Nudged elastic band (NEB) approaches to locating transition states are efficient alternatives to evaluating the PES on a uniform grid over some coordinates of interest. The implementation in autodE follows that in [Henkelman and H. Jónsson, J. Chem. Phys. 113, 9978 (2000)]

For an image i in the nudged elastic band

$$\boldsymbol{\tau}_{i} \begin{cases} \boldsymbol{\tau}_{i}^{+} & \text{if } V_{i-1} < V_{i} < V_{i+1} \\ \boldsymbol{\tau}_{i}^{-} & \text{if } V_{i+1} < V_{i} < V_{i-1} \\ \boldsymbol{\tau}_{i}^{+} \Delta V_{i}^{max} + \boldsymbol{\tau}_{i}^{-} \Delta V_{i}^{min} & \text{if } V_{i-1} < V_{i+1} \\ \boldsymbol{\tau}_{i}^{+} \Delta V_{i}^{min} + \boldsymbol{\tau}_{i}^{-} \Delta V_{i}^{max} & \text{if } V_{i+1} < V_{i-1} \end{cases}$$

$$(1)$$

where

$$\tau_i^+ = \boldsymbol{x}_{i+1} - \boldsymbol{x}_i
\tau_i^- = \boldsymbol{x}_i - \boldsymbol{x}_{i-1}$$
(2)

and

$$\Delta V_i^{max} = \max(|V_{i+1} - V_i|, |V_{i-1} - V_i)$$

$$\Delta V_i^{min} = \min(|V_{i+1} - V_i|, |V_{i-1} - V_i)$$
(3)

and x_i are the coordinates of image i. The spring force is

$$\mathbf{F}_{i}^{s}|_{\parallel} = (k_{i}|\mathbf{x}_{i+1} - \mathbf{x}_{i}| - k_{i-1}|\mathbf{x}_{i} - \mathbf{x}_{i-1}|)\hat{\boldsymbol{\tau}}_{i}$$
 (4)

and the total force on the image

$$\boldsymbol{F}_i = \boldsymbol{F}_i^s|_{\parallel} - \nabla V(\boldsymbol{x}_i)|_{\perp} \cdot \hat{\boldsymbol{\tau}}_i \hat{\boldsymbol{\tau}}_i \tag{5}$$

where

$$\nabla V(\mathbf{x}_i)|_{\perp} = \nabla V(\mathbf{x}_i) - \nabla V(\mathbf{x}_i) \cdot \hat{\boldsymbol{\tau}}_i \hat{\boldsymbol{\tau}}_i$$
(6)

and finally $\hat{\boldsymbol{\tau}} = \boldsymbol{\tau}_i/|\boldsymbol{\tau}_i|$.