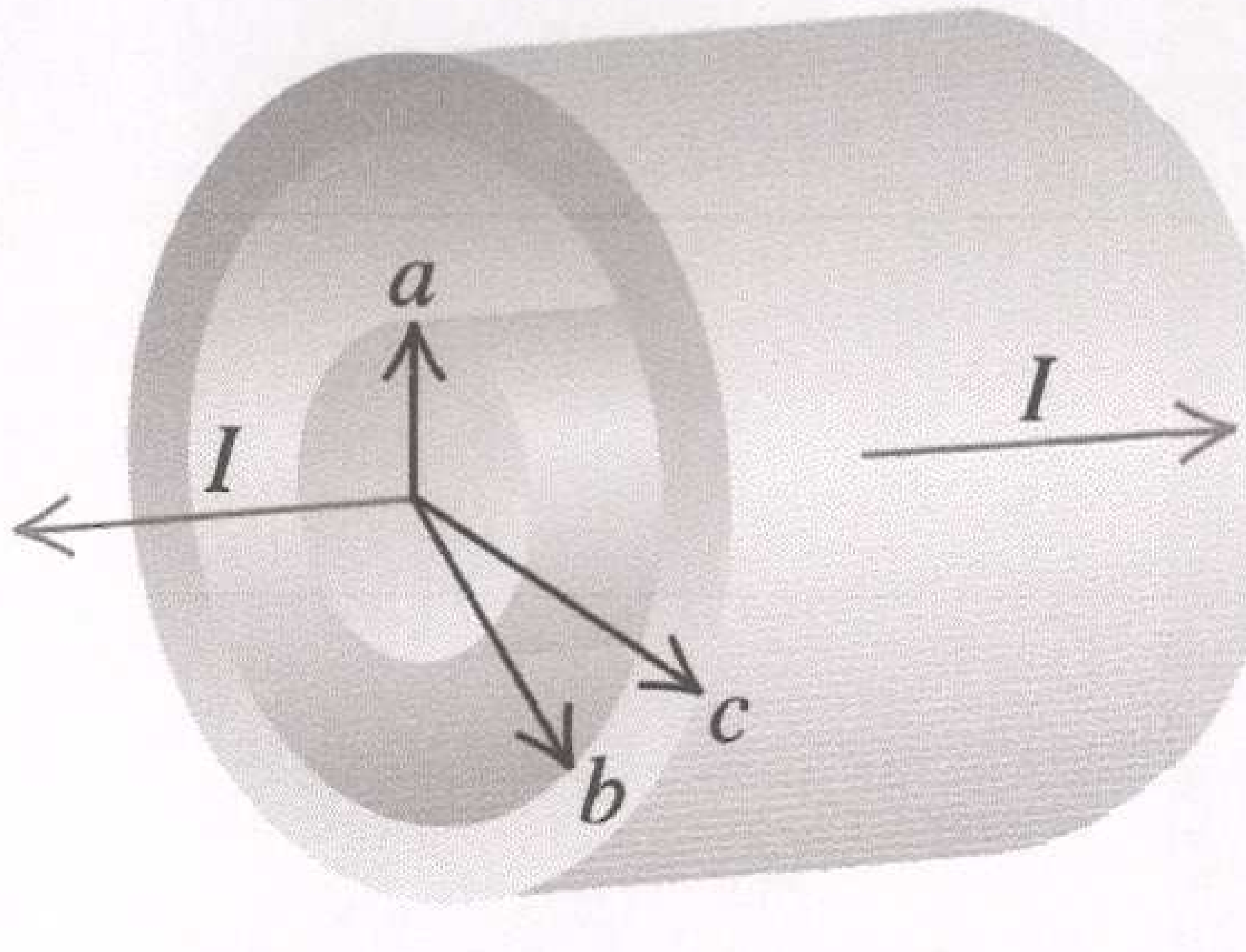
$q_{tt} = q_1 - q_2 = 240\mu C$

$\cancel{4} V' = \frac{q_{tt}}{C_{tt}} = \frac{240\mu C}{10\ \mu F} = 24V$

$q_1' = C_1 V' = 4\mu \times 24 = 96\ \mu C$

$q_2' = C_2 V' = 6\mu \times 24 = 144\mu C$

Q4) A solid conductor with radius a is surrounded by a conducting cylinder with inner radius b and outer radius c. The central conductor and the cylinder carry equal currents I in the opposite directions and the currents are distributed uniformly over the cross sections of each conductor.



