Electronic thermal conduction in graphene

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BY
Jesse D. Crossno
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Thesis advisor: Professor Philip Kim Jesse D. Crossno

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Abstract

abstract goes here

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This is the dedication.

Acknowledgments

I could not have done it without, the help and support of

0 Introduction

outline the basic idea and strict requirements of measuring temperature in 2D systems. Need something non-invasive, layer distinguishing, and electron/phonon distinguishing. Radiation thermometry meets all these requirements. Johnson noise is analogous to this

In theory, theory and practice are the same thing, but in practice...

Adam Savage

1

Johnson noise thermometry

Given any process in which an applied force generates heat, the reverse process must also exist and, as such, thermal fluctuations will be create fluctuations in that force. The idea that the same physics governing the dissipation of a object moving through some environment is responsible for the apparent random motion of that object was originally described by Einstein in the context of pollen grains[3]. The generalized fluctuation-dissipation theorem[6] quantifies this statement for linear systems* by relating the power spectral density $S_P(\omega)$ to the real part of the generalized impedance $Z(\omega)$[1].

$$S_P(\omega^2) \propto k_B T \Re[Z(\omega)] \tag{1.1}$$

Nearly a quarter of a century later, Nyquist[7] related Einsteins description of Brownian motion to the electrical noise measured by Johnson[4,5]. Although all the key components were in place it would take until 1946 for the first noise thermometer to be built[2]. The general idea is to measure the noise spectrum emitted by a device and thus determine its electronic temperature. Johnson noise thermometry (JNT) is analogous to radiation thermometry where the blackbody spectrum of an object is used to determine its temperature — in fact, both rely upon modified versions of eq 1.1

Thermal noise in resistors

Johnson noise, often referred to as Johnson-Nyquist noise, was first measured in 1927[4]. Johnson found the fluctuations in the squared voltage across a resistor was linearly proportional to both the resistance and the temperature and independent of the conductor being measured. The following year, Nyquist derived the form of this noise from thermodynamic arguments; consider two identical resistors in thermal equilibrium at a temperature T connected such that any noise emitted by one is absorbed by the other. As we are in equilibrium we know the power being absorbed per unit frequency must be $k_B T$. If we represent the Johnson noise of the first resistor as a series voltage source

*Here a linear system is one where the force acting on a particle is proportional to its velocity $F/v = constant$

we know the power dissipated in in the second resistor per unit frequency must be $I^2R = V_{JN}^2/4R$ as the total resistance of the circuit is $2R$. Setting this equal to k_BT leads us to Nyquist's famous result.

$$S_V = 4Rk_BT\Delta f \tag{1.2}$$

This derivation holds regardless of the conductor, be it an electrolytic solution or a piece of graphene in a quantum Hall state. However, there is a glaring problem with extending this formula to high frequency; similar to the UV-catastrophe in black-body radiation, Nyquist's formula extends to infinite energies as it lacks a high frequency cutoff. This is fixed by quantum mechanics resulting in a cutoff in the noise spectrum centered at $\hbar\omega = k_BT$.

$$S_V = 4\hbar\omega\, \Re(Z) \left[\frac{1}{2} + \frac{1}{exp(\hbar\omega/k_BT) - 1}\right] \Delta f \tag{1.3}$$

This high frequency cutoff was seen experimentally by Schoelkopf, et al.[?] and is only of practical import at high frequencies ($> 1\ GHz$) and low temperatures ($< 1\ K$).

Resistor networks: The Johnson noise temperature

As noise is a random process, adding multiple resistors together into a network is not a simple matter of adding their voltages and/or currents but instead their mean squared voltages $\tilde{v}^2$ and/or mean squared current $\tilde{i}^2$. This is a property of Gaussian distributed noise: adding together two

Gaussian distributions, each with mean 0 and variance σ, with result in another Gaussian distribution with mean 0 and variance $2\sigma^\dagger$.

