Numerical Computing Homework 1

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14. Use synthetic division (Horner's method) to find $P(c)$.

(a) $P(x) = x^4 + x^3 - 13x^2 - x - 12$, $c = 3$

(b) $P(x) = 2x^7 + x^6 + x^5 - 2x^4 - x + 23$, $c = -1$

Solution:

(a)

	a_4	a_3	a_2	a_1	a_0
Input	1	1	-13	-1	-12
$c = 3$		3	12	-3	-24
	1	4	-1	-4	$-24 = P(3) = b_0$
	b_4	b_3	b_2	b_1	*Output*

Therefore, $P(3) = -24$.

(b)

	a_7	a_6	a_5	a_4	a_1	a_0
Input	2	1	1	-2	-1	23
$c = -1$		-2	1	-2	4	-3
	2	-1	2	-4	3	$20 = P(3) = b_0$
	b_7	b_6	b_5	b_4	b_1	*Output*

Therefore, $P(-1) = 20$.

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1. Find the error E_x and relative error R_x. Also determine the number of significant digits in the approximation.

(a) $x = 2.71828182$, $\hat{x} = 2.7182$

(b) $y = 98,350$, $\hat{y} = 98,000$

(c) $z = 0.000068$, $\hat{z} = 0.00006$

Solution:

(a) The error is

$$E_x = |x - \hat{x}| = |2.71828182 - 2.7182| = 0.00008182,$$

and the relative error is

$$R_x = \frac{|x - \hat{x}|}{|x|} = \frac{0.00008182}{2.71828182} = 0.00003010 < \frac{10^{-4}}{2},$$

Therefore, $\hat{x}$ approximates x to 5 significant digits.

(b) The error is

$$E_y = |y - \hat{y}| = |98350 - 98000| = 350,$$

and the relative error is

$$R_y = \frac{|y - \hat{y}|}{|y|} = \frac{350}{98350} = 0.003559 < \frac{10^{-2}}{2},$$

Therefore, $\hat{y}$ approximates y to 3 significant digits.

(c) The error is

$$E_z = |z - \hat{z}| = |0,000068 - 0.00006| = 0.000008,$$

and the relative error is

$$R_z = \frac{|z - \hat{z}|}{|z|} = \frac{0.000008}{0.000068} = 0.117647 < \frac{10^{0}}{2},$$

Therefore, $\hat{z}$ approximates z to 1 significant digits.

3.

(a) Consider the data $p_1 = 1.414$ and $p_2 = 0.09125$, which have four significant digits of accuracy. Determine the proper answer for the sum $p_1 + p_2$ and the product $p_1 p_2$.

(b) Consider the data $p_1 = 31.415$ and $p_2 = 0.027182$, which have five significant digits of accuracy. Determine the proper answer for the sum $p_1 + p_2$ and the product $p_1 p_2$.

Solution:

(a)

$$p_1 + p_2 = 1.414 + 0.09125 = 1.505$$

$$p_1 p_2 = 1.414 \times 0.09125 = 0.1290$$

(b)

$$p_1 + p_2 = 31.415 + 0.027182 = 31.442$$
$$p_1 p_2 = 31.415 \times 0.027182 = 0.85392$$

9. Given the Taylor polynomial expansions

$$\frac{1}{1-h} = 1 + h + h^2 + h^3 + \boldsymbol{O}(h^4)$$

and

$$cos(h) = 1 - \frac{h^2}{2!} + \frac{h^4}{4!} + \boldsymbol{O}(h^6),$$

determine the order of approximation for their sum and product.

Solution:

For the sum we have

$$\begin{aligned}\frac{1}{1-h} + cos(h) &= 1 + h + h^2 + h^3 + \boldsymbol{O}(h^4) + 1 - \frac{h^2}{2!} + \frac{h^4}{4!} + \boldsymbol{O}(h^6) \\ &= 2 + h + \frac{h^2}{2} + h^3 + \boldsymbol{O}(h^4) + \frac{h^4}{4!} + \boldsymbol{O}(h^6).\end{aligned}$$

Since $\boldsymbol{O}(h^4) + \frac{h^4}{4!} = \boldsymbol{O}(h^4)$ and $\boldsymbol{O}(h^4) + \boldsymbol{O}(h^6) = \boldsymbol{O}(h^4)$, this reduces to

$$\frac{1}{1-h} + cos(h) = 2 + h + \frac{h^2}{2} + h^3 + \boldsymbol{O}(h^4),$$

and the order of approximates is $\boldsymbol{O}(h^4)$.

The product is treated similarly:

$$\begin{aligned}\frac{1}{1-h} \times cos(h) &= \left(1 + h + h^2 + h^3 + \boldsymbol{O}(h^4)\right)\left(1 - \frac{h^2}{2!} + \frac{h^4}{4!} + \boldsymbol{O}(h^6)\right) \\ &= \left(1 + h + h^2 + h^3\right)\left(1 - \frac{h^2}{2!} + \frac{h^4}{4!}\right) \\ &\quad + \left(1 + h + h^2 + h^3\right)\boldsymbol{O}(h^6) + \left(1 - \frac{h^2}{2!} + \frac{h^4}{4!}\right)\boldsymbol{O}(h^4) \\ &\quad + \boldsymbol{O}(h^4)\boldsymbol{O}(h^6) \\ &= 1 + h + \frac{h^2}{2} + \frac{h^3}{2} - \frac{11h^4}{24} - \frac{11h^5}{24} + \frac{h^6}{24} + \frac{h^7}{24} \\ &\quad + \boldsymbol{O}(h^6) + \boldsymbol{O}(h^4) + \boldsymbol{O}(h^4)\boldsymbol{O}(h^6)\end{aligned}$$

Since $\boldsymbol{O}(h^4)\boldsymbol{O}(h^6) = \boldsymbol{O}(h^{10})$ and

$$-\frac{11h^4}{24} - \frac{11h^5}{24} + \frac{h^6}{24} + \frac{h^7}{24} + \boldsymbol{O}(h^6) + \boldsymbol{O}(h^4) + \boldsymbol{O}(h^{10}) = \boldsymbol{O}(h^4),$$

the preceding equation is simplified to yield

$$\frac{1}{1-h} \times cos(h) = 1 + h + \frac{h^2}{2} + \frac{h^3}{2} + \boldsymbol{O}(h^4),$$

and the order of approximates is $\boldsymbol{O}(h^4)$.