Derive an expression for the magnitude of the magnetic field for:

a) $r<a$ (points inside the central conductor)
b) $a<r<b$ (points between the two conductors)
c) $r>c$ (points outside the cylinder)

a) inner : $J = \frac{I}{\pi a^2}$

$r<a$: $\oint \vec{B}\, d\vec{\ell} = \mu_0 I_{enc} = \mu_0 J \pi r^2$

$\vec{B} \parallel d\vec{\ell}$ $\quad 2\pi \cancel{r} B = \mu_0 \frac{I}{\cancel{\pi} a^2} \cancel{\pi} r^{\cancel{2}} \Rightarrow B = \frac{\mu_0 I r}{2\pi a^2}$

b) $a<r<b$ $\quad \oint B\, d\ell = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi r}$

c) $r>c$; $\quad \oint \vec{B}\, d\vec{\ell} = \mu_0 (I - I) = 0 \Rightarrow \underline{B = 0}$

Name:

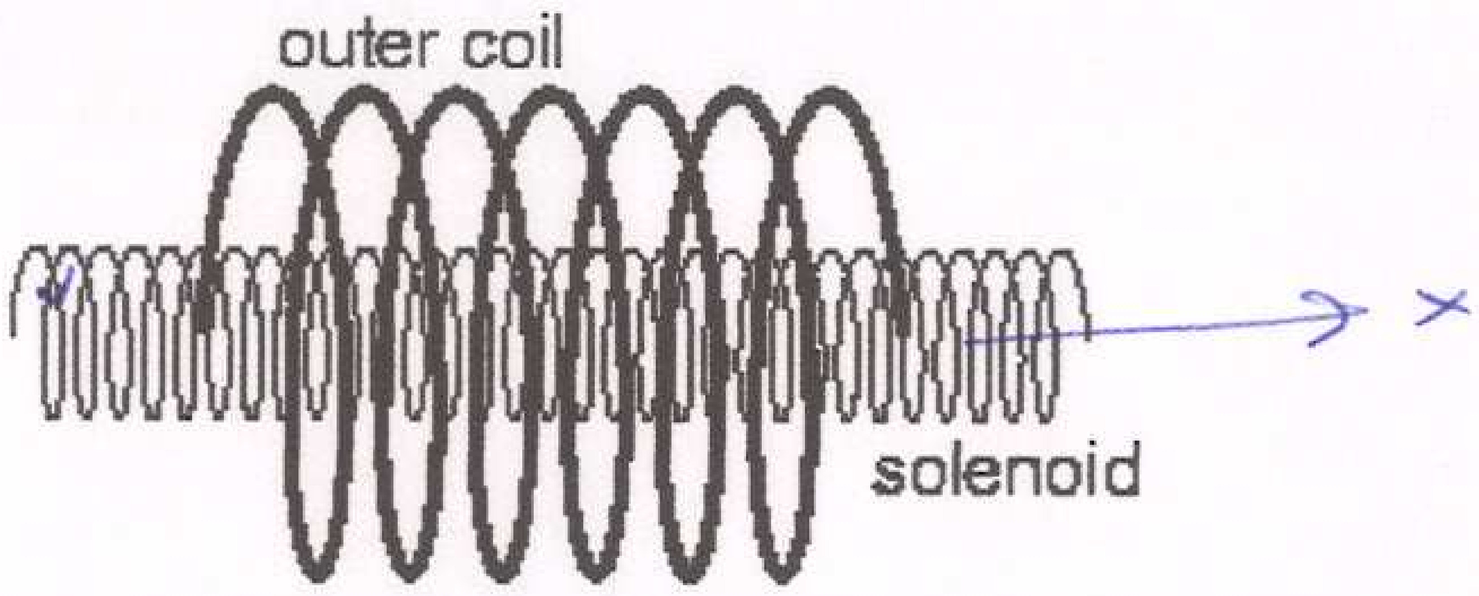
Q5) A very long solenoid of radius r made from n_1 turns of wire per unit length carries a time dependent current $I = I_0 e^{-\alpha t}$ where α is a positive constant.

There is another circular coil outside with N_2 turns and it is far away from the ends of the solenoid.

The coil with N_2 turns has a resistance of R.

a) Write down the time dependent magnetic field inside the solenoid. (4 points)
b) Calculate the total flux on the circular coil. (4 points)
c) Calculate the emf induced on the circular coil. (4 points)
d) Determine the direction of the induced current on the circular coil. (4 points)
e) Show that the magnitude of the induced current on the circular coil is given by:

$$I_{ind} = \frac{n_1 N_2 \alpha \mu_0 I_0 e^{-\alpha t} \pi r^2}{R} \quad \text{(4 points)}$$



a.) $B = \mu_0 . n_1 . I = \mu_0 n_1 I_0 e^{-\alpha t}$

b) $\Phi = BA = \mu_0 n_1 I_0 e^{-\alpha t} \pi r^2$

c) $\mathcal{E} = -N_2 \frac{d\Phi}{dt} = -N_2 \alpha \mu_0 n_1 \underbrace{I_0 e^{-\alpha t}}_{I} \pi r^2$