To find the noise emitted by two resistors in series with resistance R_1 and R_2 and temperature T_1 and T_2, we add their mean squared voltages.

$$\tilde{v}^2 = 4k_B(R_1T_1 + R_2T_2)\Delta f \tag{1.4}$$

While in the case of the same two resistors in parallel we must add their mean squared currents.

$$\tilde{i}^2 = 4k_B\left(\frac{T_1}{R_1} + \frac{T_2}{R_2}\right)\Delta f \tag{1.5}$$

This process can be extended to any network of discrete, two-terminal resistors.

An effective "Johnson noise temperature" for a given resistor network can be defined as the temperature, T_{JN}, such that the total noise emitted between two given terminals of the network is:

$$\tilde{v}^2 = 4k_BR\Delta f * T_{JN} \tag{1.6}$$

where R is the two-terminal resistance. For an arbitrary network with many terminals, T_{JN} will differ depending upon which two-terminals the noise is measured between. For resistors in series we can

† This is why mean squared error is often a useful metric. If errors are unbiased and Gaussian distributed then summing their variance is appropriate

see from eq. 1.4

$$\tilde{v}^2 = 4k_B R \left(\frac{R_1}{R} T_1 + \frac{R_2}{R} T_2 \right) \Delta f \tag{1.7}$$

and thus we can define the Johnson noise temperature for this network as:

$$T_{JN}^{series} = \sum_i \frac{R_i}{R} T_i \tag{1.8}$$

Similarly from eq.1.5 we see that for resistors in parallel

$$\tilde{v}^2 = \tilde{i}^2 \times R^2 = 4k_B R \Delta f(\frac{R}{R_1} T_1 + \frac{R}{R_2} * T_2) \tag{1.9}$$

$$T_{JN}^{parallel} = \sum_i \frac{R}{R_i} T_i \tag{1.10}$$

These equations are unified by considering the relationship between the power dissipated in a particular resistor $\dot{Q}_i$ from a voltage across the two terminals of the network (or equally a current across the network) compared to the total power dissipated over the entire network $\dot{Q}_0$. For the resistors in series $\dot{Q}_i/\dot{Q}_0 = R_i/R$ and for resistors in parallel $\dot{Q}_i/\dot{Q}_0 = R/R_i$. Thus in both cases:

$$T_{JN} = \sum_i \frac{\dot{Q}_i}{\dot{Q}_0} T_i \tag{1.11}$$

In fact this is quite general and holds for any combination of resistors. It stems from the statement: The voltage created on any given two terminals of a resistor network due to the power fluctuations of a given element are exactly given by the power dissipated in that element due to a voltage on those terminals.

In the continuous limit, eq. ?? can be used to find the noise emitted by a device with a spatially non-uniform temperature profile $T(\vec{r})$ by solving for the spatial power dissipation profile $\dot{q}(\vec{r})$.

$$T_{JN} = \frac{\int \dot{q}(\vec{r}) * T(\vec{r}) d\vec{r}}{\int \dot{q}(\vec{r}) d\vec{r}} \tag{1.12}$$

where $\vec{r}$ is over the spatial dimensions of the device. Eq. 1.12 is the main result of this section.

Johnson noise in RF circuits

When measuring Johnson noise at high frequency, it can be useful to reformulate the problem into the language of microwave circuits. The Nyquist theorem, eq. 1.2, can be rewritten to describe the average power, $\tilde{\mathrm{P}}$, absorbed by an amplifier coupled to the device with reflection coefficient Γ^2:

$$\tilde{\mathrm{P}} = k_B T \Delta f \left(1 - \Gamma^2\right) \tag{1.13}$$

and

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} \tag{1.14}$$

where Z is the complex impedance of the device and Z_0 is the impedance of the measurement circuit — typically 50 Ω. In this form is it quite easy to see the thermodynamic origins of the Nyquist equation; A device at temperature T radiates a power of $k_B T$ per unit frequency, then some of that power is absorbed by the measurement circuit, and some is reflected back to the sample. All the resistance

Figure 1.1: High level schematic of a typical Johnson noise thermometry measurement circuit. Noise from an impedance matched sample is amplified and a measurement bandwidth is selected using a homodyne mixer and low-pass filter. The noise power is then measured with a power diode or linear multiplier. In this example, an RF switch acts as a chopper and the signal is measured using an lock-in amplifier.

