Numerical Computing Homework 3

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3. Solve the upper-triangular system and find the value of the determinant of the coefficient matrix.

$$\begin{aligned} 4x_1 - x_2 + 2x_3 + 2x_4 - x_5 &= 4 \\ -2x_2 + 6x_3 + 2x_4 + 7x_5 &= 0 \\ x_3 - x_4 - 2x_5 &= 3 \\ -2x_4 - x_5 &= 10 \\ 3x_5 &= 6 \end{aligned}$$

Solution:

We find that all the diagonal elements are non-zero. So we could solve the upper-triangular system by the method of back substitution.

The coefficient matrix $\mathbf{A}$ and the matrix $\mathbf{B}$ are:

$$\mathbf{A} = \begin{bmatrix} 4 & -1 & 2 & 2 & -1 \\ 0 & -2 & 6 & 2 & 7 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 4 \\ 0 \\ 3 \\ 10 \\ 6 \end{bmatrix}.$$

From Theorem 3.5's equation(6), we could calculate x_1, x_2, x_3, x_4, x_5:

$$x_k = \frac{b_k - \sum_{j=k+1}^{N} a_{kj}x_j}{a_{kk}}, \text{for } k = N-1, N-2, \ldots, 1.$$

Solving for x_5 in the last equation yields

$$x_5 = \frac{6}{3} = 2$$

Then we could obtain:

$$\begin{aligned} x_4 &= \frac{10 - (-1 \times 2)}{-2} = -6 \\ x_3 &= \frac{3 - (-1 \times (-6) + (-2) \times 2)}{1} = 1 \\ x_2 &= \frac{0 - (6 \times 1 + 2 \times (-6) + 7 \times 2)}{-2} = 4 \\ x_1 &= \frac{4 - ((-1) \times 4 + 2 \times 1 + 2 \times (-6) + (-1) \times 2)}{4} = 5 \end{aligned}$$

And we could calculate $\det(\mathbf{A}) = 4 \times (-2) \times 1 \times (-2) \times 3 = 48$

4(a). Consider the two upper-triangular matrices.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & b_{23} \\ 0 & 0 & b_{33} \end{bmatrix}.$$

Show that their product $\mathbf{C} = \mathbf{AB}$ is also upper triangular.

Solution:

Using Definition 3.1's equation(7), we could obtain $\mathbf{C}$:

$$\mathbf{C} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} + b_{13}b_{33} \\ 0 & a_{22}b_{22} & a_{22}b_{23} + a_{23}b_{33} \\ 0 & 0 & a_{33}b_{33} \end{bmatrix}$$

We could see that $\mathbf{C}$ is upper triangular.

9. What will happen if the bisection method is used with the function $f(x) = 1/(x-2)$ and

(a) the interval is $[3, 7]$?

(b) the interval is $[1, 7]$?

Solution:

(a) Since $f(3) \cdot f(7) > 0$, the initial condition is not satisfied.

(b) After infinite iterations, c will be infinitely close to 2, but $f(x)$ is undefined at 2.

11. Suppose that the bisection method is used to find a zero of $f(x)$ in the interval $[2,7]$. How many times must this interval be bisected to guarantee that the approximation c_N has an accuracy of 5×10^{-9}?

Solution:

Use equation(15) to get the number N, where $a = 2, b = 7, \delta = 5 \times 10^{-9}$.

$$\begin{aligned} N &= \operatorname{int}\left(\frac{\ln(b-a) - \ln(\delta)}{\ln(2)}\right) \\ &= \operatorname{int}\left(\frac{\ln(7-2) - \ln(5 \times 10^{-9})}{\ln(2)}\right) \\ &= \operatorname{int}(29.89735285) \\ &= 29 \end{aligned}$$

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4. Let $f(x) = x^3 - 3x - 2$.

(a) Find the Newton-Raphson formula $p_k = g(p_{k-1})$.

(b) Start with $p_0 = 2.1$ and find p_1, p_2, p_3 and p_4.

(c) Is the sequence converging quadratically or linearly?

Solution:

(a) According to Theorem 2.5,

$$\begin{aligned} g(x) &= x - \frac{f(x)}{f'(x)} \\ &= x - \frac{x^3 - 3x - 2}{3x^2 - 3} \\ &= \frac{2x^3 + 2}{3x^2 - 3} \end{aligned}$$

Thus,

$$p_k = g(p_{k-1}) = \frac{2p_{k-1}^3 + 2}{3p_{k-1}^2 - 3}$$

(b) We could use the formula above to calculate:

$$p_1 = g(p_0) = \frac{2 \times 2.1^3 + 2}{3 \times 2.1^2 - 3} = 2.0060606061$$

$$p_2 = g(p_1) = \frac{2 \times 2.0060606061^3 + 2}{3 \times 2.0060606061^2 - 3} = 2.0000243398$$

$$p_3 = g(p_2) = \frac{2 \times 2.0000243398^3 + 2}{3 \times 2.0000243398^2 - 3} = 2.0000000004$$

$$p_4 = g(p_3) = \frac{2 \times 2.0000000004^3 + 2}{3 \times 2.0000000004^2 - 3} = 2.0000000000$$

(c) Assume $R = 2$ and use equation(19) to examine the quadratical convergence, We find that $A \approx \frac{2}{3}$.

Thus the sequence converging quadratically.

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8. Use the secant method and formula (27) and compute the next two iterations p_2 and p_3. Let $f(x) = x^2 - 2x - 1$. Start with $p_0 = 2.6$ and $p_1 = 2.5$.

Solution:

$$\begin{aligned} p_2 &= p_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)} \\ &= 2.5 - \frac{(2.5^2 - 2 \times 2.5 - 1)(2.5 - 2.6)}{(2.5^2 - 2 \times 2.5 - 1)(2.6^2 - 2 \times 2.6 - 1)} \\ &= 2.41935484 \end{aligned}$$

$$\begin{aligned} p_3 &= p_2 - \frac{f(p_2)(p_2 - p_1)}{f(p_2) - f(p_1)} \\ &= 2.41935484 - \frac{(2.41935484^2 - 2 \times 2.41935484 - 1)(2.41935484 - 2.5)}{(2.41935484^2 - 2 \times 2.41935484 - 1)(2.5^2 - 2 \times 2.5 - 1)} \\ &= 2.41436464 \end{aligned}$$