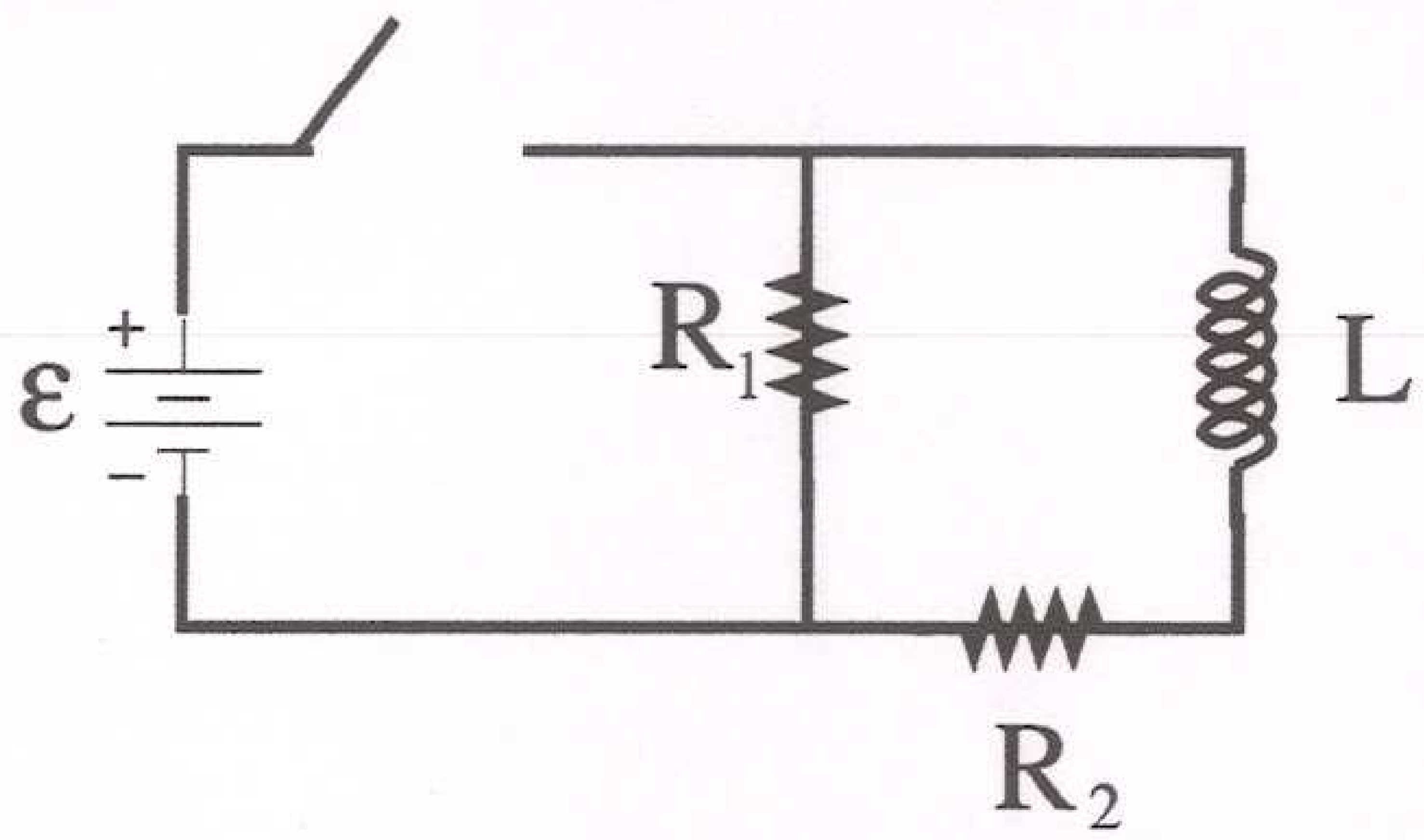
d) Same as I (because I is decreasing)

e) $\mathcal{E} = IR \rightarrow I = \frac{|\mathcal{E}|}{R}$

Q6) In the circuit drawn below, $\varepsilon = 60V$, $R_1 = 40\Omega$, $R_2 = 25\Omega$, $L = 0.3H$. The switch is closed at t = 0.