dependence of the noise power is captured by Γ[‡]. With this new formulation the importance of minimizing Γ become apparent. For effective high frequency Johnson noise thermometry we must match the impedance of the device to the measurement circuit. For devices with two-terminal resistances far from 50 Ω, it is beneficial to add impedance matching circuits to transform the device to match Z_0 — in practice resistances less than $\sim$ 10 Ω or grater than $\sim$ 250 Ω benefit from matching circuits. As can be seen from eq. 1.13, the larger the measurement bandwidth Δf the larger noise signal. In practice, measurement bandwidths are often limited by either the impedance matching circuitry or the amplifier bandwidth; operating at higher frequencies increases typically increases both these limiting bandwidths.

An autocorrelation RF noise thermometer

Fig. 1.1 shows an example of a typical, Dicke style, radiometer used to measure the temperature of a 50 Ω sample. Radiation from the resistor is coupled into a transmission line terminated in a low noise amplifier (LNA). Even though Johnson noise has a flat ”white” spectrum, it is important to filter out unwanted $1/f$ low frequency noise ($\lesssim 10$ kHz) as well as high frequencies where the amplifier gain begins to roll off. This can be done using high- and low- pass filters, or with a homodyne mixer

[‡]this is also a nice proof for why Γ in any 2 port device must be symmetric, $\Gamma_{12} = \Gamma_{21}$. If this was not true, we could place the device between two resistors in thermal equilibrium and one would heat the other. Two-port devices which report asymmetric coefficients often include internal terminated third ports.

and low-pass filter combo.

add all the details from the APL

Uncertainty in noise measurements

Even noise has noise. There are 2 main areas of uncertainty in a noise measurements: The first comes from the fact that noise is stochastic and deals with how well you know the variance of a Gaussian after measuring some amount of time. If the measurements you take are discrete and uncorrelated then we get the usual 1/sqrt(n) but what to do if we are measuring a continuous signal. Problem stems back to (year) (cite bell labs paper) and n is given by the time bandwidth product. The second source of uncertainty comes from external noise sources such as amplifiers and boils down to the question of the noise you measure, what amount comes from the sample. This can be seen as a constant offset to the sample temperature Tn. Show plot of noise vs T with offset. When we extrapolate the sample temperature to zero we find an offset due to other noise sources. This external noise modifies the uncertainty equation to (noise formula). Show plots of 2 signals with different bandwidths. Show fixed bandwidth over time.

Impedance matching

impedance matching mesoscopic devices has a unique set of challenges: 2-terminal resistance can vary significantly with gate and field, matching circuit must be constant down to cryogenic temperatures, insensitive to strong fields.

LC tank circuits

theory of LC tank matching. Show equations for Z(f). plot real an imaginary impedance verse frequency with crossing at 50+0i. Show equations finding L and C given fo and R. Show S11 for fixed R sweeping C. Show fixed LC sweeping R. To make components temperature and field insensitive must use non ferrite. Give component names and numbers. Show a plot of temperature stability

Multi-stage matching

If resistance will vary by multiple orders of magnitude (e.g. magneto resistance) multi-stage matching networks can be used. Multi-stage LC networks allow you to cover a wider area of the resistance-frequency space by giving you multiple solutions of Z=50. Cell phones use many stages to capture the full range of the human voice (need source). If the resistance of the device is fixed we can use multi-stage matching to increase the bandwidth over which we are matched. Show real and imaginary components for single vs double stage matching (subtract off 50 to show how these are zeros). In this plot there are 2 solutions to Z = 50. In the R-f plane this looks like (show color plot of solution) However, if instead we want to match to larger range of resistances, we can move one of these zeros to a higher resistance. This increases the dynamic range of the matching circuit at the expense of bandwidth.

Effective noise temperature

The effective noise temperature is the sum of all (unwanted) noise in your system in units of the sample noise temperature. Your signal to noise ratio is given by T/Tn. Tn is a function of gamma integrated over all frequencies. Show formula for. In practice the frequencies integrated over are determined by filters.

Calibration

Since the gain and noise temperature of the circuit are a function of many complicated factors (stray, loss, gain profiles, noise profiles) it is necessary to calibrate. Furthermore its strongly dependent on the device resistance and therefore must be calibrated for every device/measurement circuit. Goal is to focus here on general calibration not graphene specifics,calibration of graphene will get its own chapter.

Cross-correlated noise thermometry

One challenge in noise measurements is identifying the noise you wish to measure from the unwanted background noise. In the right circumstances, cross-correlation can used to average out unwanted noise leaving behind. The idea is to send the noise from the device into 2 independent channels and if the noise on each channel is uncorrelated then you win. Describe the idea in the perfect world. Show schematic (APL fig 1b). using this you can remove the offset from the amplifiers (APL fig 2). However, this does not reduce the precision of the measurement. The integration time needed to attain a given uncertainty is not reduced (APL fig 3)

MULTI-TERMINAL CROSS-CORRELATION

If cross correlation is used on multiple terminals the overlapping noise can be found similar to the Johnson noise temperature.

This is some random quote to start off the chapter.

Firstname lastname

2

Electronic cooling mechanisms in graphene

Wiedemann-Franz

basic mechanism. Electron carry heat away through conduction. Each particle carries a fixed charge and fixed heat capacity. Include equation. Discuss assumptions. Show plot of experimental agreement.

Linearization

Hot electron shot noise

Electron-Phonon coupling

general form including delta and Sigmaelph.

Linearization

Bloch-Gruneisen temperature

Acoustic phonon

Intrinsic optical phonons

Remote optical phonons

include some table keeping track of the power laws

Photon cooling

This is Johnson noise and its what you measure

Heat transfer equations

total heat transfer for the entire system Q=... formulate kappa equations for general steady state heating profile

3

Thermal conductance via electrical noise

from eq above we see we need apply some sort of known heating profile and then measuring the resulting temperature rise. Define thermal conductance as Q/TJN

RECTANGULAR DEVICE

start by just walking through joule heating a rectangular device where the JN temp is the mean temp.

Electronic conduction only

in the absence of phonons the problem can be solved analytically. Derive temperature profile. Show temperature profile. The mean temperature is the JN temperature. The thermal conductance has the usual geometric factors but with an extra factor of 12. If the WF law is formulated as the 2-terminal resistance it gives you the thermal conductance but for this 12. This 12 is related to how electric R is being measured by applying a voltage difference between the terminals while the thermal conductance is via this parabolic profile.

Phonon cooling

If phonons dominate the profile is flat and we cannot effectively measure kappa. However in this limit we can measure the Sigmaelph and delta.

Johnson noise temperature vs mean temperatures: wedge device

unlike the simple rectangle where the Johnson noise temperature was simply the mean temperature, for more complicated geometries we have to compute what effective temperature we will measure in a noise experiment given the temperature profile. The wedge can be solved analytically. If we use the definition G_{th} above we see again that in the limit of no phonons, the thermal conductance relates to kappa the same way as electrical conductance relates to sigma but with an extra factor of 12. In the phonon limit however, the temperature is no longer flat as the current density is no longer uniform

Arbitrary shapes: the geometric factor α

For arbitrary shapes where analytic solutions do not exist we can turn to finite element simulations.

We find this factor of 12 to be a universal property of 2-terminal devices, independent of geometry.

circuitry

4

Hydrodynamic framework

explain the problem of transport. Build up historically. Drude. Fermi liquid. Interaction become dominate.

Hydrodynamic framework

inspiration from Subir's old papers and andy's papers

Experimental evidence

GaAs. Other 2 papers.

5

RF cryostats and circuitry

Janis

Oxford

Leiden

JNT circuits

6

Calibration of graphene devices

7

Thermal conductance in high density graphene

all the data on Aria

Device characteristics

Circuitry

Low temperature Wiedemann-franz

High temperature electron-phonon

quote

Quoteauthor Lastname

8

The Dirac fluid

BLAH BLAH, testing

Since the dawn of time, man hath sought to make things smaller.

Eric Bachmann

9

Magneto-thermal transport

There's something to be said for having a good opening line.

10

Conclusions and future work

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