What is the current and the potential difference of the inductor L:

a) just after the switch is closed? (7 points)
b) after a very long time? (7 points)
c) at any time? (6 points)



a) $i_L(t=0) = 0$

$\mathcal{E}_L = -\mathcal{E}$

b.) $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = 15.4\Omega \rightarrow i = \frac{\mathcal{E}(R_1+R_2)}{R_1 R_2} = 3.9A$

$i_1 = \mathcal{E}/R_1$, $i_2 = i - i_1$

$i_2 = \frac{60.65}{40.25} - 60/40$

$i_2 = 2.4A$

$\mathcal{E}_L = 0$

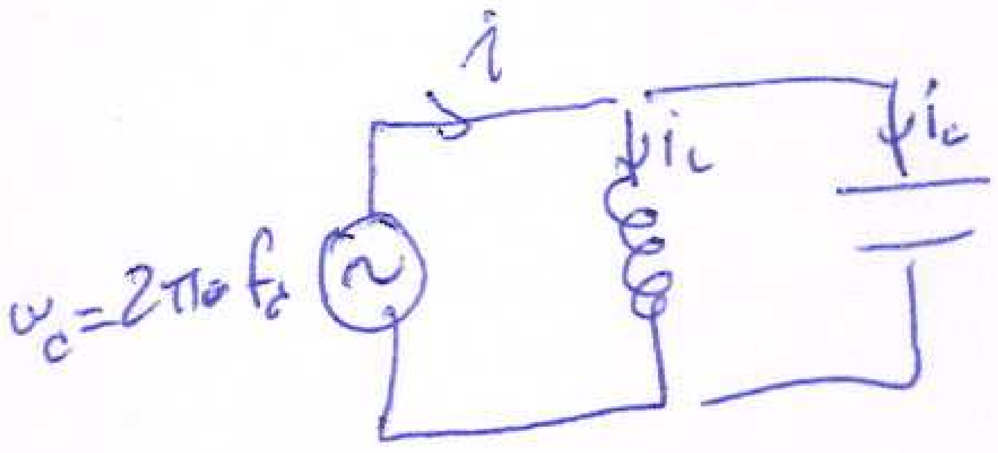
c.) $i(t) = \frac{\mathcal{E}}{R_2}\left(1 - e^{-t/L/R_2}\right)$

Name:

Q7) a) What is the impedance Z of a circuit section consisting of an inductor L = *4.96* H and a capacitor C = 10 μF connected **in parallel**, driven at a frequency f = 50Hz ? (10 points)

b) Draw the phasors diagram for V, I_C & I_l (5 points)

c) At which angular frequency does the system enters into resonance? (5 points)



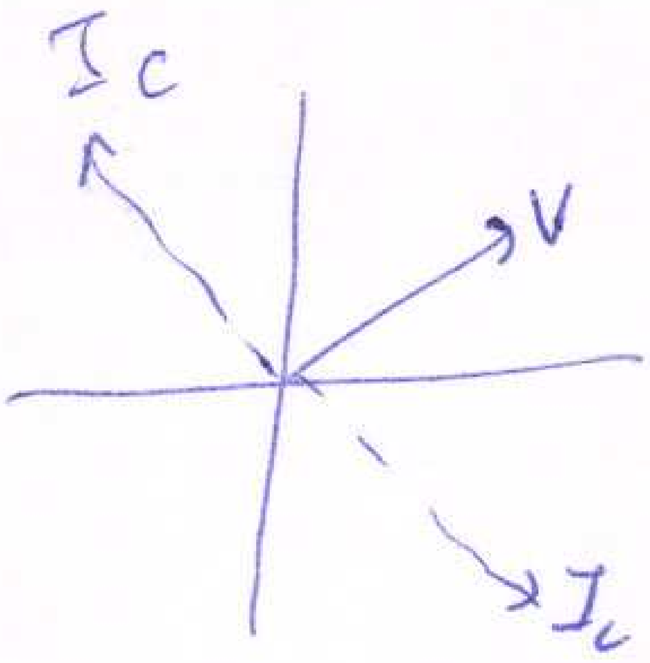
$V = V_L = V_C$

$i = i_L + i_C$

$i_L = \frac{V}{\omega_d L} \rightarrow$ lags V by $\pi/2$

$i_C = \omega_d C V \Rightarrow$ leads V by $\pi/2$

b.)



$\omega_d = 2\pi\, 50 = 314.16\, rad/s$

$I = I_C - I_L$

$I = V\omega_d C - V\frac{1}{\omega_d L}$

$I = \frac{V}{Z}$

$\frac{1}{Z} = \omega_d C - \frac{1}{\omega_d L}$

a.) $Z = \frac{\omega_d L}{\omega_d^2 C L - 1} = 400.3\ \Omega$

c.) $\omega_d^2 CL = 1 \rightarrow \omega_d = \frac{1}{\sqrt{CL}} = 141.99\ rad/s$